TRACKING STORMS BY FORERUNNERS OF SWELL

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ABSTRACT

Long, low waves preceding the arrival of the visible swell from a storm have been recorded off Pendeen, England, and Woods Hole, Massachusetts, by means of new instruments for the measurement and analysis of ocean waves. These forerunners provide storm warnings of practical value. Expressions giving the forerunner's distance from the storm system and its travel time as functions of recorded period and rate of change of period are derived from very general assumptions. The expressions are suitable for simple graphical representation. The application of the method to tracking storms across the ocean is illustrated by means of a few actual examples, and the computed storm tracks are shown to be in good agreement with the information contained on weather maps. Certain features of the wave records may eventually make it possible to compute not only the location but also the size, intensity, and general character of the storm.

1. Introduction

The use of swell as a sea-borne storm warning is quite natural and has been recognized by seafaring men since ancient times. In the early days of weather forecasting in New Zealand, particular attention was paid to observations of swell. However, the interpretation of swell appears to have been intuitive, based upon experience gained in a lifetime at sea.

Here it has been attempted to develop a *quantitative* method for using wave observations to track storms at sea. The method depends on the use of special instruments for recording waves and on the interpretation of the records in the light of hydrodynamics.

Another quantitative method, which differs entirely from the one to be described here, was suggested by the relationships established for forecasting sea and swell from weather maps (Sverdrup and Munk, 1947, p. 28). These relationships contain a quantitative formulation of the common experience that relatively high waves of short period come from a near-by storm, while low waves of long period come from a distant storm. However, when these relationships are applied in a reverse manner, that is, in the computation of storm properties from wave observations, the problem is no longer completely determinate. A number of other inherent difficulties further limit the use of this method. Even with these limitations the method has found practical application.

The limitations can be overcome in part by dealing with the low, long, invisible forerunners of the swell, rather than with the swell ordinarily observed. This makes it impossible to apply simple, visual methods of observation, but the advantages involved appear to justify the use of special instrumental techniques. In the first place, the forerunners travel considerably faster than the visible swell, as the speed of waves in deep water is proportional to their period. On the basis of instrumental observations now available (Deacon, 1946), the period of the forerunners may reach values up to 30 sec, in contrast with the average swell period of about 10 sec. The corresponding speeds with which the disturbance created by a storm is effectively propagated by wave motion (group velocity) are of the order of 1100 nautical miles per day for the forerunners, in contrast with 400 nautical miles per day for the swell. In the case of a fairly distant storm, information carried by the visible swell therefore pertains to wind conditions which have existed a number of days before the swell is observed. In the case of a fast-moving storm system, the visible swell may give little or no advance warning. The speed of the forerunners, on the other hand, although slow compared with the speed of microseisms, radio waves, and other phenomena providing auxiliary weather data, may be sufficiently rapid to permit practical use of the information carried by the forerunners.

The other advantage in dealing with the forerunners rather than the visible swell is that the forerunners seem to have the characteristics of *conservative* waves, that is, waves whose crests maintain their identity over great distances. The crests of ordinary swell maintain their identity over much shorter distances. Although the difference is largely one of degree, it is important because the treatment of visible swell must depart from methods of classical hydrodynamics and take into account the statistical[2] nature of the

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[2] C.-G. Rossby (verbal communication) has suggested that visible swell may be treated from a point of view closely akin to that of statistical mechanics.

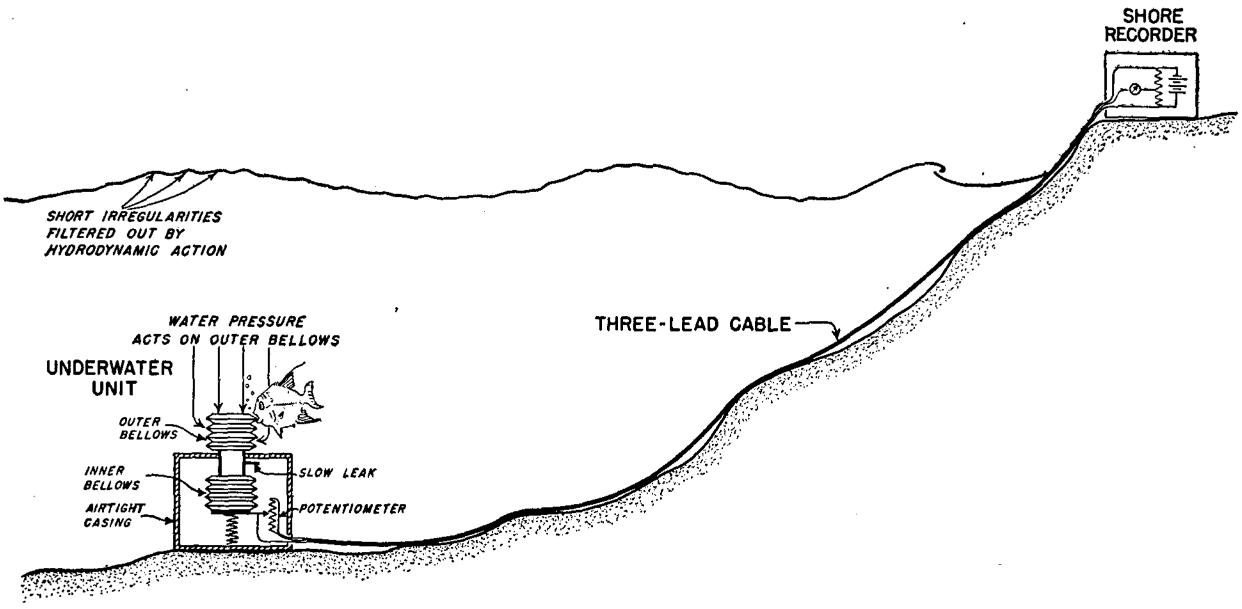

FIG. 1. Recording mechanism. The pressure fluctuation at the sea bottom is due to surface waves; it is picked up by the underwater unit and recorded on shore.

problem (Sverdrup and Munk, 1947, p. 12). The height and period of the visible swell do not suffice to determine uniquely the principal unknowns, travel time and storm distance; additional assumptions must be introduced, such as a relationship between storm intensity and storm size. The problem dealing with the conservative forerunners is much simpler. Travel time and storm distance can be uniquely determined without further assumptions. Furthermore, it is possible to carry out a purely kinematic analysis whereby the need for measuring wave height is eliminated and only a knowledge of wave period is required. In a determination of the deep-water wave height it is necessary to take into account, by means of specially constructed 'refraction diagrams' (Munk and Traylor, 1947), the effect of bottom topography and of the configuration of the coastlines. The response characteristics of the instruments also enter as an important factor. Wave periods, on the other hand, are relatively easy to ascertain from wave records and are not subject to the complex changes in wave pattern that occur as waves come into shallow water.

2. Instrumentation

The development of wave-recording instruments has been carried out simultaneously in Great Britain and the United States. An early model developed by the Mine Design Department, British Admiralty, has been in operation at Pendeen, near Lands End, England, for over two years (Deacon, 1946). Records taken by this instrument have been kindly placed at our disposal by the Oceanographic Research Group, Admiralty Research Laboratory, Teddington, England, and form the basis for the examples given in this report.

In the United States the development of wave instruments is chiefly due to M. Ewing of the Woods Hole Oceanographic Institution. In the early models the recording apparatus was contained directly in the underwater unit. Later units recording on shore were installed near Cuttyhunk, Massachusetts, and at the island of Bermuda.[3] A number of models have been developed at the College of Engineering, University of California,[4] and some of these will be used in the installation at the Scripps Institution.

Although the various instruments differ in many important aspects, the following principles are common to all and should be understood for a proper interpretation of the final records. Surface waves induce pressure fluctuations in the entire column of water between the surface and the sea bottom (fig. 1). For any given depth of water and wave height, the amplitude of these fluctuations depends on the wave period in such a manner that waves of very short period are virtually eliminated. The underwater unit measures the fluctuating pressure at the sea bottom. A 'slow leak' in the underwater unit eliminates the effect of tides and other very long waves. The pressure fluctuations are converted into electrical modulations, which are transmitted through a cable to a shore recorder. Finally the records are subjected to a har-

[3] A. A. Klebba, "A summary of shore recording wavemeters, preliminary report," Woods Hole Oceanographic Institution, 1945.

[4] College of Engineering, University of California, "Shore wave recorder drawings," Navy Department, Bureau of Ships, Contract Report No. 219, 1946.

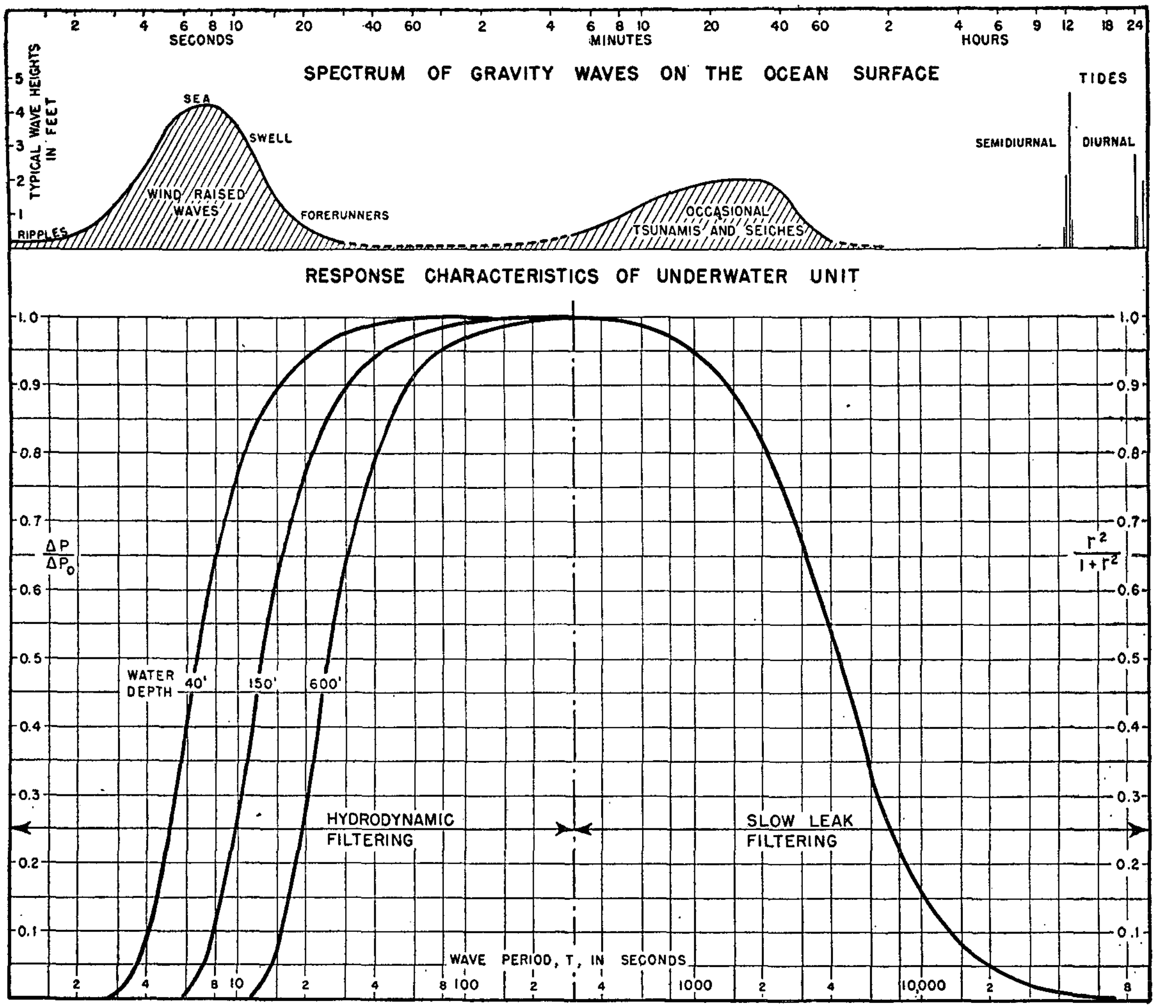

FIG. 2. Upper portion, a typical distribution of energy among various types of ocean waves, ranging from wind ripples having a period of one second to tides having periods of 12 and 24 hours; lower portion, the over-all response characteristics of the underwater instruments. Both graphs are drawn to the same period scale.

monic analysis by a special instrument. The natural wave spectrum is therefore modified in three stages: (a) waves of short period are removed by hydrodynamic filtering; (b) waves of very long period are removed by the slow leak in the underwater unit; (c) the remaining record is broken down further by harmonic analysis.

Hydrodynamic filtering.—Owing to their low amplitude, the forerunners must be extracted from the complex pattern of the sea surface by means of special techniques. One technique uses the ocean itself as a highly effective low-pass filter. The response characteristics of this filter can be expressed by the ratio $\Delta P/\Delta P_0$ of the amplitude of the pressure fluctuation at the bottom to that just beneath the surface. Let h designate the water depth, L the wave length, T the wave period, and g the acceleration due to gravity. The depth L can be eliminated between the equation

$$\frac{\Delta P}{\Delta P_0} = \frac{1}{\cosh (2\pi h/L)}$$

and the well known relationship

$$\left(\frac{2\pi}{T}\right)^2 = \frac{gL}{2\pi} \tanh \frac{2\pi h}{L},$$

so that the hydrodynamic filtering effect $\Delta P/\Delta P_0$ can be computed as a function of depth and wave period.[5]

The left sides of the curves in the lower part of fig. 2 have been computed in this manner for three selected depths. Thus, at depths of 40, 150, and 600 feet, waves of periods less than 4, 8, and 15 sec respectively are practically eliminated. The underwater unit of the Woods Hole Oceanographic Institution is situated at an average depth of 80 feet, the unit of the British Admiralty Research Laboratory (Barber, Ursell, Darbyshire and Tucker, 1946) at an average depth of 120 feet. Comparison of the upper and lower parts of fig. 2 shows that at these depths the visible swell is not effectively reduced; therefore neither instrument

[5] W. H. Munk, "Measurement of waves from pressure fluctuations at ocean bottom," Scripps Institution Wave Project Report No. 5, 1944.

is particularly suited for the recording of the forerunners, as would be an instrument at 600 feet. The proposed wave station of the Scripps Institution of Oceanography provides for three underwater units at depths of 40, 150, and 600 feet. According to fig. 2, the deep unit will be out of reach of the visible swell and can be designed to record the low forerunners with great sensitivity. Fortunately a submarine canyon enables the deep instrument to be placed only 3000 feet from the end of a pier. In most localities it will be cheaper, though hardly more effective, to suppress the short-period waves instrumentally.

The underwater unit.—Fig. 1 gives a schematic view of one type of underwater unit and serves to illustrate the desired characteristics. The pressure P_w is transmitted through an outer bellows into a second bellows inside the instrument casing. Thus the pressure of the air inside the two bellows always equals the pressure in the water outside. The air inside the bellows can pass through a slow leak into the instrument casing, so that the *average* pressure inside the casing equals the average pressure in the water, that is, the hydrostatic pressure P_h. The leak is so slow that the pressure inside the instrument does not change appreciably during one wave period.

The total displacement of the inner bellows depends on the *difference* in pressure between the inside and the outside of the bellows and therefore measures the deviations of pressure from the hydrostatic mean. These deviations have an amplitude ΔP.

For waves of very long periods the fluctuation of air pressure in the instrument casing cannot be neglected. Indeed waves of tidal period are almost completely eliminated from the record, as the slow leak is able to compensate for the gradual rise and fall of sea level during a tidal cycle. In the general case the amplitude response of the instrument is proportional to $r^2/(1 + r^2)$, where $r = T_r/T$; T_r is the 'resonance' period, which depends on the size of the slow leak, the air volume of the instrument, and the water depth. In fig. 2, T_r has been assumed to equal about one hour. Owing to the combined effect of hydrodynamic filtering and the slow leak, waves of short and very long period are effectively eliminated. Maximum sensitivity is achieved in the desired range by a proper choice of depth for the underwater unit and of size for the 'slow leak.'

Frequency analyzer.—The following description refers to the analyzer developed at the Admiralty Research Laboratory (Barber, Ursell, Darbyshire and Tucker, 1946). The pressure deviations are recorded photographically on shore. The finished record is half black and half white, and the deflection of the black-white boundary is proportional to the pressure deviation. Each record covers an interval of about 20 minutes. The record is placed in the analyzer by wrapping it around the circumference of a heavy wheel, which is rotated and permitted to slow down by friction. A narrow band of light is thrown on the record, and the record is scanned photoelectrically. The current fluctuation in the photoelectric-cell circuit follows the fluctuation on the record. The current output is amplified and fed into a vibration galvanometer with a sharp resonance characteristic of about 100 cycles, and the output of the galvanometer is mechanically plotted against the angular velocity of the wheel. A set of period analyses is shown in fig. 6.

Wave direction.—Two methods for measuring wave direction are being studied. One depends on the measurement of the time interval between the passage of a wave crest over two underwater units placed equally distant from shore. The other method depends on the use of a Rayleigh disk. The development of a reliable method for measuring wave direction is an essential requirement in making use of wave records for tracking storms.

3. Kinematics of conservative waves

Certain properties peculiar to conservative wave motion are derived in this section; these properties form the basis of the method for tracking storms by forerunners of swell. Wave length is defined as the horizontal distance between two adjacent wave crests, wave period as the time interval between the appearance of two successive crests at a fixed spot. It should be noted that these definitions refer not to the underlying regular wave trains of a Fourier spectrum but to the physical deformations of the sea surface (Munk, 1947). The following analysis deals with a train of waves in which the wave length L and the wave period T are slowly varying functions of time and distance.

The wave length L of a wave traveling with (phase) velocity C along the positive x-axis changes at the rate[6]

$$\frac{DL}{Dt} = \frac{\partial L}{\partial t} + \frac{\partial L}{\partial x} C. \tag{1}$$

This change can also be found from the rate at which the wave 'stretches' due to the difference in speed of two adjoining crests:

$$\frac{DL}{Dt} = \frac{\partial C}{\partial x} L = \frac{dC}{dL} \frac{\partial L}{\partial x} L. \tag{2}$$

The latter substitution is possible in a dispersive

[6] The notation D/Dt denotes the rate of change with respect to an observer moving with a *fixed wave*, in analogy with a convention introduced by Stokes according to which D/Dt denotes the 'material' derivative, that is, the rate of change with respect to an observer moving with a *fixed particle*. The partial derivatives $\partial/\partial x$ and $\partial/\partial t$ have their usual meaning. The derivative dC/dL designates the total derivative in the usual sense, where it is considered that C is a function of L only and does not involve either x or t explicitly. Rossby (1945) uses d/dt to denote the rate of change with respect to an observer moving with group velocity.

medium where C is a function of L only. Combining (1) and (2) gives

$$\frac{\partial L}{\partial t} + \left(C - L\frac{dC}{dL} \right) \frac{\partial L}{\partial x} = 0. \qquad (3)$$

Let the group velocity V be defined by the equation

$$V = C - L(dC/dL). \qquad (4)$$

According to (3) and (4) *an observer traveling with group velocity records a constant wave length.* If dC/dL is positive, then $C > V$, and the observer is continually overtaken by individual waves.

This elegant derivation of the group velocity was first given by Lamb (1932, p. 382). Rossby (1945) derives equation (4) by equating the rate of increase of wave crests within a certain distance Δx to the excess of waves entering at x over those leaving at $x + \Delta x$.

For the case of low waves in deep water the classical equations

$$C \equiv \frac{L}{T} = \left(\frac{gL}{2\pi} \right)^{\frac{1}{2}} = \frac{gT}{2\pi} \qquad (5)$$

apply, since the period is assumed to be a slowly varying function of time and distance. Equations (4) and (5) lead to the well known formulas

$$V = C/2 = gT/(4\pi). \qquad (6)$$

Equations (3) and (5) give

$$\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial x} = 0, \qquad (7)$$

so that an observer traveling at group velocity records constant wave period. In a wave train in which individual waves (or crests) are neither created nor destroyed, the period as a function of time and distance must satisfy equation (7).

Equation (7) can be solved by the method of Lagrange. One solution,

$$T = \text{constant},$$

is of no further interest; the other solution,

$$T = f(t - x/V), \quad \text{or} \quad t - x/V = F(T), \quad (8a, b)$$

contains an arbitrary function, which depends upon the choice of coordinate axes.

According to equation (7) the period T, but not necessarily the group velocity V, remains constant relative to an observer traveling at group velocity. For surface waves in deep water, however, the group velocity depends on period only (equation 6), and, as long as T is constant, V must be constant.

Assume that a train of waves is generated by a sudden disturbance,[7] and let t and x designate the time and radial distance relative to this disturbance (fig. 3). Then the path of an observer traveling from the initial disturbance with constant group velocity V is given by

$$x = Vt \qquad (9)$$

and the arbitrary function $F(T)$ in equation (8b) must vanish. The solid lines in fig. 3 represent the paths of two observers traveling at group velocities V_1 and V_2, respectively, and recording constant wave periods $T_1 = 4\pi V_1/g$ and $T_2 = 4\pi V_2/g$ (equation 6). The dashed lines represent the paths of individual waves. Fig. 3 shows that the individual wave increases continuously in period and velocity.[8]

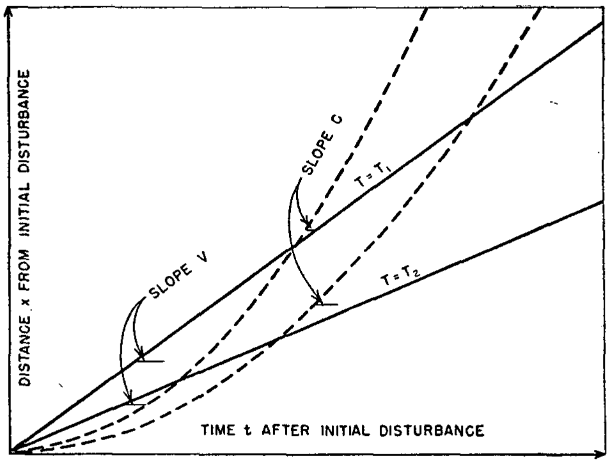

FIG. 3. Solid lines, paths of two observers each traveling with constant group velocity; dashed lines, paths of two individual waves. At the intersection of dashed and solid lines the slope of the dashed line equals twice the slope of the solid line, in accord with the equation $C = 2V$.

Combining (6) and (9) gives

$$T = \frac{4\pi}{g}\frac{x}{t}, \qquad (10)$$

which has been derived independently by Rossby (1945). This solution is the same as that for the Cauchy-Poisson wave problem dealing with impulsive generation of waves (Lamb, 1932, pp. 384–398).

According to equation (10) the fast, long-period waves travel in front of the wave train and are followed by waves of gradually diminishing periods. The change of period with time at a fixed locality is found by partial differentiation:

$$\frac{\partial T}{\partial t} = -\frac{4\pi}{g}\frac{x}{t^2} = -\frac{T}{t}. \qquad (11)$$

[7] The exact nature of the initial disturbance need not be considered, as long as the initial surface deformation can be assumed to resolve quickly into a train of regular waves. Such an assumption appears justified by an exact analysis involving Fourier integrals (Lamb, 1932, pp. 384–394).

[8] A line integral between the origin and any point in the t,x-plane can be taken either along the curves denoting the travel of individual waves or along the straight lines denoting the path of an observer traveling with constant group velocity. The former alternative may appear to have greater physical reality; the latter alternative is preferable from a mathematical point of view. The straight lines are the 'rays' in the Hamilton-Jacobi wave theory.

Combining (10) and (11) leads to the interesting equations

$$x = \frac{gT^2}{4\pi(-\partial T/\partial t)}, \quad t = \frac{T}{(-\partial T/\partial t)}, \quad (12a, b)$$

so that the distance x and time t can be determined if two quantities are known: the period T and the decrease of period with time, $-\partial T/\partial t$. Both quantities can be obtained from wave records. Equations (12) form the basis of the method for locating and tracking storms by means of the forerunners. They have been derived under the assumptions (a) that the effective distance and duration of generation are large compared with the total travel distance and travel time, (b) that individual waves are neither created nor destroyed, and (c) that the resulting wave pattern varies slowly with time and distance. The third assumption must be modified if the existing wave pattern is the result of two or more disturbances.

4. Conservative waves from multiple disturbances

Let t_s designate the time of a disturbance, t_w the time when waves of period T are recorded. Substituting

$$t = t_w - t_s \quad (13)$$

into equation (10) gives

$$T = \frac{4\pi}{g}\frac{x}{t_w - t_s}, \quad (14)$$

which is shown by the curve AA′ in fig. 4.

A second disturbance, at a time $t_s + \Delta t$ and distance $x - \Delta x$ from the wave station, generates a second train of waves. Periods of the second train are given by the equation

$$T' = \frac{4\pi}{g}\frac{x - \Delta x}{t_w - (t_s + \Delta t)} \quad (15)$$

and are represented by the curve BB′ in fig. 4.

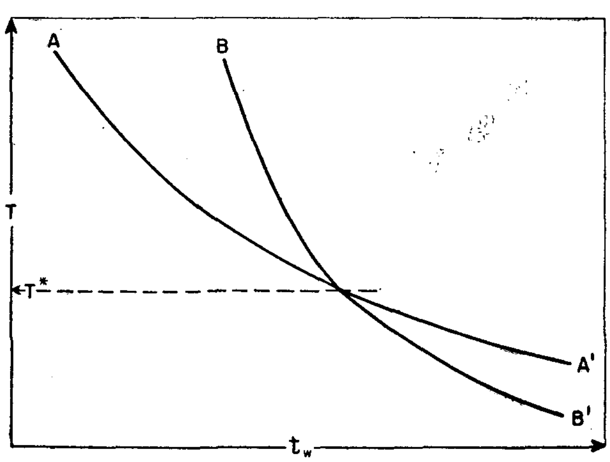

FIG. 4. Wave period T as a function of time t_w at a fixed station. At some instant the wave trains due to two distinct impulses have the same period, designated by T^*.

At any time t_w,

$$T' - T = \frac{4\pi}{g}\frac{\Delta t}{t - \Delta t}(V - \bar{U}) \quad (16)$$

where

$$V = -x/t \quad (17)$$

is the group velocity and

$$\bar{U} = -\Delta x/\Delta t. \quad (18)$$

The minus sign appears in the last two equations because the x-axis is now drawn opposite to the direction of wave propagation. At the point where the curves AA′ and BB′ intersect one another

$$T = T' \equiv T^*, \quad \bar{U} \equiv V^* = gT^*/(4\pi), \quad (19a, b)$$

the last term following from equation (6); furthermore T' is longer than, equal to, or shorter than T according as V^* is larger than, equal to, or smaller than $\bar{U}$.

Equation (10) applies separately to each of the two wave trains simultaneously present at the wave station but does not apply to the resulting interference pattern, as the wave period and wave length of the interference pattern are no longer slowly varying functions of x and t. The definitions of L and T given in the previous section must be interpreted to apply to each of the wave trains, which can be separated from the interference pattern by a suitable method of harmonic analysis.

The reasoning can be extended to an infinite number of disturbances, separated by infinitesimal time intervals and distances, each sending out a wave train whose periods obey equation (10). As a result, neither one wave period nor two distinct wave periods are present at the wave station at any given time, but a *band* of wave periods.

5. Conservative wave theory applied to forerunners

It will be assumed that a storm system acts as a continuous, traveling source, applying an infinite number of impulses to the sea surface, each impulse giving rise to a train of waves in the sense described above. The speed $\bar{U}$ defined in equation (18) can then be identified with the rate of movement of the storm system toward the wave station.

The harmonic analyzer at the wave station draws a spectrogram of the resulting interference pattern. The limits of the band and conspicuous features within the band can be identified on consecutive records. If such a feature can be traced over a number of hours, say six hours, and if it is found to agree with equation (10), it may reasonably be attributed to a definite conservative wave train, generated essentially by a single impulse. The location of the impulse is then readily determined from equations (12).

To decompose the action of a storm upon the sea surface into separate impulses may appear highly arbitrary, yet, in the author's opinion, a more rigorous analysis, perhaps following methods of statistical mechanics, would lead to similar results.

Care must be exercised in interpreting the significance of the limits of the period band. The shortest wave period is determined by the *earliest* storm action from which waves gathered sufficient energy to be perceptible at the wave station. The longest wave period is determined by the *latest* storm action from which waves are perceptible at the wave station. If a storm emits a broad spectrum of waves, as may indeed be the case, waves of period five minutes or longer, traveling as shallow-water waves similar to tsunamis (long ocean waves generated during submarine earthquakes and usually referred to as 'tidal waves'), constitute the theoretical upper limit of the period band (fig. 2).

The longest recorded waves do not reach this limiting period because the storm energy apportioned to the very long waves is small and because the response characteristics of the underwater unit are unfavorable. Equation (10) is therefore not applicable to the recorded upper limit as such, but in practice conspicuous features near the upper limit can be identified and used in computations. The lower limit, however, does not depend on the characteristics of the instrument, and equation (10) may be applied directly.

The use of forerunners for tracking storms is based on equations which were first derived in a study dealing with a different problem, namely, the increase in the period of waves traveling over large distances (Munk, 1947). It was shown that the same general theory can account for the observed increase in period of such widely different phenomena as swell, seismic surface waves, and tsunamis. Thus the fundamental notions appear to have a truly geophysical significance applicable to all waves traveling for long times and over long distances. The wide applicability derives from the general nature of equation (3), to which Rossby (1945) has applied the descriptive term 'wave equation of continuity.' Carl Eckart of the Institute of Geophysics, University of California, recently pointed out[9] that some of the results could also have been derived on the basis of the Hamilton-Jacobi wave theory. In the present paper, the only equations derived are those essential to the problem of storm tracking. A general development of the theory may be found in the paper on the period increase (Munk, 1947).

6. A graphical method for tracking storms

Equation (14) can be written in the form

$$\frac{4\pi}{g}\frac{1}{T} = \frac{1}{x} t_w - \frac{t_s}{x}. \qquad (20)$$

[9] Verbal communication.

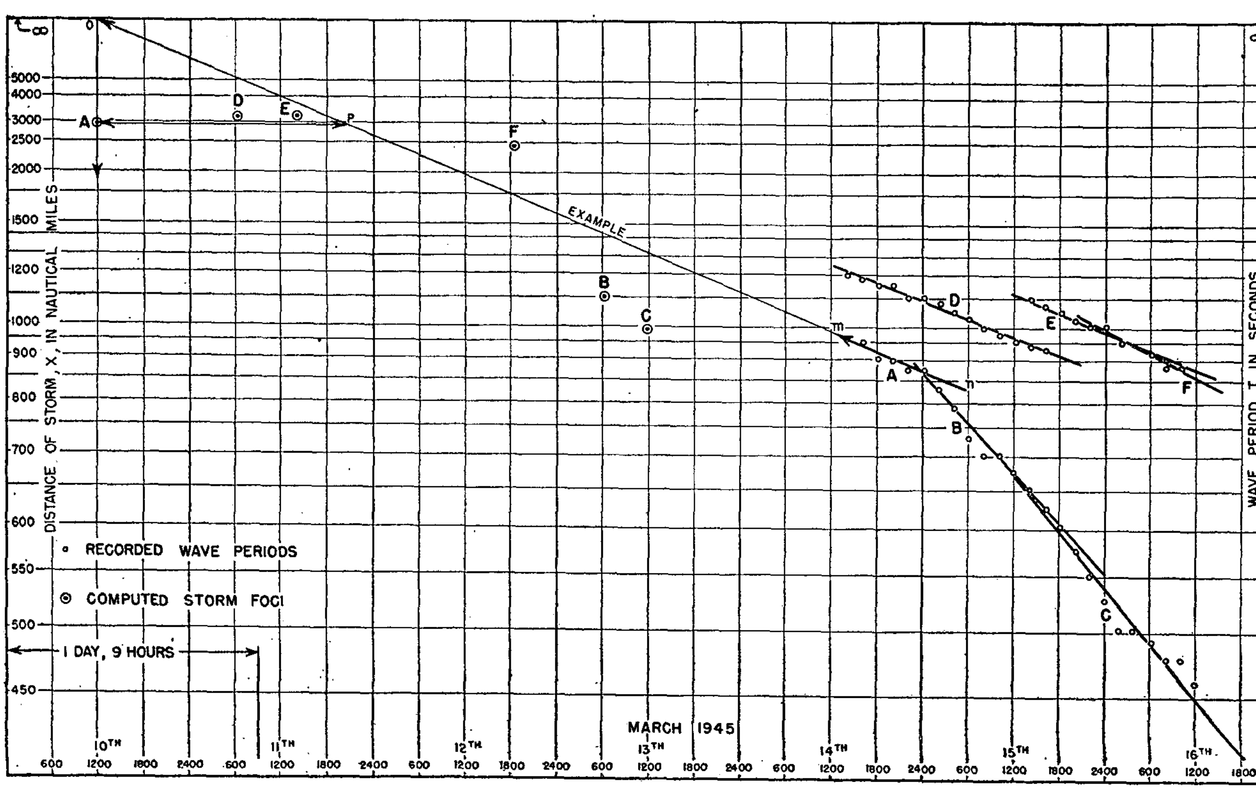

FIG. 5. Graph for computing storm foci (see section 6).

The problem is to evaluate x and t_s from a knowledge of T at various times t_w. If $1/T$, the inverse of the recorded period, is plotted against the recording time, t_w, then for any fixed wave train all points must fall along a straight line. The slope of the line equals $1/x$, and $t_w = t_s$ at the intercept of the line with $1/T = 0$. These relationships permit a simple graphical determination of x and t_s; the recorded wave periods and the computed storm positions are plotted on a single graph with the abscissa representing both t_s and t_w, the ordinate $1/T$ and $1/x$ (fig. 5). For convenience, the parameters $1/T$ and $1/x$ are designated by an inverted scale, so that T and x can be read off directly. The procedure is:

(a) Identify, in the recorded period spectrum, outstanding features such as the limits of the period band. The period T associated with each feature is plotted against t_w, the time when the wave was recorded. The open circles on the right side of fig. 5 were obtained in this manner for the upper and lower limits of the period band shown in fig. 6.

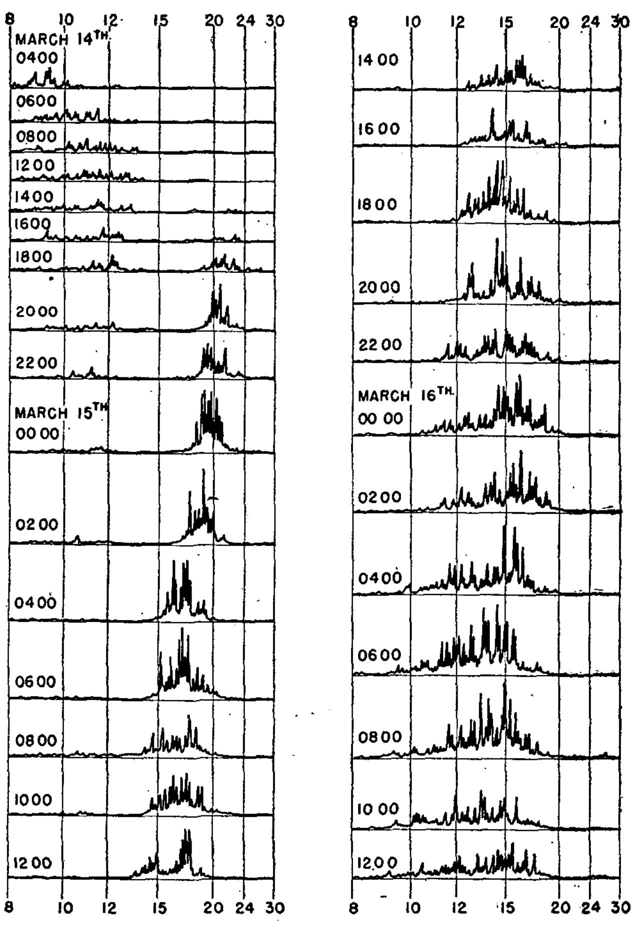

FIG. 6. Spectrograms of waves recorded during 14–16 March 1945 at Pendeen, England. Each spectrogram gives the distribution of energy for a 20-minute record. The records were taken at 2-hourly intervals. (From Deacon, 1946.)

(b) Connect any features which can be identified on at least three records and over at least six hours and which fall approximately along straight lines (lines A, B, $\cdots$ F in fig. 5).

To each of these lines on the x,t_w-diagram there corresponds a 'focal point' which gives the distance x of the storm at the time t_s. The foci are designated by the large circles on the left side of fig. 5, and each is marked according to the line with which it is associated. Thus the waves whose periods fall along line A were generated at the focus A. The six foci in fig. 5 indicate the movement of a storm system from a distance of almost 3000 miles at noon 10 March to a distance of about 1000 miles at noon 13 March. Fig. 5 illustrates the method whereby focus A is located:

(c) Extend line A until it intersects the line $T = \infty$, and draw a vertical. The vertical immediately gives t_s. The value of x is found by following a vertical downward until the horizontal distance AP between the vertical and the extended line through the observations corresponds to a time interval of 1 day, 9 hours. This time interval is the same for all foci and can be measured off in the lower left corner of the figure. Focus A is plotted, and the value of x is read from the scale on the left.

The time interval depends on the ratio of the two ordinate scales, which was arbitrarily chosen to be (50 nautical miles):(1sec). Point P, representing a period T and time t_w, has the same ordinate as point A, representing the distance x and time t_s; consequently x/T is equal to the ratio of scales. From equation (14),

$$t_w - t_s = \frac{4\pi}{g}\frac{x}{T} = \frac{4\pi}{g}\frac{50 \text{ nautical miles}}{1 \text{ sec}}$$
$$= 1.17 \times 10^5 \text{ sec} = 1 \text{ day, 9 hours.}$$

The entire derivation is completely determinate, and no arbitrary constants or functions have been introduced to fit the data. The examples in the next section will show that the calculations not only give the correct order of magnitude but also permit the location of storm positions to a considerable degree of accuracy.

The following additional rules follow from equations (19):

(d) The average speed of the storm system between the positions represented by foci A and B equals

$$\bar{U} = V^* = gT^*/(4\pi) \qquad (19b)$$

where T^* is the period at the intersection of the two straight lines A and B. Numerically $\bar{U} = 1.5\ T^*$ if $\bar{U}$ is given in knots and T^* in seconds. Any *abrupt* change in slope of the line representing a boundary of the period band is the result of an intersection of two such

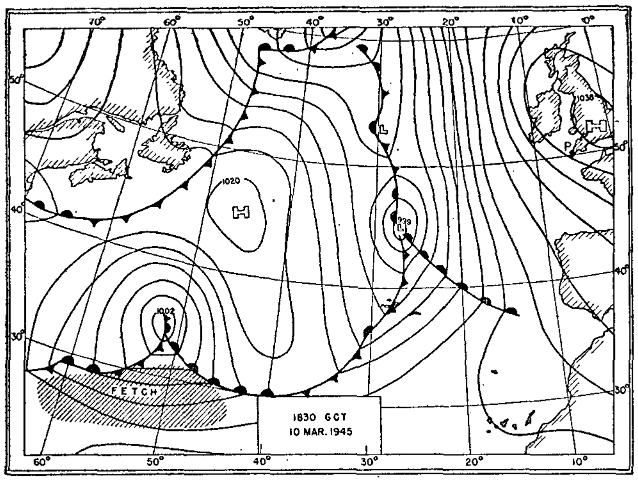

FIG. 7. Surface weather map for 10 March 1945. Fronts are represented in the conventional manner, and isobars are labeled in millibars. Point P denotes the wave station at Pendeen, England. Figs. 7–10 show the passage across the Atlantic Ocean of the storm for which the wave spectrograms are shown in fig. 6.

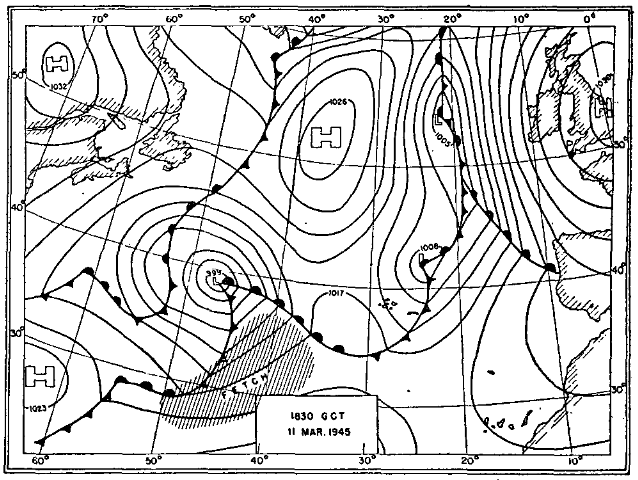

FIG. 8. Surface weather map for 11 March 1945.

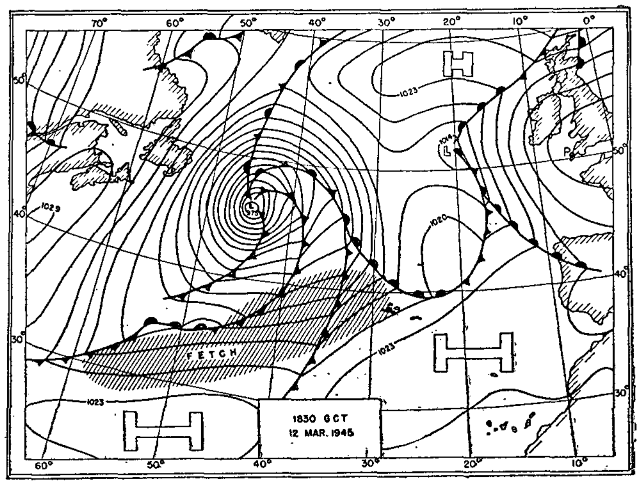

FIG. 9. Surface weather map for 12 March 1945.

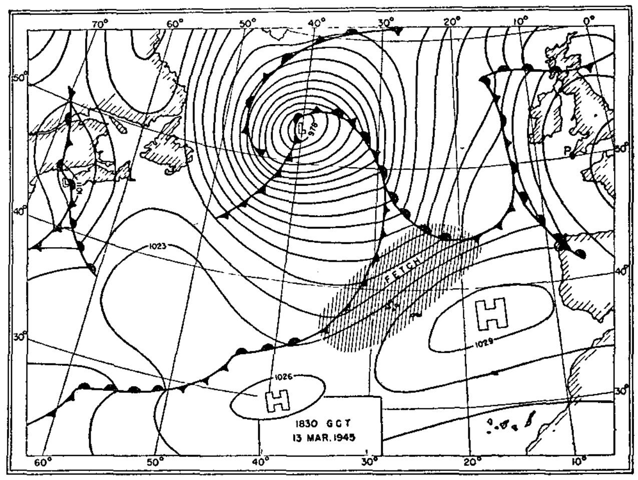

FIG. 10. Surface weather map for 13 March 1945.

lines and can at once be interpreted in terms of the rate of movement of the storm system.

(e) A *gradual* increase of slope can be formed only by an envelope from a family of lines from adjoining foci. This situation arises only when the storm center is decelerating, $\bar{U}$ decreasing according to equation (19b), where T^* is now the recorded period along the envelope.

7. Examples based on records from Pendeen, England

14–16 March 1945.—Fig. 6 shows a set of period analyses obtained by methods discussed in section 2. The band of gradually diminishing periods is the result of an intense storm which formed on 10 March 1945 off the coast of Cuba, 3000 miles from Pendeen, and traveled in a general northeast direction, passing west of the British Isles. Four representative weather maps are shown in figs. 7–10.[10] The positions indicated for the 'fetches' were determined according to the rules developed for wave forecasting.[11] It should be noted that the fetch lengthened during the time interval between 18^h30^m 11 March and 18^h30^m 12 March.

The first indications of the storm on the wave record are some 'low-frequency noises' at 14^h 14 March. At that time visual observations at Pendeen revealed only the presence of 10-sec and 12-sec waves from another source; the long forerunners of the swell were too low to be visible. The upper and lower limits of the period band of these forerunners have been read from the record and plotted in fig. 5. The positions of the six foci A, B, $\cdots$ F have been determined by means of the graphical method described in the preceding section.

The information taken from the weather maps and the periods recorded at the wave station have been replotted in fig. 11 to emphasize the similarity, with

[10] The weather maps were drawn and analyzed at the U. S. Navy Weather Central, Washington, D. C.

[11] Hydrographic Office, U. S. Navy, "Wind waves and swell; principles in forecasting," H. O. Misc. No. 11,275, 61 pp., 1944.

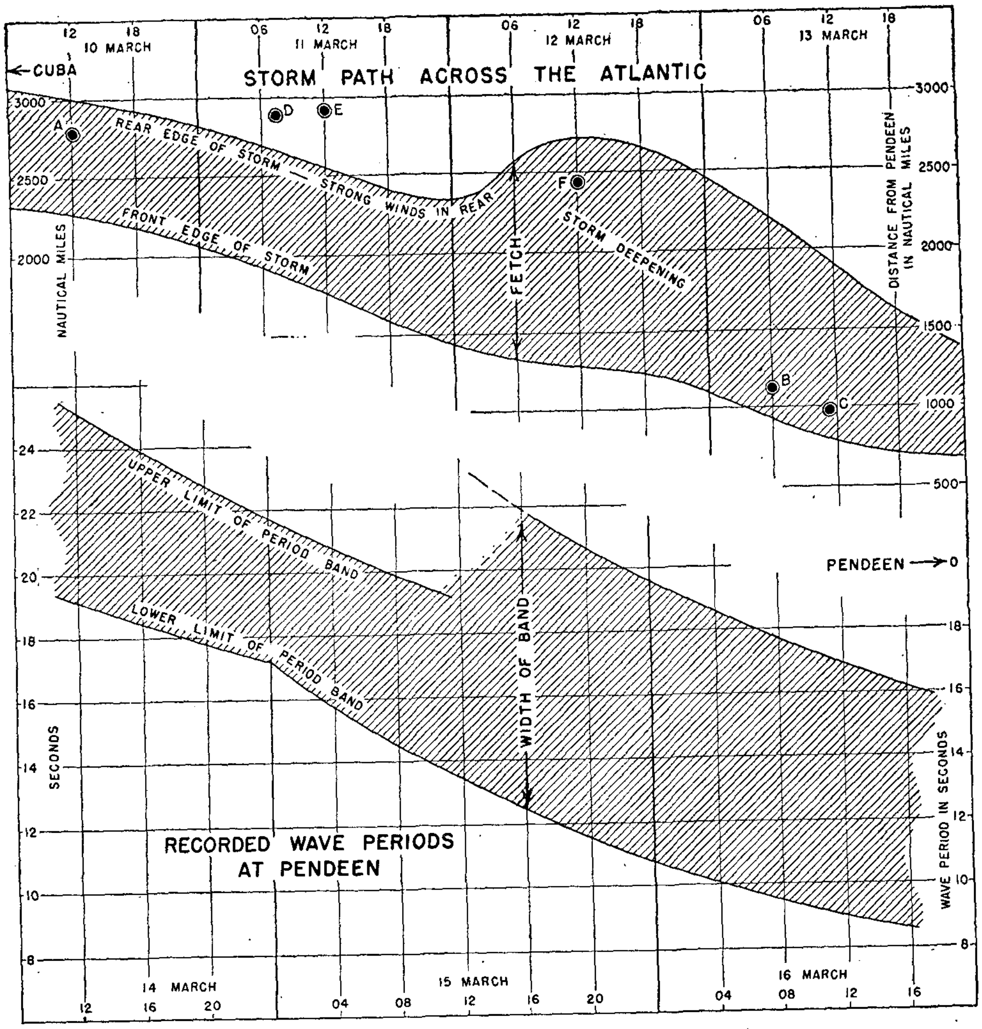

FIG. 11. Comparison of storm path with period band. The shaded band in the upper part shows the movement of the storm system toward the wave station. The forward and rear edges of the storm were determined from 6-hourly weather maps, of which every fourth map is shown in figs. 7–10. The computed storm positions (foci) are marked A, B, ··· F. The limits of the period band in the lower part were determined from the wave spectrograms shown in fig. 6.

regard to both shape and width, between the storm path and the period band. The computed positions, designated by A, B, ··· F, agree fairly well with the information on the weather maps, especially if it is remembered that the foci were obtained from theory without the introduction of any arbitrary constants. The foci A, B, and C were computed from the lower limit of the period band and seem to correspond to the forward edge of the fetch; D, E, and F were computed from conspicuous features near the upper limit of the period band and give the approximate position of the rear edge of the fetch. It can be seen, however, that focus A in the upper part of fig. 11 is located 500 miles behind the storm front and the other two foci about 200 miles behind the front. These discrepancies must be expected, since waves generated at the front itself could not gather sufficient energy to be perceptible at the wave station; furthermore, the distance between the point of wave origin and the storm front must be larger for the furthest disturbance, which is almost 3000 miles from Pendeen.

27–29 April 1945, fig. 12.—The waves are the result of a cyclone which centered over Newfoundland early on 23 April, when an 800-mile fetch in the warm sector extended from Bermuda toward east-northeast. By noon on 24 April the storm had moved to mid-Atlantic, where it was reinforced by a secondary cold front. The strong winds were blowing toward Africa, so that the fetch toward Pendeen was considerably weakened. This condition persisted for about a day until noon of 25 April, when a secondary trough merged with the main trough, and an intensification of the storm occurred, strong winds blowing toward the wave station (see map in fig. 12). This intensification, which followed an interval of relatively weak generation of waves, is reflected in the spectrograms as a new feature, marked F in fig. 12. This feature is quite distinct from the main period band. It will be noted from the lower

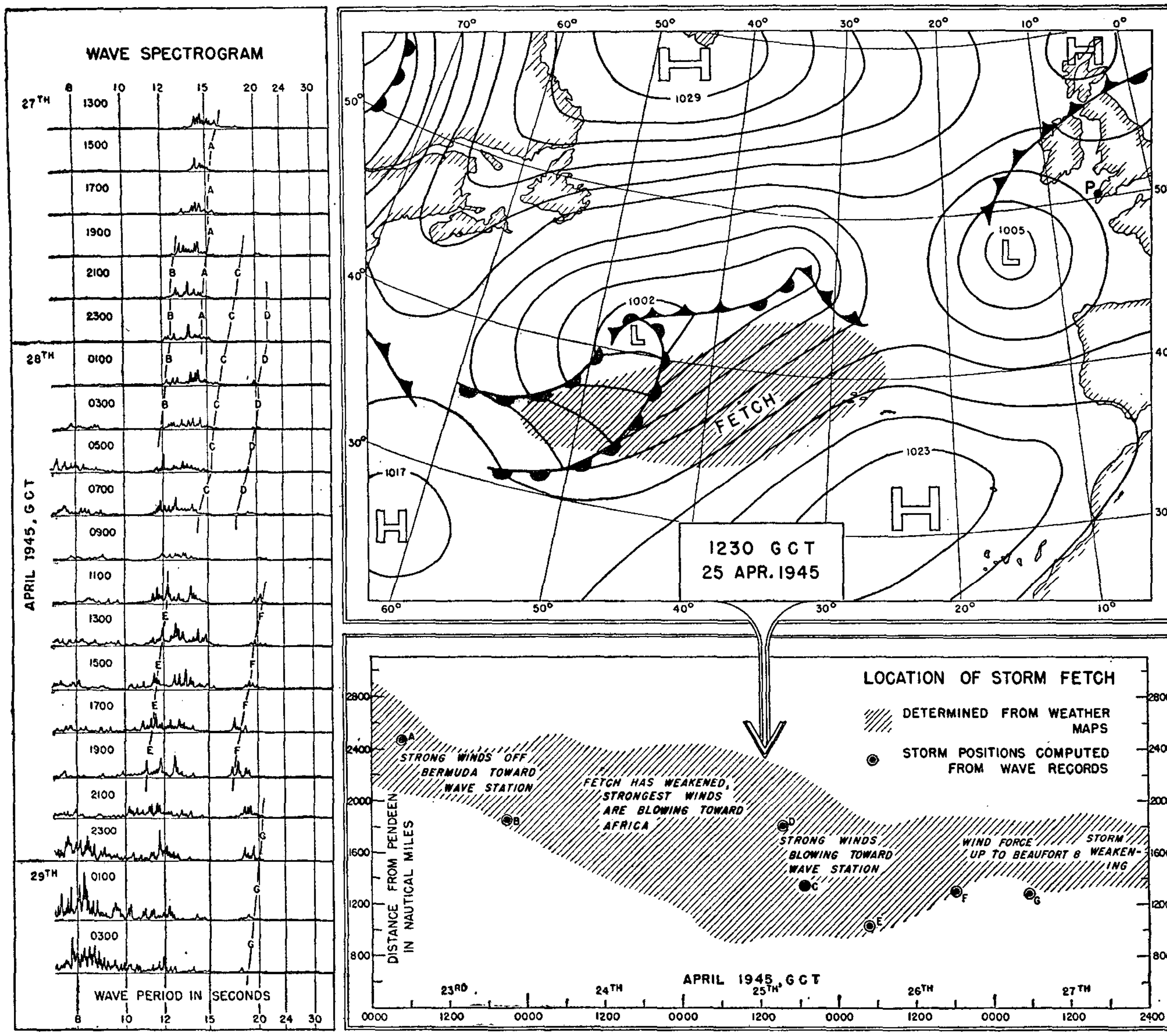

FIG. 12. Application of storm-tracking method to wave records of 27–29 April 1945. The period spectrograms recorded at Pendeen are shown on the left. Significant features on adjoining records are connected by dashed lines, marked A, B, ··· G. The corresponding foci are shown by the circles in the diagram at the lower right, where the shaded band gives the path of the storm determined from weather maps. One of these weather maps is shown at the upper right.

right part of fig. 12 that the focus F falls in the fetch at the time of intensification.

The storm then remained stationary and intensified further, the wind attaining a force up to Beaufort 8 early on 27 April. On the spectrograms the feature marked G can be associated with the region of highest wind. On 27 April the cyclone started to move northward, weakening and undergoing frontolysis.

According to fig. 12 the computed foci are in good agreement with information contained on the weather maps. All seven foci fall into intervals of strong winds, and no features which would fall into time intervals of weak winds could be identified on the wave records.

18–21 February 1945, fig. 13.—This example illustrates the application of the method to a more complex meteorological situation. Two storm systems, which existed simultaneously, could be identified and traced separately.

On 15 February the development of a cyclone started off the east coast of the United States. On 17 February the center of the cyclone had reached mid-Atlantic, and an unusually well defined 1000-mile fetch in the warm sector was pointing directly toward the wave station. At the same time a new low-pressure area had just moved off the east coast of the United States and formed a second fetch immediately north of Bermuda (fig. 13). As the first storm stagnated and veered northward toward Iceland, Pendeen came under the influence of the fetch in the cold sector, which remained at about the same distance from Pendeen. In the meantime the fetch in the warm sector

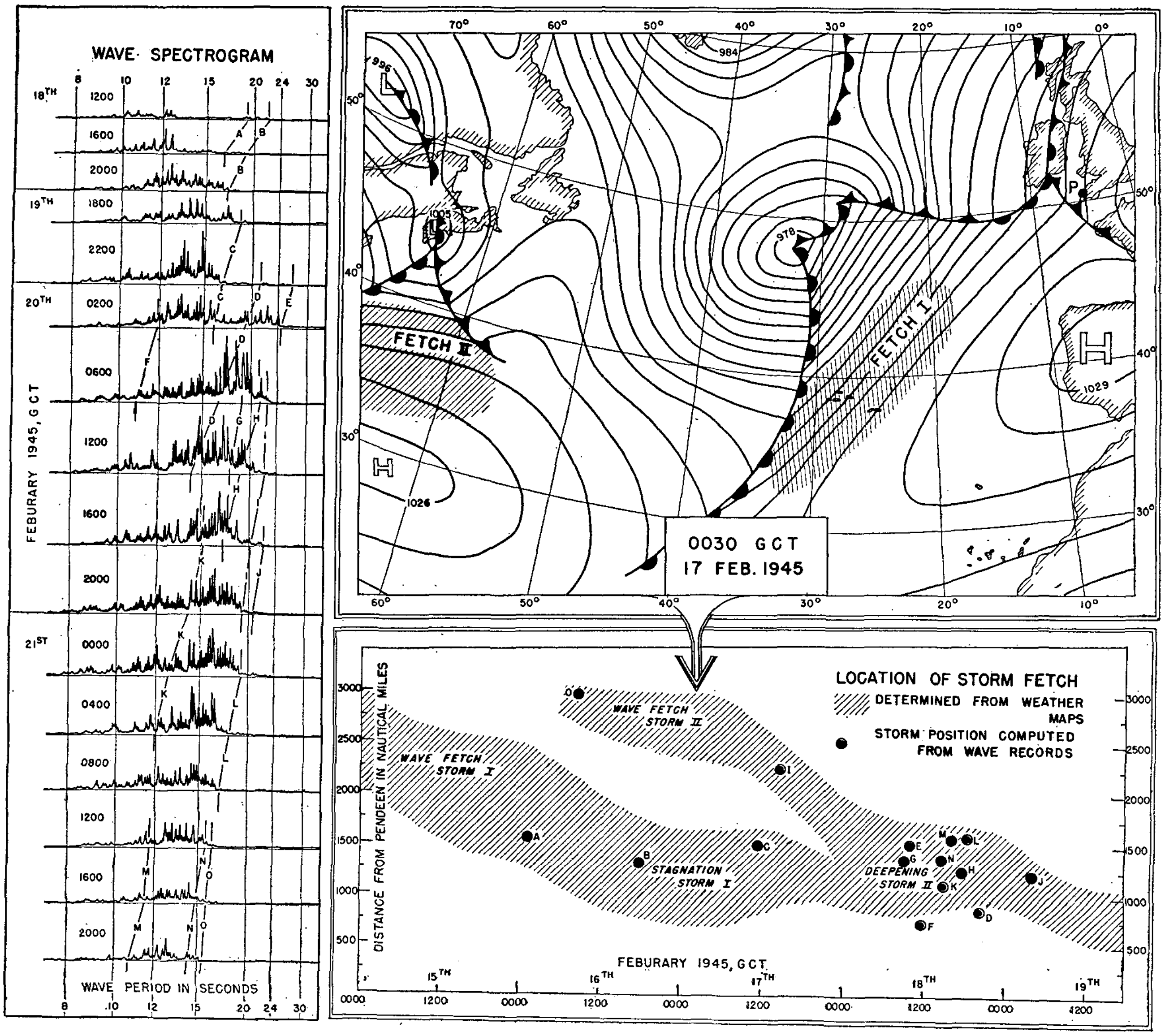

FIG. 13. Application of storm-tracking method to wave records of 18–21 February 1945. The presentation of the data is the same as that used in fig. 12, except that only alternate spectrograms are shown in the left part of the figure.

of the second storm moved slowly eastward, and on 18 February this and the fetch remaining from the first storm appeared as a single generation area. By 19 February the first storm had undergone complete frontolysis. The only remaining fetch was that in the cold sector of the second storm, which was moving northward toward Iceland.

The spectrograms shown in fig. 13 are more complicated than those in the preceding examples, and the interpretation is much more difficult. The first three foci fall in the first storm as it moved across the Atlantic. Foci D to H are associated with wave trains that were formed as the first storm veered northward and its fetch was joined with the fetch of the second storm. Only by that time had the main period band moved sufficiently into the lower periods to permit identification of a wave train, F, from the distant second storm. Foci J to N originated from the joint fetch existing from 18 to 19 February, but focus O had its origin in the second storm system, when it was still 3000 miles from the wave station.

8. Conclusions

Altogether eleven meteorological sequences have been studied by means of the Pendeen wave records. Of these the three best cases have been selected for examples in this report, but in all instances the agreement between computations based on the wave records and the information on the weather maps was encouraging. In one instance, 26 June 1945, waves from a hurricane off the coast of Florida were faintly recognizable above the disturbance caused by a moderate local storm. Attempts have also been made to analyze wave spectrograms from Woods Hole, Massachusetts, but there the time interval between records equaled six hours. To identify the same features on subsequent records was consequently almost impossible. For this reason it is not known at this time whether the methods of this report can be applied to wave records taken at stations located to the rear of the storms.'

It is hoped that theoretical studies dealing with the propagation of *energy* by the forerunners (Sverdrup, 1947) will lead to an interpretation of the spectrogram *ordinate*, which has received no attention so far but must be related to the storm intensity. Considerable progress can also be expected from the use of a deeper underwater unit. It may be possible to record forerunners with periods up to 60 sec and corresponding group velocity of 2200 nautical miles per day. Information transmitted by these very early forerunners would not only reach the wave station sooner but, according to experience gained so far, would also be more accurate.

The relatively long interval between the time the waves are generated and the time they are recorded at the wave station is a disadvantage in the application of the method. Therefore the practical use will be largely to verify or modify the interpretations already made on the basis of meteorological information. This should be particularly useful for regions where the network of observing stations is widely spaced. In the case of compact meteorological disturbances, such as young hurricanes, the waves may well provide the first clue as to the existence of the storms. The application of the method is further enhanced by the possibility of determining not only the location of the storm but also its size, intensity, acceleration, and some of its other characteristics. In this connection it should be remembered that the wave pattern provides a complete picture, though a distorted one, of the entire wind distribution over the ocean, and our interpretation of this picture should improve with the advance of instruments, of theoretical studies, and of empirical experience. Not until many more situations have been analyzed will an evaluation of the usefulness of the method be possible. The results so far obtained demonstrate the feasibility of locating and tracking storms at distances exceeding 3000 miles and of making rough estimates regarding the size and character of these storms.

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