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Bottom friction beneath random waves

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Abstract

The effect of random waves on the bottom friction is studied by assuming that the wave motion is a stationary Gaussian narrow-band random process. The approach is also based on simple explicit friction coefficient formulas for sinusoidal waves. The probability distribution functions of the maximum bottom shear stress for laminar flow as well as smooth turbulent and rough turbulent flow are presented. The maximum bottom shear stress follows the Rayleigh distribution for laminar flow and the Weibull distribution for smooth turbulent and rough turbulent flow. Some characteristic statistical values of the maximum bottom shear stress for the three flow regimes are also given.

1. Introduction

The wave boundary layer affects many phenomena in coastal and offshore engineering as well as in oceanography, e.g., sediment transport, pipeline stability, etc. The wave boundary layer has been studied by itself and also in combination with the current boundary layer as the flow from waves combined with current represents the most common flow condition on the seabed for shallow and intermediate water depths, i.e. in coastal zones and on continental shelves.

The combined wave and current boundary layer on the seabed has been investigated by many. Reviews are given in Grant and Madsen (1986) and Soulsby et al. (1993b). Results from theoretical models and laboratory and field experiments show that the presence of waves increases significantly the bottom roughness parameter for the current boundary layer, i.e., the roughness parameter depends strongly on the seastate. In contrast, it has been found that the current has little effect on the wave boundary layer.

The wave boundary layer by itself has also been studied with experiments and theoretical models. There have been many laboratory experiments on the wave boundary layer, among which Jensen (1989) represents the most recent and detailed experimental investigation (see also Jensen et al., 1989). Results from measurements in the ocean have been reported by Lambrakos (1982) and Myrhaug et al. (1992). Theoretical modelling of the wave

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boundary layer is based on simple eddy viscosity models or Prandtl's mixing length hypothesis. More recent models involve a refined turbulence modelling technique (Justesen, 1988; Thanh and Temperville, 1991). Reviews of wave boundary layers are given in Myrhaug (1986) and Nielsen (1992). However, few studies on the effect of the randomness of the wave motion on the bottom friction is available in the open literature. Recently Zhao and Anastasiou (1993) presented a theoretical study on bottom friction effects for random waves plus currents. Ockenden and Soulsby (1994) presented a method of predicting sediment transport for the case of currents plus irregular waves. They specified an equivalent sinusoidal wave which was shown to give the same mean transport rate as the irregular wave to within $\pm 20\%$ for most cases. Davies (1994) used a full boundary layer model to predict the bedload transport rate for currents plus irregular waves for some of the same conditions considered by Ockenden and Soulsby (1994). By using an equivalent sinusoidal wave his model predicted the same bedload transport rate as the irregular wave to within $\pm 10\%$ for a limited number of cases.

This paper presents the bottom friction beneath random waves. The waves are described as a stationary Gaussian narrow-band random process. Further, the approach is based on simple explicit friction coefficient formulas for sinusoidal waves. The probability distributions of the maximum bottom shear stress for laminar flow as well as smooth turbulent and rough turbulent flow, together with some characteristic statistical values of the maximum bottom shear stress for the three flow regimes are presented.

2. Explicit friction coefficient formulas for sinusoidal waves

The maximum bottom shear stress for sinusoidal waves is given as

$$\frac{\tau_m}{\rho}=\frac{1}{2}f_w U^2 \tag{1}$$

where U is the orbital velocity amplitude at the seabed, f_w is the wave friction coefficient, and ρ is the density of the fluid.

For laminar flow (Stokes' second problem; Schlichting, 1979)

$$f_w=2Re^{-0.5} \tag{2}$$

where

$$Re=\frac{UA}{\nu} \tag{3}$$

is the Reynolds number associated with the oscillatory wave motion, A is the orbital displacement amplitude at the seabed, and ν is the kinematic viscosity of the fluid, see Fig. 1.

For smooth turbulent flow Jonsson (1980) suggested the friction coefficient formula $f_w=0.0465Re^{-0.1}$, but since then new data for smooth turbulent flow have been published (Jensen et al., 1987). They also found that the flow becomes fully turbulent after Re reaches the value of approximately 3×10^6, see Fig. 1. Based on these results the following friction coefficient for smooth turbulent flow is suggested, obtained as a best fit by eye,

$$f_w = 0.0450 Re^{-0.175} \tag{4}$$

which is shown in Fig. 1.

For rough turbulent flow Soulsby et al. (1993a) proposed the following friction coefficient formula, obtained as the best fit to the data in Fig. 2,

$$f_w = 1.39 \left(\frac{A}{z_0} \right)^{-0.52} \tag{5}$$

where z_0 is the seabed roughness parameter.

Thus Eqs. (2) and (4) have the form

$$f_w = rRe^{-s} \tag{6}$$

while Eq. (5) has the form

$$f_w = c \left(\frac{A}{z_0} \right)^{-d} \tag{7}$$

where r, s, c and d are constants.

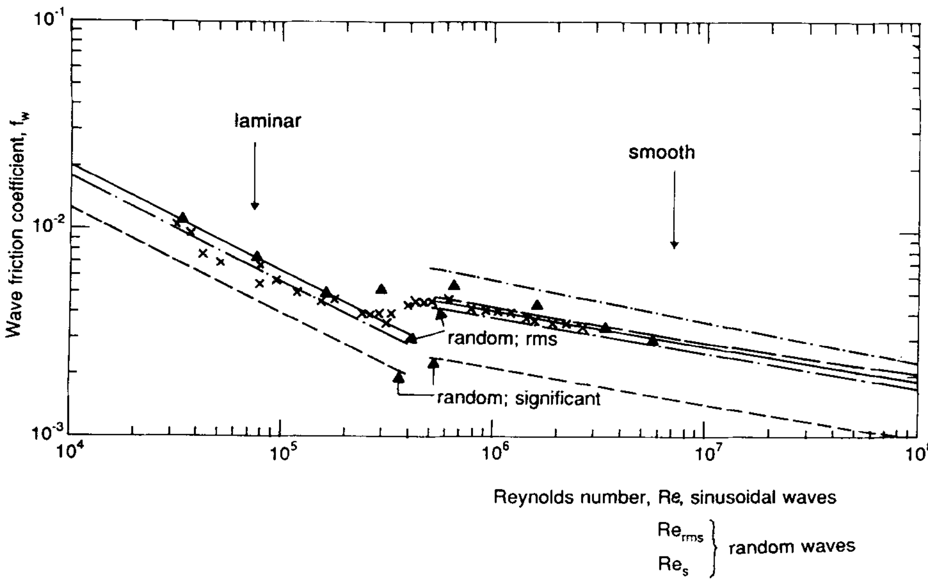

Fig. 1. Wave friction coefficient vs. Reynolds number for laminar and smooth turbulent flow. Laminar flow: (——) Eq. (2) for sinusoidal waves; (—·—·—) Eq. (44) with $\alpha_L = 1.772$ for random waves with rms value; (– – –) Eqs. (43) and (47) for random waves with significant value. Smooth turbulent flow: (–·–·) Jonsson (1980) for sinusoidal waves; (— — —) Myrhaug (1989) theory for sinusoidal waves; (——) Eq. (4) for sinusoidal waves; (—·—·—) Eq. (45) with $\alpha_S = 0.0422$ for random waves with rms value; (– – –) Eqs. (43) and (48) for random waves with significant value. All the data are for sinusoidal waves/oscillations: (×) Kamphuis (1975); (▲) Jensen et al. (1987).

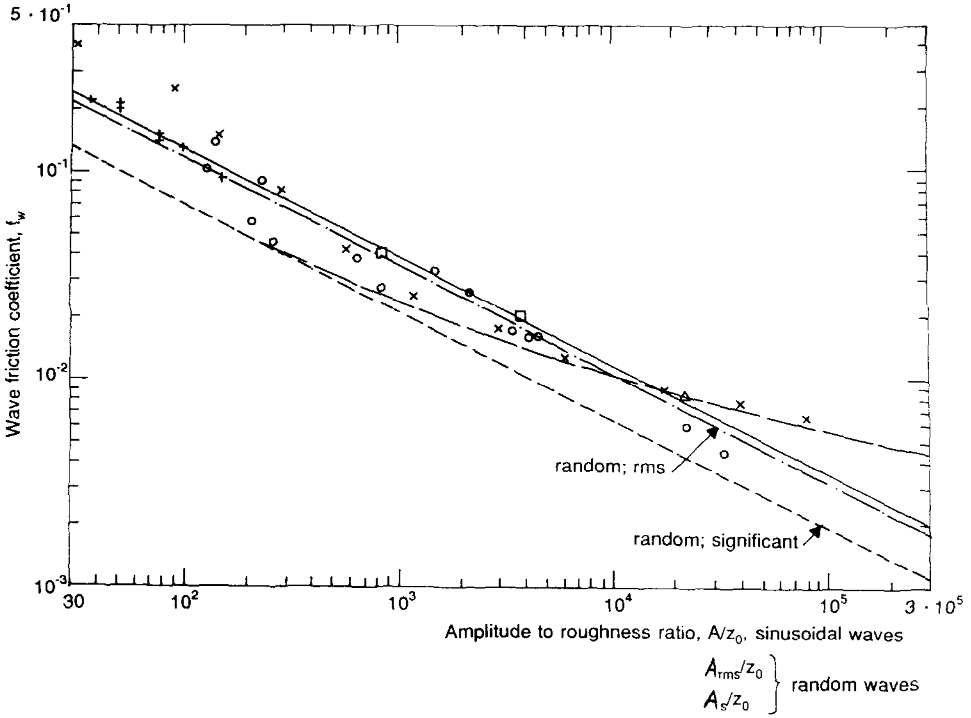

Fig. 2. Wave friction coefficient vs. amplitude to roughness ratio for rough turbulent flow: (— — —) Myrhaug (1989) theory for sinusoidal waves; (——) Eq. (5) (Soulsby et al., 1993a) for sinusoidal waves; (– · – · –) Eq. (46) with $\alpha_R = 1.27$ for random waves with rms value; (– – –) Eqs. (43) and (49) for random waves with significant value. All the data are for sinusoidal waves/oscillations: (+) Bagnold (1946); (×) Kamphuis (1975); (□) Jonsson and Carlsen (1976); (△) Sumer et al. (1987); (○) Sleath (1987).

3. Bottom friction for random waves

3.1. Probability distribution of maximum bottom shear stress

The basis for the present approach is that the maximum bottom shear stress for sinusoidal waves given in Eq. (1) combined with Eqs. (2) to (5), is valid for random waves as well. Consequently it is assumed that each wave can be treated individually. The accuracy of this assumption should be validated by using a full boundary layer model to calculate the shear stress under random waves. However, the preliminary results by Davies (1994) suggest that this assumption can be used to predict integrated effects such as the bedload transport rate with a reasonable degree of accuracy. Thus this assumption is considered to be adequate as a first approximation. Further it is assumed that the free surface elevation $\zeta(t)$ is a stationary Gaussian narrow-band random process with zero expectation and the one-sided spectral density $S_{\zeta\zeta}(\omega)$, where ω is the cyclic wave frequency. Thus the present approach should be applicable for the description of the bottom friction beneath irregular waves occurring in wave groups in intermediate water depths. However, when the water depth decreases the waves will begin to shoal and the waves become nonlinear. Consequently both the Gaussian and the narrow-band assumption will no longer be valid. The accuracy of the narrow-band assumption will be discussed subsequently.

Based on the present assumptions the bed orbital displacement $a(t)$ as well as the bed orbital velocity $u(t)$ will be stationary Gaussian narrow-band random processes with zero expectations and with the one-sided spectral densities

$$S_{aa}(\omega)=\frac{S_{\zeta\zeta}(\omega)}{\sinh^2 kh} \tag{8}$$

and

$$S_{uu}(\omega)=\omega^2 S_{aa}(\omega)=\frac{\omega^2 S_{\zeta\zeta}(\omega)}{\sinh^2 kh} \tag{9}$$

respectively, where k is the wave number determined from the dispersion relationship $\omega^2=gk\tanh kh$, h is the water depth, and g is the acceleration of gravity.

Let $z(x,y)$ denote a general function of two random variables x and y. Then the probability distribution of z is given as the non-exceedence of the level z' by

$$P(z')=\text{Prob}\,[z(x,y)\le z']=\int_{z'}\int_{(x',y')} p(x,y)\,\mathrm{d}x\mathrm{d}y \tag{10}$$

where $p(x,y)$ is the joint probability density function of x and y, and the integration is over a region R_z, including all the points of the x,y-plane where $z(x,y)\le z'$. Thus the probability distribution of τ_m/ρ can be obtained if $p(A,U)$ is known.

For a narrow-band process the waves are specified as a "harmonic" wave with cyclic frequency ω and with slowly varying amplitude and phase. Then the bed orbital displacement is given as (see e.g. Sveshnikov, 1966)

$$a(t)=A(\epsilon t)\cos[\omega t+\Phi(\epsilon t)] \tag{11}$$

where $\epsilon\ll 1$ is introduced to indicate that the bed orbital displacement amplitude A and the phase Φ are slowly varying with t. Then the bed orbital velocity is given as

$$u(t)=\frac{\mathrm{d}a(t)}{\mathrm{d}t}=\omega A(\epsilon t)\sin[\omega t+\Phi(\epsilon t)-\frac{\pi}{2}]+\mathrm{O}(\epsilon) \tag{12}$$

where the term $\mathrm{O}(\epsilon)$ represents terms of order ϵ. As a first approximation, which is consistent with the narrow-band assumption, the terms of $\mathrm{O}(\epsilon)$ are neglected, and accordingly the bed orbital velocity amplitude is related to the displacement amplitude by $U=\omega A$, where U is slowly varying with t as well.

The accuracy of the approximate relation obtained by neglecting the terms of $\mathrm{O}(\epsilon)$ in Eq. (12) is discussed in Sveshnikov (1966). A test of the accuracy is the error in the variance of the derivative of the random function $a(t)$. It appears that this error is small in the case of a narrow-band spectrum. Overall some of the main features are covered by using the narrow-band approximation.

For a narrow-band process the conditional probability density function of U given A is given as

$$p(U|A)=\delta(U-\omega A) \tag{13}$$

where Dirac's delta-function is defined as

$$\int_{-\infty}^{\infty} f(x)\,\delta(x-\xi)\,\mathrm{d}x = f(\xi) \tag{14}$$

Thus the joint A,U-distribution is given as

$$p(A,U) = p(U|A)p(A) = p(A)\,\delta(U-\omega A) \tag{15}$$

Now A and U will both be Rayleigh-distributed with the probability density functions

$$p(A) = 2\frac{A}{A_{\mathrm{rms}}^2}\exp\left(-\frac{A^2}{A_{\mathrm{rms}}^2}\right) \tag{16}$$

and

$$p(U) = 2\frac{U}{U_{\mathrm{rms}}^2}\exp\left(-\frac{U^2}{U_{\mathrm{rms}}^2}\right) \tag{17}$$

respectively. A_{rms} and U_{rms} are the root-mean-square (rms) values of A and U, respectively, and are related to the zeroth moments m_{0aa} and m_{0uu} of the amplitude and velocity spectral densities, respectively, or corresponding to the variances of the amplitude (σ_{aa}^2) and the velocity (σ_{uu}^2), given by

$$A_{\mathrm{rms}}^2 = 2m_{0aa} = 2\sigma_{aa}^2 = 2\int_0^{\infty} S_{aa}(\omega)\,\mathrm{d}\omega = 2\int_0^{\infty} \frac{S_{\zeta\zeta}(\omega)}{\sinh^2 kh}\,\mathrm{d}\omega \tag{18}$$

and

$$U_{\mathrm{rms}}^2 = 2m_{0uu} = 2\sigma_{uu}^2 = 2\int_0^{\infty} S_{uu}(\omega)\,\mathrm{d}\omega = 2\int_0^{\infty} \omega^2 S_{aa}(\omega)\,\mathrm{d}\omega = 2\int_0^{\infty} \frac{\omega^2 S_{\zeta\zeta}(\omega)}{\sinh^2 kh}\,\mathrm{d}\omega \tag{19}$$

From Eq. (19) it also appears that $m_{0uu} = m_{2aa}$, where m_{2aa} is the second moment of the amplitude spectral density.

It is noticed that $p(U)$ in Eq. (17) is obtained from Eq. (15) by integration of A. A reasonable choice for ω is the mean zero-crossing wave frequency, which is obtained from the spectral moments of $a(t)$ as

$$\omega = \omega_{m02} = \left(\frac{m_{2aa}}{m_{0aa}}\right)^{1/2} = \left(\frac{m_{0uu}}{m_{0aa}}\right)^{1/2} = \frac{U_{\mathrm{rms}}}{A_{\mathrm{rms}}} \tag{20}$$

where Eqs. (18) and (19) have been used.

By using Eqs. (1), (6), (10), (15), (16), (20) and (14) the probability distribution function of the normalized maximum bottom shear stress for laminar and smooth turbulent flow is given by

$$P(t) = 1-\exp(-t^{1/(1-s)}); \quad t \geq 0 \tag{21}$$

where

$$t=\frac{\tau_m}{\rho u_{*L,S}^2}=\left(\frac{A}{A_{rms}}\right)^{2-2s} \tag{22}$$

$$u_{*L,S}^2=\frac{1}{2}\,rRe_{rms}^{-s}U_{rms}^2 \tag{23}$$

$$Re_{rms}=\frac{U_{rms}A_{rms}}{\nu} \tag{24}$$

Similarly, by using Eqs. (1), (7), (10), (15), (16), (20) and (14) the probability distribution function of the normalized maximum bottom shear stress for rough turbulent flow is given by

$$P(t)=1-\exp(-t^{2/(2-d)});\quad t\geq 0 \tag{25}$$

where

$$t=\frac{\tau_m}{\rho u_{*R}^2}=\left(\frac{A}{A_{rms}}\right)^{2-d} \tag{26}$$

$$u_{*R}^2=\frac{1}{2}\,c\left(\frac{A_{rms}}{z_0}\right)^{-d}U_{rms}^2 \tag{27}$$

By using the results in Eqs. (21) to (27) and substituting the actual values for r, s, c and d from Eqs. (2), (4) and (5), the results for the three flow regimes are given by

$$P(t)=1-\exp(-t^2);\quad t=\frac{\tau_m}{\rho u_{*L}^2}\geq 0,\ \text{laminar} \tag{28}$$

$$P(t)=1-\exp(-t^{1.212});\quad t=\frac{\tau_m}{\rho u_{*S}^2}\geq 0,\ \text{smooth} \tag{29}$$

$$P(t)=1-\exp(-t^{1.35});\quad t=\frac{\tau_m}{\rho u_{*R}^2}\geq 0,\ \text{rough} \tag{30}$$

where

$$u_{*L}^2=\frac{1}{2}\cdot 2Re_{rms}^{-0.5}U_{rms}^2 \tag{31}$$

$$u_{*S}^2=\frac{1}{2}\cdot 0.0450Re_{rms}^{-0.175}U_{rms}^2 \tag{32}$$

$$u_{*R}^2=\frac{1}{2}\cdot 1.39\left(\frac{A_{rms}}{z_0}\right)^{-0.52}U_{rms}^2 \tag{33}$$

It appears that the maximum bottom shear stress is Rayleigh distributed for laminar flow, while τ_m/ρ is Weibull-distributed for smooth turbulent and rough turbulent flow. Fig. 3 shows the probability distribution functions $P(t)$ in Weibull scale for the three flow regimes.

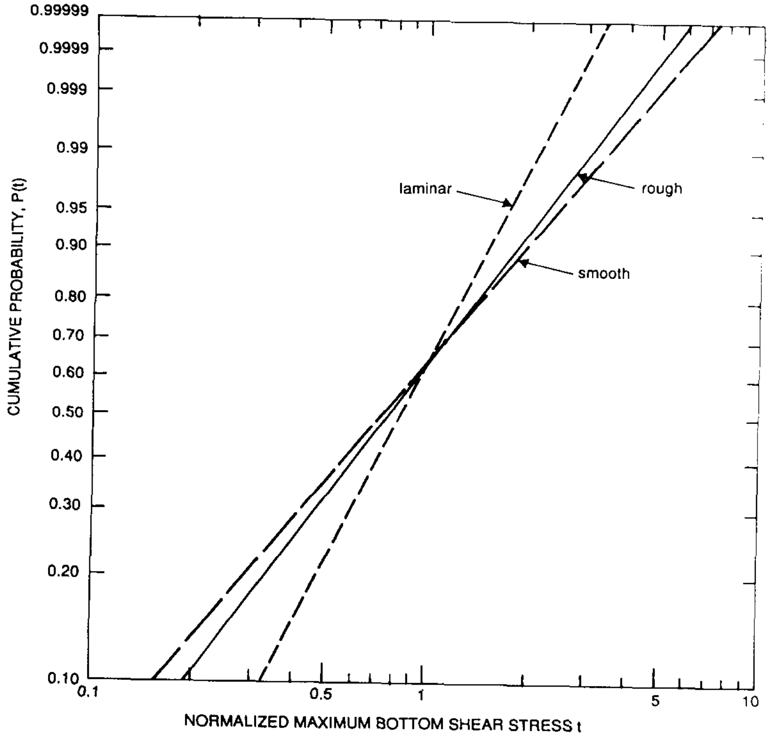

Fig. 3. Probability distribution function of normalized maximum bottom shear stress in Weibull scale: (– – –) laminar flow, Eq. (28); (— — —) smooth turbulent flow, Eq. (29); (——) rough turbulent flow, Eq. (30).

3.2. Some characteristic statistical values

When the probability distributions are known, the characteristic statistical values of the maximum bottom shear stress in random waves can be obtained. For a Weibull-distributed random variable x with the distribution function

$$P(x) = 1 - \exp(-x^{\beta}); \quad x \geq 0, \beta > 0 \tag{34}$$

the expected (mean) value and the variance of the random variable are given by, respectively (see e.g. Bury, 1975)

$$E[x] = \Gamma(1 + \frac{1}{\beta}) \tag{35}$$

$$\mathrm{Var}[x] = \Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right) \tag{36}$$

where Γ is the Gamma-function. By using these results, $E[\tau_m/\rho]$ and the standard deviation $\sigma_{\tau_m/\rho} \equiv (\mathrm{Var}[\tau_m/\rho])^{1/2}$ can be calculated, and the results are given in Table 1.

Table 1
Some characteristic statistical values for the three flow regimes. u_{*L}^2, u_{*S}^2 and u_{*R}^2 are defined in Eqs. (31), (32) and (33), respectively

	Laminar	Smooth turbulent	Rough turbulent
$E\left[\frac{\tau_m}{\rho}\right]$	$0.886u_{*L}^2$	$0.938u_{*S}^2$	$0.917u_{*R}^2$
$\frac{\sigma_{\tau_m/\rho}}{E\left[\frac{\tau_m}{\rho}\right]}$	0.52	0.83	0.75
$\left(\frac{\tau_m}{\rho}\right)_{1/n}$; $n=3$	$1.048u_{*L}^2$	$1.081u_{*S}^2$	$1.072u_{*R}^2$
$\left(\frac{\tau_m}{\rho}\right)_{1/n}$; $n=10$	$1.517u_{*L}^2$	$1.990u_{*S}^2$	$1.855u_{*R}^2$
$\left(\frac{\tau_m}{\rho}\right)_{1/n}$; $n=100$	$2.146u_{*L}^2$	$3.526u_{*S}^2$	$3.100u_{*R}^2$
$E\left[\left(\frac{\tau_m}{\rho}\right)_{1/3}\right]$	$1.416u_{*L}^2$	$1.818u_{*S}^2$	$1.700u_{*R}^2$

Other statistical quantities of interest are the value of the maximum bottom shear stress which are exceeded by a certain percentage. Let $(\tau_m/\rho)_{1/n}$ denote the value of τ_m/ρ which is exceeded by the probability $1/n$, which is determined by

$$Q\left[\left(\frac{\tau_m}{\rho}\right)_{1/n}\right]=1-P\left[\left(\frac{\tau_m}{\rho}\right)_{1/n}\right]=\frac{1}{n} \tag{37}$$

By using Eqs. (37) and (21) to (27), the following results are obtained for:

laminar and smooth turbulent flow

$$\left(\frac{\tau_m}{\rho}\right)_{1/n}=(\ln n)^{1-s}u_{*L,S}^2 \tag{38}$$

rough turbulent flow

$$\left(\frac{\tau_m}{\rho}\right)_{1/n}=(\ln n)^{1-d/2}u_{*R}^2 \tag{39}$$

As examples the values which are exceeded by 33, 10 and 1%, i.e. for $n=3$, $n=10$ and $n=100$, respectively, for the three flow regimes are given in Table 1.

Further, the expected value of the $1/n$ highest values of τ_m/ρ is given by

$$E\left[\left(\frac{\tau_m}{\rho}\right)_{1/n}\right]=n\int_{(\tau_m/\rho)_{1/n}}^{\infty}p\left(\frac{\tau_m}{\rho}\right)\mathrm{d}\left(\frac{\tau_m}{\rho}\right) \tag{40}$$

Table 2
Probability of exceeding characteristic statistical values, see also Table 1

$$Q_1 = 1 - P\left(E\left[\frac{\tau_m}{\rho}\right] + n\sigma_{\tau_m/\rho}\right) = 1 - P(t_n)$$

$$Q_2 = 1 - P\left(E\left[\left(\frac{\tau_m}{\rho}\right)_{1/3}\right]\right)$$

	Laminar	Smooth turbulent	Rough turbulent
t_n	$0.886(1+n\cdot 0.52)$	$0.938(1+n\cdot 0.83)$	$0.917(1+n\cdot 0.75)$
$Q_1(t_n)$; $n=0$	0.456	0.396	0.411
$Q_1(t_n)$; $n=1$	0.163	0.146	0.151
$Q_1(t_n)$; $n=2$	0.0381	0.0484	0.0467
$Q_1(t_n)$; $n=3$	0.00583	0.0149	0.0127
$Q_1(t_n)$; $n=4$	0.000583	0.00429	0.00309
Q_2	0.135	0.127	0.129

By using Eqs. (40) and (21) to (27), the following results are obtained for:

laminar and smooth turbulent flow

$$E\left[\left(\frac{\tau_m}{\rho}\right)_{1/n}\right] = n\Gamma(2-s)Q(\chi^2 = 2\ln n \mid \nu = 4-2s)u_{*\mathrm{L,S}}^2 \tag{41}$$

rough turbulent flow

$$E\left[\left(\frac{\tau_m}{\rho}\right)_{1/n}\right] = n\Gamma\left(\frac{4-d}{2}\right)Q(\chi^2 = 2\ln n \mid \nu = 4-d)u_{*\mathrm{R}}^2 \tag{42}$$

where $Q(\chi^2 \mid \nu)$ is the χ^2 probability exceedence function with ν degrees of freedom (Ch. 26.4, Abramowitz and Stegun, 1972).

As an example the values for $n=3$, i.e. corresponding to the significant value of the maximum bottom shear stress, are given in Table 1 for the three flow regimes.

The probabilities of exceeding the various characteristic statistical values for the three flow regimes are given in Table 2. The statistical values considered here are the expected value, the expected value plus one to four standard deviations, as well as the probabilities of exceeding the significant value. It appears that the probabilities of exceeding the significant values and the expected value plus one standard deviation are about the same, i.e. in the range 13 to 16%.

3.3. Friction coefficient

The friction coefficient for random waves is defined from Eq. (1) as

$$f_w = \frac{\tau_m/\rho}{\frac{1}{2}U^2} \tag{43}$$

where τ_m/ρ is represented by an appropriate statistical value, e.g. one of the values given in Table 1, and U is represented by U_{rms}. The friction coefficients associated with the various statistical values are given by

$$f_w = \alpha_L Re_{rms}^{-0.5}, \quad \text{laminar} \tag{44}$$

$$f_w = \alpha_S Re_{rms}^{-0.175}, \quad \text{smooth} \tag{45}$$

$$f_w = \alpha_R \left(\frac{A_{rms}}{z_0}\right)^{-0.52}, \quad \text{rough} \tag{46}$$

where α_L, α_S and α_R are constants depending on the statistical relationship of τ_m/ρ used in Eq. (43). The actual values of these constants together with the appropriate relationship used for τ_m/ρ, are given in Table 3. As an example the friction coefficients based on $E[\tau_m/\rho]$, i.e. Eqs. (44) to (46) with $\alpha_L = 1.772$, $\alpha_S = 0.0422$ and $\alpha_R = 1.27$, are shown in Figs. 1 and 2 as the curves marked "random; rms".

Alternatively the random waves can be represented by the significant values. By using that the significant values are given by $U_s = \sqrt{2}\,U_{rms}$ and $A_s = \sqrt{2}A_{rms}$, the expected value of the maximum bottom shear stress is given by

Table 3
The constants α_L, α_S and α_R in Eqs. (44), (45) and (46), respectively, and the appropriate relationships for τ_m/ρ they are associated with, see also Table 1

	Laminar α_L	Smooth turbulent α_S	Rough turbulent α_R
	$2\cdot 0.886(1+n\cdot 0.52)$	$0.0450\cdot 0.938(1+n\cdot 0.83)$	$1.39\cdot 0.917(1+n\cdot 0.75)$
$E\left[\frac{\tau_m}{\rho}\right]+n\cdot\sigma_{\tau_m/\rho};\ n=0$	1.772	0.0422	1.27
$E\left[\frac{\tau_m}{\rho}\right]+n\cdot\sigma_{\tau_m/\rho};\ n=1$	2.693	0.0722	2.22
$E\left[\frac{\tau_m}{\rho}\right]+n\cdot\sigma_{\tau_m/\rho};\ n=2$	3.615	0.112	3.18
$E\left[\frac{\tau_m}{\rho}\right]+n\cdot\sigma_{\tau_m/\rho};\ n=3$	4.536	0.147	4.13
$E\left[\frac{\tau_m}{\rho}\right]+n\cdot\sigma_{\tau_m/\rho};\ n=4$	5.458	0.182	5.08
$\left(\frac{\tau_m}{\rho}\right)_{1/n};\ n=3$	2.096	0.0486	1.49
$\left(\frac{\tau_m}{\rho}\right)_{1/n};\ n=10$	3.034	0.0896	2.58
$\left(\frac{\tau_m}{\rho}\right)_{1/n};\ n=100$	4.292	0.159	4.31
$E\left[\left(\frac{\tau_m}{\rho}\right)_{1/3}\right]$	2.832	0.0818	2.36

$$E\left(\frac{\tau_m}{\rho}\right)=\frac{1}{2}\cdot\sqrt{\frac{\pi}{2}}Re_s^{-0.5}U_s^2,\quad \text{laminar} \tag{47}$$

$$E\left(\frac{\tau_m}{\rho}\right)=\frac{1}{2}\cdot 0.0238Re_s^{-0.175}U_s^2,\quad \text{smooth} \tag{48}$$

$$E\left(\frac{\tau_m}{\rho}\right)=\frac{1}{2}\cdot 0.76\left(\frac{A_s}{z_0}\right)^{-0.52}U_s^2,\quad \text{rough} \tag{49}$$

where

$$Re_s=\frac{U_sA_s}{\nu} \tag{50}$$

The results for $\sigma_{\tau_m/\rho}/E[\tau_m/\rho]$ given in Table 1 are also valid in this case. Similar expressions can also be found for the other statistical quantities. Thus the friction coefficient for random waves can alternatively be obtained from Eq. (43) where U is taken as U_s and by using the appropriate statistical relationship for τ_m/ρ. As an example the friction coefficients using Eqs. (47) to (49) are shown in Figs. 1 and 2 as the curves marked "random; significant".

3.4. Discussion

Ockenden and Soulsby (1994) presented a method for predicting sediment transport for the case of currents plus irregular waves. Their approach uses the wave friction coefficient for rough turbulent flow given in Eq. (5). In their case the maximum bottom shear stress for waves only is given by Eqs. (1) and (5) where A is replaced by $A_p=\sigma_{uu}T_p/2\pi$, that is, an amplitude associated with the peak period T_p in the spectrum and the standard deviation of the velocity σ_{uu} (see Eq. 19). U is taken as $U_{rms}=\sqrt{2}\,\sigma_{uu}$. In order to compare this with the present approach a mean JONSWAP spectrum with peakedness factor $\gamma=3.3$ is considered. In this case $T_p=1.28T_{m02}$ (see e.g. Fig. 11 in Myrhaug and Kjeldsen, 1987) where $T_{m02}=2\pi/\omega_{m02}$. By using Eqs. (19) and (20) the Ockenden and Soulsby version of Eq. (5) gives the factor $\alpha_R=1.39(1.28/\sqrt{2})^{-0.52}=1.46$, which is close to the factor 1.49 associated with $(\tau_m/\rho)_{1/3}$ in Table 3. Thus it appears that the Ockenden and Soulsby version of Eq. (5) for irregular waves is close to the value of the maximum seabed shear stress exceeded by the probability 1/3. The probability of exceedence of Ockenden and Soulsby's expression of the maximum bottom shear stress by using the distribution in Eq. (30) is 0.34.

The most appropriate statistical value to use will depend on the problem dealt with. The mean value of the maximum bottom shear stress might be a relevant quantity to use to represent the dissipation of irregular surface water waves in e.g. physical models for predicting coastal and ocean flow circulations. However, in other applications, such as in suspended sediment calculations beneath irregular waves, the maximum bottom shear stress which is exceeded by a certain percentage might be a more appropriate value to use.

No data are available in the open literature at present, and therefore no conclusion can be drawn on the ability of this approach to describe measured data. However, although these friction coefficient formulas are simple, they are believed to be adequate as a first approximation to represent the bottom friction beneath random waves.

4. Conclusions

The paper presents bottom friction beneath random waves by assuming that (1) the waves are described as a stationary Gaussian narrow-band random process, and (2) simple explicit friction coefficient formulas for sinusoidal waves are valid for random waves as well. The probability distribution functions of the maximum bottom shear stress for laminar flow as well as smooth turbulent and rough turbulent flow are presented. The maximum bottom shear stress follows the Rayleigh distribution for laminar flow and the Weibull distribution for smooth turbulent and rough turbulent flow. Some characteristic statistical values for the three flow regimes are also given. However, which of the statistical values to be used will depend on the problem dealt with. It appears that the friction coefficient formulas for sinusoidal waves can be used for random waves as well, if the wave parameters are represented by the rms- or significant values, and that the constants in the formulas are changed. Although these friction coefficient formulas are simple, they should be adequate as a first approximation to represent the bottom friction beneath random waves.

5. Nomenclature

a	bed orbital displacement
A	bed orbital displacement amplitude
c, d	constants
$E[\ ]$	expectation of random variable
f_w	wave friction coefficient
g	acceleration of gravity
h	water depth
k	wave number
m_{0aa}, m_{2aa}	zeroth and second moment of bed orbital displacement spectral density
m_{0uu}	zeroth moment of bed orbital velocity spectral density
p	probability density function
P	probability distribution function
Q	probability exceedence function
r, s	constants
Re	Reynolds number
$S_{aa}(\omega)$	spectral density of bed orbital displacement
$S_{uu}(\omega)$	spectral density of bed orbital velocity
$S_{\zeta\zeta}(\omega)$	spectral density of free surface elevation
t	time coordinate; also normalized maximum bottom shear stress
T_{m02}	mean zero-crossing period
T_p	peak period

u	bed orbital velocity
u^2_{*L}, u^2_{*S}, u^2_{*R}	factors used for normalization of maximum bottom shear stress, see Eqs. (31) to (33)
U	bed orbital velocity amplitude
Var[ ]	variance of random variable
z_0	seabed roughness parameter
α_L, α_S, α_R	constants used in friction coefficient, see Eqs. (44) to (46)
Γ	Gamma-function
δ	Dirac's delta-function
ϵ	small parameter
ζ	free surface elevation
ν	kinematic viscosity of fluid; also degrees of freedom in χ^2 probability distribution
ρ	density of fluid
σ^2_{aa}	variance of bed orbital displacement
σ^2_{uu}	variance of bed orbital velocity
$\sigma_{\tau_m/\rho}$	standard deviation of maximum bottom shear stress
τ_m	maximum bottom shear stress
χ^2	probability distribution
ω	cyclic wave frequency
ω_{m02}	mean zero-crossing wave frequency

Indices

L	laminar
rms	root-mean-square value
R	rough
s	significant value
S	smooth

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