# Measurements of southern swell at Guadalupe Island*

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**Abstract**—We have obtained some energy spectra of pressure fluctuations on the sea bottom at a depth of 60 fathoms off Guadalupe Island, Mexico. The spectra consist of peaks whose frequency increases by about 10 per cent per day ; sequences repeat once every three or four days. The waves come from the southwest quarter, and the distance of generation is estimated at 8000 nautical miles. The signals are attributed to cyclones in the storm belt of the Southern Hemisphere. The earliest arrivals are of the order of 1 mm high and 1 km long. They may have originated in the Indian Ocean and passed just east of New Zealand, or to the west through the Tasman Sea. Later arrivals were generated in the South Pacific. Measurements taken at many other occasions at various localities in the north-east Pacific indicate that this radiation from the Southern Hemisphere is present at all seasons. Apparently it constitutes the principal background in the ocean wave spectrum for the low frequencies under consideration.

## A HISTORICAL NOTE

THE photograph shown in Fig. 1 was the first indication of southern swell reaching the coast of California. Allowing for refraction in shallow water the off-shore direction was computed to be south south-west, and it was inferred that the waves had originated in a storm area in the southern oceans (HYDROGRAPHIC OFFICE, 1944). The photographed wave pattern is remarkably regular; it looks almost like the single-frequency, long-crested train of sine waves used in elementary oceanographic texts and advanced mathematical treatises to portray ocean waves. A distant origin could account for the regularity. The wave direction would be nearly uniform (i.e. the waves are long-crested) inasmuch as a distant storm subtends a small angle, and the range of frequency would be small because of the sorting of frequencies by dispersion. Fig. 1 was the first of a long series of aerial photographs we took during the war, and it must be added that in no subsequent photograph did we observe quite as regular a wave pattern.

The problem of the propagation of ocean waves was not at all well understood until BARBER and URSELL (1948) succeeded in obtaining spectra of ocean waves. In nearly all of forty case histories they were able to trace prominent frequency bands to storms in the north Atlantic Ocean, but on at least two occasions they found unmistakable evidence that the swell had been generated in storms occurring beyond the limits of this ocean. In one case they first noted a spectral band peaked at a period of 21 sec at 1100 hr on 14 May 1946, and this period decreased uniformly (except for a frequency modulation by tidal currents, which confirmed the southern origin) to 15 sec on 19 May. We shall make references to the following quotation from BARBER and URSELL's (1948) paper :

" . . . there appears to be at least one significant discontinuity in the trend

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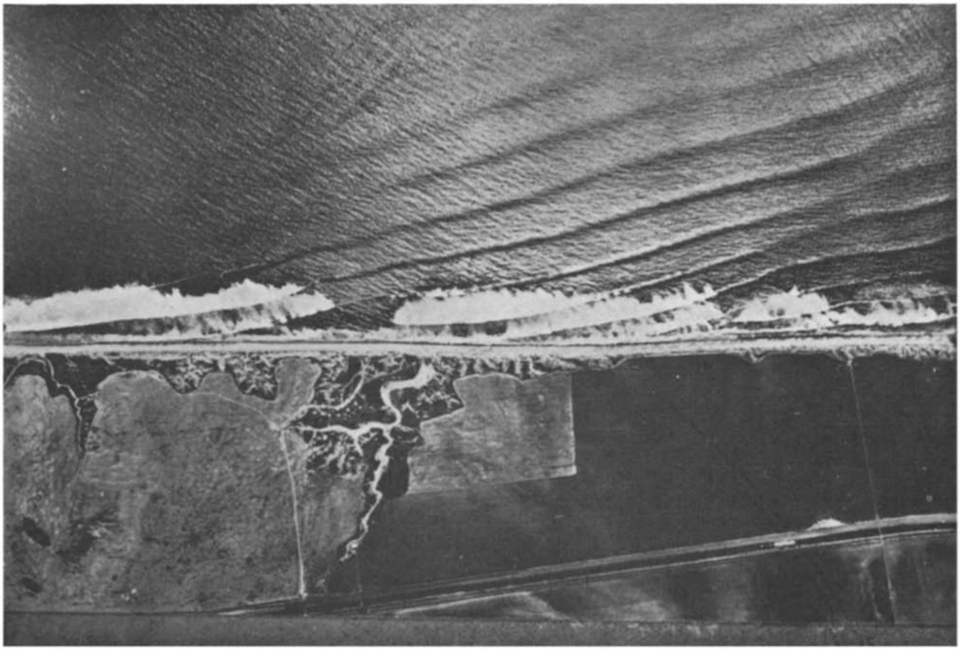

Fig. 1. Aerial photograph of waves at Camp Pendleton, California, on 16 January, 1944. The coast trends north-west – south-east.

of maximum periods ; after 1700 hr on 16 May the spectra suggest that the frequency is widened by the arrival of swell whose maximum period is 2 to 3 sec longer than the mean maximum of the previous spectra. A second widening at the upper end of the frequency band possibly occurs at 1900 hr on 19 May when there is some, though not very reliable, indication of activity at periods up to 23 sec."

BARBER and URSELL compute the distance of origin from the observed frequency shift, and arrive at 6000–7000 nautical miles : " the storm must, therefore, be sought in the southern ocean, probably in the Falkland sector to which there is an open great circle course from Land's End". The meteorological evidence available was too scanty to be very helpful. The maximum height of the southern swell was 30 cm.

DONN (1949) has discussed wave spectra taken at Cuttyhunk and Bermuda. He notes that

> " The periodograms frequently show a unique wave signature exactly at the 15 sec mark. This group may show a range of from 14 to 18 sec, with occasional extensions slightly beyond these limits. From a study of many records it appears that this group is nearly always present, but at times may become too small to be detected, even by harmonic analysis."

Allowing for appropriate travel time, DONN has compared the Cuttyhunk and Bermuda spectra and concludes that the waves come from the south.

> " Assuming this southerly origin, weather maps of the belt of the westerlies of the Northern and Southern Hemispheres have been studied at length. No such swell source appears possible from transitory cyclones and anticyclones of either hemisphere. In addition the " window " of approach from possible storms in the South Atlantic is extremely restricted. The suggestion is thus offered here that this swell group is generated by the South-east trade winds. This seems to be the only source that would explain their constancy."

J. E. DINGER (personal communication) has remarked on a 15–16 sec hump in the spectra of waves at the Barbados. RUDNICK (1951) finds 15–20 sec bands in spectra of waves at Guam.

WIEGEL and KIMBERLEY (1950) obtained (without frequency analysis) the " significant " height and period of the predominant waves recorded at Camp Pendleton, California, and conclude that they originate in the area between 40° and 65°S and 120° and 160°W. The largest recorded period during a summer season was 22 sec.

## OFF-SHORE RECORDING

For the last two years we have used a Vibrotron transducer (SNODGRASS, MUNK and TUCKER, in press) to record fluctuations in bottom pressure at depths varying from 80 to 300 m. Twenty-four stations have been occupied on the continental shelf and borderland off southern California. At *all* stations, regardless of season, we have noted a pronounced regular oscillation, such as shown in Fig. 2. The recorded periods (mean interval between crests) varies from 14 to 20 sec, wave heights (root mean square values corrected for depth) from 10 cm to 1 m. In the majority of cases the periods were between 15 and 16 sec.

Out first attempt to determine wave direction was by comparison with simultaneous visual observations conducted at Camp Pendleton (see Fig. 1) by Warrant Officer BOYD, U.S.M.C. The visual observations include breaker angle and littoral current ; these parameters are notoriously unreliable for determining off-shore wave direction, but they were the only means available. All that can be said is that on such days when the shore observations indicated waves from the south-west quarter, their height and period were consistent with the values recorded offshore.

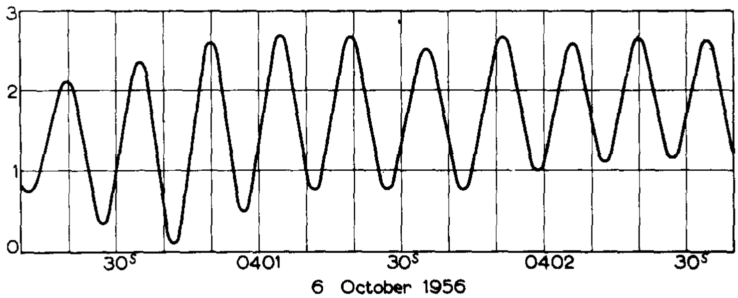


Fig. 2. Short section of Vibrotron record at the protected station of Guadalupe Island. The scale gives inches of water pressure on the sea bottom.

On 25–27 June we attempted to use the wave "shadow" behind San Clemente Island as a means of estimating wave direction. We first occupied a station at the north-west end of the island, which is exposed to waves from the south-west quarter, followed immediately by a station at the east side, which is protected from this quarter. The results were :

| | *June*, 1954 | *Period* (sec) | *Height* (cm) |
|---|---|---|---|
| San Clemente, West | 25, 1300 hr | 14·8 | 64 |
| | 25, 2300 | 14·4 | 59 |
| | 26, 0500 | 13·8 | 77 |
| San Clemente, East | 26, 1100 | 14·6 | 15 |
| | 27, 1400 | 14·1 | 10 |

On 3–8 October 1956 we occupied two stations at Guadalupe Island, 200 nauticat miles off the coast of Mexico and well off the continental borderland. The firsl station was protected from the south-west quarter, the second station was not. At the exposed station the waves were four times higher.

The evidence, while not conclusive, points to a southern origin of the low-frequency signal. Our observations differ from previous work on account of the great depth at which they were taken. Depth acts as a low-pass filter ; local disturbances are of higher frequency, and attenuated accordingly. The result is that low frequency signals were prominent at all times in these filtered records, and at such times as we could check, a southern origin was indicated.

## GUADALUPE SPECTRA

Fig. 3 shows seven successive spectra of the Guadalupe wave records. The following information is recorded :

Guadalupe Island, north station, 29°09′·30N, 118°16′·35W. Depth : 113 m

| 1956 | $m$ | $N$ | $f$ | 95% limits |
|---|---|---|---|---|
| (1) 3 Oct., 2220 – 4, 0500 | 62 | 6000 | 194 | 0·82 – 1·24 |
| (2) 4 Oct., 1020 – 4, 1700 | 62 | 6000 | 194 | 0·82 – 1·24 |
| (3) 4 Oct., 2220 – 5, 0500 | 62 | 6000 | 194 | 0·82 – 1·24 |
| (4) 5 Oct., 1020 – 5, 1700 | 62 | 6000 | 194 | 0·82 – 1·24 |
| (5) 5 Oct., 2220 – 6, 0500 | 62 | 6000 | 194 | 0·82 – 1·24 |

Guadalupe Island, south station, 28° 50′·55N, 118°16′·45W. Depth : 109 m

| | | | | |
|---|---|---|---|---|
| (6) 7 Oct., 1236 – 7, 2142 | 62 | 8190 | 261 | 0·84 – 1·21 |
| (7) 7 Oct., 2142 – 8, 0648 | 62 | 8190 | 261 | 0·84 – 1·21 |

Bottom pressure was recorded every 4 sec, and successive values were harmonically analysed on an electronic computer, following the method of TUKEY (1949). We obtained $m = 62$ estimates of energy density, of which 25 values are plotted (without smoothing). The degrees of freedom $f = 2N/m$ determines the statistical reliability of the estimates. For spectra (1) to (4) there is only one chance in twenty that the true energy density is less than 0·82 or more than 1·24 times the computed value. Tides were removed prior to analysis by numerical high-pass filtering. The plotted curves are surface spectra ; the depth of the instrument has been allowed for according to hydrodynamic theory. The many details of the analysis have been described elsewhere (MUNK, SNODGRASS and TUCKER, in press).

The characteristic feature of the spectra is the progressive shifts of individual peaks from low to high frequencies. The peaks emerge at about 50 cycles per kilosecond (period 20 sec) and their frequency increases by something like 7 per cent per day until they disappear in the locally generated spectrum. The energy of the peaks reaches a maximum around 65 c/ks. The history repeats itself at an interval of 3 to 4 days. This behaviour is in accord with the remarks by BARBER and URSELL quoted earlier. There is a sharp increase in one of the peaks from record (4) to (5). We return to this later.

The essential point is that we have analysed records from 25 to 30 times longer than those previously used (except for two isolated analyses by RUDNICK, 1951) and that the resolution is improved accordingly. Thus the spectrum between 40 and 90 c/ks is described by 25 estimates of energy density, rather than just a very few values of less reliability. The spectra available to DONN were of such poor resolution that progressive spectral peaks, if present, could not have been discerned, and the signals would appear as a steady " hum " at 15 sec. The observed steadiness has lead DONN (1949) to suggest generation by the steady south-east trades rather than by transitory storms to the south. The British results, and those obtained at Guadalupe, suggest that the Bermuda spectra could likewise reveal a fine structure of progressive peaks if the records were analysed with adequate resolution.

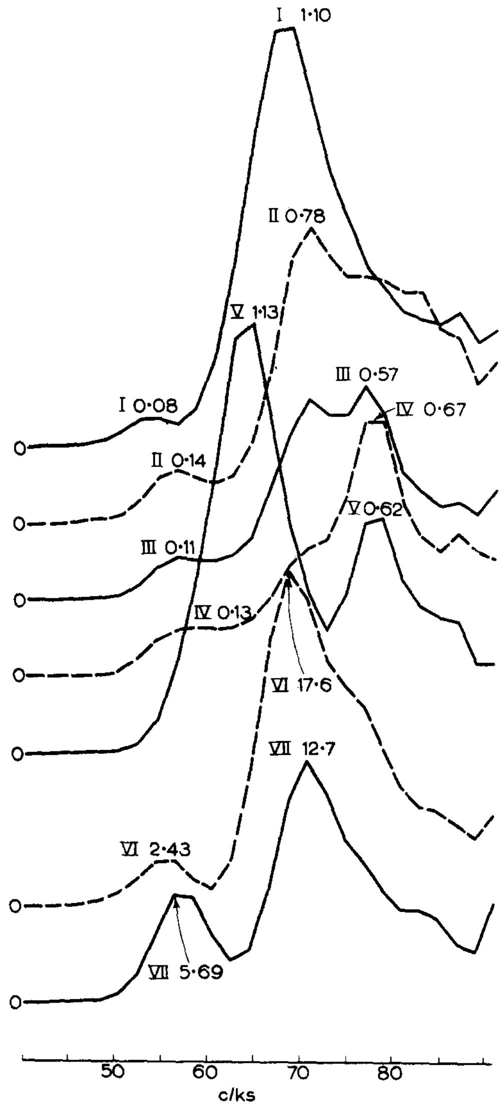


Fig. 3. Energy spectra of waves recorded at Guadalupe Island. The frequency scale is in cycles per kilosecond (or millicycles per second); the ordinate gives "energy" density in $cm^2$ per c/ks on a sliding scale. Zeroes and peak values are indicated. The last two spectra are drawn to one-twentieth scale.

WAVES FROM A SINGLE IMPULSIVE SOURCE

A distance-time diagram is useful (Fig. 4, bottom). Suppose that at some distance $r_0$ from the recording station, and at some time $t_0$, a sudden concentrated disturbance generates waves over a broad band of frequencies. Any elementary wave train of

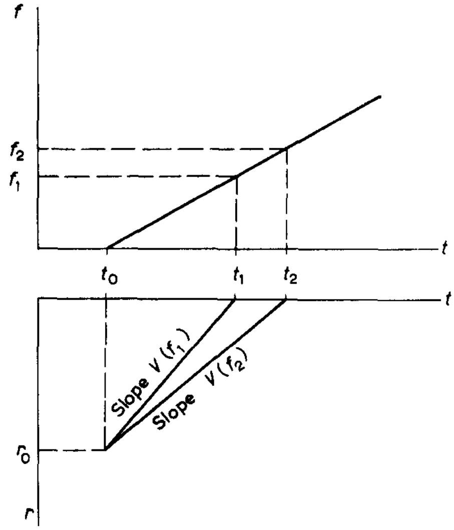


Fig. 4. Frequency-time and distance-time diagrams for an impulsive source at a distance $r$ from the wave station, and time $t_0$. Two particular frequencies, $f_1$ and $f_2$, arriving at the station at times $t_1$ and $t_2$ respectively, are characterized by the rays of slope $V(f_1)$ and $V(f_2)$ on the $r$, $t$-diagram.

frequency $f$, travelling at the appropriate group velocity $V = g/(4\pi f)$, is characterised by a " ray " in the $r$, $t$-diagram extending from the source ($r_0$, $t_0$) to the receiver ($O$, $t$) with slope $V(f)$, where

$$V = \frac{r_0}{t - t_0} = \frac{g}{4\pi f}. \tag{1}$$

The mathematical foundation of this simple geometric construction has been summarized by ECKART (1948). The requirement is that the different velocities of the Fourier components have dispersed the disturbance into a well-resolved spectrum.

BARBER and URSELL (1948) have constructed such " propagation diagrams " to locate the source of the recorded waves. Let the spectrum of waves arriving at time $t_1$ reveal a narrow band peaked at frequency $f_1$, and at a later time $t_2$ a peak at a higher frequency $f_2$. Then the rays drawn through the points 0, $t_1$ and 0, $t_2$ with the slopes $V_1$ and $V_2$ as given by equation (1) intersect at a point which determines the distance and time of the source. In one case five rays covering the interval 14–18 May 1946 give a good intersection for a point at a distance 6000 miles from the recording station, at a time 6 May 1946.

An equivalent construction has been used by MUNK (1947). Equation (1) can be written in the form

$$f = \frac{g}{4\pi r_0}(t - t_0) \tag{2}$$

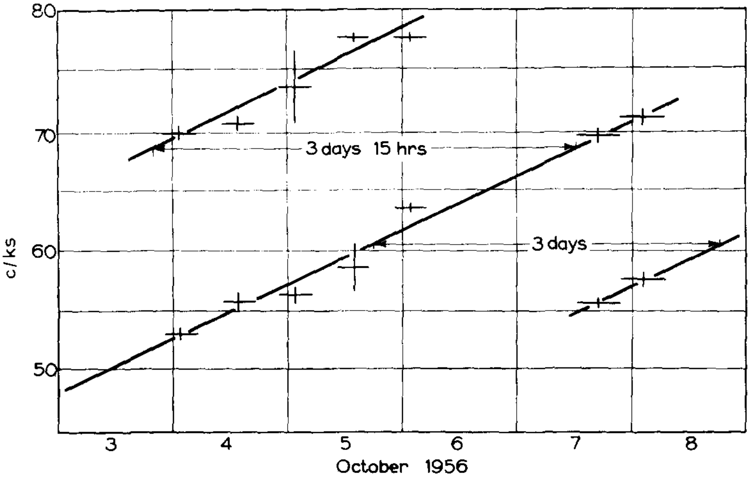


Fig. 5. Frequency-time diagram for the Guadalupe Spectral peaks, Fig. 3. Each frequency band is indicated by a cross. The vertical lines give a measure of the possible range of values of peak frequency; the horizontal lines give the duration of the analysed record.

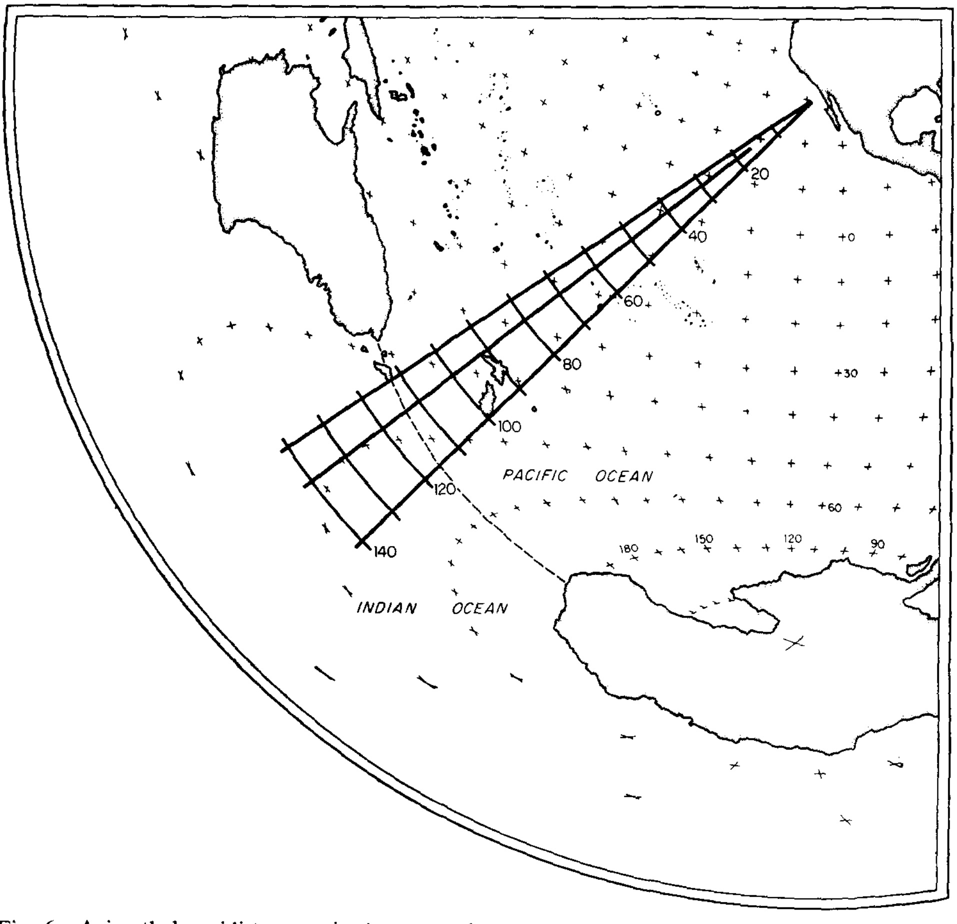


Fig. 6. Azimuthal-equidistant projection centred on San Diego. Distances from Guadalupe are given in units of degrees (1° equals 111 km).

so that the frequencies from a point source lie along a straight line in a frequency-time diagram (Fig. 4, top). The intercept determines $t_0$ and the slope gives $r_0$. The Guadalupe spectral peaks have been fitted by straight parallel lines on an $f$, $t$-diagram, (Fig. 5). There are three sequences, separated by 3 days 15 hr ; and by 3 days respectively. If we insist in fitting all sequences by parallel lines, we obtain a single value, $r_0 = 135°$ (15,000 km). This places the source in the Indian Ocean (Fig. 6) The values of $t_0$ are 18 Sept. 1956, 1900 hr ; 22 Sept., 1000 hr ; 25 Sept., 1000 hr. Conceivably we might ignore the last two values inasmuch as they were taken at another station, and draw a steeper line. The smallest value of $r_0$ is about 110°, and the $t_0$ times are later, but by no more than 1 day 17 hr.

But one is led immediately to question the validity of seeking an instantaneous point source. A travelling storm will be many days along an unobstructed path from the time it passes south of Australia. We are naturally led to generalize the construction.

## WAVES FROM A MOVING POINT SOURCE

A storm is moving eastward at some constant rate $W$ along colatitude $c$ (Fig. 7). Then

$$\phi R \sin c = Wt \tag{3}$$

is the storm's distance from a point $P$ just south of the station. $R$ is the Earth's radius. The storm's longitude $\phi$ and the time $t$ are taken as negative when the storm

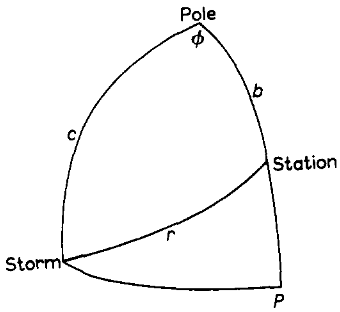


Fig. 7. $b$ and $c$ are polar angles of station and storm referred to the pole nearest the station. $P$ is a point south of the station at co-latitude $c$.

is west of the station, as shown in the figure. The great circle distance from storm to station* is $rR$, where

$$\cos r = \cos b \cos c + \sin b \sin c \cos \phi. \tag{4}$$

It is convenient to interpret $\phi$ also as dimensionless time according to (3). Fig. 8 shows the storm track on an $r$, $\phi$-diagram. The storm is nearest the station when $\phi = 0$ and furthest when $\phi = 180°$. The inflection point is of particular interest. Setting $d^2 r/d\phi^2 = 0$ gives two solutions :

$$\cos r_i = \frac{\cos b}{\cos c}, \qquad \cos \phi_i = \frac{\tan c}{\tan b}, \qquad \frac{dr}{dt} = \frac{W}{R}, \qquad \text{for } b + c > 180° \tag{5a}$$

*This presumes that waves travel along great circles. Eckart finds (personal communication) that because of the earth's rotation there will be an angular deflection of the order (wave period/½ day)$^2$ or $10^{-7}$ for 20 sec waves. The deviation from the great circle route is then of the order of 1 m.

$$\cos r_i = \frac{\cos c}{\cos b}, \qquad \cos \phi_i = \frac{\tan b}{\tan c}, \qquad \frac{dr}{dt} = \frac{W}{R}\frac{\sin b}{\sin c}, \qquad \text{for } b + c < 180° \tag{5b}$$

In the first case the wave station is no further north of the equator than the storm is south of the equator (or vice versa). There exists then a great circle route through the station and tangent to the storm's latitude. At the point of tangency the storm moves east with the same speed as it moves towards the station ; $R \cdot dr/dt = W$. In the second case the station is further north than the storm is south, and the rate of approach of the storm towards the station is always less than $W$.

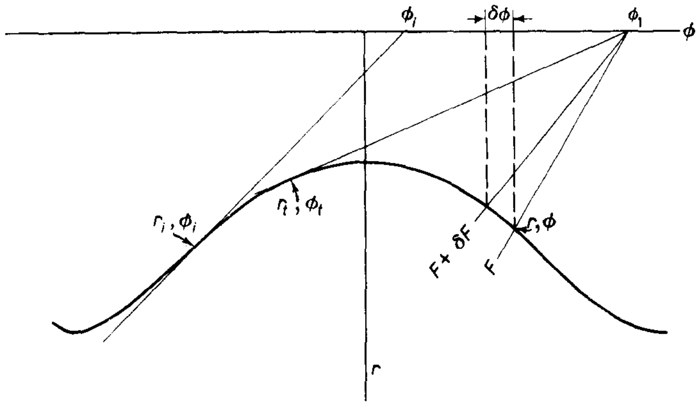


Fig. 8. A distance-time diagram for a moving point source.

Consider a " ray " from $\phi$, $r$ to $\phi_1$, 0 (Fig. 8). This ray represents the travel of an elementary wave train having left the storm at time $t = (R \sin c/W)\,\phi$ and arrived at the station at $t_1 = (R \sin c/W)\,\phi_1$, thus having covered a distance $rR$ with a group velocity

$$V = \frac{rR}{(R \sin c/W)\,(\phi_1 - \phi)}.$$

A convenient notation is $f_0 = g/4\pi\, W$ for the frequency corresponding to a group velocity $W$. In view of (1)

$$F = \frac{f}{f_0} = \frac{W}{V} = \frac{(\phi_1 - \phi) \sin c}{r}. \tag{6}$$

For any (dimensionless) frequency $F$ recorded at time $t_1$ the distance and longitude of generation are determined by the intersect of the ray with the storm's track on the $r$, $\phi$-diagram. There may be zero, one or three intersections. For the tangent ray (Fig. 8)

$$F_t = -\frac{\sin r_t}{\sin b \sin \phi_t} = \frac{(\phi_1 - \phi_t) \sin c}{r_t}. \tag{7}$$

Let $E(F)$ designate the " energy " spectrum of the wave record. The contribution to the mean square elevation from frequencies in the range $F$ to $F + \delta F$ is denoted by $E(F)\,\delta F$. This contribution $E(F)\,\delta F$ comes from a segment of the storm track that is subtended by the rays associated with $F$ and $F + \delta F$, respectively (Fig. 8).

Let $\delta\phi = [W/(R \sin c)]\, \delta t$ be the projection of this segment upon the $\phi$-axis. The storm is presumed to radiate energy at a uniform rate*, and $E(F)\, \delta F$ is proportional to $|\delta t|$, and hence to $|\delta\phi| \sin c$ ; it also varies as the energy density $S(F)$ of the generation spectrum, inversely with $r$ because of dispersion, and inversely with $\sin r$ because of angular spreading (see, for example, LAMB, 1932, pp. 432-433). Nothing is known about attenuation, and we neglect it. Hence

$$E(F)\, \delta f \sim S(F)\, |\delta\phi| \sin c/(r \sin r)$$

$$E(F) = \mu S(F), \qquad \mu = \frac{|\delta\phi/\delta F| \sin c}{r \sin r} \tag{8}$$

where $\mu(F)$ may be regarded as the energy amplification, operating on the generation spectrum $S(F)$ to give the recorded spectrum $E(F)$. The evaluation is a problem in differential geometry (see Appendix), but in general terms the result is evident from Fig. 8 : (1) For any given time $t_1$ (hence $\phi_1$) $\mu$ is sharply peaked for the particular ray through $r_t$, $\phi_t$ tangent to the storm track ; (2) the tangent ray through the inflection point $r_i$, $\phi_i$ gives a peak of all such peak values. The foregoing argument is related to the method of stationary phase.

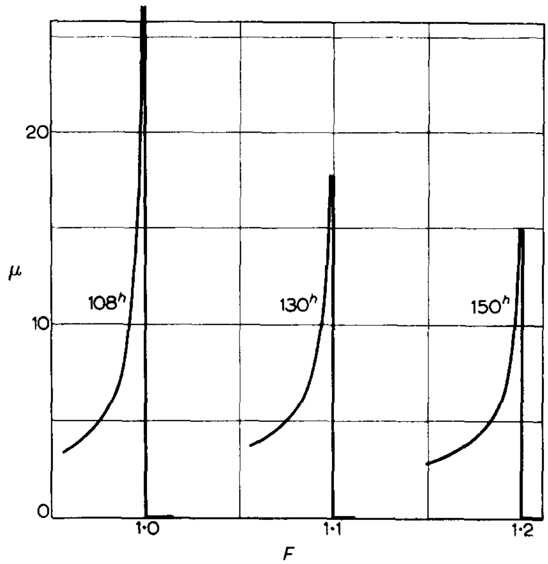


Fig. 9. The amplification factor at various times *after* the passage of the moving point source to the south of the station (point $P$, Fig. 7), the first " tangent arrival " through the inflection point is more amplified than successive tangent arrivals.

Fig. 9 has been drawn according to equation (15). Near the tangent ray $\mu$ approaches infinity as $(F - F_t)^{-\frac{1}{2}}$ except near the inflection point. Then $F_t = 1$, and $\mu$ $(= \mu_i)$ approaches infinity as $(F - 1)^{-\frac{2}{3}}$. Though the spikes are infinite, the area under the spikes is finite. If the generation spectrum $S(F)$ is a smooth function, estimates of the energy density in the frequency interval $F_t - \Delta F$ to $F_t$ can be written

*One may think of the storm as a series of momentary disturbances, each sending out an identical dispersive train. The *number* of disturbances contributing to $E(F)\, \delta F$ is then proportional to $|\delta t|$.

$$\int_{F_t-\Delta F}^{F_t} E\mu \, dF = E \int_{F_t-\Delta F}^{F_t} \mu \, dF = 2h\,(\Delta F)^{\frac{1}{2}}\,E, \qquad \text{(tangent ray)}$$

$$= 3h_i\,(\Delta F)^{\frac{1}{3}}\,E \qquad \text{(inflection ray)}$$

as compared to something of the order $(\Delta F)\,E$ away from the spikes. The *recorded* peak amplifications

$$2h\,(\Delta F)^{-\frac{1}{2}}, \qquad 3h_i\,(\Delta F)^{-\frac{2}{3}} \tag{9}$$

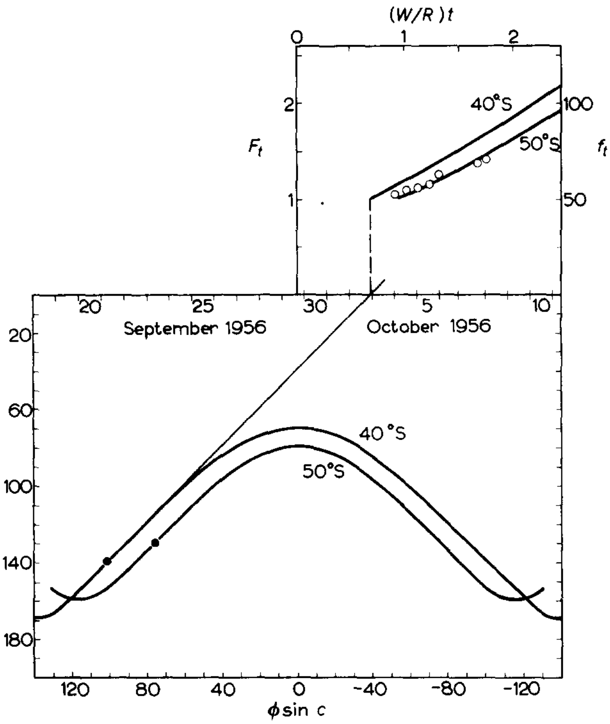


Fig. 10. The $f_t$, $t$- and $r$, $t$-diagrams for a spherical earth. The polar distance of the station is taken as $b = 61°$, corresponding to Guadalupe Island. The diagrams are drawn for the travelling point source at 40° and 50°S. The inflection points are marked and one of the tangents is drawn. The curves in the upper figure gives $F_t$ against $(W/R)\,t$ for the two assumed latitudes of generation. The scales in real time and in frequency units c/ks have been drawn for $W = 30$ knots. The observed points are for the middle sequence of spectral peaks (Figs. 4 and 5).

depend upon the frequency *resolution* $\Delta F$ and will be large if the resolution is high, i.e. $\Delta F$ is small. In our work $\Delta f$ is 4 c/ks, $f_0 = 50$ c/ks, hence $\Delta F = \Delta f/f_0 = 0{\cdot}08$ (equation (6)). Fig. 9 is drawn for $b = 61°$ (co-latitude of Guadalupe), $c = 140°$, $W = 30$ knots. For the tangent ray through the inflection point, and two subsequent tangent rays, we have

| | | | |
|---|---|---|---|
| $F_t$ | 1·0 | 1·1 | 1·2 |
| $t_1$ | $108^h$ | $130^h$ | $150^h$ |
| $r_t$ | 130° | 107° | 100° |
| $\phi_t$ | − 118° | − 82° | − 70° |
| $h_i$, $h$, $h$ | 0·41 | 0·78 | 0·64 |
| Equation (9) | 6·7 | 5·7 | 4·7 |

The conclusion is that at *any* time the energy density of the frequency corresponding to the tangent ray is amplified, but that the earliest of all possible tangent rays (the inflection point ray) is amplified more than subsequent tangent rays. We might then expect the observed spectral peaks to correspond to these tangent rays. In physical terms, the prominent waves are those whose group velocity equals the rate of approach of the storm toward the station somewhere along its path.* The existence of such a " group velocity peak " has been noted by ECKART (1953) for waves generated by a moving gust, and is implicit in his equation (35).

The constructions in Fig. 10 are for storms 40°S and 50°S and the wave station at 29°N. The computed $f_t$, $t$-curves in the upper figure are obtained by drawing tangents to the storm paths to where they intersect $r = 0$. The cotangent of the angle between $r = 0$ and the tangent line equals $F_t$ at the time given by the intersection. The observed points correspond to the central sequence of spectral peaks. For a storm path along 50°S the storm was at the inflection point on 23 Sept., 0800 hr at a distance of 130°. The waves were recorded 4½ days *after* the storm had passed to the south of the station. This is hardly synoptic information ! The computed distance of generation is similar to that derived for an assumed point source. We conclude that according to the present hypothesis of a moving point source the earliest arrivals of swell observed off North America originated in the Indian Ocean at a distance of 120–140° (Fig. 6).

## DISCUSSION

We have attempted to interpret the Guadalupe spectra on the basis of two hypotheses, both severely oversimplified : (1) generation by an instantaneous point source ; (2) by a point source moving with constant speed along a fixed latitude. Both hypotheses lead to wave trains of gradually increasing frequency but for altogether different reasons. In the case of (1) the increase in frequency is due to dispersion; i.e. low frequencies travel faster than high frequencies, and arrive earlier. The essence of the moving-point hypothesis is that one frequency is amplified over all others : that frequency whose group velocity equals the rate at which the storm approaches the station (tangent rays on a distance-time graph).

The gravest observed frequency (from the first arrivals) are likewise determined by quite different considerations. In hypothesis (1) the limit is imposed by the generation spectrum : winds of velocity $U$ cannot effectively generate waves whose *phase* velocity $C$ exceeds $U$. In the latter case the *group* velocity $\frac{1}{2}C$ of the first arrivals is determined by the storm's rate of approach. In general this is less than the storm's rate of travel, $W$, unless the storm is moving along a great circle route towards the station. For an east-west movement there is a point where a great circle through the station is tangent to the storm's latitude circle, and the amplified frequency, $g/(4\pi W)$, is the gravest possible (the " inflection ray " on the distance-time graph). Suppose the storm is at 50°S ; the great circle through the station can be drawn only if the station is south of 50°N. If it is north of 50°N then the gravest frequency is somewhat less than $g/(4\pi W)$ in accordance with equation (5b). Thus the gravest frequencies are of the order

$$g/(2\pi U) \text{ or } g/(4\pi W)$$

*OLIVER and EWING (1957) apply similar considerations to account for the observed increase in the frequency of waves and microseisms from hurricane Dolly. They refer to LAMB's discussion of the ship wake problem, being able to neglect the earth's curvature on account of the proximity of the storm.

according to the two hypothesis. Typical values for Antarctic cyclones are $U = 50$ knots, $W = 30$ knots. Both criteria lead to frequencies of the order of 1 cycle in 20 sec, as observed.

All of this discussion depends, of course, upon the existence of unobstructed great circle routes. One unobstructed great circle can be drawn from Guadalupe passing just east of New Zealand and tangent to 55°S at a point in the Indian Ocean 130°E longitude (see Fig. 6). Another great circle can be drawn through the Tasman Sea tangent to 50°S at a point in the Indian Ocean south of Western Australia. These regions are among the stormiest in existence. The window through the Tasman Sea is narrow, and the waves must be scattered by South Pacific islands. A storm moving eastward would be " seen " only briefly through the Tasman Sea window, and a proper hypothesis might be some combination of the instantaneous and moving source hypotheses. In all events, the observed frequency shift by less than 10 per cent per day is consistent only with very distant areas of generation, regardless of which of the two hypotheses is followed, and the estimated distance of 130° is consistent with either of the two routes. It is therefore tentatively suggested that the very low, long early arrivals came from the Indian Ocean. We are puzzled by the sharp increase in the spectral peak on the morning of November 6 (see Fig. 3), corresponding to the fifth point of the central sequence (Figs. 5 and 10). The corresponding tangent ray gives a distance 100° for the storm at 50°S. Could this represent an arrival of an unobstructed wave train from a storm passing over New Zealand into the Pacific Ocean ?

N. F. BARBER has kindly examined the weather maps drawn by the New Zealand Meteorological Office for the period in question. Unfortunately the meteorological situation was not clear-cut. The computed travel times are in doubt by, say $\pm$ 20 per cent or $\pm$ 2 days, and no definite correlations can be made. A 30–35 knot wind over a fetch of 500 nautical miles on 17 Sept. 1957 centred at 140°E is consistent with a route to the east of New Zealand, and may have been responsible for the central sequence. We prefer to postpone any detailed comparisons with meteorological data until the International Geophysical Year. For that purpose we are establishing a wave station west of San Clemente Island which will include a measure of wave direction and beam width in addition to the features discussed in this paper.

For the British records there is no unobstructed great circle corresponding to the inflection point arrival. Setting $b = 40°$, $c = 140°$, $\phi = -50°$ gives $r = 108°$,

$$(dr/dt)/W = \sin b \sin c \sin \phi = 0{\cdot}47$$

for the earliest unobstructed tangent arrivals. The corresponding frequencies of something like 100 c/ks are much higher than those observed for the first arrivals. In the case of the Bermuda records any unobstructed southern storm is east of the station, and receding from it. The tangent ray hypothesis is therefore inapplicable to the Atlantic records, and this detracts from its attractiveness. There is the possibility that reflection from ocean boundaries is an important factor, especially for the very long, low initial arrivals.

There has been no discussion as to why the spectral bands should reach their maximum development at frequencies of about 65 c/ks, or concerning the existence of a steady 15–16 sec "hum", if such a hum does in fact exist. In an average taken over many storms, the effect of the amplification spikes (Fig. 9) is erased, and an explanation must be sought along different lines. The generation spectrum is peaked at frequencies significantly higher than 65 c/ks, but it is possible that the shift toward lower frequencies resulting from selective attenuation may account for the observed values.

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APPENDIX

The problem is to evaluate

$$\mu(F) = \frac{\sin c}{r \sin r F'} \tag{10}$$

near the tangent ray

$$r_t' = r_t/x_t, \qquad x = \phi - \phi_1. \tag{11}$$

The accent denotes differentiation with respect to $\phi$ (or $x$), and the subscript " $t$ " refers to evaluation at the tangent point. $\phi_1$ and $\phi_t$ are constants. Differentiating $F = -x \sin c/r$ (equation (5)) and using $r_t' x_t = r_t$ leads to

$$F_t' = 0, \qquad F_t'' = x_t r_t^{-2} r_t'' \sin c, \qquad F_t''' = x_t r_t^{-2} r_t''' \sin c. \tag{12}$$

To evaluate $F'$ we eliminate $\xi$ from the Taylor expansions

$$F = F_t + \xi F_t' + \tfrac{1}{2}\xi^2 F_t'' + \tfrac{1}{6}\xi^3 F'''_t,$$

$$F' = F_t' + \xi F_t'' + \tfrac{1}{2}\xi^2 F_t'''.$$

For the ordinary tangent ray the last term for each series is dropped, and $F_t' = 0$. The result is

$$F' = [2(F - F_t) F_t'']^{\frac{1}{2}}. \tag{13}$$

But for the tangent ray through the inflection point $r_t'' = F_t'' = 0$. Eliminating $\xi$ now gives

$$F' = (9/2)^{\frac{1}{3}} (F - F_t)^{\frac{2}{3}} (F_t''')^{\frac{1}{3}}. \tag{14}$$

The admittance factor $\mu$ can now be evaluated in terms of $x$ and $r$ making use of (12) :

$$\mu^{-2} = 2 \sin^2 r_t (F - F_t)\, x_t\, r_t''/\sin c,$$

$$\mu_i^{-3} = (9/2) \sin^3 r_t (F - F_t)^2\, x_i\, r_t\, r_t'''/\sin^2 c.$$

Differentiating equation (4) and using (7) and (5a) leads to

$$r_t' = -\sin c/F_t$$

$$r_t'' = \sin b \sin c \operatorname{cosec}^2 r_t (\sin r_t \cos \phi_t - r_t' \cos r_t \sin \phi_t)$$

$$r_i''' = r_i' (r_i'^2 - 1), \qquad r_i' = \sin c,$$

so that finally

$$\mu = h (F - F_t)^{-\frac{1}{2}}, \qquad \mu_i = h_i (F - 1)^{-\frac{2}{3}} \tag{15}$$

where

$$h^{-2} = 2 r_t \sin b (\cos r_t \sin \phi_t + F_t \sin r_t \cos \phi_t/\sin c)$$

$$h_i^{-3} = (9/2) \cot^2 c\, r_i^2 \sin^3 r_i.$$