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SURF BEATS

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Abstract--Irregular oscillations of the near-shore water level, with periods of the order of several minutes, are correlated with fluctuations in the height and period of incoming waves.

An instrument for recording long-period ocean waves [MUNK, IGLESIAS, and FOLSOM, 1948] has been in operation since January 1948 at the end of the Scripps Institution pier, La Jolla, California. The instrument, which is sensitive to waves whose frequencies fall between those of the wind-generated waves and those of the astronomic tides, is called the tsunami recorder, as it is especially adapted to the period range of tsunamis. In the presence of high wind waves the tsunami recorder reveals irregular oscillations of several minutes' period. These oscillations have a broad frequency spectrum, which reaches a maximum in amplitude for periods of the order of two minutes, and drops off for longer and shorter periods by about six decibels per octave.

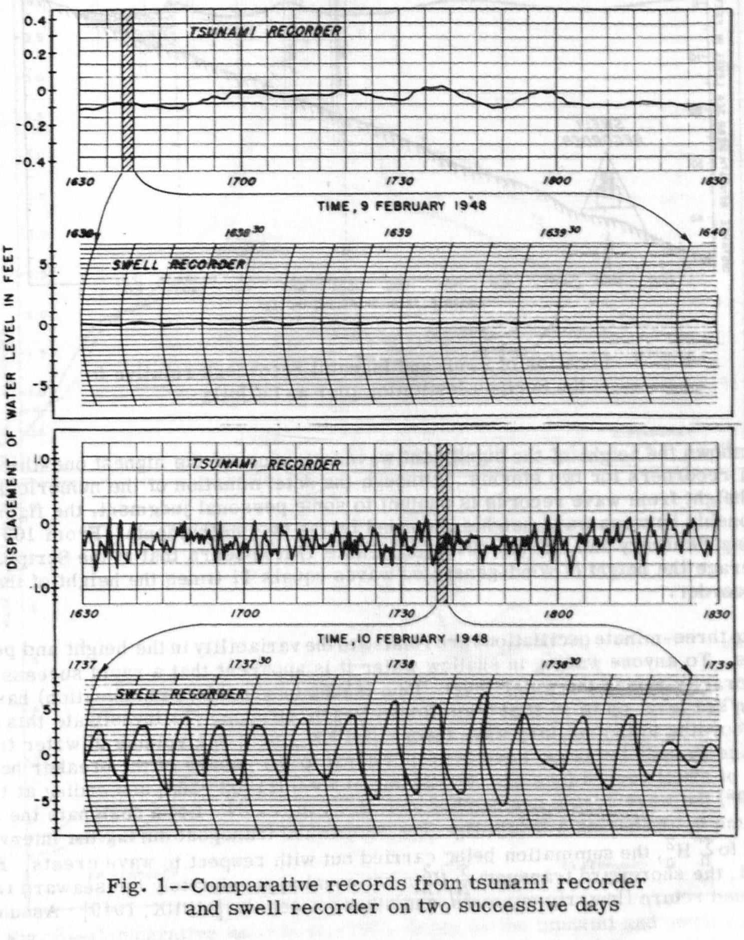

Fig. 1--Comparative records from tsunami recorder and swell recorder on two successive days

Figure 1 shows comparative records taken on two successive days by the tsunami recorder and by a swell recorder [FOLSOM, 1949]. The time scales bear a ratio of 60:1, and the shaded band on the tsunami record shows the duration of the swell record. The height scales on the tsunami and swell records refer to periods of two minutes and eight seconds, respectively. The swell recorder is about equally responsive to waves of eight-second and two-minute periods, whereas the tsunami recorder is forty times as responsive to the two-minute waves as to the eight-second waves. The swell recorder is located 1000 ft off the end of Scripps Pier in water of about 40-ft depth, (see Fig. 2); it records pressure fluctuations at the sea bottom against time, and these can be interpreted in terms of the oscillations of the sea surface. Both instruments are connected by electric cables to a building ashore where the waves are recorded. On the afternoon of February 9 the sea was glassy calm (wave heights less than three inches), and the tsunami record smooth, except for two-inch undulations of approximately 20-min period, which are typical of certain meteorological conditions off the coast of California. Twenty-four hours later the swell recorder showed waves of seven- to nine-second period up to ten feet in height, and the tsunami recorder revealed irregular two- to three-minute oscillations up to 0.8 feet in height.

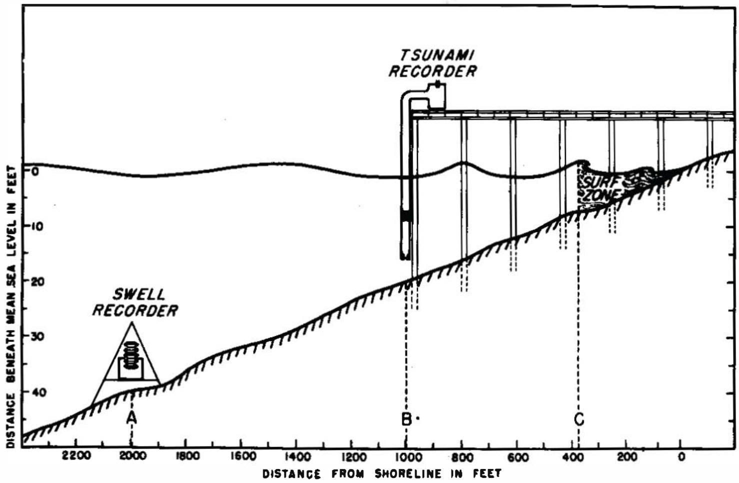

Fig. 2--Location of swell and tsunami recorders relative to the Scripps Institution pier at La Jolla

Figure 3 shows the height of the significant waves (average of the highest one-third) on the tsunami and swell recorders for two storms. Although the determination of the numerical values of significant wave height from wave records is subject to some personal judgment, the figure indicates a definite relationship between wave heights recorded by the two instruments. From 103 measurements conducted during February and March, 1948, (see Table 1) it appears that at the Scripps Institution pier on the average the height of wind-generated waves equals 11 times the height of the waves on the tsunami recorder.

The two- to three-minute oscillations are related to the variability in the height and period of the incoming waves. To anyone wading in shallow water it is apparent that a rapid succession of high breakers temporarily raises the water level. John Isaacs (personal communication) has observed such changes in sea level up to 16 ft at Twin Rocks Beach, Oregon. To investigate this quantitatively, we note that according to the solitary wave theory [MUNK, 1949] the volume of water transported into the surf zone by each breaking wave is proportional to the square of the breaker height. On a 20-min section of swell record (see Fig. 4) starting at $t = 0$ ($11^h02^m20^s$) and ending at $t = 1080$ sec ($11^h20^m20^s$) the wave crests are numbered 1, 2, . . .n, . . .N. Let n designate the nth wave arriving at a time t_n and having a height H_n. The shoreward transport during the interval O to t_n is proportional to $\sum_n H_n^2$, the summation being carried out with respect to wave crests. For the entire 20-min interval, the shoreward transport $\sum_N H_n^2$ must equal approximately the seaward transport due to a superimposed return flow, rip currents, and other processes [MUNK, 1949]. Assume the seaward transport to be constant with time. Then

$$\sum_{n} H_n^2 - (n/N)\sum_{N} H_n^2$$

Table 1--Comparative heights of waves on tsunami recorder (period two to three minutes) and swell recorders (period seven to nine seconds)

Range in wave height	Number of measurements	Significant wave height: Tsunami recorder	Significant wave height: Swell recorder	Ratio Col 4 / Col 3
1	2	3	4	5
ft		ft	ft	
<1	15	.085	0.8	9.5
1 - 2	36	.15	1.6	10.7
2 - 3	24	.20	2.6	13.0
>3	28	.35	4.6	13.1
Weighted mean				11.4

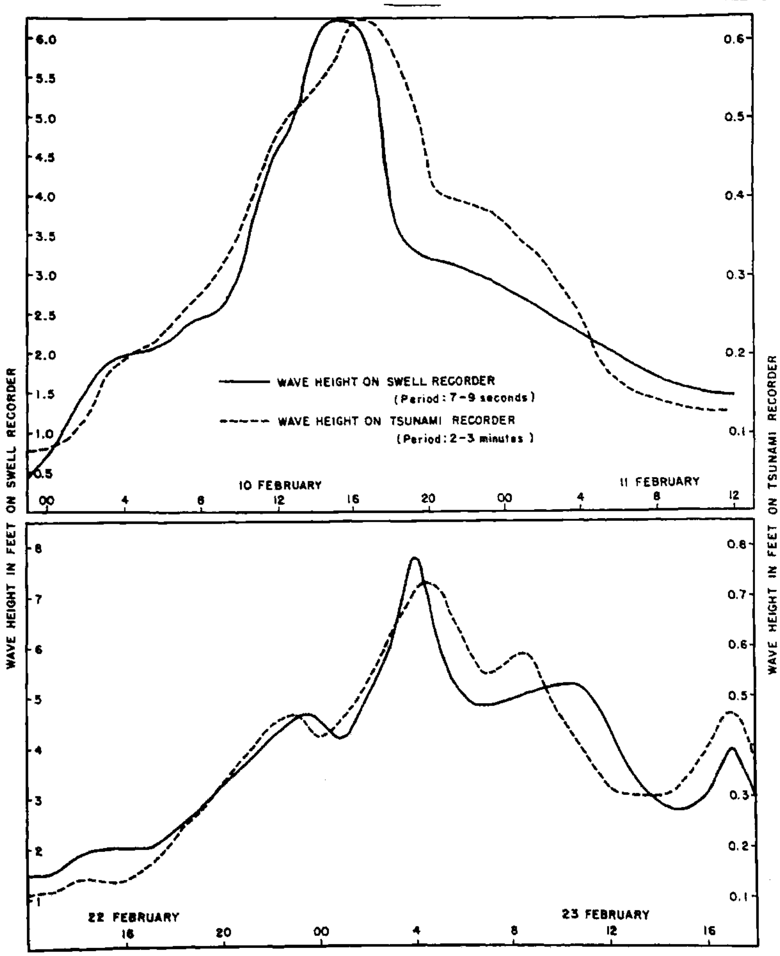

Fig. 3--Comparative heights of waves shown by the tsunami and swell recorders for two storms; the height scale on the tsunami record is exaggerated ten times relative to the height scale on the swell records

represents the excess (in arbitrary units) of the shoreward transport over the seaward return between the times 0 and t_n. This quantity is marked "integrated swell record" in Figure 4.

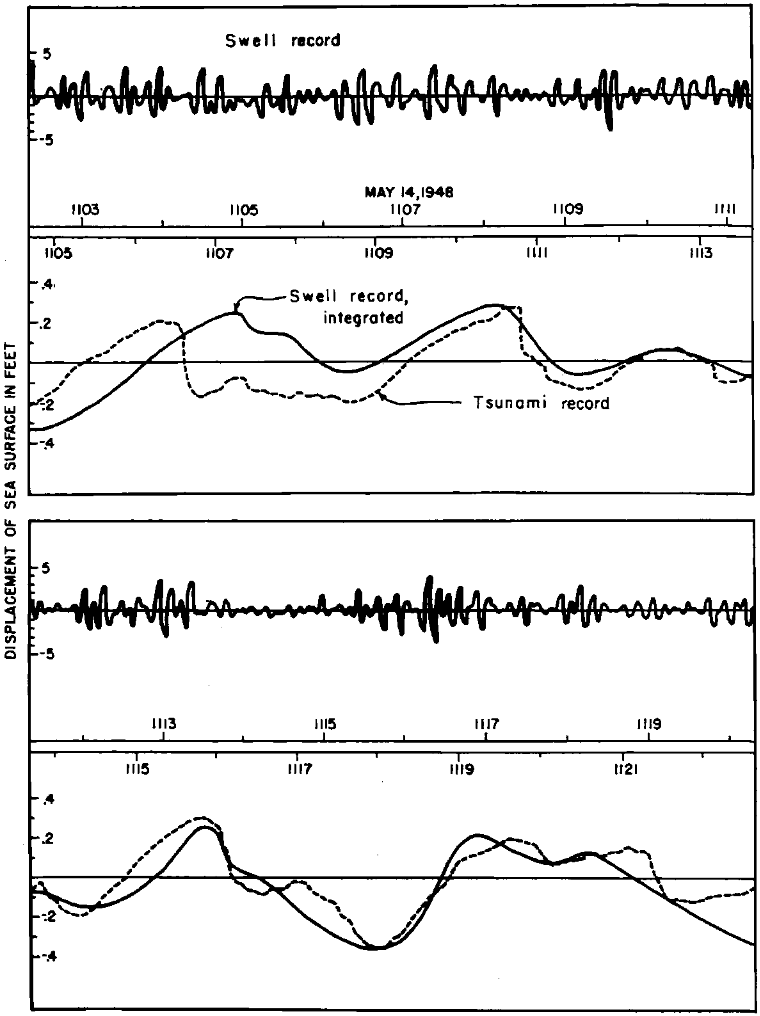

Fig. 4--Detailed comparison of waves on swell and tsunami recorders; the upper time scale refers to swell record and integrated swell record; the lower time scale refers to tsunami record, and is displaced 140 sec relative to upper time scale

During the 20-min interval shown in Figure 4 the tsunami recorder was run at 60 times its normal speed in order for the time scales on the two records to be identical. The integrated swell record resembles the tsunami record, provided the two records are displaced by 140 sec.

This lag Δt of the tsunami record relative to the integrated swell record results from the time involved for the waves to travel from the swell recorder past the tsunami recorder to the surf zone, for the two- to three-minute oscillations to travel from the surf zone back to the tsunami recorder (Fig. 2), and the phase lag of the tsunami recorder, about 90° for waves of two-min periods. Thus

$$\Delta t = \int_A^B (1/V)dx + \int_B^C (1/V)dx + \int_C^B (1/V)dx + (90^\circ/360^\circ)120$$
$$= 42 + 34 + 30 + 30 = 136 \text{ seconds}$$

Table 2--Computed horizontal orbital velocities, displacements, and pressure fluctuations at ocean bottom associated with waves of eight-second period and 4.5-ft height in deep water, and with surf beats of two-minute period caused by these waves

Depth	Height		Horizontal orbital velocity		Horizontal displacement		Pressure	
	Waves	Surf beat	Waves	Surf beat	Waves	Surf beat	Waves	Surf beat
ft	ft	ft	ft/sec	ft/sec	ft	ft	ft/sea water	ft/sea water
7.5[a]	6.0	0.6	5.9	0.62	15	24	5.6	0.60
10	5.2	0.5	4.4	0.45	11	17	4.7	0.50
20	4.5	0.47	2.5	0.30	6.4	11	3.7	0.47
50	4.2	0.39	1.1	0.16	2.8	6.1	2.4	0.39
100	4.3	0.31	4.7×10^{-1}	0.088	1.2	3.4	1.2	0.31
200	4.5	0.26	8.0×10^{-2}	0.052	2.0×10^{-1}	2.0	1.9×10^{-1}	0.26
500	4.5	0.21	1.2×10^{-4}	0.027	3.1×10^{-4}	1.0	3.0×10^{-4}	0.21
1000	4.5	0.18	8.1×10^{-9}	0.016	2.0×10^{-8}	0.6	2.1×10^{-8}	0.18

[a]Breaking point

compared to a measured time lag of 140 sec. The values have been obtained by numerical integration, setting eight seconds and two minutes for the periods of incoming and outgoing waves, respectively, and substituting

$$V = \sqrt{(g \tanh kh)/k}\ (1 + 2kh/\sinh 2kh)/2$$

for the group velocity of waves of length $2\pi/k$ at depth h. The distances are those shown in Figure 2. The seaward boundary of the surf zone was determined by the wave dimensions at $11^h\ 00^m$ on May 14, 1948; period, eight seconds; deep water height, 4.5 feet; breaker height, six feet; depth of breaking, 7.5 feet. If the fluctuations of the tsunami record had been the result of a non-linearity in response of the instrument, the time lag would have been 42 + 30, or 72 sec, approximately half the measured time lag.

In view of the 1000 ft distance separating the swell and tsunami recorders and of other uncertainties, the agreement between the integrated swell record and the tsunami record (see Fig. 4) and between the computed and observed time lag are as good as one might expect. The two- to three-minute oscillations shown by the tsunami recorder appear to be caused by the variability of water transport into the surf zone, and are therefore termed "surf beat." Fluctuations in the speed of longshore currents and rip currents [PUTMAN, MUNK, and TRAYLOR, 1949; SHEPARD and INMAN, in press] are due to the same condition. The variability of the breaking waves results from the variability in the height and period of the waves offshore, and this variability in turn depends essentially on the ratio of storm size to storm distance, and is relatively small for compact storms at large distances.

The shore line acts therefore as a radiating line source which returns approximately one per cent of the incoming wave energy in the form of long-period waves. This is characteristic of the reaction of non-linear systems to high incoming waves. Indeed the surf zone may be considered an extreme case of non-linearity.

The amplitudes of orbital velocity and pressure fluctuation and the double amplitude of horizontal displacements at the sea bottom have been computed for the conditions shown in Figures 2 and 4, assuming the surf beat to travel seaward and being modified according to the usual theory [LAMB, 1932, and U. S. HYDROGRAPHIC OFFICE, 1944]. Results are summarized in Table 2. For depths exceeding 200 ft the greater length of the surf beat more than compensates for its smaller height; at a depth of 500 ft, a typical depth for the outer edge of the continental shelf, the surf beat is associated with 200 times the orbital velocity, and 70 times the pressure fluctuation of that of the incoming waves. At mid-depth in water of 1000-ft depth the vertical accelerations associated with the surf beat and the waves are both approximately 3×10^{-6}g. The surf beat may be the cause of short-period internal waves, and of surges in harbors.

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