Wave-forced motion of submerged single-stem vegetation

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[1] We derive an analytical model for the wave-forced movement of single-stem vegetation and test the model against observed vegetation motion in a natural salt marsh. Solutions for constant diameter and tapered stems are expanded using normal mode solutions to the Euler-Bernoulli problem for a cantilevered beam. These solutions are compared with motion of water and of the sedge *Schoenoplectus americanus* observed (using synchronized current meters and video) in a shallow salt marsh (depth < 1 m). Consistent with theory, sedge motion led water motion, with the phase decreasing (from 90 to 0 degrees) with increasing wave frequency. After tuning of a single free parameter (Young's modulus), the theory successfully predicted the transfer function between measured water and stem motion. Formulae predicting frequency-dependent wave dissipation by flexible vegetation are derived. For the moderately flexible stems observed, the model predicted total dissipation was about 30% of the dissipation for equivalent rigid stems.

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1. Introduction

[2] Submerged vegetation can dissipate waves and tidal currents [*Dalrymple et al.*, 1984; *Fonseca and Cahalan*, 1992; *Shi et al.*, 1995; *Möller et al.*, 1999; *Nepf*, 1999; *Ghisalberti and Nepf*, 2002; *Mendez and Losada*, 2004; *Nepf*, 2004; *Lowe et al.*, 2007; *Augustin et al.*, 2009; *Bradley and Houser*, 2009], and can alter transport of contaminants, sediments and nutrients [*Ward et al.*, 1984; *Phillips*, 1989]. Furthermore, intertidal salt marshes provide crucial habitat for many species of fish, insects, birds and other aquatic life, some of which are threatened or endangered [*Zedler et al.*, 2001; *Greenberg et al.*, 2006].

[3] Simulation of vegetation-induced wave dissipation is complicated by the need to integrate over the length of each stem and sum over all stems in a canopy [*Dalrymple et al.*, 1984; *Mendez and Losada*, 2004]. In contrast, dissipation by bottom drag can be parameterized simply using the velocity at a single near-bed elevation [*Tolman*, 1994].

[4] Questions remain regarding the ability of different types of vegetation, with varying length and flexibility, to dissipate wave energy. Rigid vegetation can dissipate waves rapidly [e.g., *Dalrymple et al.*, 1984; *Kobayashi et al.*, 1993], whereas highly flexible vegetation, including certain giant kelp, can move with the flow, causing little dissipation [*Elwany et al.*, 1995]. However, even flexible kelp does not move with surrounding water near the seabed [*Stevens et al.*, 2001, 2002], possibly leading to near-bed dissipation.

[5] Several simple models for wave-forced vegetation motion have been proposed. *Asano et al.* [1993] and *Mendez*

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et al. [1999] simulate mobile vegetation using rigid stems hinged at the seabed, whose tilt is proportional to applied forces. A more complete model of stem motion was developed by *Gaylord and Denny* [1997] using the theory for deformable, linearly elastic cantilever beams (for discussion of applications to engineering structures see *Blevins* [1990]), with the simplifying assumption that the fluid drag (responsible for bending stems) was applied entirely at the stem tip.

[6] Here an analytical model for the movement of singlestem vegetation is developed based on cantilever theory. Stem deformation and along-stem variations in fluid drag and stem diameter are simulated. The model is based on a balance between drag forces, which tend to bend vegetation, and elastic forces, which tend to hold vegetation straight. The model equations and analytical solutions (section 2) highlight the importance of a dimensionless parameter we will call the stiffness S. This parameter incorporates material and geometric properties of the vegetation, and is also a function of wave parameters (including frequency and water speed). As $S \rightarrow 0$, stems move with surrounding flow except in a thin near-bed elastic boundary layer. As $S \rightarrow \infty$, stem motion approaches zero and leads the surrounding flow by 90° (section 2.4). Formulas for vertically integrated wave dissipation, derived in section 3, predict that dissipation increases with stiffness, ranging from zero dissipation when S = 0, to the rigid value [Dalrymple et al., 1984] when $S \rightarrow \infty$.

[7] To test the model, motion of an intermediate-stiffness sedge *Schoenoplectus americanus* was measured in a natural salt marsh using video. Associated water motion was measured using an acoustic current meter (section 4). Theoretical predictions of stem motion were in agreement with observations (section 5). The simulated dissipation by the flexible stems was about 30% of the dissipation that would result from equivalent rigid stems. The predicted reduction in dissipation was greatest at 1 Hz, with stems effectively more rigid at

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higher and lower frequencies. Results are summarized in section 6.

2. Model

2.1. Governing Equations

[8] The dimensionless time (*t*), along-stem distance measured from the free end of the stem (*z*), stem radius (*r*), stem cross-sectional area (*A*), second moment of stem area (*I*), stem density (ρ_s), flow speed ($|\mathbf{u}|$), and stem-normal displacements of water (*W*) and stem (*X*), are defined by

$$t = \frac{t_*}{t_{0*}},\tag{1}$$

$$z = \frac{z_*}{l_{0*}},\tag{2}$$

$$r = \frac{r_*}{r_{0*}},$$
 (3)

$$A = \frac{A_*}{r_{0*}^2},$$
 (4)

$$I = \frac{I_*}{r_{0*}^4},$$
 (5)

$$\rho_s = \frac{\rho_{s*}}{\rho_*},\tag{6}$$

$$|\mathbf{u}| = \frac{|\mathbf{u}_*| \ t_{0*}}{2W_{0*}},\tag{7}$$

$$W = \frac{W_*}{W_{0*}},\tag{8}$$

$$X = \frac{X_*}{W_{0*}},\tag{9}$$

where t_{0*} , W_{0*} , l_{0*} , r_{0*} and ρ_* are typical values of wave period, water displacement (i.e., particle excursion length), stem length, stem radius and water density (dimensional quantities are denoted with * throughout this paper), $|\mathbf{u}_*|$ is a characteristic velocity magnitude (see section 5), and I_* , a geometrical parameter playing a key role in determining stem stiffness [*Nayfeh*, 2000], is

$$I_* = \int x_*^2 dA_*, \tag{10}$$

where x_* is the displacement from the stem center. For simplicity, we will select W_{0*} so that $|\mathbf{u}| = 1$.

[9] We approximate the hydrodynamic drag on stem, per unit stem length, by the linearized formula

$$F_* = r_* \rho_* \frac{C_D}{2} \left| \mathbf{u}_* \right| \frac{\partial \left(W_* - X_* \right)}{\partial t_*}, \tag{11}$$

where the stem is assumed sufficiently thin that inertia is negligible (Appendix A). We assume negligible stem buoyancy (Appendix A), small deformations (strain $\ll 1$) and thin ($r_* \ll l_{0*}$) near-vertical (tilt angle $\ll 1$) stems. In this case, stem motion is governed by an Euler-Bernoulli equation [e.g., *Karnovsky and Lebed*, 2004] expressing a balance between elastic restoring forces and drag forces,

$$S\frac{\partial^2}{\partial z^2}\left(I\frac{\partial^2 X}{\partial z^2}\right) = r\frac{\partial(W-X)}{\partial t},\tag{12}$$

where *S* is the "dimensionless stiffness,"

$$S = \frac{E_* r_{0*}^3 t_{0*}^2}{\rho_* C_D l_{0*}^4 W_{0*}},$$
(13)

 E_* is Young's modulus, and C_D is the drag coefficient. The stem is fixed at the seabed, so

$$X = \frac{\partial X}{\partial z} = 0 \text{ at } z = 1.$$
 (14)

At the stem's free end, boundary conditions are

$$\frac{\partial^2 X}{\partial z^2} = \frac{\partial^3 X}{\partial z^3} = 0 \text{ at } z = 0.$$
(15)

Ghisalberti and Nepf [2002] give scaling analysis and governing dimensionless parameters for a case when buoyancy is not negligible. Similar Euler-Bernoulli equations have been used to analyze forces on artificial structures such as bridges or offshore platforms [e.g., *Timoshenko*, 1953; *Blevins*, 1990].

2.2. Normal Mode Expansion of Solutions

[10] For simplicity, assume stems have circular cross section (for which $I = \pi r^{4}/4$), and consider a time series discretely sampled at times $t_m = m\Delta t$, with *m* between -N and *N*. Water and stem motions (*W* and *X*, respectively) are expanded as Fourier modes in time *t* and normal modes in the vertical coordinate *z*,

$$W = \sum_{m=-N}^{N} \sum_{n=0}^{\infty} \langle W \rangle_{m,n} e^{i\omega_m t} \phi_n(z), \qquad (16)$$

$$X = \sum_{m=-N}^{N} \sum_{n=0}^{\infty} \langle X \rangle_{m,n} e^{i\omega_m t} \phi_n(z), \qquad (17)$$

where, for any variable β , $\langle \beta \rangle$ denotes the complex amplitude of β , the radian frequencies $\omega_m = m\Delta\omega$ (where $\Delta\omega =$



Figure 1. The first four normal modes for motion of stem with (a) constant diameter (section B1) and (b) tapered diameter (section B2): modes 0 (solid thick), 1 (solid thin), 2 (thick dashed), and 3 (thin dashed).

 $(2\pi)/[(2N + 1)\Delta t])$, and the normal modes $\phi(z)$ satisfy the orthogonality condition

$$\int r(z)\phi_n(z)\phi_m(z)dz = \begin{cases} 1 & n=m\\ 0 & \text{otherwise,} \end{cases}$$
(18)

together with the eigenvalue problem (14), (15) and

$$S\frac{\partial^2}{\partial z^2}\left(I\frac{\partial^2\phi_n}{\partial z^2}\right) = r\lambda_n\phi_n.$$
 (19)

equations (12)–(15) have the solution

$$\langle X \rangle_{m,n} = \frac{\langle W \rangle_{m,n}}{1 - i\lambda_n/\omega_m} = \frac{\langle W \rangle_{m,n}}{1 - i\pi S \alpha_n^4/(4\omega_m)},$$
 (20)

where the

$$\alpha_n = \left(4\lambda_n/(\pi S)\right)^{1/4} \tag{21}$$

are independent of ω_m (section 2.3). The α_n values, which are almost linearly proportional to *n* (section 2.3), are raised to the fourth power in (20), so the amplitudes of high modes are small unless $S/\omega \ll 1$ (high frequency results in low effective stiffness).

2.3. Normal Modes for Sample Stem Geometries

[11] The exact form of the normal modes depends on the stem geometry. Figure 1 shows modes zero to three for constant diameter and tapered ($r = z^{1/4}$) stems. The associated α_n are listed in Table 1 for n = 0 to 3 (for n = 0 to 9, excellent approximations for straight and tapered stems are given by $\alpha_n = 3.13n + 1.67$ and $\alpha_n = 2.54n + 1.87$, respectively), and analytic expressions for these solutions are detailed in Appendix B.

2.4. Limiting Cases of Stiff and Flexible Stems

[12] Consider the limiting cases of stiff and flexible stems, that is (from 20)

Stiff limit :
$$S \to \infty$$
, $\langle X \rangle_{m,n} = \frac{\langle W \rangle_{m,n}}{\pi S \alpha_n^4 / (4\omega_m)} i$, (22)

Flexible limit :
$$S \to 0, \langle X \rangle_{m,n} = \langle W \rangle_{m,n}.$$
 (23)

In the stiff limit (22), the vegetation strongly resists deformation and moves much less than the water (so $|\langle X \rangle_{m,n} / \langle W \rangle_{m,n}| \ll 1$). Motions are dominated by the first few modes ($\langle X \rangle_{m,n}$ proportional to α_n^{-4}), with vegetation and water in quadrature (90° out of phase: $\langle X \rangle_{m,n} / \langle W \rangle_{m,n}$ is imaginary). Quadrature results from a balance between the forces of elasticity (proportional to the displacement *X*) and drag (proportional to the velocity $\partial (W - X)/\partial t$, which roughly equals $\partial W / \partial t$ because *X* is small in the rigid limit). Many researchers have calculated wave dissipation caused by such essentially rigid vegetation [*Dalrymple et al.*, 1984; *Kobayashi et al.*, 1993].

[13] In the flexible limit (23), the vegetation simply moves with the surrounding water ($\langle X \rangle_{m,n} = \langle W \rangle_{m,n}$, so vegetation and water motions have the same magnitude and phase), and frictional dissipation is weak. For realistic (nonzero) *S*, the factor α_n^4 in (20) limits the amplitude of very high modes,

Table 1. First Four α Values (Proportional to Fourth Root of Eigenvalues, Equation (21)) for Constant Diameter and Tapered Stems

Stem Geometry	α_0	α_1	α_2	α_3
Constant diameter $r = z^{1/4}$ tapered	1.875	4.694	7.855	11.00
	1.989	4.359	6.899	9.444



Figure 2. (a) Amplitude and (b) phase of stem motion in the elastic boundary layer (section 2.4).

so vegetation does not follow water motion at very small scales. For small but nonzero *S*, stem motion reduces smoothly to zero in a thin near-bed elastic boundary layer, where elastic forces are significant. For an introduction to elastic boundary layers in beam bending theory, see *Nayfeh* [2000]. From (12), the elastic boundary layer (where *W*-*X* is order *W*) extends a dimensionless distance order $S^{1/4}$ (a dimensional distance $S^{1/4}l_{0*}$) above the bed.

[14] A simple solution can be obtained if the elastic boundary layer is much thicker than the viscous bottom boundary layer (where friction with the seabed affects water motion), so water displacement is essentially a constant (W_b) throughout the elastic boundary layer. Expanding solutions in Fourier modes,

$$W_b = \sum_m W_m e^{i\omega_m t}, \qquad (24)$$

$$X = \sum_{m} X_m(\zeta) e^{i\omega_m t},$$
(25)

where the boundary layer coordinate

$$\zeta = \left(\frac{4\omega_m}{\pi S}\right)^{1/4} (1-z) \tag{26}$$

equals zero at the bed, and noting $r \approx 1$ and $I = \pi/4$, reduces the Euler-Bernoulli equation (12) to

$$\frac{\partial^4 X}{\partial \zeta^4} = \frac{\partial (W_b - X)}{\partial t},\tag{27}$$

which has the solution

$$X_m = W_m \bigg\{ 1 - \frac{1}{r_1 - r_2} [r_1 \exp(r_2 \zeta) - r_2 \exp(r_1 \zeta)] \bigg\}, \qquad (28)$$

where $r_1 = e^{\frac{-t^*5\pi}{8}}$ and $r_2 = e^{\frac{-t^*7\pi}{8}}$. The solution (28) is shown in Figure 2 for $W_m = 1$. In the elastic bottom boundary layer for stem motion, as in the viscous bottom boundary layer for water motion [*Lamb*, 1993], near bed motion leads freestream motion by 45°.

2.5. Predicted Water-Stem Transfer Function

[15] To facilitate comparison with observations, we calculate the theoretical transfer function between water and stem motion, $\Gamma^{\rho}_{\omega}(z)$, defined by

$$\Gamma^{p}_{\omega}(z) = \frac{\langle X(z) \rangle_{\omega}}{\langle W(0) \rangle_{\omega}},\tag{29}$$

where $\langle X(z) \rangle_{\omega}$ is the frequency- ω complex amplitude of stem displacement X at elevation z, and $\langle W(0) \rangle_{\omega}$ is the complex amplitude of horizontal water displacement at the water surface. When the transfer function is expressed as a sum of normal modes

$$\Gamma^p_{\omega}(z) = \sum_{n=0}^{\infty} A^p_{\omega,n} \phi_n(z), \qquad (30)$$

and linear wave theory depth dependence is assumed for the water velocity, the amplitude of the *n*th normal mode is (Appendix C)

$$A^{p}_{\omega,n} = \frac{\int_{0}^{1} r\phi_{n}(z') \cosh[k(h-z'-z_{0})]dz'}{(1-i\lambda_{n}/\omega)\cosh(kh)},$$
(31)

where k is the wave number at frequency ω , h is the water depth, and z_0 is the depth of the top of the stem below



Figure 3. Friction factors (36) calculated using *S* estimated for the longer of the two stems measured in the field experiment for modes zero (solid thick black line), one (solid thin black line), and two (thick dashed line). The thin vertical dashed line indicates the peak forcing frequency. The thick gray line is the ratio of the dissipation (summed over the first 10 modes) to the dissipation for an equivalent rigid stem (38).

the water surface (all dimensionless: $k = l_{0*}k_*$, $h = h_*/l_{0*}$, $z_0 = z_{0*}/l_{0*}$).

3. Estimating Dissipation of Waves by Vegetation

[16] The mean depth-integrated rate of wave dissipation resulting from drag on a single stem is

$$\epsilon_* = \overline{\int_0^{l_{0*}} F_* \frac{\partial W_*}{\partial t_*} dz_*},\tag{32}$$

where the force on the stem is given by (11) and the overbar () denotes a time average. The dimensional water velocity u_* is expanded as Fourier modes in time and normal modes in z,

$$u_{*} = \sum_{m=-N}^{N} \sum_{n=0}^{\infty} \langle u_{*} \rangle_{m,n} e^{i\omega_{m}t} \phi_{n}(z).$$
(33)

From (11), (16), (17), (20) and (32)

$$\epsilon_{*} = r_{0*} l_{0*} \rho_{*} \frac{C_D}{2} |\mathbf{u}_{*}| \sum_{m=-N}^{N} \sum_{n=0}^{\infty} f_{m,n} |\langle u_{*} \rangle_{m,n}|^2, \qquad (34)$$

where we have used the result

$$\langle u_* \rangle_{m,n} = \frac{i\omega_m}{t_{0*}} \langle W_* \rangle_{m,n},\tag{35}$$

and the dimensionless friction factor for the *m*th frequency and *n*th mode

$$f_{m,n} = \frac{\left(\pi S \alpha_n^4 / 4\omega_m\right)^2}{1 + \left(\pi S \alpha_n^4 / 4\omega_m\right)^2}.$$
(36)

Since the vegetation may extend over much of the depth, dissipation depends on depth dependence of the flow, as represented by the mode-dependent friction factor $f_{m,n}$. In the rigid limit $S \to \infty$, $f_{m,n} \to 1$ and (34) reduces to

$$\epsilon_{*} = r_{0*} l_{0*} \rho_{*} \frac{C_D}{2} |\mathbf{u}_{*}| \sum_{m=-N}^{N} \sum_{n=0}^{\infty} |\langle u_{*} \rangle_{m,n}|^2, \qquad (37)$$

which, except for the linearization of the drag expressed by (11), is equivalent to the rigid vegetation model of *Dalrymple et al.* [1984]. The friction factors for modes 0 to 2 are shown in Figure 3 for an intermediate stiffness stem (the longer of the two sedges measured in the field, $S \approx 0.27$, section 5). At frequencies between 0.5 Hz and 1.5 Hz, this intermediate stiffness vegetation is essentially flexible with respect to the first vertical mode, but nearly rigid for all higher modes. ϵ_* for the intermediate flexibility stem considered in Figure 1 is 30% of ϵ_* for an equivalent rigid stem (the rigid stem ϵ_* is calculated by setting $f_{m,n} = 1$). In the flexible limit $S \to 0$, $f_{m,n} \to 0$, vegetation moves with the flow, and dissipation tends to zero.

[17] The dimensionless ratio

$$f_m = \frac{\sum_{n=0}^{\infty} f_{m,n} |\langle u_* \rangle_{m,n}|^2}{\sum_{n=0}^{\infty} |\langle u_* \rangle_{m,n}|^2}$$
(38)

quantifies the relative reduction in depth-integrated dissipation at frequency ω_m resulting from vegetation motion $(f_m = 1 \text{ for rigid vegetation})$. For the longer stem analyzed in section 5, f_m declines with frequency to a minimum near 1 Hz, before again increasing (thick gray line, Figure 3). The decline at frequencies < 1 Hz reflects the decline in stiffness with increasing frequency discussed in section 2.2 ($f_{m,n} \rightarrow 0$ as $m \to \infty$ for fixed *n*, equation (36)). The increase in f_m at frequencies > 1 Hz reflects the rapid depth attenuation of high-frequency waves; strongly depth-attenuated waves excite high modes of vegetation motion, and these high modes are effectively more rigid than lower modes (i.e., $f_{m,n} \rightarrow 1$ for large *n*, equation (36) and see also Figure 3). Physically, the increase in f_m at high frequencies reflects the fact that short sections of stem are more difficult to bend than long sections [effective stiffness \propto (section length)⁻⁴, (12)], and that depth-attenuated high-frequency waves bend only the short upper sections of stems. The frequency dependence of f_m indicates that vegetation can act as a band-pass filter, rapidly dissipating high and low frequencies, while more slowly dissipating intermediate frequencies.

[18] For very small S, dissipation is concentrated in the elastic boundary layer. In this case the complex dependence of dissipation on vertical flow structure expressed by the normal-mode expansion (33) can be replaced with a simpler dependence on only the near-bed water velocity. Substituting the boundary layer solution into (32) yields the approximation,

$$\epsilon_* = r_{0*} l_{0*} \rho_* \frac{C_D}{2} |\mathbf{u}_*| \sum_{m=-N}^N f_m^{\text{bl}} |\langle u_{b*} \rangle_m|^2, \qquad (39)$$



Figure 4. Coastline of Puget Sound showing the location of Skagit Bay.

where $\langle u_{b*} \rangle_m = i\omega_m \langle W_{b*} \rangle_m$ is the amplitude of frequency ω_m near-bed velocity, and the effective friction factor for the elastic boundary layer is

$$f_m^{\rm bl} = \Re \left\{ \frac{1}{(r_1 - r_2)} \left(\frac{\pi S}{4\omega_m} \right)^{\frac{1}{4}} \left(\frac{r_2}{r_1} - \frac{r_1}{r_2} \right) \right\} \approx 1.23 \left(\frac{S}{\omega_m} \right)^{\frac{1}{4}}.$$
 (40)

To calculate dissipation by a canopy, the above formulas for single-stem dissipation, (34) or (39), are simply summed over all stems. Given \mathcal{N}_* identical stems per m² of bed, the depth-integrated wave dissipation per m² is $\mathcal{N}_*\epsilon_*$.

4. Field Measurements

4.1. Site Description and Instrument Locations

[19] Skagit Bay is a mesotidal bay within northern Puget Sound, Washington (Figure 4). We focus on a salt marsh bounded by a small (~1.5 m deep) channel. The channel runs approximately north-south, with tidal flats to the west and salt marsh to the east (Figure 5). The marsh is populated by the sedge *Schoenoplectus americanus*. During August 2009, stem heights near the instrument site ranged up to 1.5 m (mean 0.8 m). The mean diameter at the stem base was 5 mm and the mean density was 650 stems m⁻² [*Dallavis et al.*, 2010]. Measurements were made on 30 and 31 August 2009, when spring tides were sufficiently high (depths ~0.9 m) to submerge most salt marsh vegetation.

[20] Instruments were deployed in a line (Figure 6) 11 m from the channel edge (Figure 5). An eastward-flowing sea breeze (mean 2.7 m s⁻¹ on 30 August and 5.4 m s⁻¹ on 31 August) generated low-energy wind waves (period ~2 s) which propagated into the salt marsh. A Sony DCR-HC32 handycam video camera in a waterproof housing was mounted on a pulse-coherent 2 MHz Nortek Aquadopp ADCP and focused on two sedge stems (stem lengths 0.81 and 0.45 m) displaced about 0.1 m horizontally from the ADCP head. The camera was mounted a further ~100 mm

away from the head of the ADCP to minimize flow interference. A Nortek Vector Velocimeter was placed on the other side of (approximately 0.175 m from) the stems, with measurement volumes 0.32 m and 0.25 m above the bed on 30 August and 31 August. Nearby stems were removed to improve resolution of the imaged stems.

[21] The two imaged stems were marked with thin strips of red tape at 50 mm intervals. The ADCP and camera were moved vertically to record video of the movement of the stem at up to five different elevations (Figure 6). At each elevation, the video recorded stem motion for about 4 min with vertical field of view ~150 mm. On the 31 August 2009, after the full length of the stems was recorded, the stems were cut to successively shorter lengths, with vertically offset 4 min video segments captured at each length. In this manner, movement was measured for five different lengths of each stem (the ratio of stem length to water depths varied from 0.91 to 0.41 during this process).

4.2. Data Acquisition and Processing

[22] The Nortek Vector velocimeter recorded velocity continuously at 16 Hz. Postprocessing removed and interpolated over times with low (<70%) correlations (<1% of data). Velocities were rotated into components perpendicular (*u*) and parallel (*v*) to the axis of instrumentation and video. Velocities recorded by the pulse-coherent ADCP are not used here (but were consistent with velocimeter velocities). Pressure data recorded by the ADCP was used to determine water depth (the velocimeter pressure sensor was faulty).

[23] To synchronize the instruments, water velocities were calculated from the video using manual particle tracking velocimetry (following small floating debris) and matched to velocimeter velocities (ADCP velocities were also aligned with velocimeter velocities). The video velocimeter time offset error, estimated from inconsistencies between time offsets obtained using several PTV particles (likely owing to wave propagation not being exactly perpendicular to the instrumentation axis, Figure 6b), was 0.05 s.

[24] Time series of sedge displacement were calculated (at the video frame rate of 29.97 Hz) using an algorithm to



Figure 5. Aerial view of deployment site (source is Google Earth). The instrument location is shown by the white circle. The arrows show the mean wave direction on 30 August (dashed) and 31 August (solid). Google Earth imagery ©Google Inc. Used with permission. North is to the top of the image.



Figure 6. Schematic of instrument and sedge layout: (a) side view and (b) top view. Approximately to scale.

identify the position of the lower left corner of strips of red tape on the sedge (Figure 7). For each frame the algorithm first normalized the red component of each pixel by the sum of the RGB components, and then identified the lower left corner of the red tape using a simple critical gradient condition. Missing or bad data points (such as caused by debris or fish in the image) were replaced by interpolation (approximately 4% of all data points). Pixel discretization error was reduced using a smoothing spline. Tests showed that lens distortion could cause up to about 5% errors in stem displacement.

[25] Spectra were calculated from displacements and velocities using Hanning-windowed data segments with 70% overlap (37 degrees of freedom). When calculating near-surface horizontal water displacements and velocities using near-bed measurements and linear wave theory, instrument noise was amplified at high frequencies, so only frequencies less than 1.5 Hz were analyzed.

5. Model-Data Comparison

[26] Observed sedge motion led water motion, particularly near the bed (Figure 8). The theoretical transfer function $\Gamma^p_{\omega}(z)$ (30) can be compared against the observed transfer function,

$$\Gamma^o_{\omega}(z) = \frac{\Phi_{\omega}[X(z), W(0)]}{\Phi_{\omega}[W(0), W(0)]},\tag{41}$$

where $\Phi_{\omega}[a, b]$ is the cross spectrum between *a* and *b*. This empirical transfer function is a frequency domain regression coefficient for the best linear fit between the observed sedge and water motion.

[27] Caliper measurements of the imaged stem show a reasonable fit for $r \propto z^{1/4}$ (Figure 9). Drag coefficient should vary with stem radius and therefore Reynolds number [*Batchelor*, 1967]. For oscillating flow, large Keulegan-Carpenter number and very high Reynolds numbers (Re \geq

10000), C_D is around 1-2 [Sarpkaya, 1976], however, no equivalent measurements have been made for our moderate Reynolds numbers (80 < Re < 240). Therefore, we estimate C_D ranging from 1 to 3 based on the values for steady flows [Batchelor, 1967]. A 300%–400% along-stem variation in equivalent Young's modulus has been observed in terrestrial sedge [Ennos, 1993], and Young's modulus also varies across the sedge cross section. To estimate a single representative value of E_* , the squared error between observed and predicted transfer functions was minimized for the longest sedge. This minimization yielded values of $E_* = 1.1 - 3.4 \times 10^8$ Pa (for $C_D = 1 - 3$) for the tapered stem model and $E_* = 1.3 - 3$ 3.9×10^8 Pa ($C_D = 1 - 3$) for the constant diameter stem model (the same value of E_* was used for both observed stems). These values are a factor of 3-8 smaller than values inferred from measurements of a terrestrial sedge by Ennos [1993],



Figure 7. Still frame from the video. The black circles show where the algorithm identifies the lower left corner of red tape markings, which are positioned at 50 mm increments.



Figure 8. Time series of sedge (thick lines) and water (thin lines) across-video velocities at (a) 0.75 m above bed and (b) 0.2 m above bed (stem length, 0.81 m). Water velocities measured by the velocimeter were transformed to the height of the sedge measurements using linear wave theory (with a cutoff frequency of 1.2 Hz).

but are similar to values measured for a seagrass [Folkard, 2005], and an order of magnitude larger than values measured for some seaweeds [Gaylord and Denny, 1997; Harder et al., 2006]. Other parameters used to calculate S were measured stem length ($l_{0*} = 0.81$ m, 0.45 m for two stems), basal radius ($r_{0*} = 2.7$ mm, 1.6 mm for two stems), and $t_{0*} =$ 2, 2.13 s (30, 31 August). For simplicity, W_{0*} was chosen such that $|\mathbf{u}| = 1$. $|\mathbf{u}_*|$ was defined as $\sqrt{8/\pi} \times \text{depth-averaged}$ RMS speed of water relative to sedge (where the factor $\sqrt{8/\pi}$ is chosen to ensure that the dissipation calculated from (34) and the linearized drag (11) is consistent with fully nonlinear expressions in the case of a Gaussian velocity distribution [Dalrymple et al., 1984]). These values yielded $S \approx 0.27$ for the longer stem and $S \approx 0.71$ for the shorter stem (tapered stem model), so both full-length stems are transitional between fully stiff $(S \rightarrow \infty)$ and flexible $(S \rightarrow 0)$ limits.

[28] The observed transfer functions for the two stems (Figures 10a, 10d, 11a, and 11d) agreed well with both the tapered stem model (Figures 10b, 10e, 11b, and 11e, section B2) and the constant diameter model (Figures 10c, 10f, 11c, and 11f, section B1). The tapered stem model resulted in slightly higher amplitudes very near the top of the stem.

[29] The observed complex amplitude of mode *n*, normalized by surface motion $\langle W(0) \rangle_{\omega}$,

$$A^{o}_{n,\omega} = \int r \Gamma^{o}_{\omega}(z) \phi_n(z) \mathrm{d}z, \qquad (42)$$

is estimated from Γ_{ω}^{o} at a finite set of elevations z_i by trapezoidal integration. Both tapered and constant diameter models predict the dominant mode 0 amplitudes (Figure 12) with the tapered stem model having slightly higher skill (an RMS error of 0.002, compared to 0.005 for the constant diameter model). Amplitudes are presented only for mode 0, because tests showed that the limited number of observed depths were insufficient to resolve the integral (42) for higher modes. Predicted mode zero amplitudes always exceeded highermode amplitudes by a factor of at least 7 (for $f \le 1.5$ Hz).



Figure 9. Stem radius r_* as a function of stem position (z_* measured from the free end of the stem) for the two stems used in the field experiment: 0.81 m stem (thin circles) and 0.45 m stem (thick circles). Dashed lines show $r_* \propto z_*^{1/4}$ fits to the data.



Figure 10. Amplitude of transfer functions between stem motion and surface water motion for two sedge stems ((a–c) stem 1, $l_{0*} = 0.81$ m; (d–f) stem 2, $l_{0*} = 0.45$ m) computed from observations (Figures 10a and 10d) using (41), together with corresponding theoretical transfer functions (30) for a tapered stem (Figures 10b and 10e) and a constant diameter stem (Figures 10c and 10f). Observed amplitudes are shown only when the squared coherency was >0.3 (this region is marked by the white outline in Figures 10b, 10c, 10e, and 10f). Black contour lines indicate amplitudes of 0.4, 0.8, and 1.2.

[30] Stems were cut to successively shorter lengths to investigate the effect of stem length on stem motion. The peak-frequency transfer function, evaluated using the measurement nearest the end of the stem, is shown as a function of stem length (Figure 13). The corresponding solutions to (12) using the constant diameter stem model (which allowed correct boundary conditions at the cut stem end) match the observed qualitative behavior (increasing amplitude with stem length), but simulations under predict the magnitudes significantly (consistent with the underprediction of amplitudes at the end of stems by the constant diameter model, Figure 10). Potential sources of error include the neglect of buoyancy (Appendix A), and the assumed elevationindependence of drag coefficient and Young's modulus.

[31] The phases in Figure 13 show good agreement for longer stems, but the theoretical values of phases for the shorter and stiffer stems do not exceed 90°, whereas the observations reveal phases of around 120° for the shorter



Figure 11. Phases between stem motion and surface water motion for two sedge stems ((a–c) stem 1, $l_{0*} = 0.81$ m; (d–f) stem 2, $l_{0*} = 0.45$ m) computed from observations (Figures 11a and 11d) using (41), together with corresponding theoretical phases (30) for a tapered stem (Figures 11b and 11e) and a constant diameter stem (Figures 11c and 11f). Observed phases are shown only when the squared coherency was >0.3 (this region is marked by the white outline in Figures 11b, 11c, 11e, and 11f). Black contour lines indicate phases of 0, 30, and 60°.

stems. This unexpected observation may indicate a timing error, or neglected nonlinear effects.

6. Summary

[32] Theoretical analysis highlights the importance of a dimensionless stiffness parameter in controlling vegetation motion under waves. Low stiffness stems move with the surrounding water (except in a thin near-bed elastic boundary layer). In contrast, the motion of high-stiffness stems is

minimal, and leads the surrounding water by 90° . Stiffness depends on properties of the stem, and on properties of the wave motion; low stiffness values are associated with long thin stems, with low Young's modulus, and with high-energy, high-frequency waves. These theoretical predictions were confirmed by measuring wave-forced sedge motion in a natural salt marsh. Most measured relationships between water and sedge motion were in good agreement with the theory.



Figure 12. Observed (circles) and theoretical (thick lines) ((a and b) tapered stem model; (c and d) constant diameter stem model) magnitudes (Figures 12a and 12c) and phases (Figures 12b and 12d) of mode 0 stem motion.

[33] Formulas for wave dissipation by mobile vegetation predicted that dissipation increases with increasing stiffness. In the salt marsh studied here dissipation predicted for the observed flexible stems was about 30% of the dissipation that would be predicted for rigid stems. The predicted reduction in dissipation, relative to rigid stems, was frequency dependent, with a maximum reduction at around 1 Hz. Consequently, vegetation can act as a band-pass filter, preferentially damping both high- and low-frequency waves, while most easily passing intermediate frequencies.

[34] The theory presented here might be applicable to species other than the sedge *Schoenoplectus americanus*. However, applicability will in some cases be limited by the neglect of stem inertia and buoyancy, the assumption of nearly vertical stem orientation, and the assumed simple, single-stem geometry.

Appendix A: Derivation of Governing Equations and Scaling

[35] We assume small tilt of thin submerged stems ($r_* \ll l_{0*}$, where r_* and l_{0*} are stem radius and length). For dis-

cussion of the case of large tilt (e.g., for more flexible species such as algae) [see *Denny and Gaylord*, 2002; *Alben et al.*, 2004; *Gosselin and de Langre*, 2009]. The motion of the stems is governed by the Euler-Bernoulli equation [e.g., *Karnovsky and Lebed* 2004] (dimensional quantities are denoted by *),

$$\rho_{s*}A_*(1+M)\frac{\partial^2 X_*}{\partial t_*^2} + \frac{\partial^2}{\partial z_*^2} \left(E_*I_*\frac{\partial^2 X_*}{\partial z_*^2}\right) + F_{B^*} = F_*, \quad (A1)$$

where ρ_{s*} is stem density (kg m⁻³), *M* is an added mass coefficient, and the stem-normal component of the buoyancy force per unit stem length is

$$F_{B^{*}} \approx -\left(\rho_{*} - \rho_{s*}\right)g_{*}A_{*}\frac{\partial X_{*}}{\partial z_{*}},\tag{A2}$$

where g_* is gravitational acceleration. In terms of the dimensionless variables (1)–(9) and

$$g = \frac{g_* t_{0*}^2}{l_{0*}},\tag{A3}$$



Figure 13. Observed (open circles) and predicted (solid circles, constant diameter stem model) (a and b) amplitude and (c and d) phase of peak-frequency transfer function between water motion at the surface and stem motion near the stem's free end. Two stems (Figures 13a and 13c, stem 1; Figures 13b and 13d, stem 2) were cut to successively shorter lengths. Error bars give upper and lower bounds for phases using an estimated time synchronization error of ± 0.05 s (see section 4.2).

equation (A1) is

$$\frac{\rho_s A}{C_D K_C} \left[(1+M) \frac{\partial^2 X}{\partial t^2} - \frac{g(1-\rho_s)}{\rho_s} \frac{\partial X}{\partial z} \right] + S \frac{\partial^2}{\partial z^2} \left(I \frac{\partial^2 X}{\partial z^2} \right)$$
$$= r \mid \mathbf{u} \mid \frac{\partial (W-X)}{\partial t}, \tag{A4}$$

where the importance of inertia is determined by the Keulegan-Carpenter number

$$K_C = \frac{W_{0*}}{r_{0*}}.$$
 (A5)

For many cases of practical interest (including sedges and Spartina grasses in sheltered estuaries, and kelp forests exposed to energetic ocean swell, but not including sheltered mangrove forests), water particle displacements are much greater than the stem diameter, so $K_C \gg 1$ (C_D and M are of order 1). We assume the density of the sedge to be close

to that of water (see *Folkard* [2005], in which the density of a seagrass is given as 910 \pm 110 kg m⁻³), so ρ_s and $g(1 - \rho_s)/\rho_s$ are also of order 1 and so (A4) reduces to (12).

Appendix B: Normal Mode Solutions for Sample Stem Geometries

B1. Constant Diameter Stem

[36] For a stem with constant diameter r = 1, the α_n satisfy

$$1 + \cos(\alpha_n)\cosh(\alpha_n) = 0, \tag{B1}$$

and associated eigenfunctions are

$$\psi_n(z) = \sin(\alpha_n z) + \sinh(\alpha_n z) - \gamma [\cos(\alpha_n z) + \cosh(\alpha_n z)],$$
 (B2)

where

$$\gamma = \frac{\sin(\alpha_n) + \sinh(\alpha_n)}{\cos(\alpha_n) + \cosh(\alpha_n)}.$$
 (B3)

Analogous solutions apply to vibration modes of a beam [*Volterra and Zachmanoglou*, 1965]. The unnormalized eigenfunctions $\psi_n(z)$ are related to $\phi_n(z)$ by

$$\phi_n(z) = \frac{\psi_n(z)}{\left[\int_0^1 r\psi_n^2(z)dz\right]^{1/2}}.$$
 (B4)

B2. Tapered Stem

[37] For a tapered stem with $r = z^{1/4}$, the α_n satisfy

$$-{}_{0}F_{3}\left(-;\frac{5}{13},\frac{9}{13},\frac{9}{13};\frac{256\alpha_{n}^{4}}{28561}\right) + \frac{64}{405}\alpha_{n}^{4}{}_{0}F_{3}$$

$$\cdot \left(-;\frac{18}{13},\frac{22}{13};\frac{22}{13};\frac{256\alpha_{n}^{4}}{28561}\right) - \frac{64}{1989}\alpha_{n}^{68/13}\gamma_{0}F_{3}$$

$$\cdot \left(-;\frac{22}{13},2,\frac{30}{13};\frac{256\alpha_{n}^{4}}{28561}\right) = 0$$
(B5)

and associated eigenfunctions are

$$\psi_n(z) = {}_0F_3\left(-;\frac{5}{13},\frac{9}{13},\frac{9}{13};\frac{256\alpha_n^4 z^{13/4}}{28561}\right) - \left(\alpha_n^{16/13} z\right)\gamma_0F_3$$
$$\cdot \left(-;\frac{9}{13},1,\frac{17}{13};\frac{256\alpha_n^4 z^{13/4}}{28561}\right),\tag{B6}$$

where $_{j}F_{k}$ is the generalized hypergeometric function [*Ambramowitz and Stegun*, 1972], and

$$\gamma = \frac{{}_{0}F_{3}\left(-;\frac{5}{13},\frac{9}{13},\frac{9}{13};\frac{256\alpha_{n}^{4}}{28561}\right)}{\alpha_{n}^{16/13} {}_{0}F_{3}\left(-;\frac{9}{13},1,\frac{17}{13};\frac{256\alpha_{n}^{4}}{28561}\right)}.$$
 (B7)

Appendix C: Derivation of Theoretical Transfer Function

[38] By definition,

$$\Phi_{\omega}[X(z), W(0)] = \frac{E[\langle X(z) \rangle_{\omega} \langle W(0) \rangle_{-\omega}]}{d\omega}.$$
 (C1)

From (17),

$$\langle X(z) \rangle_{\omega} = \sum_{n=0}^{\infty} \langle X \rangle_{\omega,n} \phi_n(z).$$
 (C2)

From (C1) and (C2),

$$\Phi_{\omega}[X(z), W(0)] = \sum_{n=0}^{\infty} \frac{E[\langle X \rangle_{\omega, n} \langle W(0) \rangle_{-\omega}]}{d\omega} \phi_n(z).$$
(C3)

From (20) and (C3),

$$\Phi_{\omega}[X(z), W(0)] = \sum_{n=0}^{\infty} \left(\frac{1}{1 - i\lambda/\omega}\right) \frac{E\left[\langle W \rangle_{\omega,n} \langle W(0) \rangle_{-\omega}\right]}{d\omega} \phi_n(z).$$
(C4)

Substituting

$$\langle W \rangle_{\omega,n} = \int_{z'} r \langle W(z') \rangle_{\omega} \phi_n(z') dz'$$
 (C5)

into (C4) and using linear wave theory yields

$$\Phi_{\omega}[X(z), W(0)] = \sum_{n=0}^{\infty} \left(\frac{1}{1 - i\lambda/\omega}\right) \int_{z'} r \frac{E[\langle W(0) \rangle_{\omega} \langle W(0) \rangle_{-\omega}]}{d\omega} \cdot \left(\frac{\cosh[k(h - z' - z_0)]}{\cosh(kh)}\right) \phi_n(z') dz' \phi_n(z).$$
(C6)

Recalling the definition,

$$\Phi_{\omega}[W(0), W(0)] = \frac{E[\langle W(0) \rangle_{\omega} \langle W(0) \rangle_{-\omega}]}{d\omega}, \qquad (C7)$$

yields

$$\frac{\Phi_{\omega}[X(z), W(0)]}{\Phi_{\omega}[W(0), W(0)]} = \sum_{n=0}^{\infty} \left(\frac{1}{1 - i\lambda/\omega}\right) \int_{z'} \cdot r\left(\frac{\cosh[k(h - z' - z_0)]}{\cosh(kh)}\right) \phi_n(z') dz' \phi_n(z),$$
(C8)

which is the combination of (30) and (31).

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