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# Effects of sea roughness and atmospheric stability on wind wave growth

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#### Abstract

The paper considers the effects of sea roughness and atmospheric stability on the wind wave growth by using the logarithmic boundary layer profile including a stability function, as well as adopting Toba et al.'s [J. Phys. Ocean. 34 (1990) 705] significant wave height formula combined with some commonly used sea surface roughness formulations. The wind wave growth is represented by the non-dimensional total wave energy relative to that for neutral stability used by Young [Coast. Engng 34 (1998) 23]. For a given velocity at the 10 m elevation, spectral peak period and stability parameter, the wind wave growth is determined. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Ocean wind generated waves; Wind wave growth; Sea roughness; Atmospheric stability

### 1. Introduction

The detailed structure of the atmospheric boundary layer is important for the wind wave evolution. The boundary layer flow over the sea surface depends on the sea surface roughness and the atmospheric stability.

The boundary layer flow over the sea surface is complicated by the description of the sea surface roughness over waves, which depends on air-sea interaction conditions. A review of the subject is given in e.g. Smith et al. (1996). The sea surface

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roughness is difficult to estimate; no consistent theory exists; many different formulas have been proposed since Charnock (1955) proposed his formula from a dimensional argument, but no firm conclusion has yet been drawn on which of the attempts is the correct to use. Essentially it is a discussion of to what extent laboratory and ocean-wave systems actually involve precisely the same physics, i.e., if extrapolation of the laboratory data to the field using non-dimensional quantities such as wave age or dimensionless fetch is feasible.

For strong winds the effect of temperature stratification is minimal due to mixing of the air. However, for weaker and moderate winds, i.e., for wind velocities at the 10 m elevation up to about 25 m/s, the presence of stratification effects due to temperature gradients has been documented (Smith, 1980; Andersen and Løvseth, 1995). Generally, in spring the stratification is mainly stable, and in autumn it is mainly unstable, since the seawater temperature is lower and higher, respectively, than the air temperature. By analyzing wind speed observations over the Southern North Sea covering a period of about seven years, Coelingh et al. (1996) quantified this. Their results are summarized in Table 1 showing that stable and unstable conditions cover 85–90% of all hours in the various seasons of the year. They showed also that on a yearly average neutral condition might be assumed for the description of the annual mean wind speed.

Young (1998) studied the effect of atmospheric stability on wind wave growth by considering the non-dimensional total wave energy defined as

$$\varepsilon = \frac{g^2 E}{U_{10}^4}; E = \frac{H_s^2}{16} \tag{1}$$

where g is the acceleration of gravity,  $U_{10}$  is the mean wind velocity at the 10 m elevation, E is the wave energy, and  $H_s$  is the significant wave height. The effect of stability was investigated by using a measure of the deviation of the non-dimensional energy from the results of neutral stability taken as

$$\xi = \frac{\varepsilon - \varepsilon_{\rm n}}{\varepsilon_{\rm n}} \tag{2}$$

where  $\varepsilon_n$  refers to neutral stability.  $\xi$  represents the wind wave growth relative to neutral stability.

This paper considers the effect of sea surface roughness and atmospheric stability on wind wave growth by using the logarithmic boundary layer profile including a

Percentage of atmospheric stability versus season of the year						
Condition	Season Spring	Summer	Fall	Winter	Annual	
Stable	40–50	30–40	10-25	20–30	30–35	
Unstable	40-45	55-60	65-80	60-75	55-60	
Neutral	10-15	~10	~10	10-15	~10	

Table 1 Percentage of atmospheric stability versus season of the year

stability function as well as adopting some commonly used sea surface roughness formulations.

#### 2. Theoretical background

#### 2.1. Sea surface shear stress

The influence of both stable and unstable stratification on the sea surface boundary layer structure usually scales with the dimensionless stability parameter  $\zeta = z/L$ . Here z is the height above the surface and L is a buoyancy length scale known as the Monin–Obukhov length defined as  $L = -u_*^3 T_0 / g \kappa Q_0$  (Arya, 1982).  $\kappa$  is the von Karman's constant (=0.4),  $u_*$  is the friction velocity equal to the square root of the vertical flux of horizontal momentum at the surface,  $T_0$  is the surface temperature, and  $Q_0$  is the surface kinematic heat flux. Physically  $\zeta$  expresses the ratio between the potential energy required at a particular level to mix the potential temperature gradient and the turbulent kinetic energy supplied by the wind shear at that level. For  $\zeta < 0, \zeta = 0, \zeta > 0$  the stratification is referred to as unstable, neutral and stable, respectively. For large values of |L|, mechanical mixing dominates buoyancy in the turbulent intensity production in the boundary layer. In general  $|\zeta| \ll 1$  close to the surface, regardless of the magnitude of L, and the influence of buoyancy on wind profiles is of secondary importance. Arya (1982) gives a review of the dimensionless parameters and their relative importance for the structure of stratified boundary layers.

The purpose here is to discuss effects of sea roughness and atmospheric stability on wind wave growth. Close to the surface Kraus and Businger (1994) give the mean wind velocity profile

$$U(z) = \frac{u_*}{\kappa} \left[ \ell n \frac{z}{z_0} - \psi(\zeta) \right]$$
(3)

where  $\psi(\zeta)$  is often referred to as the stability function, and  $z_0$  is the sea surface roughness length that will be discussed later. Following Kraus and Businger (1994), the stability function is parameterized as

$$\psi = \ell n \frac{(1+x^2)(1+x)^2}{8} - 2arctgx + \frac{\pi}{2},\tag{4}$$

$$x = (1 - 16\zeta)^{1/4}; \text{ unstable } (\zeta < 0)$$
  

$$\psi = -5\zeta; \text{ stable } (\zeta > 0)$$
(5)

One should note that Eqs. (4) and (5) are derived from observations over land, but a number of indirect results suggest that these equations are also valid over water (Kraus and Businger, 1994). According to Panofsky and Dutton (1984), Eq. (4) is the most widely used parameterization for unstable air, although other parameterizations also exist. Furthermore, for stable air all measurements suggest Eq. (5), with

the estimates of the constant in the range 4.7–5.2. More details are given in Panofsky and Dutton (1984). However, one should note that recent analysis of Norwegian coastal wind measurements by Heggem (1997) gave slightly different values of the stability function, i.e., Eq. (4) with  $x = (1-17\zeta)^{1/4}$  and  $\psi = -3.6\zeta$  for unstable and stable conditions, respectively, while the Charnock constant was found to be 0.0172 (see the next section). The shape of the atmospheric boundary layer by using Eqs. (3) to (5) for different values of *L* is shown in e.g. Young (1998, Fig. 1).

For  $z = z_{10} = 10$  m, Eq. (3) can be rearranged to

$$u_* = \frac{\kappa U_{10}}{\ell n \frac{z_{10}}{z_0} - \psi\left(\frac{z_{10}}{L}\right)}$$
(6)

 $u_*$  can be determined from Eq. (6) by iteration for given values of  $U_{10}$  and L by substituting an appropriate model of  $z_0$ . The surface drag coefficient is defined as

$$C_{\rm D} = \left(\frac{u_*}{U_{10}}\right)^2 \tag{7}$$

The surface drag coefficient for neutral stratification,  $C_{\text{Dn}}$ , is then given from Eqs. (6) and (7) as

$$C_{\rm Dn} = \kappa^2 \left( \ell n \frac{10}{z_0} \right)^{-2}$$
(8)

## 2.2. Sea surface roughness parameter

The roughness of the sea surface depends on air-sea interaction conditions and is difficult to estimate. No consistent theory exists on the relation between  $z_0$  and the roughness of the sea surface, which in case of wind waves is mobile, making the problem difficult. Since Charnock (1955) proposed his well-known formula from a dimensional argument, many different formulas have been proposed by e.g. Kitaigorodski and Volkov (1965), Toba et al. (1990), Nordeng (1991), Smith et al. (1992), Donelan et al. (1995) and Johnson et al. (1998). However, no conclusion has yet been drawn on which of the attempts is the correct to use, see e.g. the discussion by Jones and Toba (1995) and Donelan et al. (1993). However, Toba et al.'s formula seems to be the most controversial (Smith et al., 1996). Essentially it is a discussion of to what extent laboratory and ocean-wave systems actually involve precisely the same physics, i.e., if extrapolation of the laboratory data to the field using nondimensional quantities such as wave age or dimensionless fetch is feasible. Hopefully, fundamental studies in the laboratory (see e.g. Banner and Peirson, 1998) together with field investigations will contribute to clarify the matter. Here the formulas of Charnock (1955), Toba et al. (1990) and Smith et al. (1992) have been chosen as examples to show how they can be used when the effect of stratification on wind wave growth is included as well. These formulations are summarized in Table 2 and discussed briefly.

Table 2

Formulations for sea surface roughness parameter where  $z_0^* = g z_0 / u_*^2$  = dimensionless roughness;  $x = c_p / u_*$  = wave age

Authors	$z_0^* = eta x^{\gamma} \ eta \ eta$	γ	
Charnock (1955) Toba et al. (1990) Smith et al. (1992)	0.0185 0.015–0.025 0.48	$0 \\ 1 \\ -1$	

**Charnock** (1955). His formula depends only on  $u_*$  and g, and is given in Table 2, where the given  $\beta$ -value, i.e., the Charnock constant, is often used.

**Toba et al. (1990)**. Their formula is based on analysis of field and laboratory data, and is given in Table 2, representing a generalization of Charnock's formula. Their expression is valid for flow over growing waves, which are in local equilibrium with the wind, given by a form depending on the wave age  $c_p/u_*$ , where  $c_p$  is the phase speed associated with wind waves with peak frequency  $\sigma_p$ . The  $\beta$ -values, which are given within a range, are not considered conclusive. Here  $\beta = 0.020$  will be used. The criterion for wind waves is taken as  $c_p/u_* \leq 40$ . By using the dispersion relationship for linear waves in deep water, the phase speed associated with waves with peak period  $T_p = 2\pi/\sigma_p$  is given by  $c_p = gT_p/2\pi$ , and thus the wind waves criterion can be expressed as  $(g/2\pi)(T_p/u_*) \leq 40$ . By replacing  $u_*$  by  $U_{10}$ ,  $c_p/U_{10}$  is also referred to as wave age. Realistic wave age limits are often taken as  $0.03 \leq c_p/U_{10} \leq 1.0$  (Toba et al., 1990). Further, only rough turbulent flow conditions will be considered, i.e.,  $z_0u_*/v > 2.3$  (Schlichting, 1979), where v is the kinematic viscosity of the air. It should be noted that  $z_0$  increases as the wave age increases.

The criterion for local equilibrium of the wave field with the wind is consistent with the 3/2-power law between non-dimensional significant wave height  $H_s$  and significant wave period  $T_s$  normalized by  $u_*$  and g, i.e.,

$$\frac{gH_{\rm s}}{u_{*}^2} = 0.062 \left(\frac{gT_{\rm s}}{u_{*}}\right)^{3/2} \tag{9}$$

They give the following relationship between the various wave periods:  $T_s = 1.13T_z$ ,  $T_p = 1.05T_s$  where  $T_z$  is the mean zero-crossing wave period. This gives  $T_p = 1.19T_z$  which corresponds to wind waves described by a JONSWAP spectrum with a spectral peakedness factor of 7 (see Fig. 11, Myrhaug and Kjeldsen, 1987). Thus, by substituting for  $T_s$ , Eq. (9) can be expressed as

$$H_{\rm s} = 0.058g \ g^{1/2} \ u_*^2 \left(\frac{T_{\rm p}}{u_*}\right)^{3/2} \tag{10}$$

showing that  $H_{\rm s}$  increases with increasing wave age.

One should note that Belberova and Myrhaug (1996) found good correlation between Toba et al.'s roughness parameter and wind waves from a site off the south-

1137

ern Norwegian coast. Furthermore, Tulin et al. (1996) found good correlation between Eq. (9) and North Sea data representing wind waves.

Smith et al. (1992). They analyzed field data (Lake Ontario, HEXOS) and obtained the formula given in Table 2, showing the opposite trend to Toba et al.'s results, i.e.,  $z_0$  decreases as the wave age increases.

**Donelan et al. (1993).** Donelan et al. analyzed both field data (including the HEXOS data and data from a site in the Atlantic Ocean off the coast of Nova Scotia) and laboratory data. They found that younger waves in the field are generally rougher than fully developed waves, while this is not necessarily the case for laboratory data. They argue that laboratory data should be disregarded and not analyzed together with field data, as was done by Toba et al. (1990). The laboratory data are much smoother than the corresponding field data and consequently behave different than field waves. Generally Donelan et al. (1993) accepted the Smith et al. (1992) formula. However, Donelan et al. (1993) prefer the use of  $U_{10}$  instead of  $u_*$  and obtained

$$z_0 = 0.000037 \frac{U_{10}^2}{g} \left(\frac{c_{\rm p}}{U_{10}}\right)^{-0.9} \tag{11}$$

showing that  $z_0$  decreases as the wave age increases. They also found that

$$H_{\rm s} = 0.22 \frac{U_{10}^2}{g} \left(\frac{g}{2\pi} \frac{T_{\rm p}}{U_{10}}\right)^{1.7} \tag{12}$$

showing that  $H_s$  increases with increasing wave age, as Toba et al.'s formula in Eq. (10) does.

One should note that  $z_0$  and  $H_s$  in Eqs. (11) and (12), respectively, are stability invariant.

### 3. Results and discussion

It should be fairly clear from the previous section that the uncertainty related to the stability parameterization is smaller than that related to the roughness parameterization. Thus only one parameterization of the stability is considered here.

Firstly, as a reference case, some results for neutral stability will be given.

By using Eqs. (7) and (10), Eq. (1) takes the following form for the Toba et al. model

$$\varepsilon = (2\pi)^3 \frac{0.058^2}{16} C_{\rm D}^{1/2} \left( \frac{g}{2\pi} \frac{T_{\rm p}}{U_{10}} \right)^3 \tag{13}$$

By using Eq. (12), Eq. (1) takes the following form for the Donelan et al. model

$$\varepsilon = \frac{0.22^2}{16} \left( \frac{g}{2\pi} \frac{T_{\rm p}}{U_{10}} \right)^{3.4} \tag{14}$$

Fig. 1 shows the non-dimensional energy for neutral stability  $\boldsymbol{\epsilon}_n$  versus the wave



Fig. 1. Non-dimensional total energy for neutral flow versus wave age according to Toba et al. (1990) and Donelan et al. (1993).

age  $gT_p/2\pi U_{10}$  according to Toba et al. and Donelan et al. One should note that for Toba et al.  $\varepsilon_n$  is obtained by replacing  $C_D$  by  $C_{Dn}$  from Eq. (8) in Eq. (13), while Eq. (14) for Donelan et al. is stability-invariant as previously noted. It appears that both models show the same qualitative behavior. For both models  $H_s$  increases as the wave age increases, although there are differences between the actual values. According to the results in Fig. 1 it appears that the Toba et al. results give: about 70% larger  $H_s$  values than Donelan et al. for very young waves, i.e., for  $gT_p/2\pi U_{10} = 0.03$ ; and about 15% larger  $H_s$  values than Donelan et al. for fully developed waves, i.e., for  $gT_p/2\pi U_{10} = 1.0$ .

In the remaining part the effect of stability and roughness on wind wave growth will be studied by using Toba et al.'s  $H_s$ -formula combined with the roughness formulas of Charnock, Toba et al. and Smith et al. The motivation for using these sea roughness parameters is that they provide examples of wave age independent parameterizations, a parameterization in which the roughness (and drag) increase with the wave age, and a parameterization in which the roughness (and drag) decrease with the wave age, respectively.

By combining Eqs. (1) and (13), Eq. (2) takes the following form for the Toba et al.  $H_s$  model

$$\xi = \left(\frac{C_{\rm D}}{C_{\rm Dn}}\right)^{1/2} - 1 \tag{15}$$

where  $C_{\text{Dn}}$  is given in Eq. (8).

As a reference case the deviation of the non-dimensional energy from the neutral stability  $\xi$  versus the stability  $z_{10}/L$  according to Eq. (15) using the Charnock rough-

ness parameter in Table 2 is shown in Fig. 2. As previously referred to,  $\xi$  represents the wind wave growth relative to neutral stability. It appears that the wave growth is enhanced and reduced compared to neutral stability for unstable and stable conditions, respectively; and the deviation in wave growth from neutral condition increases as the magnitude of stability  $z_{10}/L$  increases. This is the same qualitative behaviour as obtained by Young (1998).

Figs. 3 and 4 show  $\xi$  versus  $z_{10}/L$  for different wave ages  $c_p/U_{10}$  according to Eq. (15) using Toba et al.'s and Smith et al.'s roughness parameters in Table 2, respectively.

For given wave age, the results are qualitatively the same as those given in Fig. 2 using the Charnock roughness parameter. However, for a given stability  $z_{10}/L$ , the Toba et al. and Smith et al. results show the opposite trend as the wave age  $c_p/U_{10}$  changes. Figs. 3 and 4 both show that the wave growth dependence on the wave age varies with atmospheric stability, that is, the dependence is larger for unstable  $(z_{10}/L<0)$  than for stable  $(z_{10}/L>0)$  conditions. For unstable conditions the dependence decreases with increasing  $|z_{10}/L|$ ; for stable conditions the dependence on the stability is generally an order of magnitude larger than the dependence on the wave age. Although only one parameterization of the stability dependence is considered here, it is believed that the conclusions drawn from Figs. 3 and 4 also hold for the other stability parameterization given in Panofsky and Dutton (1984, Table 6.3), as well as for the Heggem (1997) stability parameterization. The reason is that they have the same qualitative behaviour as the stability function in Eqs. (4) and (5).

Although the aim has not been to resolve between the three roughness expressions



Fig. 2. Deviation of non-dimensional energy from neutral stability versus stability according to Eq. (15) using Charnock's (1955) roughness parameter.



Fig. 3. Deviation of non-dimensional energy from neutral stability versus stability and wave age according to Eq. (15) using Toba et al.'s (1990) roughness parameter.



Fig. 4. Deviation of non-dimensional energy from neutral stability versus stability and wave age according to Eq. (15) using Smith et al.'s (1992) roughness parameter.

used here, the present results should be useful to make the engineer aware of the differences the various formulas might lead to.

## 4. Summary

In this paper the effects of sea surface roughness and atmospheric stability on wind wave growth is considered by using the logarithmic boundary layer profile including a stability function, as well as adopting the Toba et al. (1990) significant wave height formula combined with some commonly used sea surface roughness formulations. Stratification effects are present in 85–90% of all hours in the various seasons of the year for wind velocities referring to the 10 m elevation up to about 25 m/s. For given wind velocity at the 10 m elevation, spectral peak period and stability parameter, the wind wave growth is determined.

The sea surface roughness formulations considered here are those of Charnock (1955), Toba et al. (1990) and Smith et al. (1992). The emphasis here is to demonstrate the difference in wind wave growth by using the Toba et al. (1990) and Smith et al. (1992) formulations for different values of wave age and stability. It appears that the wind wave growth dependence on the stability is generally an order of magnitude larger than the dependence on the wave age.

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