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REFRACTION OF OCEAN WAVES: A PROCESS LINKING UNDERWATER TOPOGRAPHY TO BEACH EROSION

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ABSTRACT

Waves out at sea, though usually forming a complex pattern, have essentially the same characteristics over large distances. Upon entering shallow water, these waves are transformed under the influence of bottom features, and such transformations may be so marked that breaker heights may vary greatly over short distances along the shore. The effect of bottom features upon waves can largely be interpreted in terms of a simple physical process—wave refraction. In turn, wave refraction may be responsible for alteration of the bottom features by accumulation or removal of sediments and, in this manner, be an important factor in beach erosion.

In this report the role of wave refraction is first reviewed in the light of other processes affecting the transformation of waves in shallow water. The mechanism of refraction is illustrated by means of a few idealized examples, such as the refraction pattern along a straight uniformly sloping beach, over a submarine canyon and ridge, and around a headland. Next it is shown that extreme variations in breaker height along the beach north of La Jolla, California, can be computed for typical swell conditions, taking the complex local bottom topography and the orientation of the coastline into consideration. These changes are computed from refraction diagrams for typical swell conditions, and they compare favorably with observed changes in wave height, thus indicating that wave refraction is the primary mechanism controlling changes in wave height along a beach, and that friction, diffraction, and other processes can be of secondary importance only. Finally, it is noted that the transportation of sediments is dependent upon longshore currents, rip currents, and horizontal diffusion and that all these factors are greatly influenced by the existing refraction pattern.

INTRODUCTION

Waves may have traveled many days and for thousands of miles before they approach shore and become transformed under the influence of the ocean bottom. By that time the long, low, regular undulations, called "swell," may be all but hidden beneath a local chop. As soon as the swell enters shallow water, it "feels" bottom, and a striking rejuvenation takes place. This change occurs in water of depth less than one-half the wave length, where wave length is defined as the horizontal distance between adjacent crests. The wave velocity and length decrease while the height usually increases, so that the wave gradually steepens. As the waves approach the surf zone, this process of steepening accelerates suddenly—the crests rise sharply from the water surface, become peaked, and break.

The changes just described are most noticeable for a swell from a distant storm. Waves formed by a near-by storm, or by winds blowing directly toward the coast, have already considerable steepness out at sea; the transforma-

[1] Contribution from the Scripps Institution of Oceanography, New Series, No. 307. This work represents results of research carried out for the Hydrographic Office, the Office of Naval Research, and the Bureau of Ships of the Navy Department under contract with the University of California.

tion in shallow water, though similar, is therefore much less pronounced.

Only the period, defined as the time interval between the appearance of two successive crests at a fixed point, remains constant as waves travel into shallow water. According to theory, well substantiated by observation, wave velocity and length depend almost entirely upon wave period and depth (see the Appendix). For a regular wave system the velocity and length will therefore be constant along any given depth contour.

The discussion so far has dealt with changes in the character of the waves which are the direct result of changes in velocity and length. Except under very special conditions, the wave height will, however, not be uniform along any given depth contour, because the effect of refraction over an irregular bottom enters. This effect causes waves to bend, and the bending of the wave crests in turn affects the wave height. With irregular bottom topography, waves are bent differently at different sections of a beach, and the wave height will vary along the beach. The present study is concerned principally with a discussion of the refraction effects.

The refraction of ocean waves and other problems dealing with the transformation of waves in shallow water have been studied intensively during the war in connection with the development of a method for forecasting surf conditions for amphibious operations.[2] Practically all changes in the character of the waves in shallow water can be accounted for theoretically by assuming conservation of energy and irrotational wave motion, and it has been shown that the conclusions based on theory are in good agreement with observations.[3]

WAVE REFRACTION

THE PHYSICAL NATURE OF WAVE REFRACTION

Wave crests approaching a coastline at an angle tend to swing parallel to shore (Fig. 1). This happens because the wave velocity decreases as the depth decreases, so that the portion of the crest nearer the shore moves slowly while the portion of the crest in deeper water races ahead. To illustrate this, consider two surfboarders on the same crest, traveling along the lines *A* and *B* (Fig. 1). Surfboarder *B* always travels faster than *A*, and the two tend to become more nearly aligned with the shoreline as they travel into shallow water.

The process is similar to the one causing the bending of light rays in optical systems and has therefore been termed "refraction." In the case of straight parallel contours, the analogy can be carried further, because the changes in direction of light waves and ocean waves are governed by Snell's law:[4]

$$\frac{\sin \alpha}{\sin \alpha_0} = \frac{C}{C_0}, \qquad (1)$$

where α is the angle between the wave crest and the contours at any depth, C the wave velocity at the same depth, and the parameters with subscript o refer only to deep water where the direction and velocity are constant.

REFRACTION DIAGRAMS

In the case of a more complicated bottom topography for which the depth contours are not straight and parallel,

[2] "Breakers and Surf, Principles in Forecasting," *Hydrographic Office, U.S. Navy, No. 234* (November, 1944).

[3] H. U. Sverdrup and W. H. Munk, "Theoretical and Empirical Relations in Forecasting Breakers and Surf," *Trans. Amer. Geoph. Union*, Vol. XXVII, No. 6 (December, 1946), pp. 828–36.

[4] Pp. 6–7 of ftn. 2.

changes in wave direction and height can be found graphically by constructing a "refraction diagram," a term first used by O'Brien and Mason.[5]

A refraction diagram is one on which the position of wave crests at the sea surface is shown by a series of lines. These are evenly spaced in deep water—that is, in water of depth greater than half the wave length. Beyond this depth the effect of bottom contours on any of the wave characteristics is negligible. At shallower depths the relation between wave velocity and water depth is known (see Appendix), and a refraction diagram can be constructed by advancing various points on a crest through distances determined from any chosen time interval and the average depth. The direction of advance is drawn normal to the crest. In Figure 1, for example, the chosen time interval is one wave period, and the distance A_0—A'. The distance B_0—B' represents the advance of point B during the same time interval, but for the average depth between B_0 and B'. The lines A_0—A' and B_0—B' are normal to the wave crests. By locating a set of points, A', B', C', ... , the new crest can be found with sufficient accuracy by drawing a smooth line through these points. Advancing one wave length at a time, the crests may be carried into any depth of water desired, and the completed diagram shows the continuous change in the direction of a wave advancing from deep into shallow water.

Description of a practical procedure for constructing refraction diagrams, together with the necessary graphs and tables, can be found in the forecasting manual.[6] The completed diagram can be interpreted in one of two ways: (1) As a series of lines representing the positions of a *single* wave crest at various times as the crest advances toward shore. Crest interval is then defined as the interval between these times. In Figure 1 the crest interval equals one wave period. (2) As a series of lines representing the position of certain wave crests at a *single instant;* a crest interval of *one* wave length means that every crest is represented (Fig. 1), because the advance of any point on a crest during one wave period equals one wave length by definition. A crest interval of *two* wave lengths means that only every second wave crest is shown; a crest interval of *one-half* wave length, that the position of every crest and trough is shown.

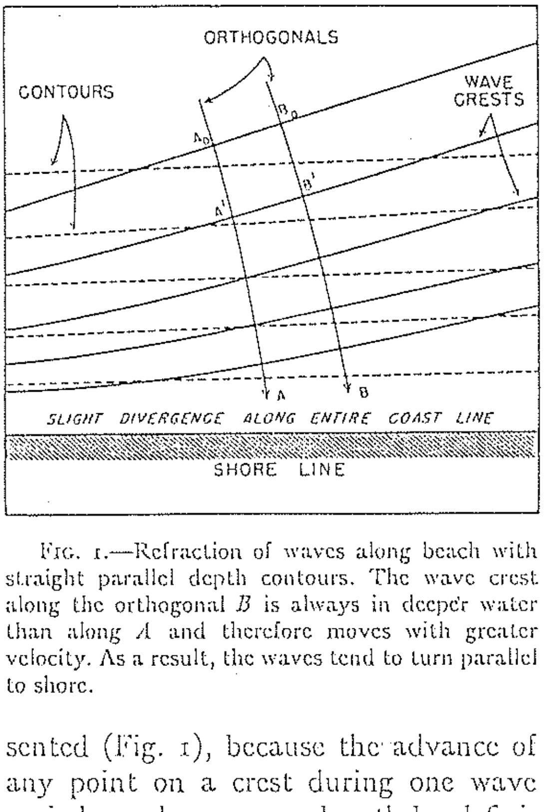


FIG. 1.—Refraction of waves along beach with straight parallel depth contours. The wave crest along the orthogonal B is always in deeper water than along A and therefore moves with greater velocity. As a result, the waves tend to turn parallel to shore.

EFFECT OF REFRACTION ON WAVE HEIGHT

Wave height is defined as the vertical distance between crest and trough. In order to evaluate the effect of refraction on wave height, a set of orthogonals, that

[5] "A Summary of the Theory of Oscillatory Waves," *Technical Report No. 2, Beach Erosion Board* (1940).

[6] Pp. 31–34 of ftn. 2.

is, a family of lines which are everywhere perpendicular to the wave crests, must be constructed. The lines A_0–A and B_0–B in Figure 1 are orthogonals; they may be visualized as the wakes behind two surfboards which are continuously oriented normal to the crest, that is, in the direction of wave motion. Assuming that the wave *energy* is transmitted in the direction of wave motion, the total flow of energy between two orthogonals must remain constant. Thus if orthogonals *converge*, the crests are *compressed*, and the energy per unit crest length is relatively *large*; if orthogonals *diverge*, the crests are *stretched*, and the energy per unit crest length is relatively *small*.

For practical purposes one must deal with wave heights rather than with wave energies, but these are related. In deep water the height of oscillatory waves varies as the *square root* of energy, and this relation holds with good approximation in shallow water until the waves "peak" just prior to breaking. The breaking waves, with their sharp crests isolated by long flat troughs, have the appearance of so-called "solitary" waves, for which the height is proportional to the *cube root* of the energy. Let H designate the wave height and s the distance between adjacent orthogonals on the refraction diagram. Parameters with subscript o again apply in deep water, those with subscripts b at the breaker point, and parameters without subscript at any depth intermediary between deep water and the breaking-point. Then

$$\frac{H}{H_0} = \gamma K , \quad \frac{H_b}{H_0} = \gamma_b K_b , \quad (2a, b)$$

where γ and γ_b have constant values along a fixed depth contour, and where

$$K = \sqrt[2]{s_0/s} , \quad K_b = \sqrt[3]{s_0/s_b} , \quad (3a, b)$$

may vary along a depth contour and will be referred to as the "refraction factors." The complete derivation of these equations is given in the Appendix.

Any *variation* in wave height along a fixed depth contour depends only upon the refraction factor. The variation can be computed from the change in distance between orthogonals according to equations (3), depending upon whether one deals with conditions in shallow water or along the breaker line. Again it should be stressed that the absolute wave height depends also upon several other factors which are implicitly contained in the parameters γ and γ_b of equations (2) (see Appendix).

Equations (2) are derived from the postulates that the energy flows along orthogonals and that energy is conserved. This in turn implies two assumptions: (1) the effect of *diffraction* (which would bring about flow of energy across orthogonals from region of high waves to regions of low waves) can be neglected and (2) the effect of bottom friction is negligible. The validity of these assumptions is borne out by the good agreement between computed changes in breaker height according to equations (2), and observations along the beach north of La Jolla, California (Fig. 16). There has been a tendency in the literature to emphasize the effect of bottom friction on wave motion and other wave characteristics. Not only the evidence presented in this paper, but also studies dealing with the generation of waves by wind,[7] and with absolute changes of wave height in shallow water and the dynamics of breaking waves,[8] indicate that wave motion in general is not appreciably affected by frictional processes.

[7] H. U. Sverdrup and W. H. Munk, "Empirical and Theoretical Relations between Wind, Sea and Swell," *Trans. Amer. Geoph. Union*, Vol. XXVII, No. 6 (December, 1946), pp. 823–27.

[8] Pp. 830–32 of ftn. 3.

REFRACTION TYPE EXAMPLES

Beach with straight and parallel depth contours.—For this simple case the change in wave direction can be expressed analytically by equation (1). Figure 1 is a schematic drawing of wave refraction along a straight coastline with parallel depth contours. Plate I, *A*,

Combining equations (1), (3), and (4) leads to

$$\left.\begin{aligned} K &= \left[\frac{\cos^2 \alpha_0}{1-\left(\frac{C}{C_0}\sin\alpha_0\right)^2}\right]^{1/4}, \\ K_b &= \left[\frac{\cos^2 \alpha_0}{1-\left(\frac{C_b}{C_0}\sin\alpha_0\right)^2}\right]^{1/6}, \end{aligned}\right\} \quad (5a,\ b)$$

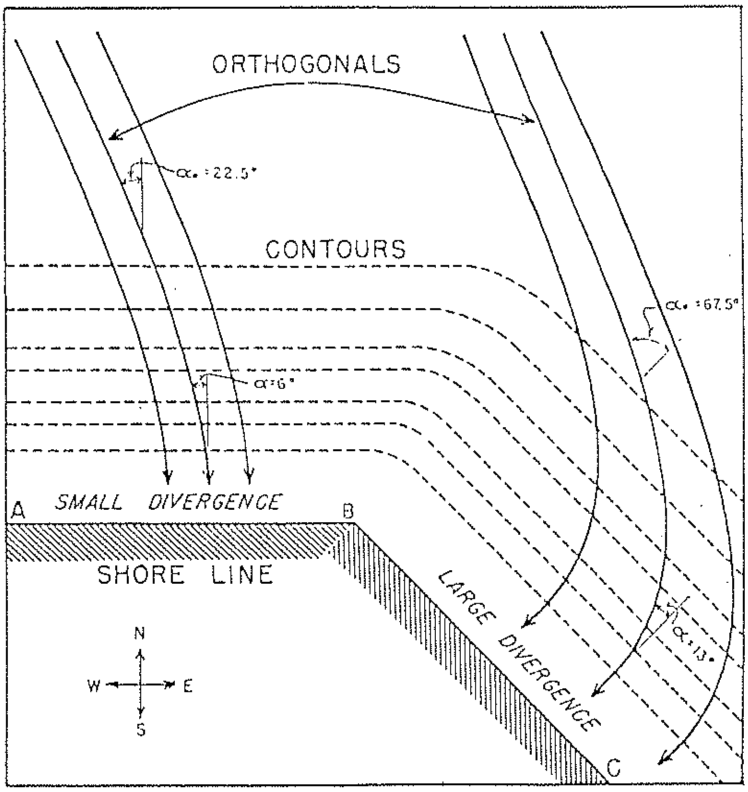


FIG. 2.—Refraction of waves along two sections of straight beaches with different exposures. The divergence along section *A–B* is less than along *B–C*, and the wave height along *A–B* exceeds therefore the wave height along *B–C*.

is an aerial photograph showing long waves from the south-southwest coming into the beach north of Oceanside, California. There is a divergence of the orthogonals indicated by the values of s computed from the equation[9]

$$\frac{s}{s_0} = \frac{\cos \alpha}{\cos \alpha_0}. \quad (4)$$

[9] Pp. 20–21 of ftn. 2.

where C/C_0 is found from equation (A7) in the Appendix.

Waves coming from an angle into a beach with straight and parallel depth contours are reduced in height according to equations (5), but the reduction is uniform along the entire beach, and no *variation* in wave height *along* the beach will result. As an example of variation in wave height, consider a coastline which forms a sharp bend at point B (Fig. 2). On both sides of point B the coastline and the depth contours are straight and parallel. Assume that waves of 14-second period come from the north-northwest. The angle α is drawn between the orthogonals and the bottom gradient, which is consistent with our earlier definition. Table 1 gives the computed direction and relative height at the 10-foot contour, which is assumed to lie outside the

TABLE 1

Section of Coast Line (Fig. 2)	A – B	B – C
Wave angle in deep water (α_0)....	22.5°	67.5°
Wave angle at 10-foot depth (eq. [1])*	6.0°	13.0°
Refraction factor (eq. [5a]).......	0.97	0.63

* The value of C/C_0 is obtained from "Breakers and Surf," *op. cit.*, Pl. I, and equals 0.25.

breaker zone. Along the beach section A–B, which is relatively exposed to the incoming swell, waves are about 50 per cent higher than along section B–C. This can also be seen from the fact that the divergence of orthogonals along A–B is less than the divergence along B–C (Fig. 2).

Submarine canyons.—Portions of wave crests over the center of a canyon are in deeper water and move ahead faster than the portions on either side (Fig. 3). Con-

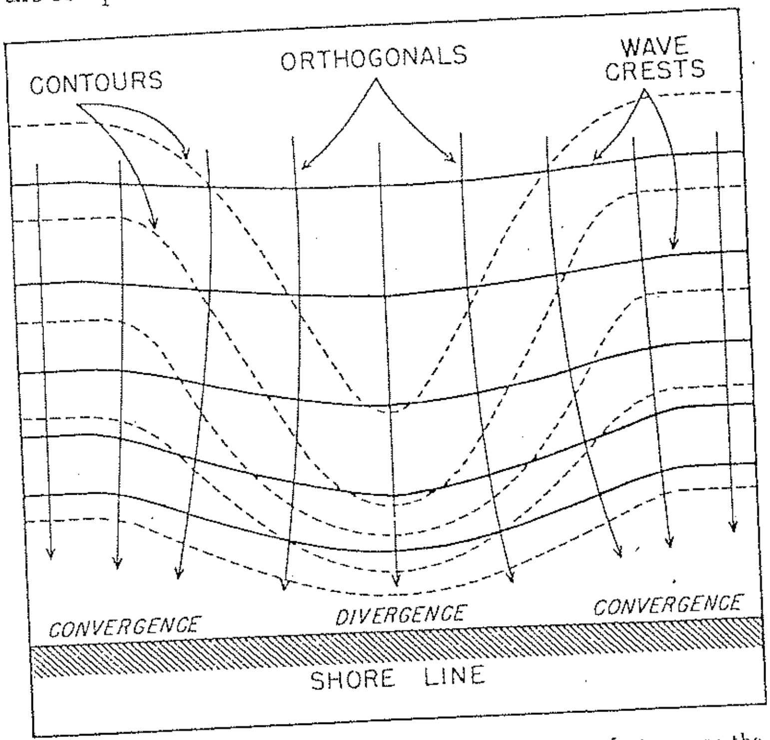


FIG. 3.—Refraction of waves by a submarine canyon. Waves move faster over the canyon than on either side of the canyon, resulting in divergence (low waves) over the mouth of the canyon and convergence (high waves) on either side.

sequently, the waves fan out, resulting in divergence (low waves) over the head of the canyon and convergence (high waves) on either side. Fishermen take advantage of this feature when they anchor near the head of a submarine canyon. Plate II is an aerial photograph of waves over the submarine canyon at Redondo, California. The approximate position of the canyon is indicated. The wave crests bend sharply over the canyon walls. Variations in wave height can be recognized by the prominence of the wave crests on either side of the canyon compared to the crests directly over the canyon and by the variation in the width of the surf zone.

Submarine ridges.—Underwater ridges near shore have an effect opposite to that of canyons. Waves passing over the ridge are in shallower water and are therefore retarded, and on either side waves move ahead, creating a convergence over the ridge (Fig. 4). This is borne out by frequent observations of unusually violent breakers over the shoal portion of underwater ridges. An aerial photograph take over the northern shore of Oahu, T.H. (Pl. I, *B*), illustrates the lagging of the wave crests over a submarine ridge and the resulting convergence. Figure 5 illustrates the same features over a sand bank opposite the mouth of Mission Bay, San Diego, California. In the case of Mission Bay the refraction pattern is somewhat affected by the tidal flow through the outlet.

Abrupt changes in depth along coral islands lead to particularly well-defined refraction patterns. Plate III shows two channels in the coral reef around Guam; the waves are seen to bend sharply ahead over the channels, where they are lower than on either side of the channels. The photograph also shows the formation of multiple crests just inside the reef, a phenomenon often observed in regions of abrupt depth changes, and one which has never been properly explained.

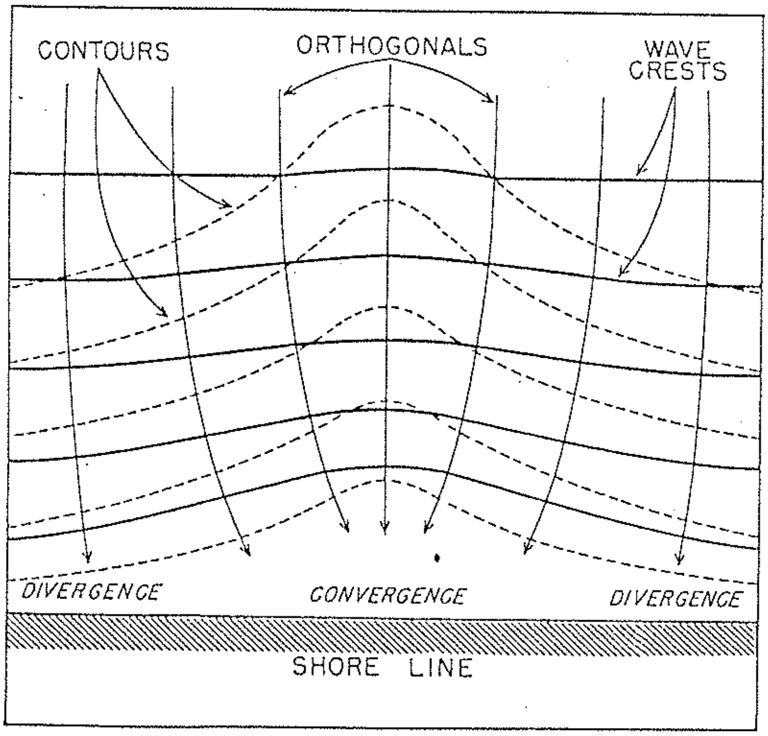


FIG. 4.—Refraction of waves by a submarine ridge. Directly over the ridge waves lag behind, and on either side waves move ahead, creating a convergence (high waves) over the ridge.

Headlands.—Since waves are retarded by shallow water, they will bend around headlands, points, and jetties. Plate IV, *A*, shows an irregular train of waves turning around Point Loma into San Diego Bay.

Under special conditions, such as portrayed in the aerial photograph of the hook of Cape Cod, Massachusetts (Pl. IV, *B*), waves may bend 180°, or even more, before they break. Such extreme refraction is associated with a very large divergence, which explains the shelter usually provided by protruding land masses.

Bays and islands.—Waves will be retarded along the sides of a bay but move relatively rapidly elsewhere. As a result, the crests will bend *into* the bay (Pl. V). The divergence will be largest at the sides of the bay, least opposite the mouth, especially if waves head directly into the bay. In the case of small islands in the path of an advancing swell the resulting refraction pattern is more complicated (Fig. 6). Offshore in the lee of the island, wave trains which have bent around both sides of the island meet and pass through each other, and a confused cross-sea results.[10] Where the bottom shoals gradually near shore, then drops off steeply, waves may be bent so effectively around an island that the lee shore offers no protection whatsoever.[11] Some coral islands have these characteristics.

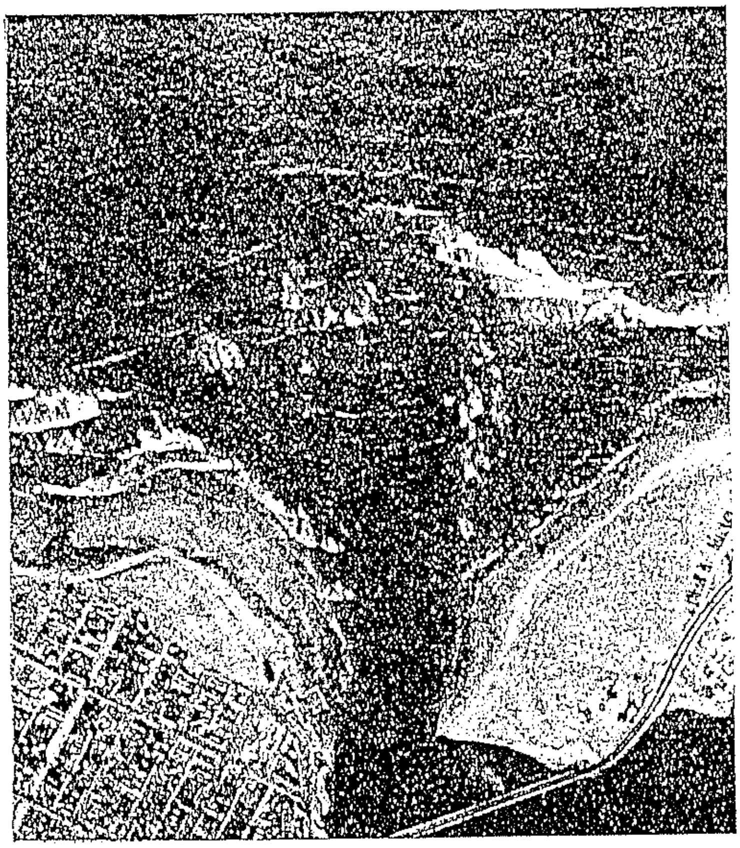


FIG. 5.—Refraction of waves over sand bar outside of Mission Bay, San Diego, California (cf. Fig. 4) The outflowing tide may have further strengthened certain features in the existing refraction pattern. (Official photograph, United States Navy.)

WAVE REFRACTION ALONG THE BEACH NORTH OF LA JOLLA, CALIFORNIA

ORIGIN OF WAVES REACHING LA JOLLA

In order to apply the principles of refraction to an actual situation, it is first necessary to determine the periods and directions of waves that will be encountered. For that purpose the character of the waves at La Jolla resulting from typical meteorological situations will be tabulated below. The paths and

[10] Pp. 12–18 of ftn. 2.

[11] Robert S. Arthur, "Refraction of Water Waves by Islands and Shoals with Circular Bottom-Contours," *Trans. Amer. Geoph. Union*, Vol. XXVII (April, 1946), pp. 168–77.

nature of the generating areas are shown schematically in Figure 7; the corresponding character of the waves at La Jolla, in Figure 7, *B*.

Situation A (winter).—Waves generated by cyclones originating at the east coast of Asia and moving along the usual cyclone path into the Gulf of Alaska:

Waves generated in the southern segments of these cyclones reach La Jolla in 2–5 days as a long-period (11–15 seconds) swell, with heights varying between 3–7 feet, according to the intensity of the cyclone. The wave energy reaching La Jolla is reduced by the offlying islands, especially San Clemente and Santa Catalina (Fig. 7, *A*). The cyclones often stagnate in the Gulf of Alaska, and the coasts of British Columbia, Washington, Oregon, and northern California will be hit by a heavy swell, whereas southern California will be protected by Point Conception, which cuts off La Jolla from waves from directions much north of WNW.

Situation B (winter).—Waves generated behind cold fronts that come into the coastal regions:

These waves have short periods (7–10 seconds), large heights (4–12 feet), and usually come from a W to NW direction.

Situation C (summer).—Waves generated by winds in the northeast periphery of the Pacific High:

Usually the period of the waves will vary from 6 to 9 seconds, the height from 2 to 5 feet. The common direction is from WNW to NW.

Situation D (summer).—Waves generated in the Southern Hemisphere:

The so-called "southern swell" comes from S to SSW, is of very long period (13–20 seconds), and is usually 3–5 feet high. This is the only direction from which waves can reach La Jolla without being broken up by offlying islands.[12] There is strong evidence that this swell has been generated by the winter storms of the Southern Hemisphere, but, owing to an almost complete lack of weather observations, this has not been verified.

Situation E (spring).—Wind waves generated in local low-pressure regions:

Local low-pressure regions, which are usually not forecast, give rise to W or NW winds with

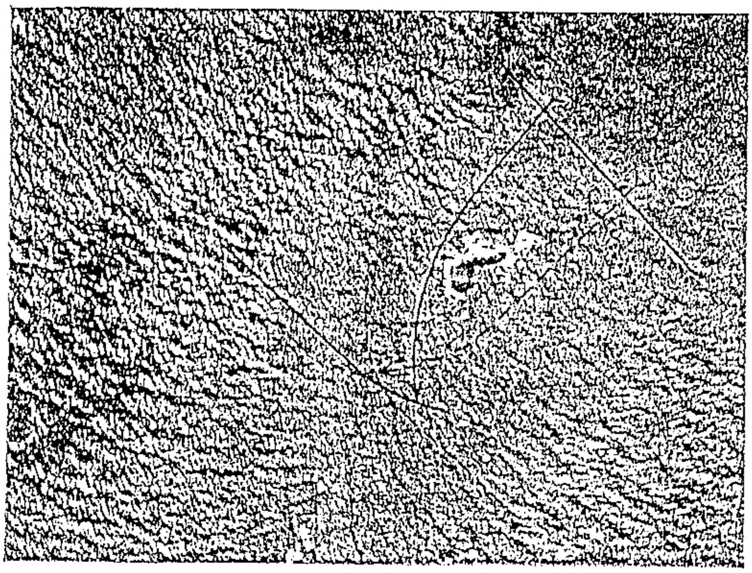


FIG. 6.—Refraction of waves behind a small island. The portion of the wave passing just to the right of the island is refracted to the left; the portion passing just to the left of the island is refracted to the right. The resulting crisscross pattern gives rise to a confused sea in the lee of the island, and the island may offer little or no shelter.

[12] Plate I, *A*, is an aerial photograph of a southern swell coming into Oceanside, Calif.

velocities up to 30 knots. These winds raise waves of 5–7 second-periods, 10–15 feet-heights. Such situations are likely to occur a number of times each spring, occasionally during winter and summer, but rarely during fall.

The character of the waves caused by these five meteorologic situations is summarized in Figure 7, *B*. Each rectangle shows typical height-period combinations corresponding to situations *A–E*.

LOCAL TOPOGRAPHY

The coast between Point La Jolla and the Scripps Institution pier is ideally suited for a study of changes in wave height. It is known locally that waves are usually low near the Beach Club, then increase sharply about 3,000 feet north of the Beach Club, and fall off a little in the vicinity of the Scripps Institution pier (Pl. VI). Breaker heights at different sections of this 1-mile stretch of beach generally differ by a factor of 3.

Such unusually large variations in breaker height can be associated with the complex submarine topography and coastline in this region. Two submarine canyons, La Jolla Canyon and its steep-walled tributary, Scripps Canyon, reach within 1,000 feet of shore (Pl. VI). These canyons have been sounded more extensively than any other submarine canyons in the world.

The walls of La Jolla Canyon are relatively gentle except locally where slopes of 100 per cent are found. Scripps Canyon on the other hand has extremely steep walls. Numerous places have been located where the slopes are so steep that the sounding lead slides down the slope after hitting bottom. The steepest measured slope has a drop of 230 feet in a horizontal distance of 30 feet. The canyon has an inner gorge which is so narrow in places that it was difficult to locate. . . .[13]

The coast line, which is oriented NNE–SSW between Scripps pier and the Beach Club, makes an almost right-angle turn 1,000 feet south of the Beach Club and projects about 3,000 feet in a WNW direction. Point La Jolla is located at the end of this break in the coast line (Pl. VI). Beyond Point La Jolla the coast continues its NNE–SSW trend. In the vicinity of the Beach Club some protection from the usual WNW and W swell is afforded by Point La Jolla, and even more by the shoal area off this point.

TABLE 2

	Figure No.				
	8	9	10	11	12
Wave direction	WNW	WNW	W	WNW	SSW
Wave period	14	12	12	8	16
Crest interval/period	1	1	1	2	1
Meteorological situation	A	A	A	B, C, or E	D

REFRACTION DIAGRAMS

The refraction diagrams (Figs. 8–12) were drawn as described in the manual on breaker and surf forecasting. Table 2 gives the assumed wave direction (direction from which the wave comes in deep water), wave period, crest interval, and the corresponding meteorological situation for each of the five figures.

[13] F. P. Shepard and K. O. Emery, "Submarine Topography off the California Coast," *Geol. Soc. Amer., Special Papers, No. 31* (May 28, 1941).

The thin dashed lines on the figures are bottom contours in fathoms; the heavy solid lines are wave crests; and the light solid lines with arrowheads are orthogonals drawn at arbitrary intervals. For waves with periods more than 10 seconds every single crest has been drawn; for waves with periods less than 10 seconds only every second crest has been drawn.

The area offshore has been divided into divergent, normal, and convergent zones, according to whether the refraction factor K (eq. [3*a*]) is less than 0.75, 0.75–1.25 inclusive, or larger than 1.25. Divergent zones are dotted, convergent zones crosshatched, and normal zones left blank.

The value of K_b (eq. [3*b*]) at various stations is proportional to the length of the rectangles at those stations and can be read off from the scale given in the legend of each refraction diagram. In order to emphasize changes in K_b, the columns have been shaded in the same manner used for the offshore area. Columns *ending* in a dotted section designate divergence (K_b less than 0.75); those ending in a crosshatched section designate convergence (K_b larger than 1.25). It should be kept in mind that values of K are proportional to the square root and values of K_b to the cube root of the relative spacing of the orthogonals. The stations where observations were made are numbered *1–12*. Other stations, designated by the letters *A–K*, have been selected because of their representative location.

Waves from WNW, period 14 seconds (Fig. 8).—A small region of relatively high waves, or convergence, is found just north of Station A. An area of intense convergence, which extends approximately northward from the Cove (Station B), is caused by the steep south wall of La Jolla Canyon. This wall is particularly effective because it lies parallel to the direction of wave travel for half a mile. The prolonged effect of the steep bottom slope caused the waves to turn sharply to the right and converge into the Cove. This probably explains the occurrence of unusually high waves at the Cove on days when waves are low along other sectors of the beach. It is well known in La Jolla that these high waves, on days when they occur, come into the Cove at intervals of 15 minutes to several hours. Perhaps the effect of La Jolla Canyon is particularly marked for certain critical combinations of wave period and direction. Although both period and direction are nearly constant for any wave train, they do show continual small variations, and certain combinations of these periods and directions may be the cause of intermittent high waves at the Cove.

The submarine canyon offers protection along the beach from Station C to Station 5 by creating a divergence. This divergence is particularly marked in the vicinity of the Beach Club, where K_b equals 0.3. A slight convergence in the vicinity of Station 6 is the result of waves turned left by the north bank of La Jolla Canyon and is analogous to the convergence near the Cove, caused by the south bank of the canyon. The divergence between Stations 8 and I is caused by the Scripps Canyon. It is less intense than the divergence due to the La Jolla Canyon because for the direction assumed waves do not travel in a line parallel to the walls of the Scripps Canyon. To the north of the Scripps Canyon, between Stations I and J, a small but sharp convergence is found.

The variation of K_b along the beach is also shown in Figure 15. The lower portion of the figure gives the location of the stations and the two submarine canyons.

The upper figure contains a curve for K_b for each of the refraction diagrams. Low wave heights near the head of the two canyons and high waves to either side are clearly shown.

Waves from WNW, period 12 seconds (Fig. 9).—The general features are similar to those caused by the 14-second WNW waves, but the divergence is less intense and covers a smaller area. Wave heights between the two submarine canyons (Stations 4–9) are relatively higher than for the 14-second wave. This difference is chiefly due to the fact that the 12-second wave is shorter than the 14-second wave and that canyons and other bottom features do not affect the 12-second waves to the same degree that they affect the 14-second waves.

Waves from W, period 12 seconds (Fig. 10).—The main features are similar to the ones already discussed, except that regions of divergence and convergence have been shifted due to the difference of wave direction. Thus the convergence on the south edge of La Jolla Canyon has shifted from the Cove to Station C, and the convergence between the canyons from Station 6 to Station 8. The convergence near Station A is more pronounced than for a west-northwesterly swell. Scripps Canyon gives rise to more extreme divergence between Stations 11 and I because the direction of wave travel in deep water is more nearly parallel to the walls of the canyon.

Waves from WNW, period 8 seconds (Fig. 11).—The length of these waves in deep water is only one-third that of the 14-second swell, and the effect of refraction is reduced even more than it was for the 12-second waves. In particular, regions of divergence have become much smaller, and only about 3,000 feet of beach are protected by the La Jolla Canyon. The region of divergence opposite the Scripps Canyon has disappeared altogether. The convergence on the south side of La Jolla Canyon has moved to Station D, thus eliminating the cause of relatively high waves at or near the Cove. Nevertheless, it can be seen from Figure 15 that the refraction pattern typical of a submarine canyon has not disappeared: waves are relatively low opposite the mouth of the canyons and high on either side.

Waves from SSW, period 16 seconds (Fig. 12).—A few selected orthogonals between deep water and shore have been entered in an inset on a smaller scale in order to show that for this particular case the effect of refraction outside the limits of the main figure cannot be neglected. By far the larger portion of Figure 12 shows values of K less than 0.75. This divergence is due to the shoal area off Point La Jolla. At only two localities in the vicinity of La Jolla can a convergence of a southern swell be found, opposite the Casa de Mañana Hotel (Station A) and to the north between Stations J and K. A divergence followed by a convergence is caused by the ridge between the submarine canyons. It is believed that this explains the occasional occurrence of relatively high, long-period waves between Stations 7 and 9 during the summer. The refraction diagram explains also why, during the summer, waves are relatively low at the Scripps Institution while they are high at localities with southern exposure, such as near Oceanside and the Coronado Strand.

OTHER METHODS OF PRESENTING REFRACTION DIAGRAMS

If an intensive study is made of a given locality, it may be of advantage to construct a single graph from which the refraction factor K_b and the breaker angle α_b can be read off directly. In general, K_b and α_b depend upon three variables—the wave direction in deep water, the wave period, and the depth of breaking—but K_b and α_b vary slowly with the depth of breaking, and an average value for the breaking depth may be introduced.

A convenient form of presentation is a polar-coordinate graph with its center placed on an outline map at the position to which the graph applies (Fig. 13). The radial lines give the direction from which the waves come in deep water; the circles give the wave period. The solid curves are lines of equal K_b, the dashed curves lines of equal α_b. The "L = LEFT" and "R = RIGHT" refer to an observer on the beach looking straight out to sea.

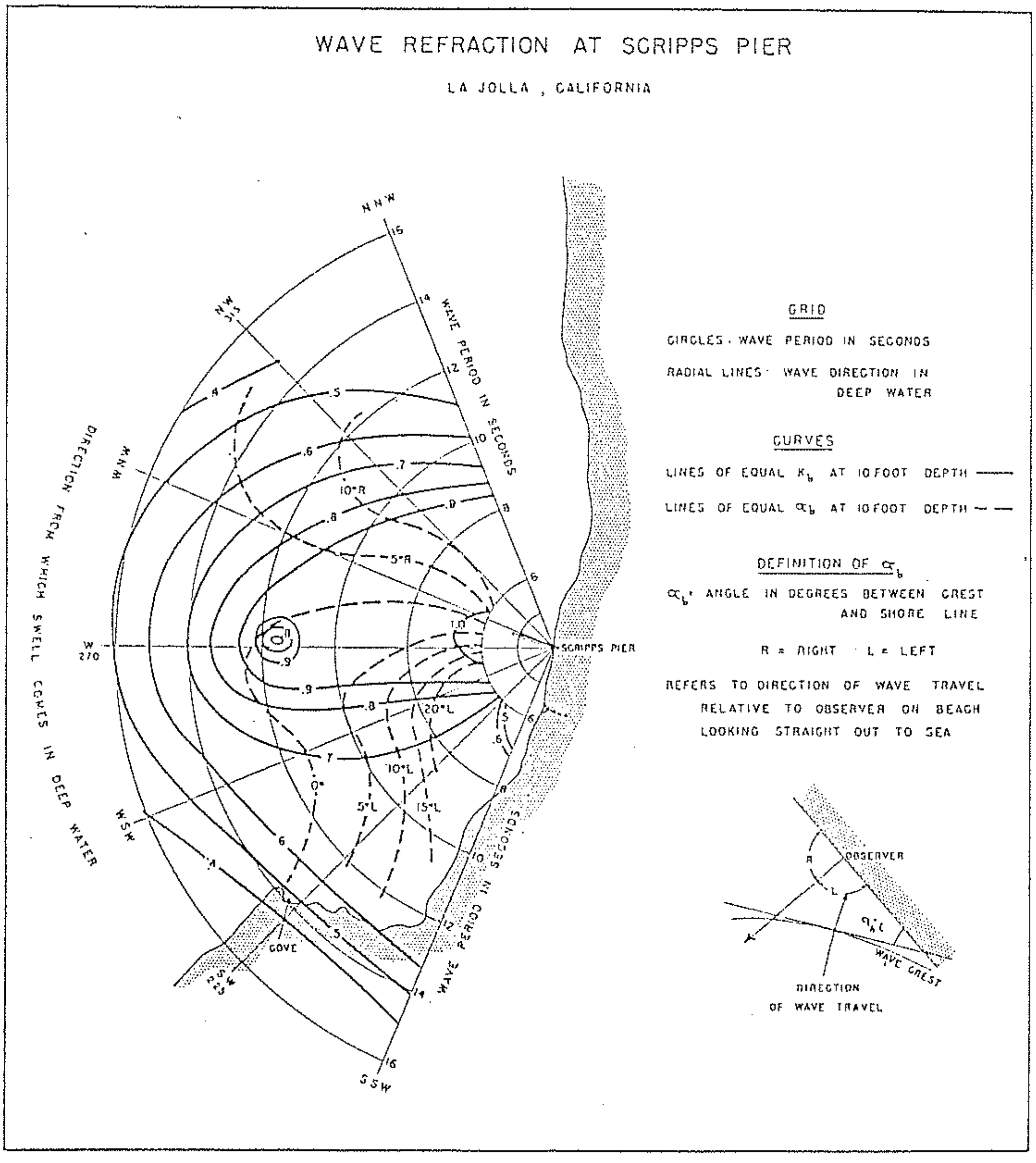


FIG. 13.—Values of K_b and α_b at a depth of 10 feet along Scripps Institution pier. The polar-coordinate presentation brings out the relative protection of the designated location from the waves coming from SSW to WSW, and the relative exposure to waves from the west.

Figure 13 is drawn for an assumed breaking depth of 10 feet along the Scripps Institution pier. The figure applies with good approximations to breakers anywhere between shore and a depth of 20 feet. The curves were drawn by interpolating between values read off Figures 8–12 and a number of other refraction diagrams drawn specially for the region in the immediate vicinity of the pier. The curve $\alpha_b = 0$ refers to breakers coming straight into shore, that is, from a WNW direction. The other dashed curves are drawn relative to this direction.

Figure 14 applies to the case of a beach with straight and parallel depth contours but is otherwise analogous to Figure 13. For this idealized case the curves for K_b and α_b are given analytically by equations (1) and (5*b*). From a comparison between Figures 13 and 14, and assuming $T = 14$ seconds, the special effects of the bottom topography and of the con-

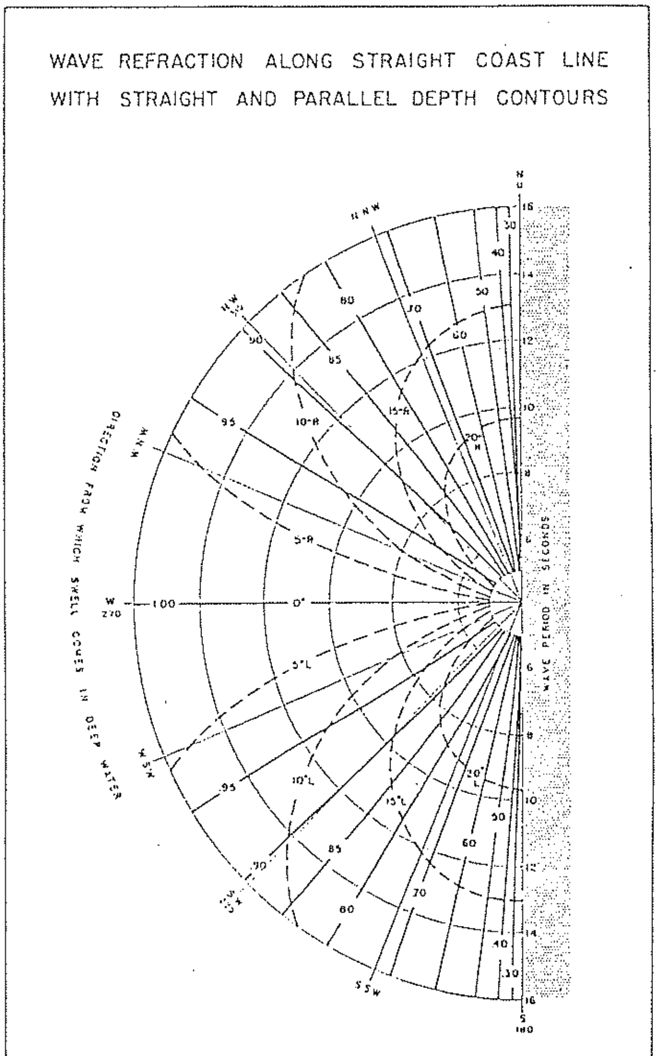


FIG. 14.—Values of K and α_b at a depth of 10 feet for a beach with straight and parallel depth contours (eqs. [1] and [5*b*]). The special effect of the coastline and the bottom topography at La Jolla can be estimated from a comparison between Figs. 13 and 14.

figuration of the coastline upon the breakers near the Scripps pier can be evaluated (Table 3).

Thus the beach along the pier is particularly well sheltered from southerly waves due to the protection afforded by Point La Jolla and is most exposed to waves directly from the west. This type of presentation gives, therefore, an over-all picture of the protection from, or the exposure to, waves of various directions and periods. The disadvantage of this presentation is that it is limited to a relatively small area. If the emphasis is to be placed upon the changes in wave characteristics along a beach for a given wave situation, the presentation shown in Figure 15 will be more practical.

TABLE 3

	Fig.	Direction from Which Waves Come in Deep Water					
		SSW	SW	WSW	W	WNW	NW
Refraction factor K_b....	13	0.51	0.45	0.57	0.70	0.60	0.45
	14	0.72	0.90	0.97	1.00	0.97	0.90
Breaker angle α_b.......	13	....	2°L	1°R	2°R	4°R	7°R
	14	13°L	10°L	4°L	0°	4°L	10°L

OBSERVATIONS

Naval officers under instruction at the Scripps Institution, after having been trained for one week in a uniform procedure of observing waves, occupied Stations 1–12 along the beach and made simultaneous observations for 15 minutes. These observations were averaged to obtain observed values. To minimize differences due to personal judgment, observers were shifted regularly from one station to another. From about seventy complete sets of observations, 7 days were selected during which the swell period was well defined and in agreement with forecasts. According to forecasts, all these waves came from WNW. Observations are summarized in Table 4. For each of the twelve stations Table 4 gives the height of the breakers in feet, the height of the breakers as percentage of the average for all stations, and the wave period in seconds. The value for H_b at Station 6 on January 5 appears to be too high, but otherwise observations are fairly consistent.

For each *station* observations made on the 7 days under consideration were averaged and entered in the last column of Table 4. For each *day* observations made at the twelve stations were averaged and entered at the bottom of the table, together with the forecast values. On the 7 days under consideration the average periods vary between 11.5 and 13.8 seconds. The over-all average for observed periods is 12.5 seconds, and for the forecast periods 14 seconds. The forecast direction is WNW. Figures 8 and 9 should therefore apply in all seven cases.

COMPARISON OF THEORY AND OBSERVATIONS

The *computed* refraction factor K_b for waves of 14- and 12-second periods from the WNW was read off at each of the twelve stations. In the case of the 14-second wave, the average value for all stations was 0.62; in the case of the 12-second wave, 0.82. At each station the individual values of K_b were then expressed as percentages of these average values and plotted in Figure 16 (solid and dashed curves).

The dots in Figure 16 give the average *observed* breaker height and the crosses the average *observed* wave periods, both expressed as percentages of the lated to a shifting sand bank about 2,500 feet south of the pier.

According to theory, the wave periods should remain constant. Observations plotted in Figure 16 show that this is nearly the case, the lowest value being 95, and the highest 105, per cent of the average. It happens that these small

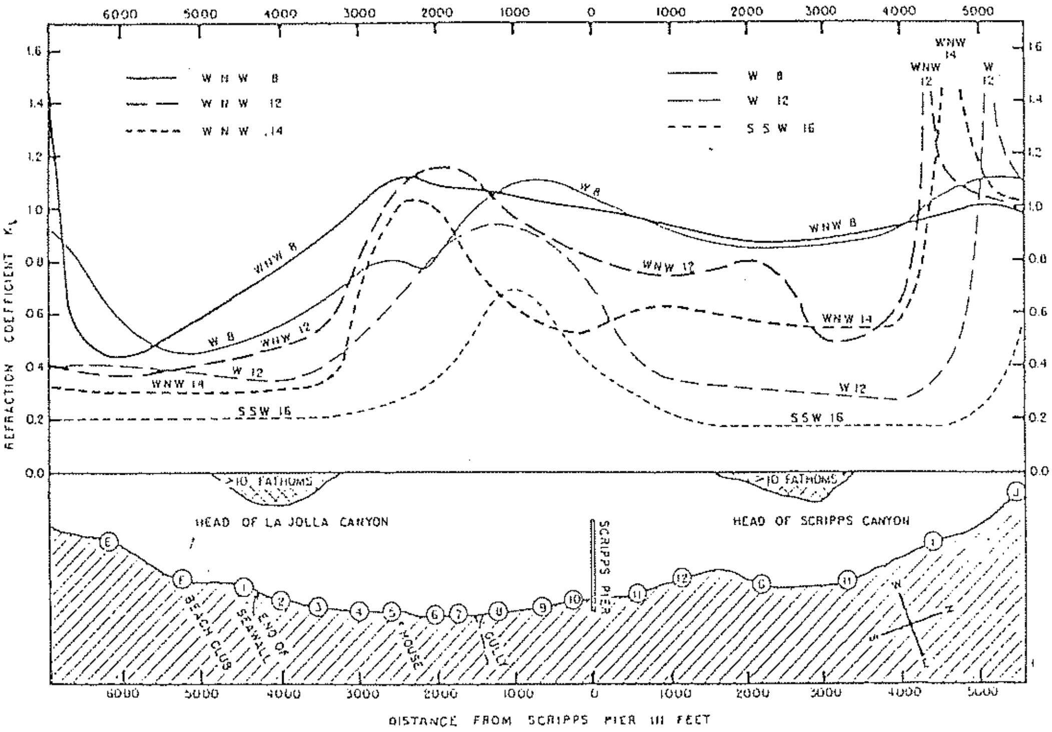


FIG. 15.—Variations of K_b along the coastline according to the refraction diagrams. The lower section of the figure shows the locations along the beach to which values of K_b in the upper section refer. For all but the southern swell, waves are generally lower near the head of the canyon than on either side. Short wave periods give smaller variations in wave height than longer periods. Changes in wave direction shift the location of the zones of convergence and divergence, but they have little effect upon the amplitude of the variation in K_b.

over-all average for all days and stations. These values were taken from the last column of Table 4. In view of the inaccuracies of the observations and the difficulty in drawing exact refraction diagrams, the agreement between theory and observation is satisfactory. The only noticeable discrepancy occurs near Station 7, where the computed wave height exceeds the observed height by about 10 per cent. This discrepancy may be re- variations in period are in phase with the variations in wave height.

The refraction diagrams can also be compared directly to aerial photographs. Plate VII was selected for an example, because it shows an unusually regular, well-defined swell. The area covered in the photograph extends between Station 5 and Station G.

The swell shown on the photograph was generated in a low-pressure area

TABLE 4

OBSERVATIONS OF BREAKER HEIGHT AND PERIOD

STA. NO.	DEC. 1, 1944			DEC. 8, 1944			JAN. 2, 1945			JAN. 3, 1945			JAN. 5, 1945			FEB. 13, 1945			FEB. 14, 1945			PER CENT AVERAGES	
	H_b		T (Sec.)	H_b		T (Sec.)	H_b		T (Sec.)	H_b		T (Sec.)	H_b		T (Sec.)	H_b		T (Sec.)	H_b		T (Sec.)	H_b	T
	Ft.	%		Ft.	%		Ft.	%		Ft.	%		Ft.	%		Ft.	%		Ft.	%			
1	0.7	33	11.5	2.0	38	11	1.0	45	.....	1.0	34	14.0	1.0	22	12.0	3.0	75	12.0	1.5	50	10.0	42	95
2	1.3	62	12.0	2.0	38	11	1.0	45	14.5	1.0	34	11.0	1.5	33	14.0	2.5	63	12.0	2.0	67	9.0	49	94
3	1.5	71	12.0	.....	.....	.....	2.5	114	12.0	1.5	52	13.5	2.0	44	14.0	1.8	45	14.0	3.0	100	12.0	71	102
4	1.5	71	12.0	5.5	104	12	2.5	114	14.0	3.5	121	11.0	4.0	87	12.0	5.0	125	12.0	3.0	100	12.0	103	97
5	2.5	119	13.0	6.0	113	12	3.0	137	14.0	5.0	173	14.0	6.0	130	14.5	6.0	150	13.0	4.0	133	12.0	136	105
6	3.5	167	12.0	4.5	85	12	3.5	159	14.5	4.0	138	14.0	11.0	240	16.0	4.5	113	12.0	4.0	133	11.0	148	103
7	3.5	167	12.0	5.0	94	13	2.5	114	14.0	4.0	138	15.0	5.0	109	15.0	5.0	125	12.0	3.5	117	11.0	123	104
8	3.0	143	12.0	6.0	113	12	2.0	91	13.0	2.5	86	10.0	4.5	98	13.0	4.0	100	12.5	3.5	117	14.0	107	99
9	2.5	119	13.0	6.5	123	13	2.5	114	13.0	3.5	121	11.0	4.5	98	13.0	3.5	88	11.5	3.5	117	11.5	111	98
10	1.5	71	13.0	8.0	151	10	.....	.....	.....	3.5	121	12.0	.....	.....	.....	4.0	100	11.5	3.0	100	11.0	100	95
11	1.5	71	13.0	6.5	123	11	2.0	91	14.0	2.5	86	12.0	5.0	109	12.0	4.0	100	12.0	3.0	100	13.0	97	99
12	.....	.....	.....	6.0	113	13	2.0	91	14.5	2.5	86	14.0	6.0	130	14.0	5.0	125	12.0	2.5	83	11.0	105	103
Av.	2.1	100	12.2	5.3	100	11.7	2.2	100	13.8	2.9	100	12.8	4.6	100	13.6	4.0	100	12.2	3.0	100	11.5	100	12.5
Forecast	3.6	(WNW)	13.0	5.8	(WNW)	12.0	2.0	(WNW)	14.0	2.4	(WNW)	15.0	3.0	(WNW)	15.0	2.6	(WNW)	14.0	3.8	(WNW)	14.0	.....	14.0

1,700 miles WNW of San Diego. The wind blew at 35 knots over a fetch of 800 miles. This situation persisted without much change from May 1 to May 3, 1944. Applying the forecasting method,[14] one obtains 6-foot breakers of 15.5-second period near the Scripps Institution pier at the time the photograph was taken. The height and period, computed from the photograph according to the method given in the surf forecasting manual,[15] equal 7 feet and 14 seconds, respectively. The breaker characteristics computed from weather maps agree with those obtained from the photograph, and the refraction diagram for 14-second waves from the WNW (Fig. 8), should be applicable.

A comparison between Figure 8 and Plate VII shows good agreement between the orientation and spacing of computed and photographed crests. To

[14] "Wind Waves and Swell, Principles in Forecasting," *Hydrographic Office, U.S. Navy, Misc. 11275* (1944). Cf. ftn. 2.

[15] Pp. 42–52 of ftn. 2.

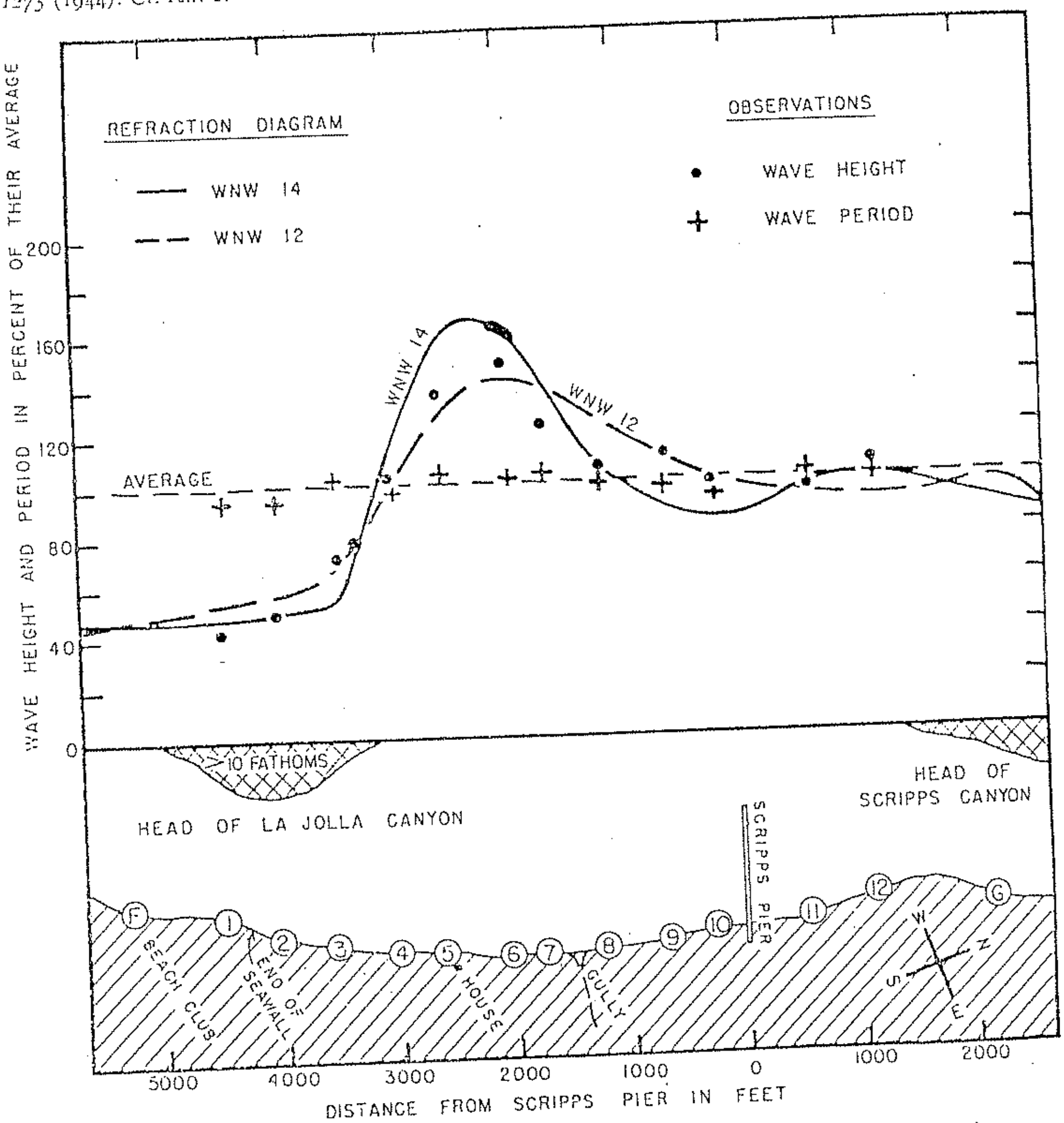


Fig. 16.—Observed versus computed changes in wave height along the beach (see text)

facilitate the comparison, some of the crests have been transferred from Figure 8 to Plate VII, where they are shown by the inked lines. Near the left edge of Plate VII the photographed crests lag, indicating a shallow bank. On a photograph taken 5 months later a similar lag occurred about 500 feet farther south. This evidence indicates the existence of a shifting sand ridge. The depth contours on Figure 8, which are based on soundings taken prior to 1928, cannot take into account such temporary features. It is quite possible that the discrepancy between observed and computed wave heights near Station 7 is also related to this shifting sand bank.

CROSSING OF WAVE TRAINS NORTH OF SCRIPPS CANYON

At the time the refraction diagrams (Figs. 8–12) were drawn, it was expected that they could give only the basic features regarding the positions of crests and the variations in wave height. Accordingly, small-scale features in the bottom topography and sharp bends in the crests or orthogonals were smoothed out, and the smoothed crest was used as a basis for the construction of the next crest. This crest was also smoothed, when necessary, and this smoothing process was continued from crest to crest as the waves were carried toward shore. In this manner a cumulative error is introduced into the refraction diagram.

If these diagrams are drawn with greater precision, and without smoothing, it is found that the steep walls on the northern side of Scripps Canyon cause a section of the crest to be bent through 90°. This section will travel north along the coast and intersect the regular wave train in a crisscross pattern.

The crisscross pattern is illustrated in Figure 17, which shows a large-scale graph of the refraction of 14-second waves from the WNW over Scripps Canyon between Stations 12 and J. The outermost crest has been transferred from Figure 8. The remaining crests were carried forward with a crest interval of one-eighth the wave period and without smoothing, but only one out of eight computed crest positions is shown on the figure (crest interval = one period). The actual construction of the refraction diagram was carried out on a detailed chart, but only two depth contours are shown on the figure.

It should be kept in mind that the shading in Figure 17 designates bottom topography and not convergence or divergence. In order to picture the degree of divergence or convergence, the *width* of the crest lines has been drawn proportional to the refraction factor K according to the scale in the legend. Wherever the refracted crests intersect the main wave train, a hump on the sea surface is formed, the height of which equals the sum of the height of the two intersecting waves.

Plate VIII, *A*, shows a photograph of this wave pattern, taken from the top of the cliffs at a position indicated on Figure 17. The wave crests in the photograph have been slightly retouched to make them appear more clearly in print. The pattern on the photograph resembles closely the pattern on the refraction diagram.

DISCUSSION AND CONCLUSIONS

METEOROLOGY AND WAVE REFRACTION

In order to draw a refraction diagram, it is necessary to know (1) the wave period and direction in deep water and (2) the bottom topography between deep water and the shoreline. The determination of typical wave periods and deep-water directions for a given locality will be discussed first, and the bottom topography and its effect will be taken up in the following section.

In the experience of the authors, existing observational data on wave characteristics are almost always inadequate for determining typical deep-water wave directions and periods for a given locality. Data on deep-water wave direction are usually lacking entirely, because they require the services of a boat or a plane. Wave period, since it remains constant, can be measured from shore, but the methods of measuring have been for the most part inconsistent and inadequate.

A better procedure is to obtain the wave characteristics from a knowledge of meteorological conditions and the relationships between wind, sea, and swell.[16] Weather maps are examined to determine all wind areas where waves affecting the locality of interest could be generated. The characteristics of the waves upon reaching the given locality are then computed by the relationships mentioned.

This method has been employed in a study for the United States Navy of the effect of wave conditions upon seaplanes. Meteorologists at the Scripps Institution carried out a day-by-day "forecast" for specified localities on the basis of historical weather maps,[17] the forecasts extending over 3 years. This method required considerable work and highly trained personnel. Fortunately, only a small number of localities have to be considered, since offshore conditions are sufficiently uniform to make the computed wave conditions applicable over long stretches of coastline.

[16] See ftn. 7.

[17] A complete set of Northern Hemisphere weather maps, dating back to 1929, were prepared during the war under the auspices of the Joint Meteorological Committee of the Army, Navy, and Weather Bureau.

The procedure described above consists essentially of computing wave characteristics from *synoptic* weather data and then applying statistical methods to the computed wave data for deriving *climatological* wave conditions. In this report a somewhat less accurate procedure has been followed by making use of average wind data obtained from climatological charts for the different seasons. An exceptionally complete set of wave observations at La Jolla, dating back to 1938, has made it possible to check the soundness of this procedure. Even so, it has been necessary to depend entirely upon local observations for information regarding the southern swell which originates in an area of the South Pacific for which weather data are lacking.

BOTTOM TOPOGRAPHY AND WAVE REFRACTION

Detailed hydrographic charts are required for an accurate construction of refraction diagrams. However, the main features in the refraction pattern can be associated with certain types of underwater topography without the need of elaborate computations (Figs. 1–6 and Pls. I–V). Quite generally it can be shown that the wave refraction pattern for any bottom topography follows a single law, according to which waves change direction and are bent in such a manner that they *tend* to assume the shape of the depth contours. Thus the wave crests on an aerial photograph draw a quasi-depth chart. This circumstance has found application during the war in determining underwater topography over enemy beaches.

Associated with the distortion of the waves are local convergences and divergences of wave energy. It is important to make a distinction between an area of *divergence* and what might be called a complete wave *shadow zone*. Waves behind a headland or to the lee of an island are usually very low because of the divergence of wave energy, yet the protected region does not lie in a wave shadow zone. Indeed, sensitive instruments have shown that waves can reach any section of a coastline, no matter how well protected, unless that section is completely cut off from communication with the open sea. By the time the waves have turned through a large angle, they are usually too low to be noticeable to the naked eye, but in some instances local convergences in so-called protected zones have led to surprising results.[18]

THE CONCEPT OF RELATIVE DEPTH

It should be emphasized that the effect of bottom features upon waves depends not upon the absolute depth of water but upon the *relative depth*, that is, the ratio of depth over deep-water wave length (h/L_0). Let h and L_0 be given in feet, and the wave period T in seconds. Setting $g = 32.2$ ft/sec² in Appendix equations (A5*b*) and (A6*a*), one obtains approximately $L_0 = 5\ T^2$, and

$$\text{Relative depth}: \quad \frac{h}{L_0} = \frac{h}{5T^2}. \qquad (6)$$

By definition shallow water has a relative depth of less than one-half. For a long 14-second swell shallow water starts therefore at a depth of 500 feet; for a short 7-second wind wave, at 125 feet. Since modification of waves occurs only in shallow water, a shoal at an average depth of 500 feet would affect the swell in a manner similar to that in which a shoal at 125 feet would affect the short-period sea. An interesting application of this principle is the *separation* of a short-period sea from a long-period swell by special types of bottom features. The bottom topography itself can be compared to an enormous lens, separating waves of different frequencies much as a lens breaks up light into the component colors. A famous illustration of this feature is provided by the surf at Waikiki beach in the Hawaiian Islands, which usually results from a NE swell refracting around Nakapuu Point and Diamond Head. Out at sea a short NE chop will usually be present in addition to the swell, but the chop is too short to be refracted effectively and thus passes on without bending into the beach.

Since ordinary waves rarely have periods much longer than 14 seconds, the effect of underwater bathymetry on these waves is virtually limited to the continental shelf. But sea and swell are not the only waves of the sea refracted by the ocean bottom. Fault movements associated with submarine earthquakes generate very long waves, "tsunamis," popularly called tidal waves (although they are not related to tides), and these have periods ranging between 10 minutes and an hour. In accordance with equation (6), these waves are always in shallow water, even over the deepest portion of the oceans. Preliminary results of an investigation by Shepard and others[19] show a good correlation between the variations

[18] An interesting application of the effect of refraction on wave height was made in connection with the liberation of the Philippine Islands. During the planning stage refraction diagrams were prepared for waves entering the Lingayen Gulf from the north, and they showed a convergence for the landing beaches, but a divergence east of the town of Lingayen. Consequently, an alternate plan was made to provide for the contingency of high waves from a typhoon. Shortly after the initial landings, high breakers prevented unloading at the main beachhead, but the breakers remained low to the east of Lingayen as indicated on the refraction diagram. A second beachhead was established according to the alternate plan, and the unloading of supplies continued without interruption.

[19] G. A. MacDonald, F. P. Shepard, and D. C. Cox, "The Tsunami of April 1, 1946, in the Hawaiian Islands (Preliminary Report)," *Pacific Sci.*, Vol. I No. 1 (January, 1947), pp. 21–37.

in the height of the April, 1946, tsunami in the Hawaiian Islands and the offshore topography and promise an interesting application of the principles discussed in this paper. A similar effect, but of even larger order of magnitude, must result from the refraction of tsunamis by large-scale bathymetric features, such as ocean basins and troughs. This might explain the curious fact that waves of the April, 1946, tsunami were recorded about twice as high at South American stations as they were at stations along California, although they had originated near the Aleutian trough and traveled more than twice as far to the South American stations.

WAVE REFRACTION AND BEACH EROSION

Krumbein[20] in his study of Half Moon Bay, California, was probably the first to consider the effect of wave refraction on sediment transport. He states that a refraction diagram indicated at least the order of change in wave energy along the coast. In the present paper it is shown that refraction diagrams also give this change quantitatively. This conclusion is of particular significance because the unusually complex bottom topography off La Jolla provides a critical test of the application of refraction theory to ocean waves. It also appears that wave refraction can be depended upon to give not only the simple main features regarding wave height and direction but also more complicated secondary effects, such as the crisscross pattern north of Scripps Canyon.

Refraction diagrams, if properly used, can become a powerful tool in the study of beach erosion. Although several processes are involved, two must be of particular importance in the effect of refraction upon beach erosion: (1) transport of sediments by *advection* associated with the secondary current system set up by the waves and (2) transport of sediments by *diffusion* associated with the turbulence set up by the breakers. The former process depends critically upon the *direction* of the waves, the latter upon the variation in *height* of the waves. The two processes depend, therefore, upon the two elements of the refraction pattern.

The term "advection" has been borrowed from meteorology or oceanography, where it denotes a transport due to a current flowing from a region of high concentration to a region of low concentration. Mathematically the effects of diffusion and convection are both contained in the vector equation

$$\frac{\partial s}{\partial t} = \nabla^2 \cdot A - \nabla \cdot sV , \qquad (7)$$

where s is any concentration, A the vector coefficient of kinematic eddy diffusivity, and V, the vector velocity. Equation (7) states that the local time change of any concentration equals the effect of diffusion minus the effect of advection. For further discussion refer to chapter v of Sverdrup *et al.*[21]

Of the currents set up by the waves, the most important are probably longshore currents, or littoral currents, and rip currents.[22] The longshore currents are set up by the transport of waves breaking at an angle with the shoreline. They are directed, therefore, *with* the longshore component of the wave motion—for example, from left to right in Figure 1. In the case of high waves and breakers forming an angle of more than 10° with

[20] "Shore Processes and Beach Characteristics," *Technical Memorandum No. 3, Beach Erosion Board* (May, 1944).

[21] H. U. Sverdrup, M. W. Johnson, and R. H. Fleming, *The Oceans, Their Physics, Chemistry and General Biology* (New York: Prentice-Hall, 1942).

[22] F. P. Shepard, K. O. Emery, and E. C. La Fond, "Rip Currents: A Process of Geological Importance," *Jour. Geol.*, Vol. XLIX, No. 4 (May–June, 1941), pp. 337–69.

the coastline, longshore currents may attain velocities of several knots. If it were not for the fact that breaker angles are greatly reduced by refraction, longshore currents would attain such high velocities that the rate of beach erosion would be many times its actual value.

The refraction pattern for submarine canyons causes the longshore currents to diverge from the center of the canyon toward both sides, leading to a removal of sediment from the canyon head. In the case of submarine ridges, the longshore currents and the sediments converge upon the ridge. Submarine canyons and ridges are associated, therefore, with a refraction pattern which sets up a current system that tends to perpetuate their existence.

Rip currents are directed from shore out to sea. Along a straight coastline they are most pronounced when the waves come straight toward shore. Usually the location of rip currents is closely associated with the refraction pattern. In the case of submarine canyons, rip currents tend to form at the zones of convergence at either side of the canyon, although frequently they turn and flow directly over the canyon just outside the breaker zone. In the case of ridges, rip currents tend to form in the convergence above the ridges. Exceptions to all these rules are, however, frequently found. The effectiveness of rip currents in transporting sand can be gauged by the fact that they have been observed to erode a channel several feet deep within a short time interval.[23]

Transportation of sediments by *diffusion* must result from the variation of wave heights along the beach. Wave heights are low opposite the mouth of the submarine canyons and high on either side. Since high breakers cause a heavy degree of turbulence, and low breakers only little turbulence, sand may be kept in suspension on the sides of the canyons while it has a chance to settle over the canyon. In the case of submarine ridges similar reasoning leads to the conclusion that the ridges tend to be eliminated by diffusion. Thus the effect of diffusion is opposite to the effect of advection. Since it is known that the heads of submarine canyons are kept fairly clean of sediments[24] and that ridges may exist for long times, it appears that the advective processes must balance the effects of diffusion and slumping.

Tidal currents are rarely of much importance along open coastlines exposed to severe wave action. The change in tide level may, however, be important, since it causes a back-and-forth shift of the surf zone, which in turn will have an effect upon the location of longshore currents, rip currents, and the zones of maximum turbulence. The rise and fall of the mean sea-level makes it impossible for any beach ever to reach a state of complete equilibrium.

It should be emphasized that the manner in which sediment is transported by wave action is not properly understood and that many exceptions have been found to the foregoing statements. Conditions, as they exist along a beach at any given time, approach a sensitive balance between erosion and deposition. The balance is profoundly disturbed by any changes in the wave and weather conditions or by the building of jetties and breakwaters. For similar reasons the manner in which the sands along a beach are shifted back and forth by wave action must depend greatly upon the refraction pattern. The resulting sediment transport may cause changes in depth which in turn affect the refraction pat-

[23] *Ibid.*, pp. 350–52.

[24] Shepard and Emery, pp. 94–103 of ftn. 13.

tern (Pl. VIII, B). This interplay between waves and bottom makes the prediction of changes in the bottom profiles extremely difficult. But by analyzing the effects of wave action from the point of view of fluid mechanics, making use also of refraction diagrams, it seems likely that a better understanding of the processes involved in beach erosion will result.

Another point of view we have tried to introduce into this report is the importance of meteorology to studies of beach erosion. A season which is unusual from the point of view of beach conditions must, in the final analysis, be the result of unusual meteorological conditions over the ocean. This does not necessarily mean that local weather conditions have been unseasonal. We have noted, for example, that some of the swell reaching southern California in the summer depends upon weather conditions in the Southern Hemisphere, perhaps 5,000 miles away. The ever changing beaches along the shorelines are a reflection of the infinite number of possible combinations and variations of storms over widely spread areas of the ocean.

APPENDIX

THEORY OF WAVE REFRACTION

Let E denote the mean energy per unit surface area, n the fraction of this energy advancing with the wave crest at velocity C, and s the distance between adjacent orthogonals. Then, if energy is conserved between orthogonals,

$$E_0 n_0 C_0 s_0 = EnCs = E_b n_b C_b s_b \,, \quad (A1)$$

where the subscript "o" refers to deep-water conditions, i.e., water whose depth exceeds one-half the wave length, and the subscript b to the breaker point. The expression without subscripts holds generally in shallow water, intermediary between deep water and the breaking-point.

In deep water the wave energy is proportional to the *square* of the wave height H_0 according to the equation

$$E_0 = \tfrac{1}{8}\rho g H_0^2 \,, \quad (A2a)$$

where ρ is density and g the acceleration due to gravity. The same relationship holds with good approximation in shallow water:

$$E = \tfrac{1}{8}\rho g H^2 \,. \quad (A2b)$$

However, just outside the breaking-point, the shape of the waves approaches that of "solitary waves," with sharp isolated crests, separated by long flat troughs. For solitary waves the wave energy is proportional to the cube of the wave height, according to the equation

$$E_b = \frac{\rho g}{L_b}\left(\frac{4h_b H_b}{3}\right)^{3/2} . \quad (A2c)$$

Combining the first two expressions of equations (A1) and (A2), we get the following general relationship for the wave height in shallow water:

$$\frac{H}{H_0} = \left(\frac{n}{n_0}\frac{C}{C_0}\frac{s}{s_0}\right)^{-1/2} . \quad (A3)$$

In deep water only half the energy, the potential energy, advances with the wave form:[25]

$$n_0 = \tfrac{1}{2} \,. \quad (A4a)$$

In the breaker zone both potential and kinetic energy travel with the waves,

$$n_b = 1 \,, \quad (A4b)$$

while for shallow water in general

$$n = \frac{1}{2}\left[1 + \frac{\frac{4\pi h}{L}}{\sinh\left(\frac{4\pi h}{L}\right)}\right] . \quad (A4c)$$

Equation (A4c) reduces properly to equations (A4a) or (A4b), depending upon whether the ratio h/L has a large or small value.

The velocity of sinusoidal irrotational waves of low amplitude is given by the classical Stokes's equation

$$C^2 = \frac{gL}{2\pi}\tanh\frac{2\pi h}{L} \,, \quad (A5a)$$

[25] H. U. Sverdrup and W. H. Munk, "Wind, Sea and Swell: Theory of Relations for Forecasting," *Hydrographic Office, Technical Report in Oceanography No. 1* (in press).

which for deep water reduces to the form

$$C_0^2 = \frac{gL_0}{2\pi}. \qquad (A5\,b)$$

Since wave period T, defined as the ratio of wave length over wave velocity, remains constant

$$T = \frac{L_0}{C_0} = \frac{L}{C} = \frac{L_b}{C_b} \qquad (A6\,a)$$

or

$$\frac{C}{C_0} = \frac{L}{L_0}. \qquad (A6\,b)$$

Dividing (A5a) by (A5b) and making use of (A6b), we get

$$\frac{C}{C_0} = \frac{L}{L_0} = \tanh\frac{2\pi h}{L}. \qquad (A7)$$

Equation (A7) can also be written

$$\frac{L}{L_0} = \tanh\left[2\pi\frac{h}{L_0}\left(\frac{L}{L_0}\right)^{-1}\right], \quad (A7')$$

which is an equation of the form

$$\frac{L}{L_0} = \text{function}\left(\frac{h}{L_0}\right). \qquad (A8)$$

The ratio h/L_0 is termed "relative depth." It follows from equations (A7) and (A8) that the ratio C/C_0 is a function of the relative depth only. In a similar manner it can be shown that the ratio n/n_0 is a function of the relative depth.

Thus equation (A3), which can be written

$$\frac{H}{H_0} = \left(\frac{n}{n_0}\frac{C}{C_0}\right)^{-1/2}\sqrt{\frac{s_0}{s}}, \qquad (A9)$$

consists of two parts: the first part, which depends upon the relative depth and does not contribute toward a *variation* in wave height along any fixed contour, and the second part, the refraction factor, according to which any variation in wave height along a beach outside the breaker zone, must be proportional to the square root of s_0/s.

To find the corresponding expression for the breakers, combine the first and third term of equations (A1) and (A2)

$$\frac{C_0}{2}\,\frac{1}{8}H_0^2 s_0 = \frac{C_b}{L_b}\left(\frac{4h_bH_b}{3}\right)^{3/2} s_b,$$

where use has been made of equations (A4a) and (A4b). In view of (A6a), this can be written

$$h_bH_b = \tfrac{3}{4}\left(\tfrac{1}{16}\right)^{2/3}\left(H_0^2L_0\right)^{2/3}\left(\frac{s_0}{s_b}\right)^{2/3}. \quad (A10)$$

According to theory substantiated by experiments with solitary waves,

$$h_b = 1.28\,H_b, \qquad (A11)$$

and equation (A10) becomes *

$$\frac{H_b}{H_0} = \frac{1}{3.3\sqrt[3]{\dfrac{H_0}{L_0}}}\sqrt[3]{\frac{s_0}{s}}. \qquad (A12)$$

Any *variation* in breaker height must be proportional to the cube root of s_0/s. The ratio H_0/L_0 is called the wave steepness in deep water. Equation (A12) and the foregoing discussion hold strictly only for waves of steepness H_0/L_0 less than 1 per cent (i.e., for swell) but is approximately true for all but the steepest storm waves.

The general equation (A9) for wave heights in shallow water neglecting refraction was first derived by Green in 1839 and extended by O'Brien and Mason[26] to include the refraction effect. The existence of solitary waves was first noticed by Russell, and their theory was developed by Boussinesque in his study of tide waves and related phenomena. The application of the solitary-wave theory to surf problems and the derivation of equation (A12) are believed to be new.

ACKNOWLEDGMENTS.—We are indebted to Dr. H. U. Sverdrup for the active role he has played in developing the theory and application of wave refraction. Dr. F. P. Shepard and Mr. D. Leipper have made many helpful suggestions. The exacting job of constructing the refraction diagrams in Figures 8–12 was undertaken by Sgt. R. Jentoft, AC, while on duty at the Scripps Institution. Mr. D. Pritchard has drawn the complex refraction diagram shown in Figure 17.

[26] Pp. 37–39 of ftn. 5.

* neglecting s_0/s

$H_0 = 6.0\,H_b^{3/2}\,L_0^{-1/2}$

PLATE I

A, Long waves from the southern hemisphere approaching the coast north of Oceanside, California. Compare refraction pattern with schematic drawing in Fig. 1. (Official photograph, United States Navy.)

B, Refraction of waves over ridge along the northern shore of Oahu, T.H. (cf. Fig. 4). Over the ridge waves lag behind and are higher than on either side. (Official photograph, United States Army.)

PLATE II

Refraction of waves over submarine canyon at Redondo, California (cf. Fig. 3). Waves bend sharply forward over canyon. Waves are higher just outside the canyon than over the canyon, and the surf zone is narrow opposite the mouth of the canyon. (Photograph by Spence Air Photos, Los Angeles.)

PLATE III

Refraction of waves over the coral reef at Guam. Wave crests move ahead over the channels and lag behind over the shoals. Wave heights are low over channels and large over shoals. Compare with schematic drawings in Figs. 3 and 4. (Official photograph, United States Navy.)

PLATE IV

A, Refraction of waves around Point Loma, San Diego, California. Solid orthogonal shows main refraction around the point; dashed orthogonal designates secondary refraction around a shoal directly off the end of Point Loma. Notice variation in width of surf zone. (Official photograph, United States Navy.)

B, Waves turn through 180° around the hook of Cape Cod but, in doing so, lose most of their height because of the very large divergence. Notice variation in width of surf zone. (Official photograph, United States Navy.)

PLATE V

Waves bend toward the famous Waikiki Beach on Oahu, T.H. (Official photograph, United States Navy.)

PLATE VI

Photograph of a three-dimensional model of the bottom topography off La Jolla, California. Two submarine canyons, La Jolla Canyon and its steep-walled tributary, Scripps Canyon, reach within a thousand feet from shore.

PLATE VII

Observed versus computed orientation and spacing of wave crests. The computed positions (*inked lines*) are parallel to the positions of the crests on the photograph, except for a small discrepancy on the left side of the figure (see text).

PLATE VIII

A, Photograph of waves north of Scripps Canyon shows the presence of the crisscross pattern which appears in the refraction diagram (Fig. 17). Peaks occur where the refracted crests intersect the crests of the main wave train.

B, Refraction of waves at boat basin, Oceanside, California. Longshore currents set up by waves have widened the beach on both sides of jetties (see also Pl. II). This in turn has caused refraction of waves near jetties. The resulting refraction pattern tends to further strengthen the convergence of longshore currents upon the jetties and may lead to accelerated deposition.

PLATE I

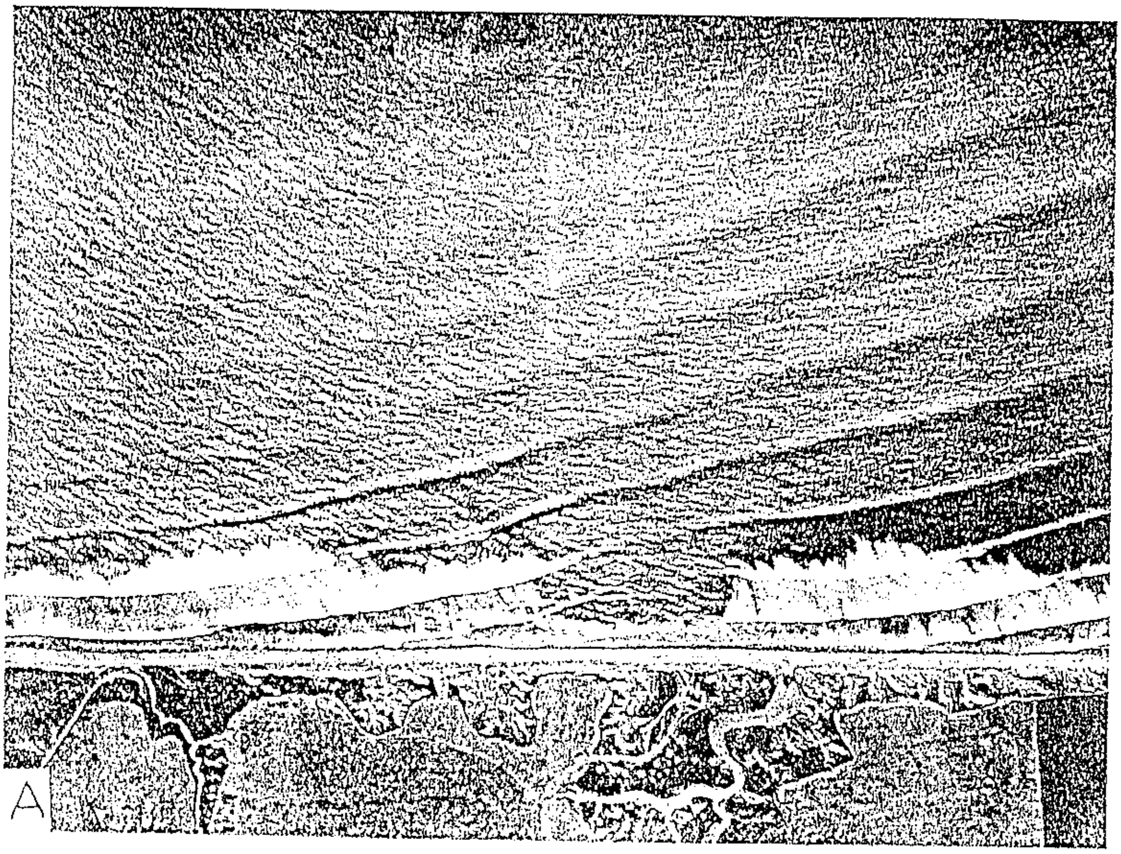


PLATE II

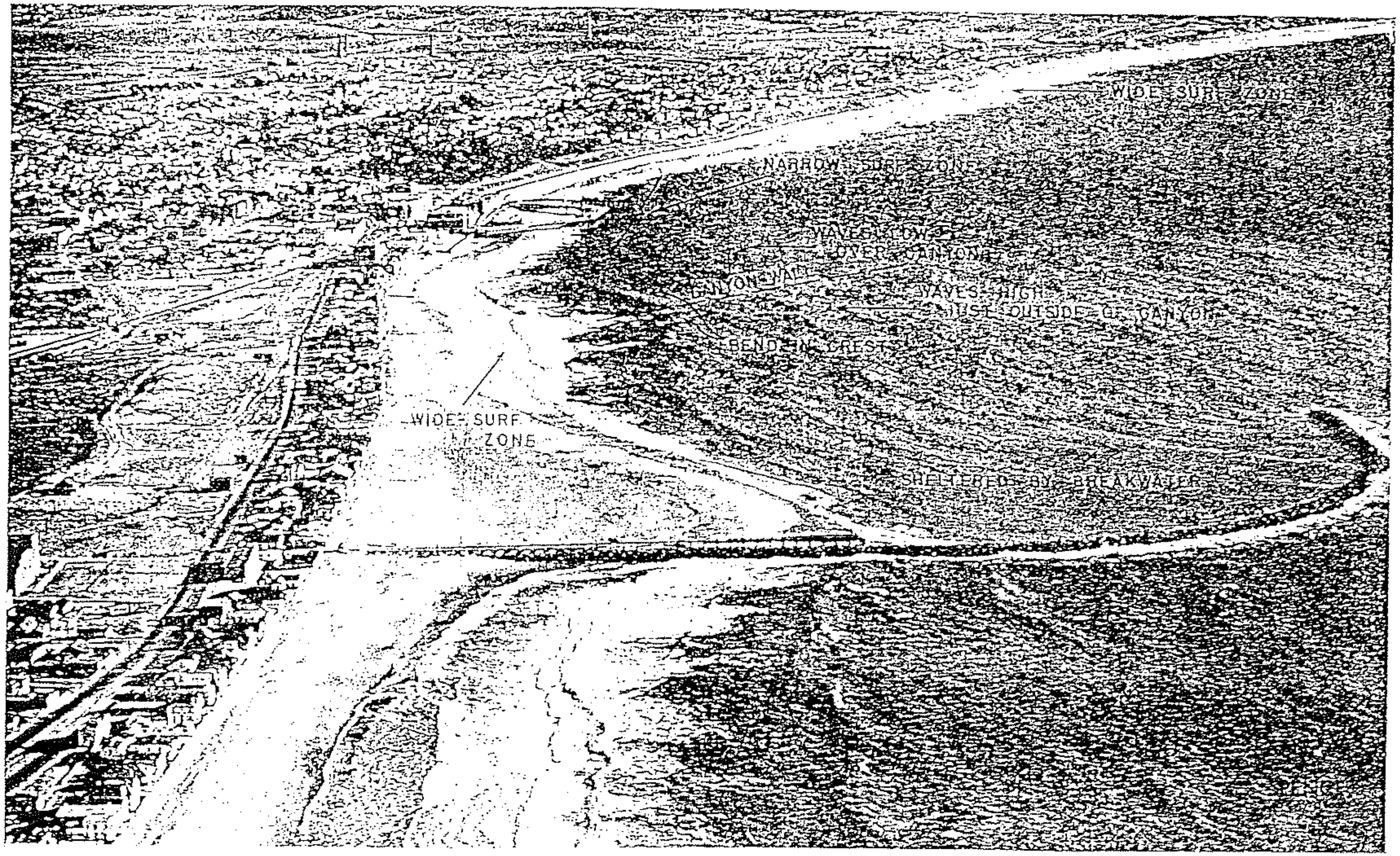
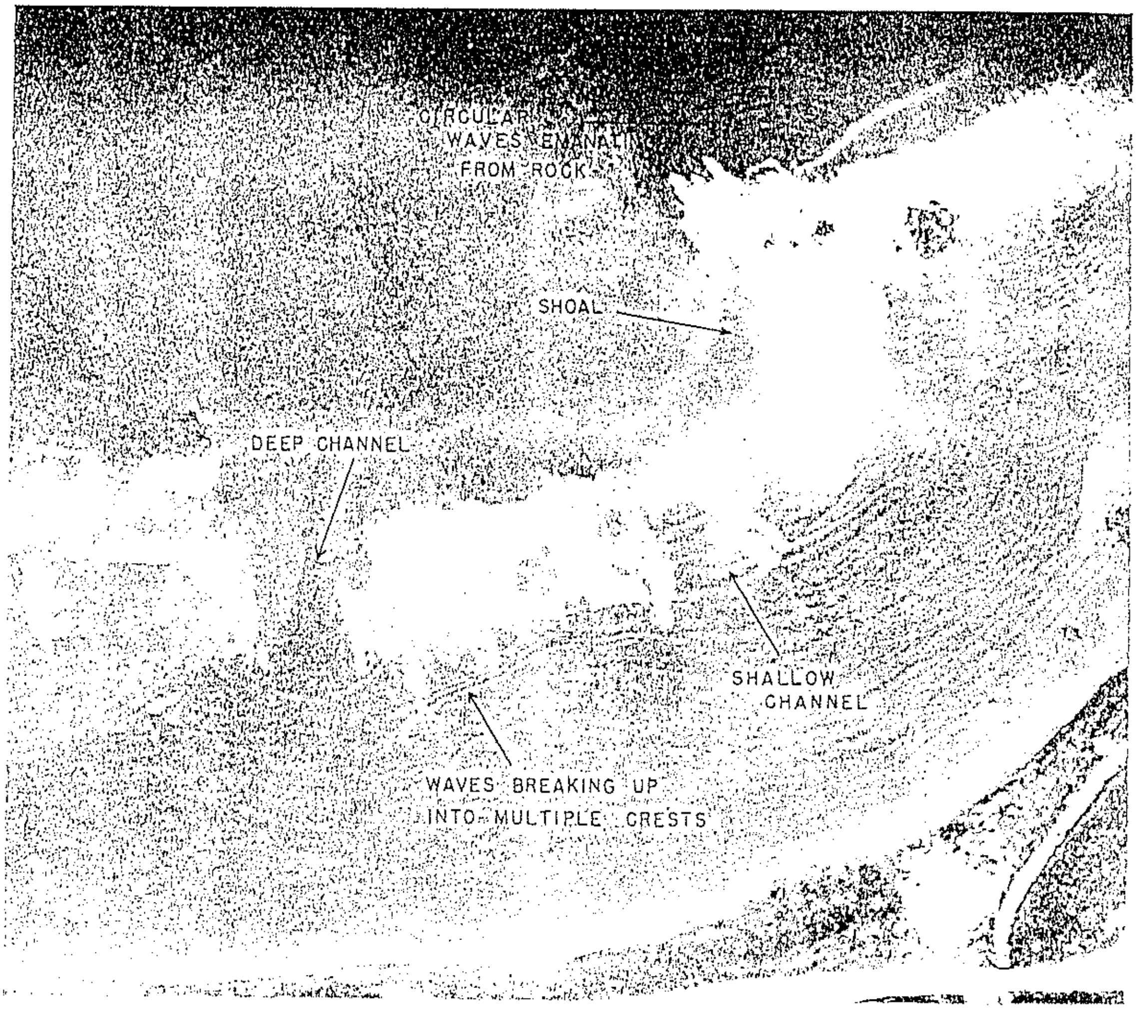

WAVES LOW
OVER CANYON
WAVES HIGH
JUST OUTSIDE OF CANYON
BEND IN CREST
WIDE SURF
ZONE
SHELTERED BY BREAKWATER

PLATE III


CIRCULAR
WAVES EMANATING
FROM ROCK
SHOAL
DEEP CHANNEL
SHALLOW
CHANNEL
WAVES BREAKING UP
INTO MULTIPLE CRESTS

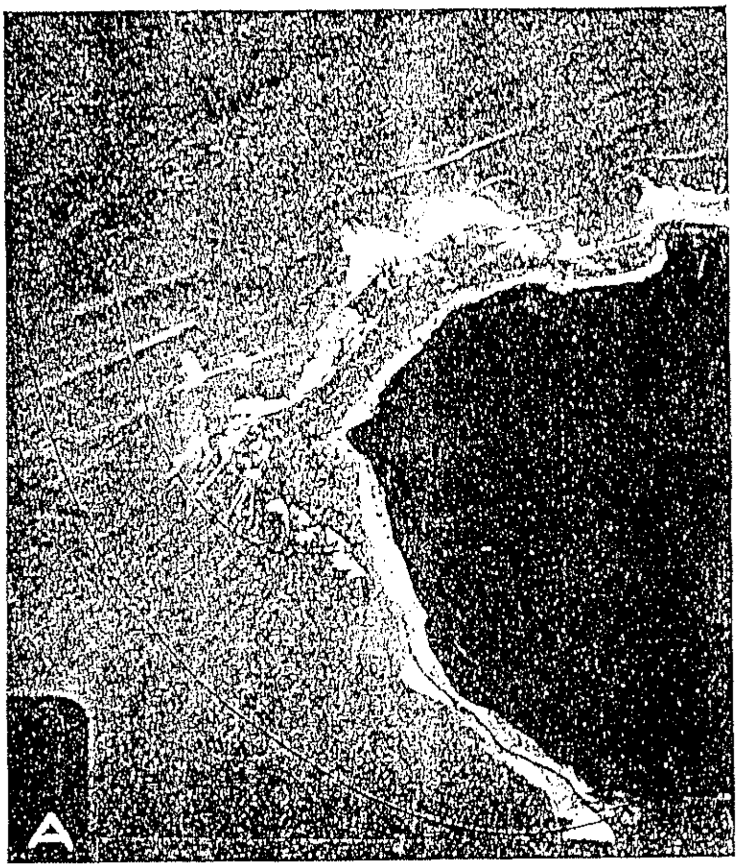


PLATE VI

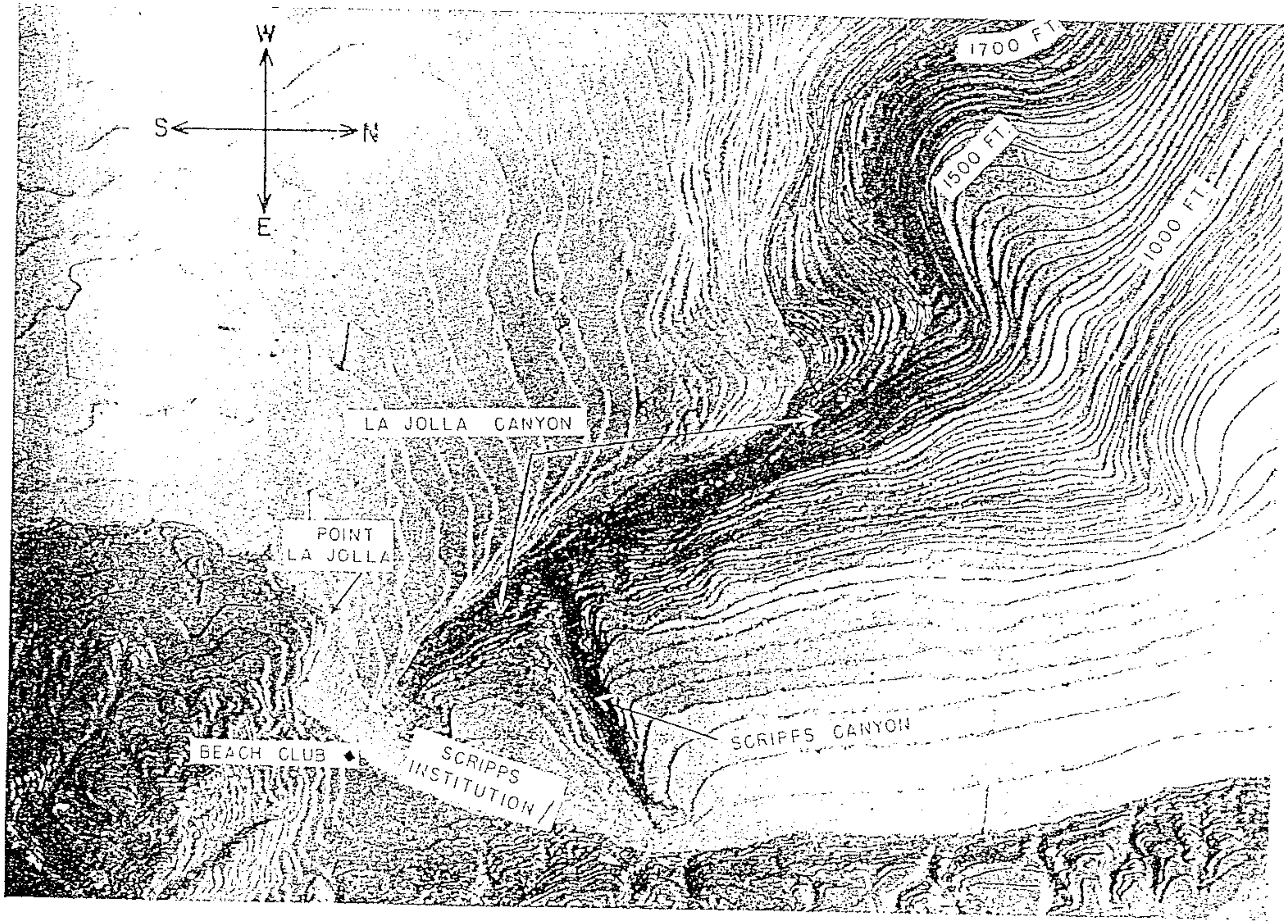

W
S
N
E
LA JOLLA CANYON
POINT LA JOLLA
BEACH CLUB
SCRIPPS INSTITUTION
SCRIPPS CANYON
1700 FT.
1500 FT.
1000 FT.

PLATE VI

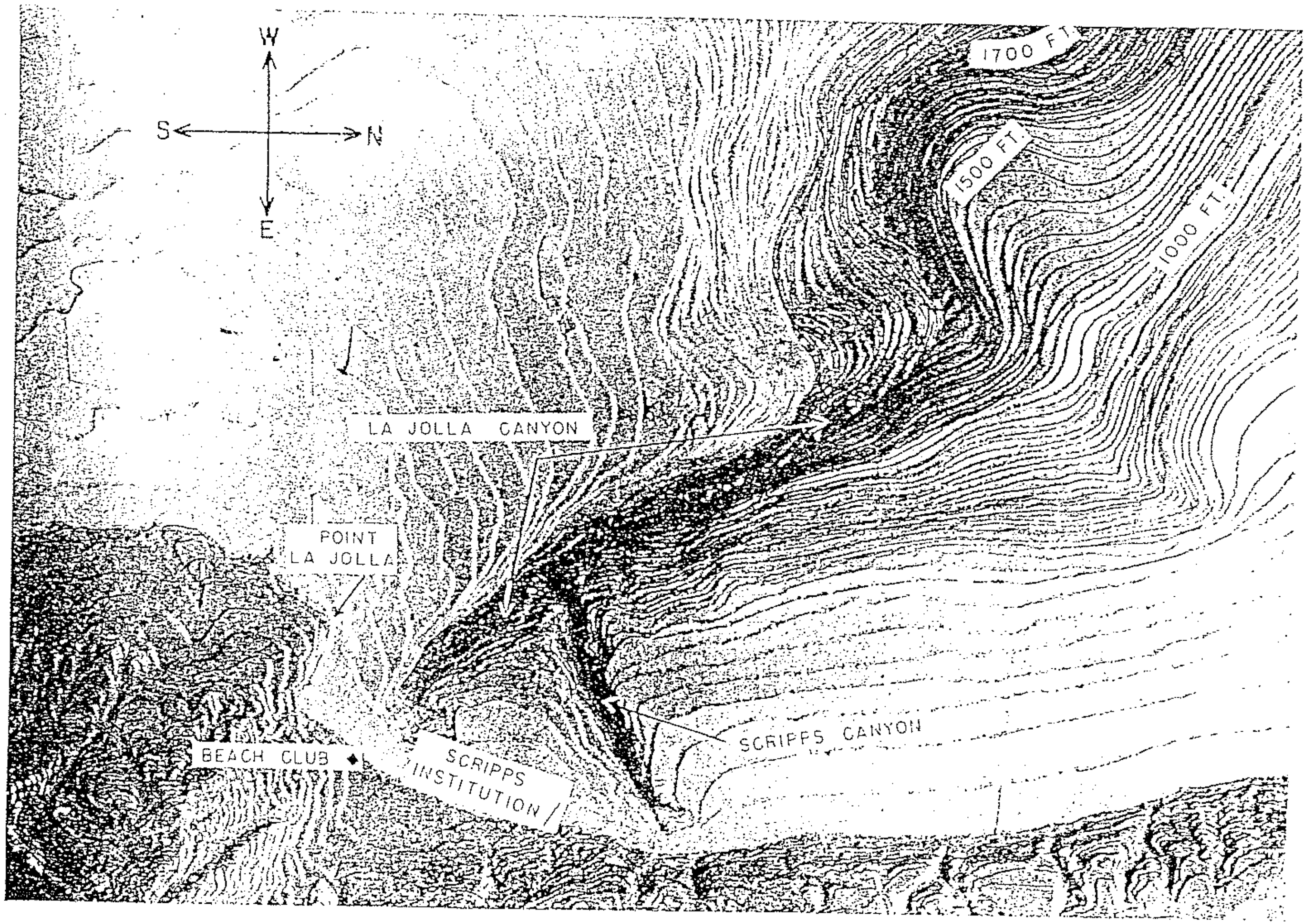
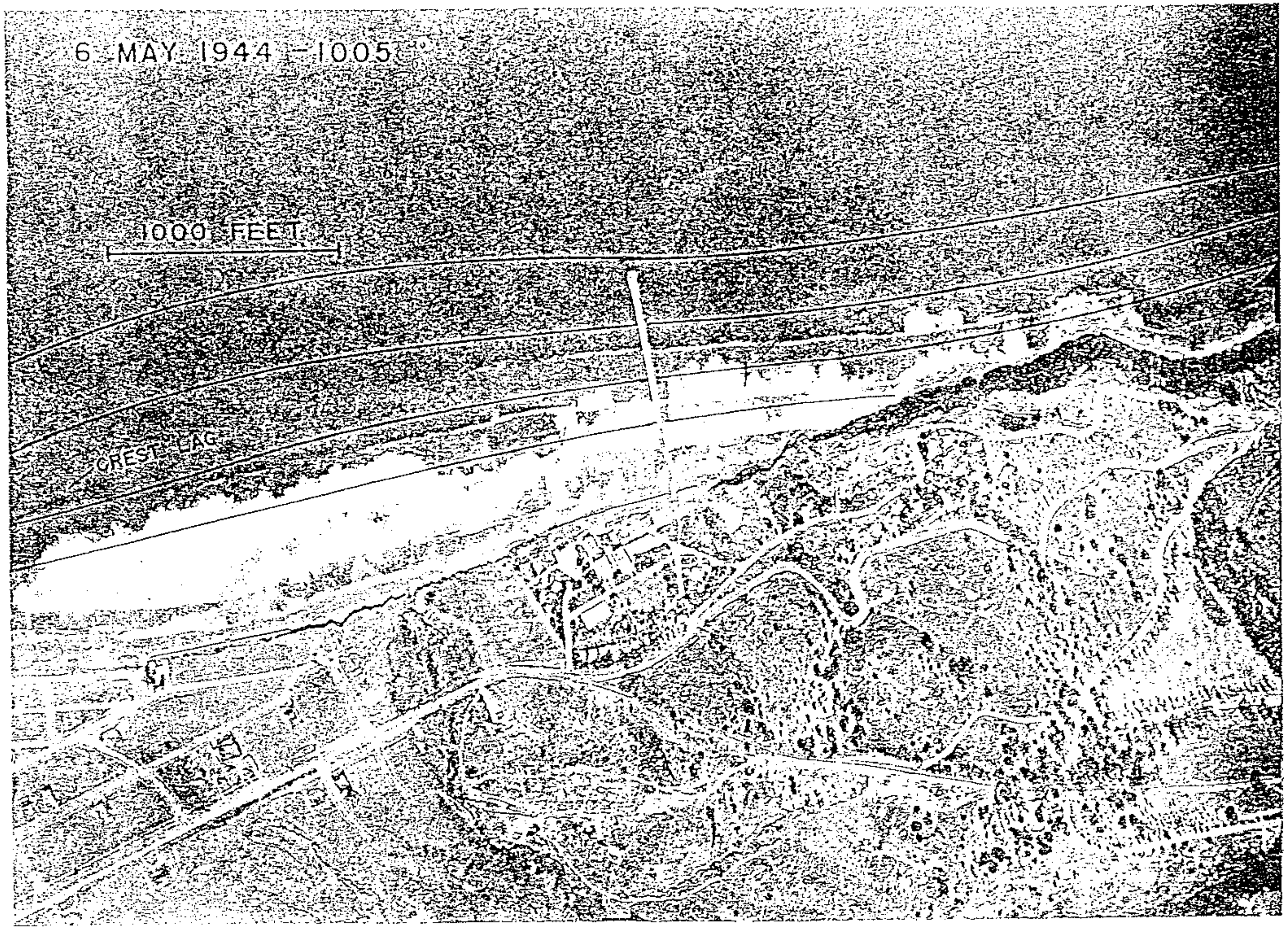
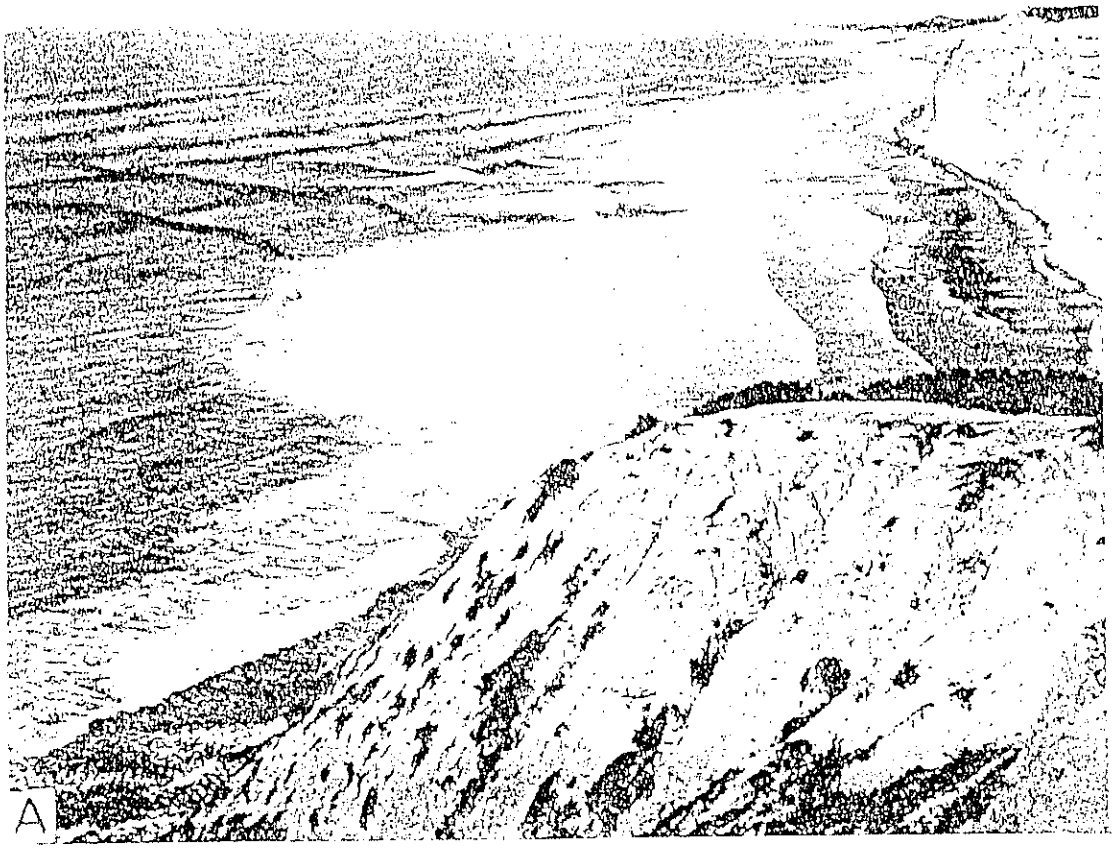
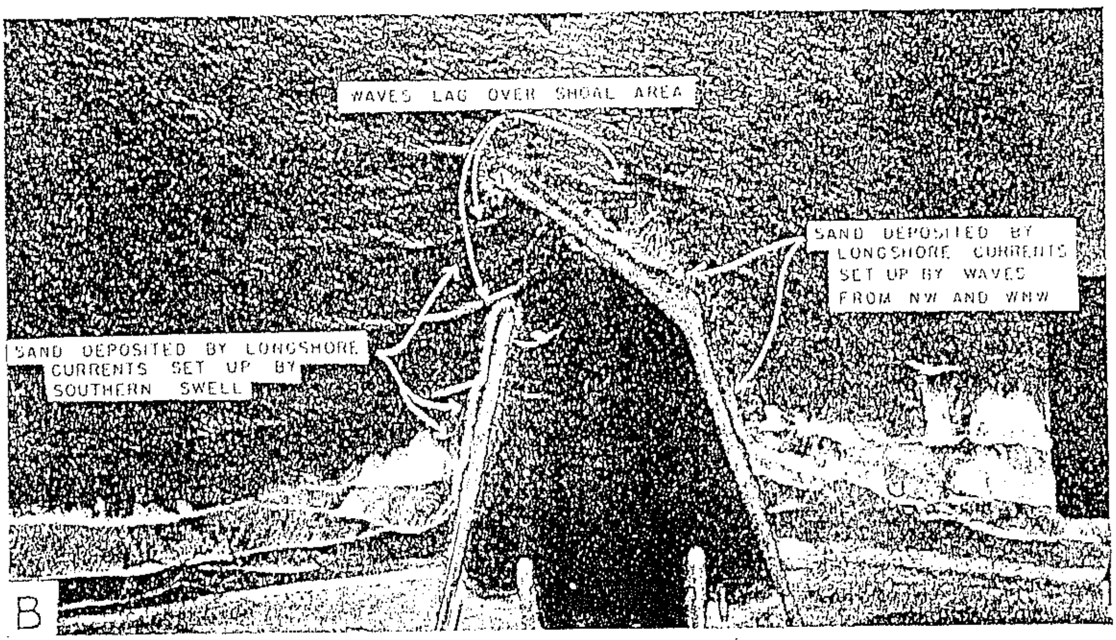

W
S
N
E
1700 FT.
1500 FT.
1000 FT.
LA JOLLA CANYON
POINT
LA JOLLA
BEACH CLUB
SCRIPPS
INSTITUTION
SCRIPPS CANYON

PLATE VII






WAVES LAG OVER SHOAL AREA
SAND DEPOSITED BY LONGSHORE CURRENTS SET UP BY WAVES FROM NW AND WNW
SAND DEPOSITED BY LONGSHORE CURRENTS SET UP BY SOUTHERN SWELL

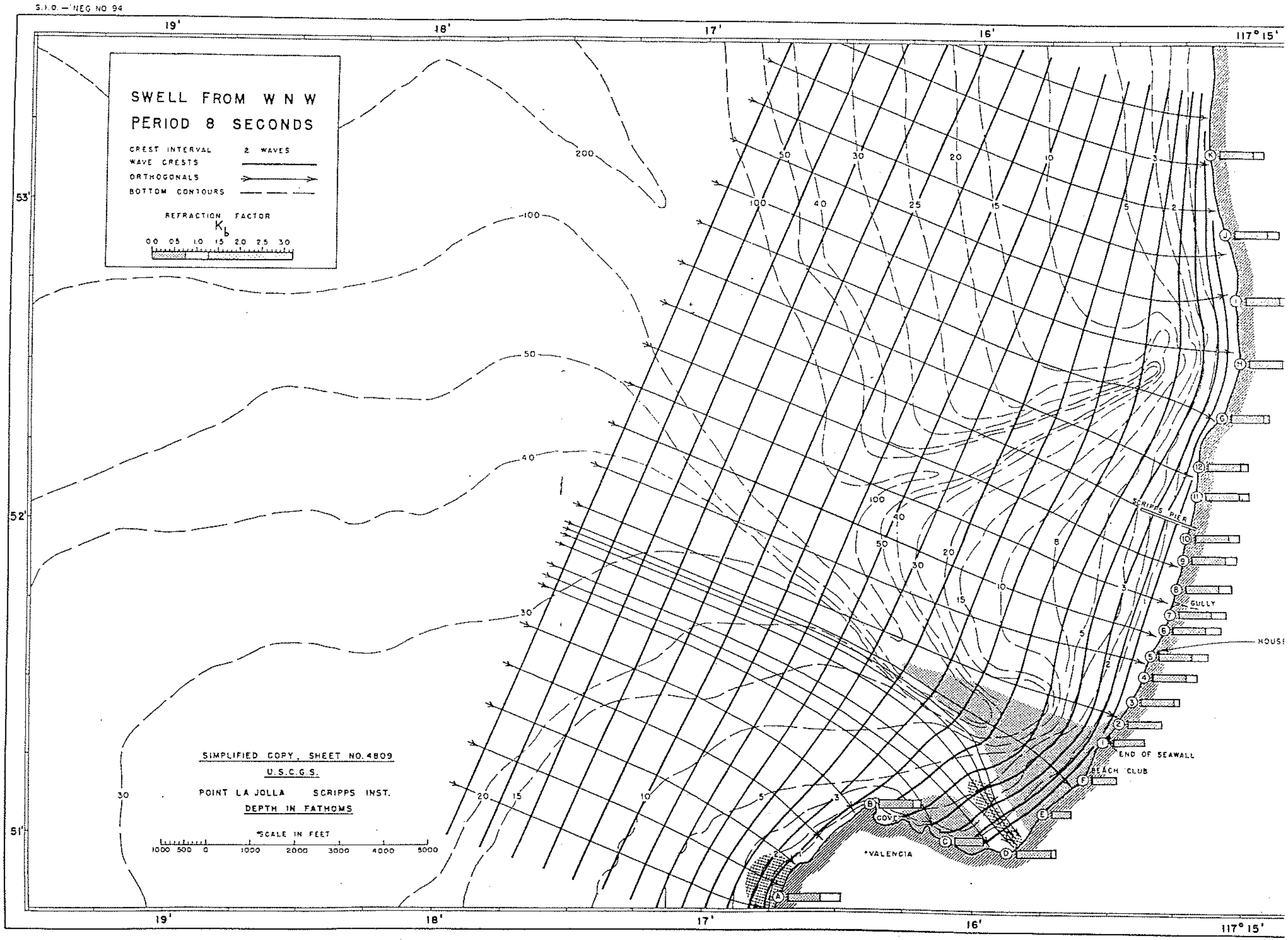


Fig. 11.—Owing to the short wave period, the effect of refraction is reduced even further than in the previous refraction diagrams

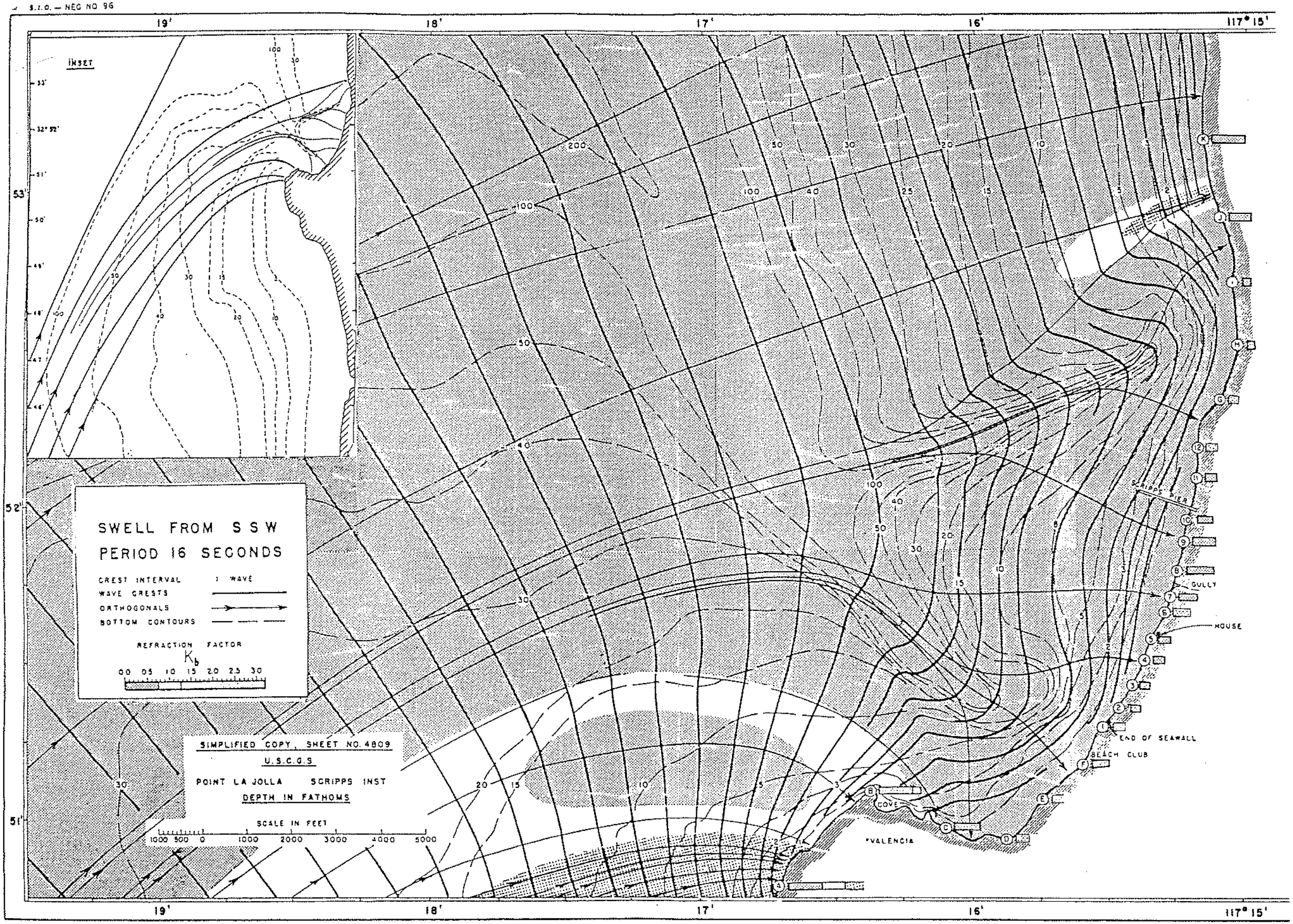


FIG. 12.—Except for two localities near Stations A and J, the entire area is protected from the southern swell. A few selected orthogonals between deep water and shore have been entered in an inset in order to show tha waves from the SSW the effect of refraction outside the limits of the main figure cannot be neglected.

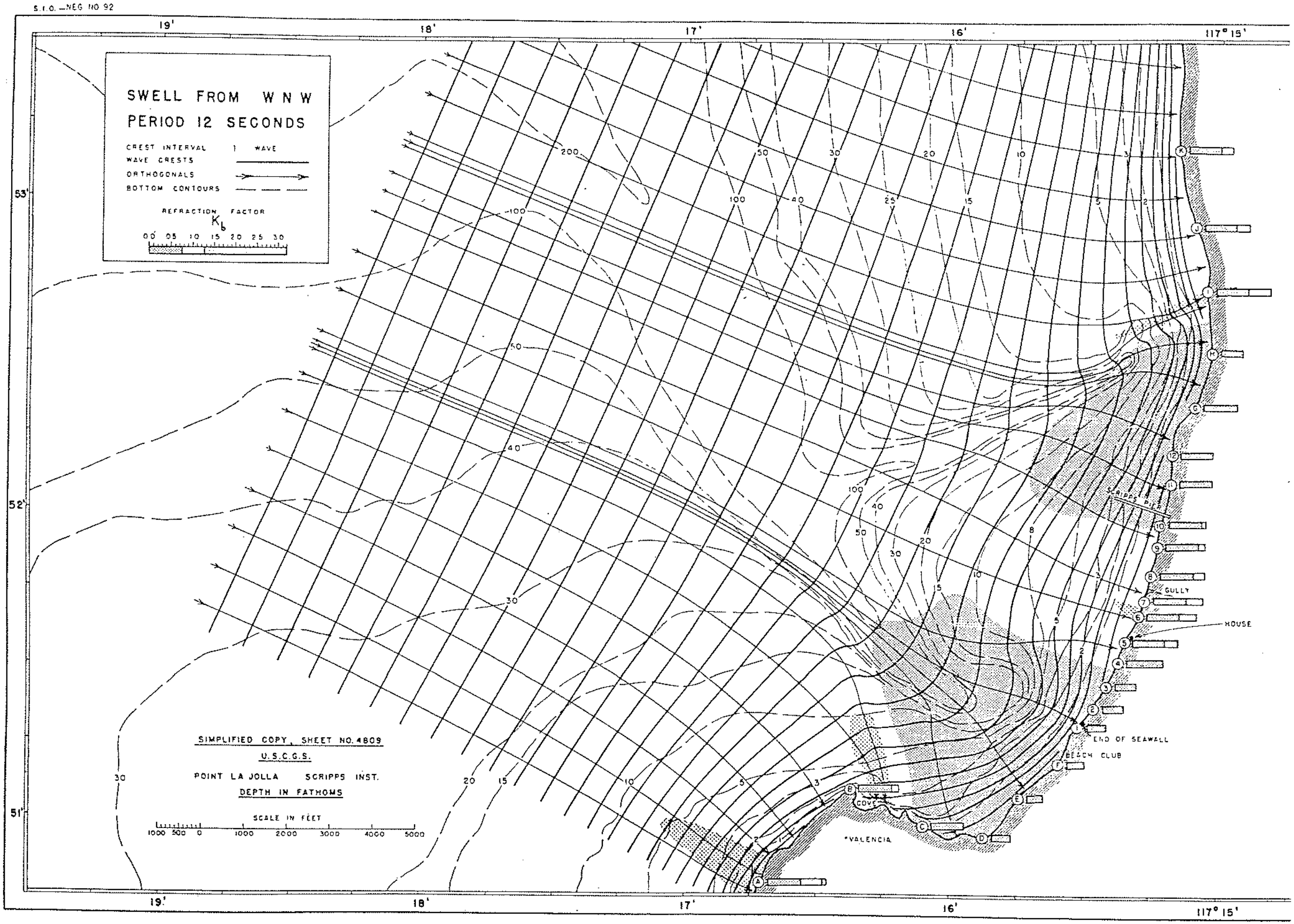


FIG. 9.—The general refraction pattern is similar to the one in Fig. 8 but, owing to the shorter wave period, variations in wave height are smaller

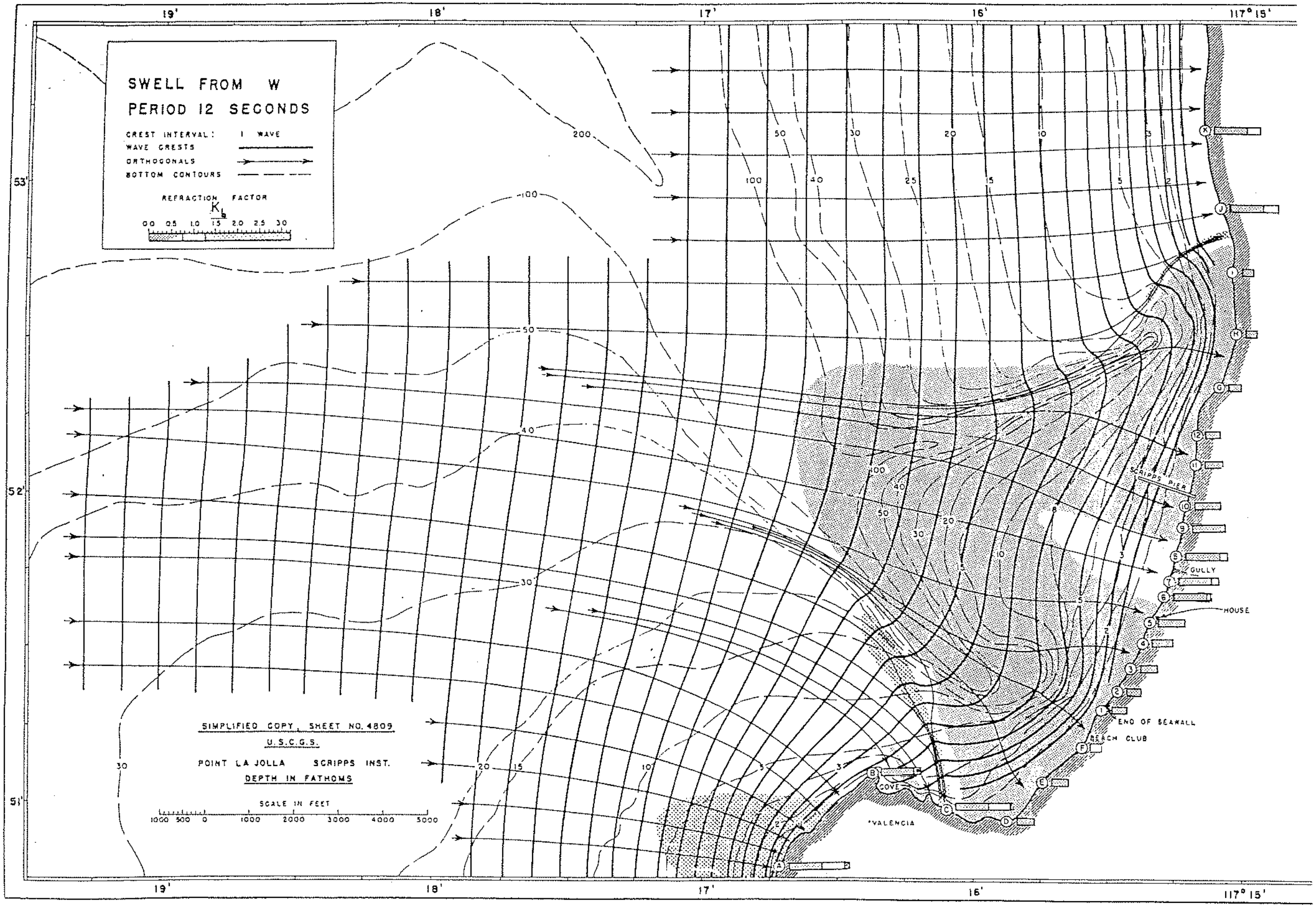


FIG. 10.—Similar to Fig. 9, but, since the waves are assumed to come from the west rather than from the west-northwest, the entire refraction pattern is rotated counterclockwise

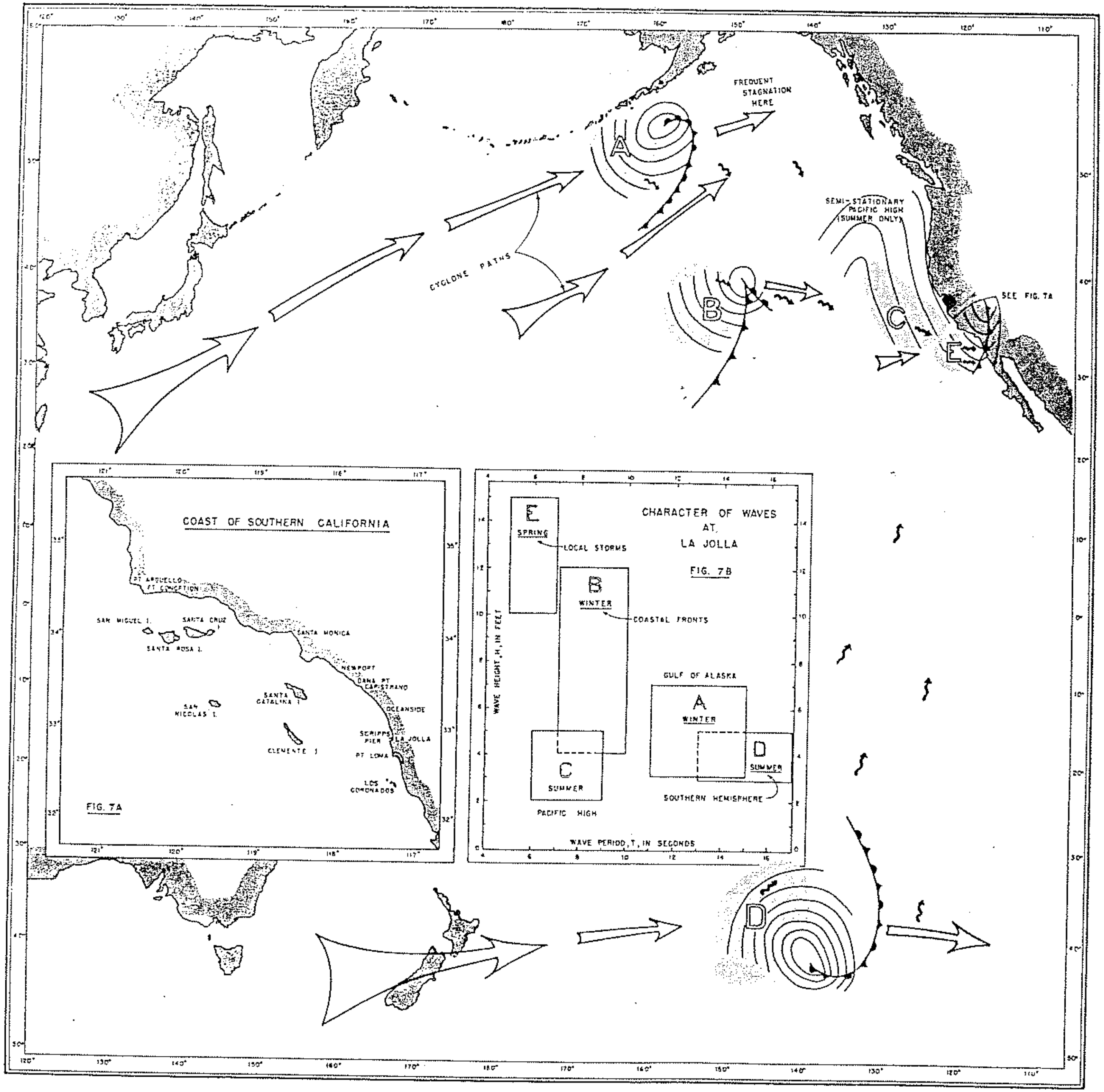


FIG. 7.—Origin of waves reaching La Jolla, California. The capital letters on the large chart represent typical meteorological situations, which give rise at La Jolla to waves of heights and periods shown in Figure 7, *B*. Figure 7, *A* is a chart of the region near La Jolla on an enlarged scale. The shaded areas on the large chart represent typical fetches, that is, areas where the waves are generated. The isobaric pattern and the nature of the meteorological fronts are also indicated.
The large open arrows represent typical ...

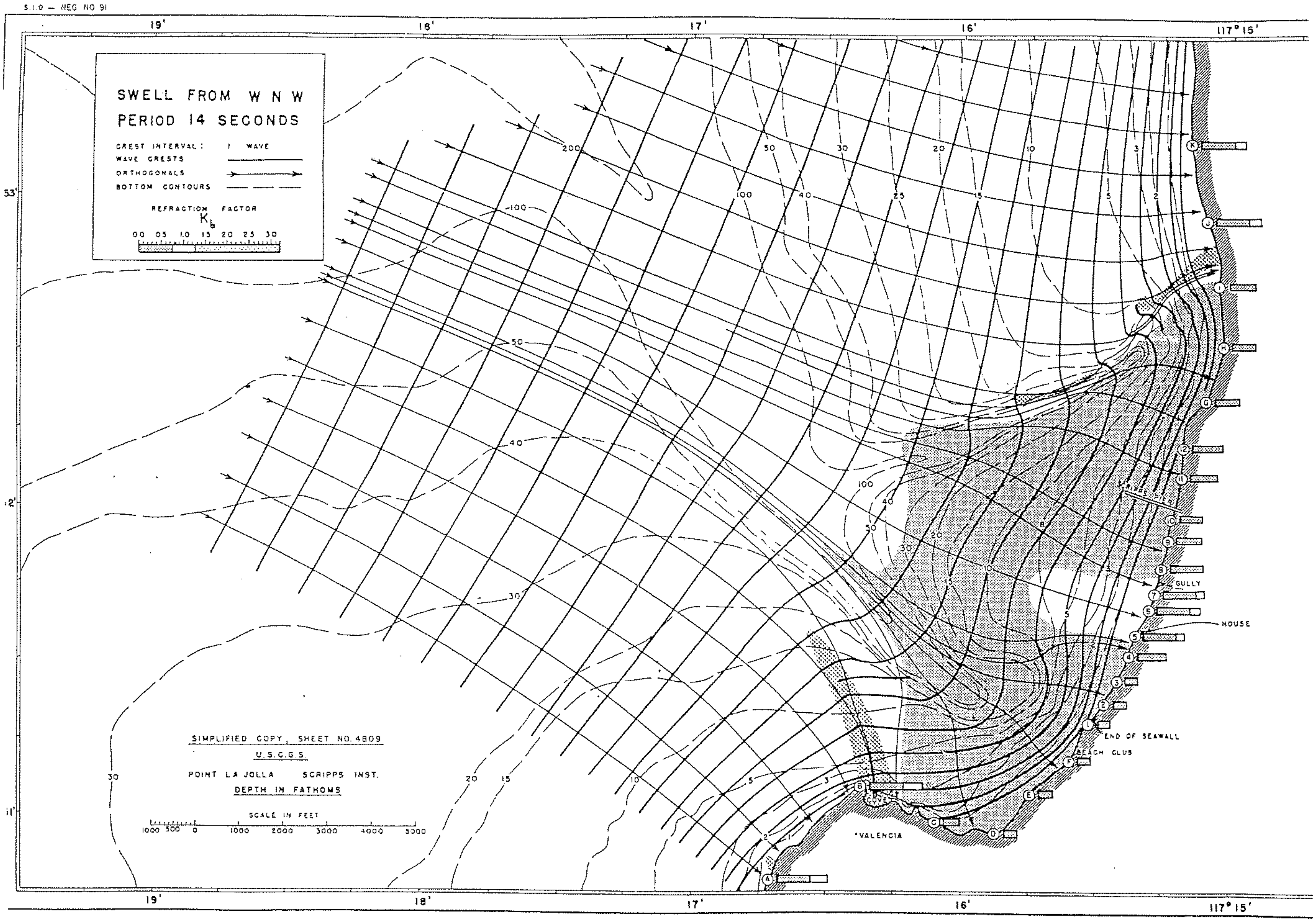


FIG. 8.—Refraction diagram for 14-second waves from the west-northwest. Heavy solid lines show positions of wave crests, the solid lines with arrowheads give orthogonals, and dashed lines are depth contours. The crosshatche[d areas ind]icate zones of convergence ($K > 1.25$); the dotted regions indicate zones of divergence ($K < 0.75$). The lengths of the rectangles opposite the shore stations are proportional to K_b and thus indicate the variation of breaker heights [along the bea]ch. The general refraction pattern conforms with the schematic drawing of Fig. 3.