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Estimates of the Power Spectrums for Fully Developed Seas for Wind Speeds of 20 to 40 Knots

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Abstract. Various criteria pertaining to the synoptic situation are presented in order to determine when a fully developed wind-generated sea might be found in the North Atlantic Ocean. Four hundred and sixty wave records, corresponding to various synoptic situations, were digitized and analyzed spectrally as a first step in the preparation of a climatology of ocean wave spectrums. The wave records were taken by the ocean weather ships of the United Kingdom by means of a Tucker shipborne wave recorder. Selected subsets from the available spectrums based on these synoptic criteria were averaged in order to produce spectrums for various wind speeds. These selected subsets were examined to see if they came from the same population by means of the Kolmogorov-Smirnov test, and the results show that, within the accuracy expected, the samples chosen represent fully developed seas. A second subset chosen at random without using these criteria was tested, and the results showed that wind speed alone does not properly characterize the sea state. A nested family of spectrums was obtained for wind speeds of 20, 25, 30, 35, and 40 knots in which the frequency of the maximum appeared to be inversely proportional to the wind speed and the significant height was proportional to the square of the wind speed. The spectrums and the results deducible from them yield results that appear to be a compromise among the various published theoretical forms for the spectrums and the equations for the significant height of a fully developed sea.

Introduction. In recent years, the study of wind-generated ocean waves and ocean wave forecasting has become important. Meteorologists and oceanographers are only now beginning to learn how energy is transferred to the waves. The use of wave spectrums, both theoretical and observed, has contributed greatly to the developments in the field of ocean waves. The wind effect on the sea surface is not simple, basically because of turbulence in both sea and air. It can be seen that waves of different heights and periods occur. The visible sea conditions are represented by a superposition of an infinite number of sinusoidal waves of different amplitudes and different periods with random phase relations and traveling in different directions. Since an infinite number of periods (frequencies) are present, a spectrum is implied.

At present there is disagreement as to the form of the wave spectrum of a fully developed sea for a particular wind speed. It is known that as the wind speed increases, if fetch and duration are adequate, the area under the spectral curve (total variance) will increase and the mode will shift toward lower frequencies (longer periods). With a constant wind speed and an adequate fetch and duration, a point will be reached where the spectrum will no longer grow. At this stage the sea is called fully developed. The spectrum will change only if additional energy is either added to or taken away from the waves.

In this paper an attempt to determine the wave spectrums for fully developed seas for various wind speeds is reported. Various synoptic conditions, for which wave records were known to exist, were chosen. The synoptic situations cover the 5-year period April 1955 through March 1960.

History. Many scientists have studied ocean waves using the concepts of a wave spectrum. Early results from these studies were not very good because the instruments were set in relatively shallow water. This tended to produce a distorted spectrum because the distribution of spectral energy in shallow water is different from that in the deep ocean. Also, inconclusive results were obtained. These studies paved the way for future developments along the lines of specifying mathematically the shape and form of the spectrum.

In 1952, *Tucker* [1956][1] developed a ship-

[1] The references cited in this paper are listed on page 5202 of this issue of the *Journal of Geophysical Research*.

borne wave recorder for use on board the ocean weather ships of Great Britain. This instrument, by a combination of pressure recorders and accelerometers, produces a continuous record of the height of the waves passing by the ship. From a wave record a spectrum may be calculated by either digital [*Blackman and Tukey,* 1958] or analog techniques.

The *Neumann* [1953] spectrum represented a great achievement in ocean wave forecasting. Neumann used many visual observations and the results of *Longuet-Higgins* [1952] to arrive at his results. The results were later incorporated in a wave forecasting manual [*Pierson et al.,* 1955] (henceforth denoted as PNJ). *Darbyshire* [1955, 1956, 1959] studied many wave records and drew conclusions that differed greatly from those of Neumann. First, Darbyshire concluded that the significant height, $H_{1/3}$ (the average of the one-third highest waves) was proportional to the square of the wind speed. Neumann concluded that the significant height was proportional to the 2.5 power of the wind speed. Darbyshire also found that the sea would grow faster and reach full development sooner than was predicted by PNJ.

The large differences between the results of Darbyshire, PNJ, and other methods were the basis for comparisons of the different wave forecasting methods [*Rattray and Burt,* 1956; *Neumann and Pierson,* 1957; *Darbyshire,* 1957; *Walden,* 1963]. *Pierson* [1959], *DeLeonibus* [1962], and *Bretschneider et al.* [1962] have studied the wave spectrums of waves generated by particular storms in the North Atlantic Ocean. A recent review [*Natl. Acad. Sci.,* 1963] shows how widely the various attempts to describe the spectrums of fully developed seas differ from one another.

Data. A search of the Daily Series Synoptic Weather Maps (U. S. Weather Bureau) for the period April 1955 through March 1960 was undertaken to find periods of time when wind direction and speed were relatively constant at the location of the English weather ships. A further study of the data chosen from the Daily Series Synoptic Weather Maps was made with the aid of the Six-Hourly Synoptic Weather Maps of the North Atlantic Ocean (U. S. Weather Bureau, April 1955–March 1960) and Table 1. This study yielded rules for the selection of wave data that were recorded at times when particular winds, as determined from the synoptic charts, maintained the same direction for as long a duration as possible.

TABLE 1. Date of Sailing to Station and Date of Return to Port of the Ocean Weather Ships

Date of Sailing	Station*	Date of Return
OWS *Weather Explorer*		
April 3, 1955	I	May 2, 1955
May 21, 1955	J	June 16, 1955
June 29, 1955	A	Aug. 2, 1955
Aug. 25, 1955	K	Sept. 27, 1955
Oct. 12, 1955	I	Nov. 10, 1955
Nov. 26, 1955	J	Dec. 23, 1955
Jan. 7, 1956	A	Feb. 11, 1956
March 5, 1956	K	April 6, 1956
April 21, 1956	I	May 20, 1956
June 5, 1956	J	July 4, 1956
July 26, 1956	I	Aug. 23, 1956
Sept. 9, 1956	J	Oct. 8, 1956
Oct. 30, 1956	I	Nov. 28, 1956
Dec. 18, 1956	I	Jan. 15, 1957
Feb. 3, 1957	I	March 4, 1957
March 20, 1957	J	April 18, 1957
May 1, 1957	A	June 4, 1957
June 27, 1957	I	July 26, 1957
Aug. 14, 1957	I	Sept. 12, 1957
Sept. 28, 1957	I	Oct. 28, 1957
Nov. 9, 1957	A	Dec. 13, 1957
Jan. 5, 1958	I	Feb. 3, 1958
Feb. 22, 1958	I	March 24, 1958
April 8, 1958	J	May 9, 1958
OWS *Weather Reporter*		
May 20, 1958	A	June 23, 1958
July 17, 1958	I	Aug. 9, 1958
Aug. 28, 1958	J	Sept. 24, 1958
Oct. 14, 1958	I	Nov. 10, 1958
Nov. 24, 1958	J	Dec. 23, 1958
Jan. 4, 1959	A	Feb. 6, 1959
March 3, 1959	K	April 4, 1959
April 18, 1959	I	May 18, 1959
June 2, 1959	J	July 2, 1959
July 15, 1959	A	Aug. 18, 1959
Sept. 11, 1959	K	Oct. 12, 1959
Oct. 27, 1959	I	Nov. 26, 1959
Dec. 11, 1959	J	Jan. 9, 1960
Jan. 23, 1960	A	Feb. 26, 1960

*The geographical locations of the stations:
A 62°N, 33°W.
B 59°N, 19°W.
J 52.5°N, 20°W.
K 45°N, 16°W.

The particular wind speeds chosen for this study begin at 20 knots and increase in intervals of 5 knots. Wind speeds less than 20 knots were omitted because of the difficulty of finding conditions when speed and especially direction were constant for long periods of time. Except for the summer months, it is difficult to say that sea conditions are truly generated by accompanying light winds. Usually the sea conditions are due to a stronger wind system that has previously passed through the area under observation. Certain synoptic criteria were prescribed for the selection of those cases for which the sea state was attributable to the local winds.

The criteria for accepting the cases used in the analysis were:

1. Winds persisted for long periods with little variation of speed or direction (less than $\pm 45°$ from the mean direction except for a few cases involving high winds).
2. Wind speeds at beginning and end of each period less than the mean wind for the period. It was hoped that a steadily increasing wind speed would produce a developing sea and would eliminate remnants of previous storms.
3. Absence of swell. Swell could not be identified from the synoptic charts, but when it was detected in the spectrum of a wave record that record was rejected from the analysis.

After the wind data had been compiled, the wave records for the selected periods were obtained from the National Institute of Oceanography. All the wave records studied were taken by either the OWS *Weather Explorer* or the OWS *Weather Reporter,* and the logs of these ships were used to determine the wind speeds to the nearest knot.

The wave records were examined to find the speed of the ship, which is also entered on every wave record. If it was found that the ship was moving at a speed greater than 2 knots, the record was not used in the final analyses. At observation time (usually every 3 hours) the ship should have been in a stopped position, but sea conditions may have forced the ship to speed up. It was believed that at a speed less than 2 knots there would be little or no frequency distortion of the spectrums [*Pierson,* 1959]. (The spectrums obtained when a ship is moving at a speed greater than 2 knots compared with the spectrums obtained when it is relatively stationary under the same conditions show a shift in the frequency of the spectral maximum. This shift is toward higher frequencies if the ship is heading against the waves and toward lower frequencies if the ship is heading with the waves.)

Approximately 1000 wave records were obtained from the National Institute of Oceanography and 420 were selected; 40 more records were available from another study [*Bretschneider et al.,* 1962], bringing the total to 460. Every wave record was approximately 15 minutes long, although several were as short as 7 minutes. Bounds were set on each record just above the highest crest and below the lowest trough, and the records were read to an accuracy of 1 part in 1000 at 1½-second intervals. Each wave record was thus represented as a time series of approximately 600 points. The time series was then spectrally analyzed on a CDC 1604 computer so as to estimate the energy spectrum of the waves at 60 points over the frequency range 0 to 0.333 cps [*Blackman and Tukey,* 1958].

The spectral estimates still had to be corrected for noise from both the original wave records and the digitization procedures. Also, at the high-frequency end a smoothing operator was applied so that our results could be compared with those of other investigators [*Bretschneider et al*, 1962] who had used this type of analysis. The noise level was determined by averaging the last ten values of the smoothed spectrum, and this average was then subtracted from each of the spectral estimates of the smoothed spectrum. To determine the final spectrum, the smoothed spectrum, less noise, was then multiplied by the calibration curve of the recorder for the particular ship that took the record. These values represent the spectral estimates in terms of the resolution of the variance of the wave record into frequency intervals.

The results of these computations should yield fairly reliable spectral estimates for frequencies ranging from 0 to 0.25 cps (the Tucker recorder is not accurate at frequencies higher than 0.25 cps).

The synoptically chosen subsets. For the evaluation of a mean spectrum for each of the wind speed subsets, the records chosen had to conform to the synoptic criteria as closely as possible. By the time all the criteria were applied, few cases at exactly the chosen wind speeds (20, 25, 30, etc.) were available. It was then decided to allow the wind speed at observation time, as determined from the logs of the ships, to vary within ±2 knots of the mean wind speed. In a very few cases the wind speed before observation time was slightly higher [lower] than two knots above [below] the mean wind speed. All the synoptically chosen spectrums were examined to be sure that there was no contamination from the presence of swell. *Darbyshire* [1959] also used spectrums that were free of swell and accepted cases of higher wind speeds before observation time. Some of the records analyzed were the same as those used by Darbyshire. It is interesting to note that different significant wave heights were obtained using the same data. Darbyshire's results were lower because he used an earlier calibration of the Tucker recorder. This calibration has been improved as a result of comparisons with the buoy described by *Longuet-Higgins et al.* [1963].

The spectrums chosen according to the synoptic criteria for each subset are given in Table 2, as well as estimates of the fetches for the synoptically chosen spectrums. As pointed out by *Walden* [1963], the fetch required for lighter winds (up to about 30 knots) to produce fully developed seas occurs frequently in most areas. Wind speeds greater than 30 knots are rarely associated with fetches great enough to produce fully developed seas. The minimum fetch necessary to produce fully developed seas for various wind speeds as given by PNJ and *Darbyshire* [1959] are:

v, kt	PNJ	Darbyshire
20	75 miles	100–200 miles
25	155 miles	100–200 miles
30	280 miles	100–200 miles
35	460 miles	100–200 miles
40	710 miles	100–200 miles

The values tabulated in Table 2 for fetch and duration represent the distance upwind over which the wind was substantially constant at the time of observation, and the duration at the ship of that wind speed. Before the number of hours indicated, the wind would not be zero, but perhaps 5 or 10 knots lower (and even lower before that) than the tabulated value, and upwind of the distance indicated, the winds would be different from the value tabulated (usually less), but not zero. Therefore, the durations and fetches do not satisfy the theoretical requirement that they describe the time required and the distance needed to generate a fully developed sea starting from zero wave conditions. For the higher winds in particular, the tabulated values often represent the time and the distance for a sea raised to a given height by a wind of lesser velocity to grow to full development at the higher velocity.

No subsets were found for wind speeds greater than 40 knots. All but two of the wave records obtained when the winds were greater than 40 knots showed waves that were no higher than the waves for 40 knots. One spectrum for a wind speed of 45 knots, JHC 41, and one spectrum for a wind speed of 50 knots, JHC 141, appear to reach the fully developed state. From all the available data it was possible to find several wave records at the lower wind speeds that, according to PNJ, represented saturated conditions. It was not possible to find any records for higher winds that approached the fully developed state as given by PNJ.

An important matter should be taken into consideration at this time, anemometer heights. The PNJ results are based on an anemometer height of 7.5 meters, whereas the anemometers

TABLE 2. Synoptically Chosen Subsets

Record No.*	Date	Time (UT)	Position	Wind Speed, kt	$H_{1/3}$, ft	Upper $H_{1/3}$, ft	Lower $H_{1/3}$, ft	$T_A - T_s$,† °F	Duration, hr	Fetch, n.m.	n
					Twenty Knots						12
JHC 015	9/19/55	3	K	21	8.1	8.3	7.2	−0.5	6	250	
JHC 016	9/19/55	9	K	21	8.0	8.4	7.5	−0.2	12	250	
JHC 063	6/19/56	14	J	20	7.1	7.3	6.2	0.7	6	175	
JHC 064	6/19/56	18	J	19	7.9	8.5	7.3	0.7	9	200	
JHC 065	6/20/56	0	J	20	7.0	7.5	6.4	0.2	15	250	
JHC 066	6/20/56	3	J	18	6.7	7.2	6.2	−0.7	18	250	
JHC 067	6/20/56	6	J	19	7.1	7.3	6.3	−0.3	21	250	
JHC 068	6/20/56	9	J	20	7.2	7.8	6.7	0.1	24	200	

TABLE 2. (Continued)

Record No.*	Date	Time (UT)	Position	Wind Speed, kt	$H_{1/3}$, ft	Upper $H_{1/3}$, ft	Lower $H_{1/3}$, ft	$T_A - T_s$,† °F	Duration, hr	Fetch, n.m.	n
JHC 069	6/20/56	12	J	19	7.3	7.5	6.4	−0.5	27	100	
DL 45	1/16/59	15	A	22	7.5	8.2	6.9	−3.6	6	250	
DL 88	10/2/59	9	K	21	8.1	8.8	7.5	−1.2	6	125	
DL 97	7/25/59	0	A	20	6.8	7.1	6.1	−0.9	18	125	
				Twenty-Five Knots							8
JHC 029	6/7/55	18	J	24	13.3	13.9	11.8	0.3	6	150	
JHC 030	6/7/55	21	J	25	14.3	15.1	12.8	0.4	9	175	
JHC 032	6/8/55	12	J	26	13.5	14.2	12.2	−0.1	21	300	
JHC 033	6/8/55	15	J	25	13.1	13.6	11.5	−0.8	24	450	
JHC 116	5/23/57	3	A	27	13.8	14.8	12.5	0.3	9	100	
JHC 117	5/23/57	6	A	25	13.4	14.6	12.4	−1.6	12	125	
JHC 127	4/18/58	6	J	24	11.5	11.9	10.3	−0.3	12	700	
JHC 128	4/18/58	12	J	25	12.0	12.5	10.6	1.1	18	350	
				Thirty Knots							12
JH 36	1/23/59	15	A	32	19.2	20.8	17.7	−4.8	6	150	
JH 37	1/24/59	18	A	32	21.0	23.1	18.9	−3.0	6	200	
DL 8	1/20/58	21	I	30	20.1	21.6	18.7	−17.9	6	200	
DL 39	10/29/59	9	I	29	19.9	21.9	18.0	−7.0	6	125	
DL 40	10/29/59	18	I	30	20.3	22.1	18.6	−6.8	15	125	
JHC 002	11/23/56	15	I	32	19.7	21.0	18.4	−2.2	6	150	
JHC 084	3/20/56	18	K	29	20.6	22.0	18.7	−3.4	12	125	
JHC 086	3/21/56	0	K	29	21.2	21.9	18.0	−1.9	18	125	
JHC 098	11/4/55	0	I	28	19.0	19.6	16.9	−1.9	9	175	
JHC 105	11/5/55	6	I	32	22.5	23.1	19.9	0.4	6	125	
JHC 106	11/5/55	15	I	30	21.8	22.2	18.5	0.8	15	100	
JHC 132	4/19/58	9	J	30	20.5	20.9	17.7	−2.1	18	225	
				Thirty-Five Knots							8
JHC 050	11/6/56	18	I	34	22.8	23.8	20.2	0.2	12	450	
JHC 104	11/5/55	3	I	35	24.4	22.8	19.4	−0.1	6	125	
JHC 147	12/11/58	18	J	35	22.2	23.7	19.9	−5.7	6	100	
DL 7	1/20/58	15	I	35	23.0	24.8	21.4	−18.4	6	225	
DL 18	4/24/58	15	J	35	24.6	26.7	22.7	−3.9	18	400	
DL 19	4/24/58	18	J	35	24.0	26.6	22.6	−3.3	21	400	
DL 20	4/24/58	21	J	35	25.2	27.4	23.1	−4.0	24	425	
DL 27	4/25/58	18	J	36	24.0	26.0	22.2	1.1	3	300	
				Forty Knots							14
JHC 012	11/25/56	3	I	42	31.6	34.3	29.1	−1.0	6	350	
JHC 013	11/25/56	9	I	38	34.5	35.6	29.3	−2.4	12	300	
JHC 076	5/11/56	9	I	40	33.3	35.5	30.0	−2.2	15	150	
JHC 077	5/11/56	12	I	42	34.4	35.6	30.7	0.6	18	175	
JHC 078	5/11/56	14	I	41	33.5	35.0	30.0	−0.2	20	175	
JHC 079	5/11/56	15	I	40	31.1	32.8	28.0	−0.2	21	175	
JHC 081	5/11/56	21	I	38	31.1	33.5	28.3	−1.1	27	150	
JHC 092	3/21/56	21	K	39	32.4	32.5	28.1	−1.5	15	150	
JHC 138	12/10/58	9	J	40	31.0	34.1	28.2	−8.3	6	175	
JHC 151	12/12/58	6	J	40	29.4	32.5	26.2	−11.8	9	700	
JHC 153	12/12/58	12	J	40	29.2	31.8	26.2	−10.6	15	450	
JHC 215	11/21/56	3	I	42	33.1	36.1	30.3	−2.3	15	100	
JHA 16	12/17/59	21	J	41	31.5	34.5	28.7		21	425	
JHA 17	12/18/59	0	J	40	35.3	39.1	31.8		24	250	

* Identification of spectrums as given in *Moskowitz et al.* [1962, 1963].
† Air-sea temperature difference.

of the *Weather Explorer* and *Weather Reporter* are at an elevation of 19.5 meters. Since wind speed varies with height, the wind speeds presented in this paper are overestimates of the wind speeds that would have been recorded at the standard anemometer height of 10 meters. Some method must be used to reduce the wind speeds at 19.5 meters to the standard level. Furthermore, if the observed wind speeds are then reduced to the 7.5-meter level, a direct comparison of forecasts can be made with the PNJ method for these new wind speeds.

The spectrums shown in Figure 1 should be more representative of wind speeds less than the average wind speeds of the subsets given in Table 3. The one case of 45 knots would be more nearly representative of a 40-knot wind and the one case of 50 knots would be more nearly representative of a 45-knot wind. Therefore the impression that wind speeds of 40 knots produce fully developed seas, which is often implied in this paper, is erroneous. The reduction of wind speeds to the standard level of 33 feet is discussed by *Pierson* [1964].

It is evident from Table 2 that only 54 spectrums were selected for further evaluation from a total of 460 spectrums. Of these, 101 were not chosen from the application of the criteria to the synoptic conditions. These were obtained for use in other studies. The 101 spectrums are composed of a set of 72 from the period December 15–28, 1959, and a set of 29 from a hurricane in September 1961. Two spectrums were chosen from the set of 72 spectrums for further consideration, and no spectrums were chosen from the set of 29 because wind speed could be estimated only in terms of the Beaufort number. When the spectrums for the higher wind speed subsets (greater than 40 knots), the spectrums pertaining to the beginning and end of the mean wind periods, and the spectrums containing extraneous swell were removed, the remaining spectrums represent fully developed seas for the various mean wind speeds according to the synoptic criteria that were used.

The synoptically chosen spectrums were then analyzed to determine a mean spectrum for each subset. The results are given in Table 3 and Figure 1. The frequency is given by dividing the lag number by 180. The values tabulated represent an estimate of the integral of spectral density over a band 1/180 sec^{-1} wide centered on the above frequency. To convert the tabulated values to units of spectral density (ft^2-sec) they should be multiplied by 180. To convert these values of spectral density to metric units (m^2-sec) multiply by 1/10.765 (0.0929). Included in Table 3 are the results of the Kolmogorov-Smirnov (K-S) test applied at each frequency for each subset. The explanation of D_n is given below.

The K-S test was applied in order to test the hypothesis that the spectral values for the synoptically chosen spectrums came from a χ^2 distribution with 19.33 degrees of freedom and a mean adjusted to the mean of each subset. The synoptically chosen subset is still a random sample of all possible spectrums that might be observed under the required synoptic weather conditions. In the original spectral analyses, 20 degrees of freedom were used (see *Moskowitz et al.* [1962, 1963]). This procedure is adequate for the spectral estimates; however, 19.33 degrees of freedom are necessary for refinement.

The K-S test determines the maximum deviation of a sample cumulative distribution $F_n(x)$ from the assumed cumulative distribution $F(x)$. That is,

$$D_n = \text{maximum } |F_n(x) - F(x)|$$

where $F_n(x)$ represents the spectral values for each frequency and $F(x)$ represents the values from the assumed χ^2 distribution. If D_n is smaller than the Kolmogorov statistic, determined by the number of samples (in this case the number of spectrums in each subset) and the probability level used, we accept the hypothesis that the samples come from the assumed population. If D_n is larger than this Kolmogorov statistic, we reject the hypothesis. The probability level used in this test was 0.90. For a detailed description of the K-S test the reader is referred to any of a number of textbooks dealing with mathematical statistics, e.g. *Whitney* [1959].

From Table 3 it can be seen that the lower and higher frequencies fail to pass the K-S test. These results are not unexpected, since the response of the Tucker recorder at the higher frequencies is in doubt. Nonlinear effects at the very low frequencies and the steepness of the forward faces of the spectrums are probably reasons for the failure of the K-S test at these frequencies.

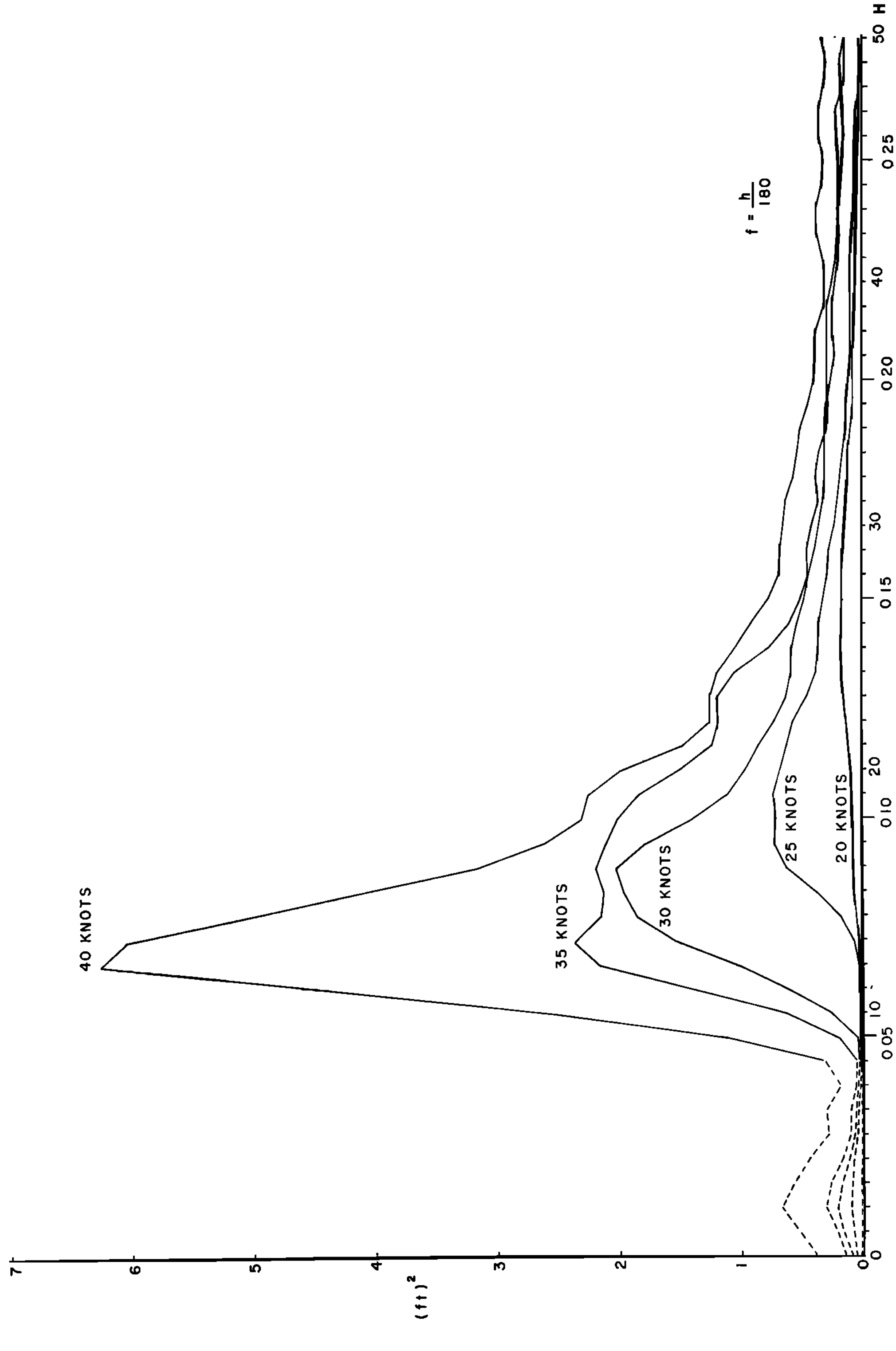

Fig. 1. Graphs of the synoptically chosen spectrums.

TABLE 3. Synoptically Chosen Spectrums

	n = 12, v = 20 (19.5) kts				n = 8, v = 25 (25.8) kts				n = 12, v = 30 (31.6) kts				n = 8, v = 35 (33.6) kts				n = 14, v = 40 kts			
h	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.
0	.0195	.0009	.7490	F	.0601	.0030	.6143	F	.0933	.0055	.8182	F	.1415	.0070	.8889	F	.3797	.1282	.8355	F
1	.0188	.0006	.6319	F	.0841	.0078	.6074	F	.1618	.0087	.3313	P	.2167	.0138	.4384	F	.5309	.1658	.3526	F
2	.0154	.0002	.5264	F	.1024	.0151	.5287	F	.2081	.0119	.3280	P	.3063	.0283	.4288	F	.6671	.1797	3612	F
3	.0101	.0001	.5208	F	.0983	.0097	.4966	F	.1753	.0098	.3930	F	.2670	.0160	.3384	P	.5790	.1509	.3531	F
4	.0061	.0000	.3479	F	.0744	.0039	.4790	F	.1160	.0049	.3103	P	.1701	.0026	.2658	P	.4484	.0599	2481	P
5	.0034	.0000	.2696	P	.0475	.0024	.5698	F	.0683	.0019	.3864	F	.1038	.0006	.2759	P	.2841	.0149	.2391	P
6	.0032	.0000	.4162	F	.0430	.0037	.6683	F	.0634	.0014	.3865	F	.1001	.0015	.3053	P	.3091	0255	.3301	F
7	.0025	.0000	.4167	F	.0226	.0009	.6758	F	.0391	.0005	.2989	P	.0568	.0006	.3621	P	.1843	.0137	.3249	F
8	.0025	.0000	.5864	F	.0133	.0003	.6863	F	.0290	.0006	.4061	P	.0528	.0011	.4777	F	.3172	.1591	.5180	F
9	.0030	.0001	.8081	F	.0086	.0001	.7744	F	.0491	.0020	.5400	P	.1909	.0412	.6250	F	1.1197	3.3802	.7480	F
10	.0075	.0004	.9000	F	.0079	.0001	.7605	F	.2413	.1288	.6802	P	.6181	.5519	.6625	F	2.4644	7.3835	.5758	F
11	.0121	.0010	.9056	F	.0101	.0001	.4865	F	.6043	.9813	.7373	P	1.3742	2.1744	.7289	F	4.3553	4.4986	.2800	P
12	.0134	.0006	.7792	F	.0228	.0003	.4259	F	1.0174	1.2829	.5225	P	2.1524	2.6901	.4978	F	6.2600	7.6949	.1581	P
13	.0229	.0011	.7585	F	.0694	.0026	.5447	F	1.5269	.6834	.2500	P	2.3646	2.0511	.4563	F	6 0404	8.4801	.2385	P
14	.0447	.0058	.8202	F	.1780	.0207	.6430	F	1.8469	.4581	.2242	P	2.1503	.9754	.3536	P	4.9884	2.8723	.2378	P
15	.0634	.0151	.8204	F	.3787	.0831	.5240	F	1.9556	.1722	.2271	P	2.1250	.5166	.2845	P	4.1104	2 2609	.2348	P
16	.0632	.0171	.8117	F	.6244	.1204	.5225	F	2.0255	.5126	.2825	P	2.1917	.5310	.3072	P	3.1776	2.1790	2670	P
17	.0673	.0150	.7697	F	.7270	.0968	.3177	P	1.7871	.4118	.2751	P	2.1138	.3631	.2271	P	2.6325	1.5151	.2147	P
18	.0827	.0189	.6770	F	.7091	.1191	.3653	P	1.3800	.1575	.2220	P	2.0063	.4546	.3486	P	2.3042	1.5944	.2216	P
19	.0881	.0106	.5589	F	.7223	.1500	.4436	F	1.0981	.0896	.1966	P	1.8438	.3531	.3051	P	2.2431	.9250	.2262	P
20	.0964	.0038	.3008	P	.6708	.0487	.3065	P	.9477	.0659	.1741	P	1.4942	.2238	.3408	P	1.9756	.3660	.1709	P
21	.1105	.0025	.2790	P	.6049	.0347	.3205	P	.8418	.0357	.1862	P	1.2218	.1023	.2080	P	1.4772	.1309	.1865	P
22	.1352	.0017	.2072	P	.5643	.0922	.4110	F	.7135	.0349	.1603	P	1.1738	.0619	.2716	P	1.2430	.1370	.2186	P
23	.1567	.0030	.2153	P	.4464	.0346	.3809	P	.6192	.0417	.2019	P	1.1850	.1202	.3475	P	1.2368	.2634	.2590	P
24	.1690	.0036	.1878	P	.3636	.0096	.2325	P	.5711	.0289	.1918	P	1.0451	.1591	.3809	P	1.1732	.4425	.3359	F
25	.1779	.0053	.2830	P	.3571	.0223	.2491	P	.5715	.0251	.2819	P	.7595	.0714	.2641	P	1.0278	.2538	.2973	P
26	.1530	.0037	.2124	P	.3430	.0194	.2968	P	.5245	.0259	.1932	P	.5834	.0385	.2427	P	.8955	.1277	.2272	P
27	.1506	.0046	.2427	P	.3015	.0085	.2326	P	.4624	.0542	.3475	F	.4928	.0313	.3071	P	.7560	.0790	.2206	P
28	.1673	.0062	.2597	P	.2775	.0040	.3397	P	.4226	.0685	.4676	F	.4305	.0363	.3734	P	.6723	.0834	.2247	P
29	.1478	.0047	.2613	P	.2649	.0071	.3506	P	.3831	.0339	.4164	F	.4490	.0446	.4504	F	.6590	.0714	.2162	P
30	.1320	.0027	.2192	P	.2196	.0041	.1993	P	.3473	.0170	.2035	P	.4052	.0409	.4227	F	.6391	.0669	.2816	P

Table 3 (Continued)

	n = 12 v = 20 (19.5) kts				n = 8 v = 25 (25.8) kts				n = 12 v = 30 (31.6) kts				n = 8 v = 35 (33.6) kts				n = 14 v = 40 kts			
h	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.
31	.1243	.0021	.2235	P	.1878	.0028	.3099	P	.3034	.0160	.2621	P	.3531	.0366	.5070	F	.6146	.0657	.2612	P
32	.1146	.0027	.2459	P	.1734	.0027	.2480	P	.2992	.0228	.3736	F	.3786	.0178	.3138	P	.5468	.0571	.3029	P
33	.1041	.0027	.2979	P	.1461	.0033	.2775	P	.3011	.0134	.2507	P	.3456	.0119	.2475	P	.5235	.0711	.3141	F
34	.0842	.0014	.2794	P	.1307	.0031	.4025	P	.2921	.0118	.3271	P	.2843	.0172	.3454	P	.4973	.0678	.2971	P
35	.0697	.0012	.2499	P	.1240	.0028	.4143	F	.2832	.0057	.1901	P	.2676	.0066	.3909	P	.4339	.0485	.3674	F
36	.0759	.0019	.3204	P	.0990	.0020	.3760	P	.2422	.0032	.1840	P	.2807	.0042	.3096	P	.3826	.0249	.2219	P
37	.0779	.0016	.2700	P	.0782	.0008	.2803	P	.2160	.0069	.1957	P	.2850	.0064	.2390	P	3790	.0223	1428	P
38	.0638	.0009	.3121	P	.0806	.0010	.3355	P	.2351	.0171	.3390	F	.2864	.0161	.2516	P	3636	.0250	.2573	P
39	.0556	.0021	.4395	F	.0947	.0037	.3890	P	.2380	.0181	.2781	P	.2833	.0092	.2340	P	.3106	.0173	.2382	P
40	.0538	.0012	.3970	F	.0998	.0044	.5557	F	.2090	.0122	.3599	F	.2454	.0093	.4162	F	.2920	.0108	.2733	P
41	.0496	.0007	.2664	P	.0942	.0032	.4763	F	.1808	.0103	.3683	F	.2092	.0128	.3832	P	.3181	.0150	.1933	P
42	.0445	.0006	.2571	P	.0792	.0015	.3825	P	.1721	.0111	.2974	P	.1942	.0095	.4322	F	.3607	.0280	.2605	P
43	.0422	.0008	.2458	P	.0668	.0014	.3707	P	.1804	.0093	.3647	F	.1913	.0041	.2393	P	.3693	.0390	.3257	F
44	.0439	.0016	.3668	F	.0646	.0020	.3750	P	.1732	.0137	.4103	F	.1816	.0051	.2905	P	.3244	.0622	.4323	F
45	.0431	.0017	.4410	F	.0624	.0024	.4849	F	.1484	.0154	.4417	F	.1878	.0071	.3611	P	.3166	.0744	.4313	F
46	.0342	.0007	.4670	F	.0580	.0024	.4994	F	.1389	.0149	.4866	F	.2181	.0084	.3511	P	.3506	.1202	.4536	F
47	.0240	.0003	.3299	P	.0471	.0012	.3750	P	.1496	.0133	.3937	F	.2129	.0041	.2465	P	.3595	.1737	.4359	F
48	.0234	.0004	.4040	F	.0227	.0002	.3749	P	.1698	.0231	.4960	F	.1569	.0076	.2818	P	.3195	.1251	.5089	F
49	.0283	.0005	.4932	F	.0150	.0005	.7229	F	.1883	.0409	.5000	F	.1357	.0205	.6119	F	.2885	.0979	.5287	F
50	.0286	.0005	.4073	F	.0314	.0019	.7213	F	.1601	.0207	.4713	F	.1472	.0500	.7204	F	.3240	.1873	.6497	F
51	.0207	.0005	.5000	F	.0457	.0039	.7401	F	.1075	.0131	.5348	F	.1047	.0174	.6231	F	.3392	.2235	.6522	F
52	.0147	.0004	.5833	F	.0753	.0064	.6250	F	.0843	.0136	.6297	F	.0551	.0070	.6250	F	.2321	.1666	.6429	F
53	.0112	.0004	.7500	F	.1063	.0181	.6250	F	.0935	.0195	.6246	F	.0886	.0157	.6250	F	.2011	.0536	.5557	F
54	.0086	.0002	.7489	F	.0788	.0066	.6196	F	.1608	.0332	.6189	F	.1306	.0242	.6248	F	2819	.3443	.7857	F
55	.0188	.0011	.7500	F	.0467	.0047	.6250	F	.1010	.0267	.6663	F	.1957	.0965	.6250	F	.4714	.8035	.7143	F
56	.0331	.0036	.8332	F	.0533	.0057	.7500	F	.0570	.0109	.7500	F	.1740	.0640	.6521	F	3270	.3067	.7143	F
57	.0190	.0011	.7222	F	.0981	.0289	.7500	F	.1419	.1012	.8873	F	.1575	.1291	.8750	F	.1875	.1506	.7120	F
58	.0047	.0003	1.0000	F	.0683	.0157	.8401	F	.1280	.0606	.8320	F	.4427	.5304	.7500	F	.4521	1.8923	.8476	F
59	.0428	.0115	.8333	F	.0232	.0043	.0000	F	.0926	.0457	.8333	F	.6326	1.7982	.7500	F	.2556	.5356	.7857	F
60	.0846	.0336	.8333	F	.0269	.0058	.0000	F	.1778	.1373	.8333	F	.7706	2.8940	.8355	F	.0835	.0282	.8571	F

TABLE 4. Randomly Chosen Subsets

Record No.*	Date	Time (UT)	Position	Wind Speed, kt	$H_{1/3}$, ft	Upper $H_{1/3}$, ft	Lower $H_{1/3}$, ft	n
				Twenty Knots				12
JHA 19	12/18/59	6	J	20	20.6	22.5	18.9	
JHB 15	12/24/59	12	J	20	17.4	19.1	15.9	
JHB 16	12/24/59	18	J	20	14.8	16.4	13.3	
JHB 18	12/25/59	6	J	20	14.6	15.9	13.3	
JHB 26	12/26/59	12	J	20	26.9	29.9	24.2	
JHC 028	6/19/55	14	J	20	11.3	12.2	10.5	
JHC 063	6/19/55	14	J	20	6.7	7.3	6.2	
JHC 068	6/20/55	9	J	20	7.2	7.8	6.7	
JHC 144	12/11/58	3	J	20	18.8	20.7	17.1	
JHC 120	4/16/58	12	J	20	6.3	6.8	5.9	
JHC 174	12/12/55	15	J	20	12.2	13.1	11.3	
JHC 205	3/6/58	18	I	20	18.4	19.8	17.1	
				Twenty-Five Knots				8
JHA 30	12/19/59	15	J	25	13.4	14.8	12.2	
JHB 20	12/25/59	15	J	25	21.2	23.4	10.2	
JHB 24	12/26/59	6	J	25	29.9	33.4	26.8	
JHB 31	12/27/59	15	J	25	16.1	17.6	14.7	
JHC 020	9/20/55	6	K	25	11.4	12.3	10.6	
JHC 115	5/22/57	0	A	25	11.4	12.3	10.5	
DL 43	1/17/59	6	A	25	15.0	16.9	13.3	
DL 99	7/25/59	6	A	25	8.7	9.4	8.0	
				Thirty Knots				12
JHA 25	12/19/59	0	J	30	16.4	17.9	15.1	
JHB 14	12/24/59	6	J	30	25.5	27.7	23.4	
JHC 083	5/12/56	3	I	30	23.2	25.5	21.1	
JHC 143	12/11/58	0	J	30	24.3	26.8	22.1	
JHC 154	12/12/58	15	J	30	30.7	34.1	27.6	
DL 3	1/19/58	12	I	30	16.4	17.8	15.1	
DL 8	1/20/58	21	I	30	20.1	21.6	18.7	
DL 29	4/26/58	0	J	30	17.7	19.3	16.3	
DL 30	4/26/58	3	J	30	16.1	17.4	14.9	
DL 108	4/6/57	15	J	30	16.0	17.2	14.9	
DL 114	4/7/57	18	J	30	16.5	18.1	15.1	
DL 115	4/8/57	3	J	30	17.2	18.8	15.8	
				Thirty-Five Knots				
JH 3	1/17/59	15	A	35	14.5	15.8	13.4	
JHC 010	11/24/56	18	I	35	31.4	34.0	29.0	
JHC 014	11/25/56	12	I	35	29.4	32.0	27.1	
JHC 087	3/21/56	3	K	35	23.7	25.8	21.8	
JHC 107	11/5/55	21	I	35	19.2	20.9	17.7	
JHC 217	11/21/56	12	I	35	30.4	33.2	27.8	
DL 7	1/20/58	15	I	35	23.0	24.8	21.4	
DL 54	11/8/59	21	I	35	25.0	27.5	22.8	
				Forty Knots				14
JH 28	1/29/59	0	A	40	28.2	30.9	25.7	
JH 43	12/19/58	0	J	40	27.1	29.7	24.7	
JHA 17	12/18/59	0	J	40	35.2	39.1	31.8	
JHB 4	12/22/59	15	J	40	38.6	42.2	35.2	
JHB 12	12/23/59	21	J	40	27.6	30.0	25.3	
JHC 011	11/25/56	0	I	40	26.0	28.2	23.9	
JHC 149	12/12/58	0	J	40	26.8	29.7	24.2	
JHC 156	6/6/59	0	J	40	18.9	20.6	17.4	

TABLE 4. (Continued)

Record* No.	Date	Time (UT)	Position	Wind Speed, kt	$H_{1/3}$, ft	Upper $H_{1/3}$, ft	Lower $H_{1/3}$, ft	n
JHC 200	3/5/58	15	I	40	20.5	22.2	18.9	
JHC 213	11/20/56	18	I	40	26 8	29.2	24.7	
DL 46	11/7/59	21	I	40	25.6	28.1	23.4	
DL 48	11/7/59	15	I	40	23.7	25 8	21 7	
DL 56	11/9/59	6	I	40	28 3	30.9	25.9	
DL 82	1/31/60	12	A	40	22.1	24.2	20 2	

* Identification of spectrums as given in *Moskowitz et al.* [1962, 1963].

The randomly chosen subsets. From all the available data, records were selected at random for which the wind speeds as determined from the ships' logs were exactly 20, 25, 30, 35, and 40 knots. In the entire set of data, there were 23 spectrums for 20 knots, 23 for 25 knots, 30 for 30 knots, 28 for 35 knots, and 35 for 40 knots. The identifications for all of the spectrums at a particular wind speed were put into a hat and the number of cases drawn was exactly the same as for the synoptically chosen subsets. No criteria were set on the synoptic conditions. The only criterion used for this selection was the exactness of the wind speed. The spectrums that were chosen at random are presented in Table 4. The averages of the randomly chosen spectrums and the results of the K-S test applied to these subsets are given in Table 5.

Comparison. In Table 6 the mean spectrums for both the synoptically chosen and randomly chosen subsets are divided into three frequency bands. The frequency bands are given in terms of the lag number h ($f = h/180$). The number of frequencies in each band which pass and fail the K-S test are also given as the percentage of passes for the dominant band of frequencies. The frequency bands are not the same for each wind speed, since the dominant range of frequencies changes with wind speed. The total number of passes for the dominant frequency bands of the synoptically chosen subsets is 119 from a total of 142. This gives a total passing percentage of 84. At the 90% level the number of expected passes is $142 \times 0.90 \pm 1.96\sigma$, where σ is the standard deviation. The standard deviation is computed by

$$\sigma = (npq)^{\frac{1}{2}} = (142 \times 0.90 \times 0.10)^{\frac{1}{2}} = 3.6$$

The minimum number of expected passes is, therefore, 128 − 7.06 or approximately 121. Although these results place the synoptically chosen spectrums outside the realm of acceptance, the dividing line is extremely thin.

The major frequency bands for the randomly chosen subsets produce only 87 passes from the same total of 142. This gives a total passing percentage of 61. The total number of passes for the random subsets far from equals the minimum number of 121 required. The number of passes for the randomly chosen subsets is more than 11 standard deviations from the expected value, and thus the odds are very small that the subsets are from a homogeneous population.

If the means of the χ^2 distribution had been adjusted to the means of each of the synoptically chosen subsets and the K-S test had then been applied to the randomly chosen subsets, the total number of passes for the randomly chosen subsets would be still lower.

In Figure 2, the mean spectrums of the synoptically chosen subsets are plotted against the mean spectrums of the randomly chosen subsets. The percentage of passes for the central range of frequencies for the two curves in this range is given in Table 6. From a comparison of the graphs in Figure 2 it may be concluded that, in general, from wind considerations only, the state of the sea may be difficult to describe. Decaying seas and swell can produce nonrepresentative spectrums, as is evident in the graphs for 20 and 25 knots. This effect is also noticeable in the graph for 30 knots, but not to the same extent. The two spectrums for 35 knots are very similar. The ocean weather ships, from which the wave records were obtained, were in those areas of the North Atlantic where wind

TABLE 5. Randomly Chosen Spectrums

	v = 20 kts				v = 25 kts				v = 30 kts				v = 35kts				v = 40 kts			
h	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.
0	.0502	.0011	.8333	F	.1082	.0362	1.0000	F	.1529	.0875	.8182	F	.2772	.0909	.7778	F	.2052	.0198	.6250	F
1	.0712	.0034	.4484	F	.1140	.0332	.7048	F	.1852	.0792	.7227	F	.4112	.1519	.6178	F	.3275	.0319	.2751	P
2	.0892	.0096	.5585	F	.1034	.0205	.6061	F	.7042	.0322	.4236	F	.5505	.2782	.6228	F	.4685	.0692	.3349	F
3	.0662	.0050	.5886	F	.0756	.0103	.6049	F	.1591	.0055	.2884	P	.4457	.1722	.7115	F	.4095	.0611	.3347	F
4	.0427	.0013	.5135	F	.0549	.0037	.5469	F	.1053	.0026	.4061	F	.2657	.0572	.6415	F	.2949	.0338	.4033	F
5	.0330	.0009	.5094	F	.0375	.0012	.5038	F	.0649	.0017	.4147	F	.1582	.0173	.4914	F	.1755	.0115	.3807	F
6	.0346	.0011	.3968	F	.0608	.0066	.7271	F	.0605	.0016	.3311	P	.1577	.0136	.4927	F	.1854	.0315	.4250	F
7	.0157	.0001	.4036	F	.0688	.0164	.7492	F	.0373	.0006	.3921	F	.0870	.0040	.4915	F	.1314	.0168	.4136	F
8	.0184	.0006	.4495	F	.1184	.0229	.7482	F	.0249	.0013	.5007	F	.0625	.0019	.4781	F	.2950	.5002	.7725	F
9	.0724	.0185	.7877	F	.5133	.8268	.8701	F	.1227	.0505	.6236	F	.1111	.0039	.3003	P	.7528	4.7431	.8477	F
10	.2917	.2912	.8324	F	1.3116	6.1066	.8750	F	.5533	.9776	.6453	F	.4194	.1446	.5244	F	1.3478	8.3053	.7318	F
11	.7311	1.8756	.8097	F	1.7224	14.1572	.8680	F	1.4092	5.3999	.6395	F	1.2507	1.4264	.6175	F	2.3671	6.9986	.5679	F
12	1.1445	4.0869	.7069	F	1.3358	6.2941	.8295	F	2.1401	7.7128	.5437	F	2.3613	5.6796	.6239	F	3.8317	11.3655	.3571	F
13	1.3629	4.0359	.5833	F	.9798	1.2423	.5074	F	2.3095	5.1930	.6178	F	3.2353	6.1293	.5589	F	4.2785	8.1211	.3437	F
14	1.3704	2.8241	.5822	F	1.2195	1.7238	.5558	F	2.0313	3.1898	.5650	F	3.5451	4.0626	.3758	P	4.2074	5.1759	.3086	P
15	1.2013	1.3379	.5579	F	1.4479	2.6825	.6242	F	1.6898	1.3953	.4479	F	3.2536	4.9054	.3503	P	3.7309	2.6643	.2237	P
16	1.0217	.6250	.4165	F	1.1775	1.6630	.6249	F	1.5112	.6863	.3051	P	2.7583	4.2451	.4112	F	3.1114	1.4975	.2270	P
17	.9817	.4986	.3395	F	.9968	1.2394	.6162	F	1.5544	.5295	.2484	P	2.1298	.9945	.3160	P	2.8154	.9372	.2563	P
18	.8343	.3554	.3333	P	.8292	.7069	.5772	F	1.3927	.2857	.2467	P	1.6890	.0913	.2827	P	2.4846	1.1907	.2838	P.
19	.0309	.2137	.3333	P	.5888	.2054	.4721	F	1.1115	.1542	.1880	P	1.6153	.1757	.2281	P	2.0227	.8949	.2398	P
20	.5216	.1711	.3333	P	.4885	.0755	.3655	P	.9966	.1753	.2391	P	1.3948	.2391	.2482	P	1.3912	.3558	.2059	P
21	.4410	.0886	.3658	F	.3883	.0253	.3325	P	.8696	.1847	.2457	P	1.2466	.2829	.4273	F	1.0702	.0777	.2677	P
22	.3668	.0495	.3801	F	.3562	.0182	.3435	P	.8179	.2051	.3561	F	1.2115	.3176	.4364	F	.9509	.0474	.1711	P
23	.3340	.0380	.3285	P	.4202	.0240	.2939	P	.7397	.1494	.3684	F	1.0580	.3120	.5256	F	.7861	.0582	.2071	P
24	.3344	.0347	.3545	F	.3919	.0212	.3243	P	.6047	.0624	.2971	P	.8990	.2615	.4428	F	.6923	.1350	.2675	P
25	.2762	.0259	.3510	F	.2974	.0147	.3979	P	.5524	.0500	.3035	P	.7559	.1664	.3922	P	.6058	.0614	.3320	F
26	.2025	.0119	.3394	F	.2213	.0091	.3717	P	.4783	.0632	.2945	P	.6534	.1388	.4150	F	.5194	.0373	.2015	P
27	.1932	.0089	.3290	P	.2483	.0161	.4475	F	.4033	.0444	.2881	P	.6171	.0915	.3721	P	.5169	.0555	.2967	P
28	.2011	.0120	.3319	P	.2630	.0149	.3119	P	.3355	.0172	.2456	P	.6298	.0959	.3569	P	.5059	.0490	.2323	P
29	.1908	.0111	.3159	P	.2077	.0067	.3909	P	.2585	.0041	.2072	P	.5483	.0614	.3196	P	.4734	.0407	.2355	P
30	.1731	.0066	.2857	P	.1716	.0054	.3229	P	.2381	.0061	.1974	P	.4175	.0344	.2500	P	.4421	.0351	.2438	P

Table 5 (Continued)

h	v = 20 kts				v = 25 kts				v = 30 kts				v = 35 kts				v = 40 kts			
	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.
31	.1518	.0037	.3214	P	.1463	.0036	.3306	P	.2434	.0079	.2941	P	.3708	.0378	.3952	P	.4114	.0406	.2675	P
32	.1562	.0044	.2618	P	.1396	.0043	.4261	F	.2798	.0121	.2946	P	.3553	.0246	.3716	P	.3674	.0252	.2764	P
33	.1497	.0051	.2994	P	.1470	.0074	.4135	F	.2910	.0153	.2977	P	.3544	.0357	.4114	F	.3101	.0232	.2441	P
34	.1268	.0045	.3559	F	.1436	.0073	.4605	F	.2459	.0113	.3081	P	.3382	.0634	.4934	F	.2556	.0146	.3013	P
35	.1312	.0030	.2619	P	.1352	.0059	.3658	P	.2048	.0042	.2158	P	.2878	.0399	.4722	F	.2522	.0206	.2539	P
36	.1450	.0057	.3727	F	.1173	.0038	.4203	F	.1929	.0045	.2463	P	.2502	.0257	.3703	P	.2708	.0302	.3401	F
37	.1293	.0046	.3529	F	.0965	.0013	.3297	P	.1960	.0160	.3259	P	.2621	.0272	.4809	F	.2882	.0335	.3143	F
38	.1098	.0036	.3394	F	.0959	.0015	.4024	P	.1831	.0241	.4283	F	.3094	.0230	.3682	P	.2467	.0192	.3275	F
39	.1027	.0041	.3965	F	.0934	.0022	.4389	F	.1592	.0198	.5814	F	.3164	.0187	.3332	P	.1974	.0134	.2573	P
40	.0886	.0040	.4533	F	.0773	.0011	.4236	F	.1517	.0106	.4175	F	.3046	.0191	.3551	P	.2125	.0103	.3385	F
41	.0784	.0029	.3717	F	.0651	.0011	.3626	P	.1489	.0078	.3411	F	.2865	.0198	.3837	P	.2549	.0345	.4026	F
42	.0804	.0026	.3639	F	.0634	.0013	.3882	P	.1400	.0086	.4038	F	.2614	.0138	.3619	P	.2770	.0367	3218	F
43	.0878	.0072	.5442	F	.0668	.0016	.5677	F	.1213	.0079	.4882	F	.2403	.0110	.3298	P	.2532	.0183	.3068	P
44	.0912	.0115	.5868	F	.0752	.0039	.5764	F	.1078	.0055	.4208	F	.2320	.0119	.3352	P	.2133	.0110	.3741	F
45	.0944	.0110	.5799	F	.0874	.0073	.6489	F	.1038	.0062	.3624	F	.2284	.0191	.4594	F	.1648	.0092	.2667	P
46	.0943	.0097	.5611	F	.0837	.0048	.5010	F	.1026	.0051	.3273	P	.2232	.0176	.4646	F	.1263	.0166	.4256	F
47	.0930	.0093	.5643	F	.0596	.0026	.5457	F	.1089	.0018	.2458	P	.2195	.0205	.4674	F	.1122	.0146	.4995	F
48	.0833	.0099	.5832	F	.0405	.0023	.4999	F	.1036	.0017	.2082	P	.2194	.0562	.6488	F	.0893	.0101	.5507	F
49	.0649	.0063	.6126	F	.0280	.0007	.4335	F	.0801	.0098	.5595	F	.1995	.0590	.6250	F	.0724	.0078	.5542	F
50	.0432	.0029	.5638	F	.0339	.0022	.6228	F	.0954	.0232	.5395	F	.0783	.0096	.7046	F	.0823	.0052	.4292	F
51	.0321	.0018	.6184	F	.0416	.0045	.8380	F	.1135	.0248	.5358	F	.0347	.0035	.7563	F	.0937	.0103	.4848	F
52	.0369	.0024	.6511	F	.0466	.0054	.8468	F	.0864	.0204	.6657	F	.0226	.0019	.8750	F	.1068	.0197	.5885	F
53	.0415	.0029	.6221	F	.0428	.0035	.6911	F	.0654	.0149	.6667	F	.0753	.0213	.7481	F	.1173	.0244	.5714	F
54	.0404	.0033	.6620	F	.0340	.0023	.6932	F	.0651	.0067	.5833	F	.2370	.0766	.6066	F	.1059	.0379	.7143	F
55	.0587	.0092	.7457	F	.0365	.0021	.6250	F	.0811	.0214	.5816	F	.3012	.1110	.6250	F	.0810	.0405	.7663	F
56	.0534	.0096	.8277	F	.0398	.0047	.7500	F	.1114	.0295	.6667	F	.2428	.1188	.7493	F	.1115	.0491	8109	F
57	.0165	.0010	.8333	F	.0307	.0034	.8750	F	.1254	.0530	.6667	F	.2111	.1362	.7500	F	.1071	.0671	.7823	F
58	.0137	.0015	.9167	F	.0456	.0166	1.0000	F	.0703	.0354	.8034	F	.3303	.4558	.8750	F	.1062	.0274	.6429	F
59	.0083	.0005	.9167	F	.0357	.0033	.7500	F	.1604	.0594	.7321	F	.2408	.1666	.8223	F	.0797	.0351	.7857	F
60	.0209	.0052	1.0000	F	.0416	.0093	.8750	F	.3226	.2436	.7500	F	.2867	.1927	.6250	F	.0943	.0777	.8571	F

TABLE 6. Results of the K-S Test for Synoptic and Random Subsets for Different Wind Speeds

Lag	V, kt	n	Pass	Fail	Pass, %
Synoptic	20	12			
0–19			1	19	
20–43			22	2	92
44–60			1	16	
Random		12			
0–19			2	18	
20–43			10	14	42
44–60			0	17	
Synoptic	25	8			
0–16			0	17	
17–42			21	5	81
43–60			4	14	
Random		8			
0–16			0	17	
17–42			16	10	62
43–60			0	18	
Synoptic	30	12			
0–12			4	9	
13–42			23	7	77
43–60			0	18	
Random		12			
0–12			2	11	
13–42			20	10	67
43–60			3	15	
Synoptic	35	8			
0–12			5	8	
13–42			24	6	80
43–60			6	12	
Random		8			
0–12			1	12	
13–42			19	11	63
43–60			2	16	
Synoptic	40	14			
0–10			2	9	
11–42			29	3	91
43–60			0	18	
Random		14			
0–10			1	10	
11–42			22	10	68
43–60			2	16	

speeds of 30 and 35 knots occur quite frequently. Since most of the records analyzed for these wind speeds had had fairly long durations, it is not too surprising that the mean spectrums for the synoptic and random subsets are similar. In the 40-knot case it is evident that the records that comprise the randomly chosen subsets did not have durations of such length as to produce a saturated sea state for this wind speed.

To summarize briefly, the synoptically chosen spectrums appear to come from a more homogeneous population than the randomly chosen spectrums. There is probably some variability in the data, not explainable, that caused the synoptically chosen spectrums to pass the K-S test fewer times than would be expected. Much of this variability can be attributed to the lack of precision in the wind reports and to the spread in the wind values that were used.

Higher-resolution spectrums. A question might arise as to the shape of the spectrums obtained from the use of the digital techniques of *Blackman and Tukey* [1958]. These techniques produce spectrums that have been convolved with a spectral window that is fairly wide. The synoptically chosen spectrums were recomputed from the original digitized wave records using 180 lags instead of 60 lags for the covariance function and the same frequency range; thus three times the resolution was obtained at the expense of the sampling variability. It was suspected that because of the use of higher resolution the forward faces of the spectrums might be steeper and the peaks might be shifted toward the lower frequencies. In Figure 3 the spectral estimates are plotted at the two different resolutions. The results tend to show that only at the higher wind speeds is it possible to detect a shift toward lower frequencies of the spectral peaks. It is also possible to detect, at these wind speeds, a slight steepening of the forward faces of the spectrums.

Additional results. From the mean spectrums given in Table 3 and some of the Russian work on ocean wave spectrums [*Kitaigorodskii,* 1961; *Kitaigorodskii and Strekalov,* 1962] an equation for the wave spectrum was determined [*Pierson and Moskowitz,* 1964]. The spectral form obtained is valid only for fully developed seas and deep water and is not valid for coastal regions. It is hoped that this equation may eventually be modified to include the effects of shorter fetch and shorter duration.

The significant height as a function of wind speed. From the tabulated spectral values given in Table 3, the significant heights for each

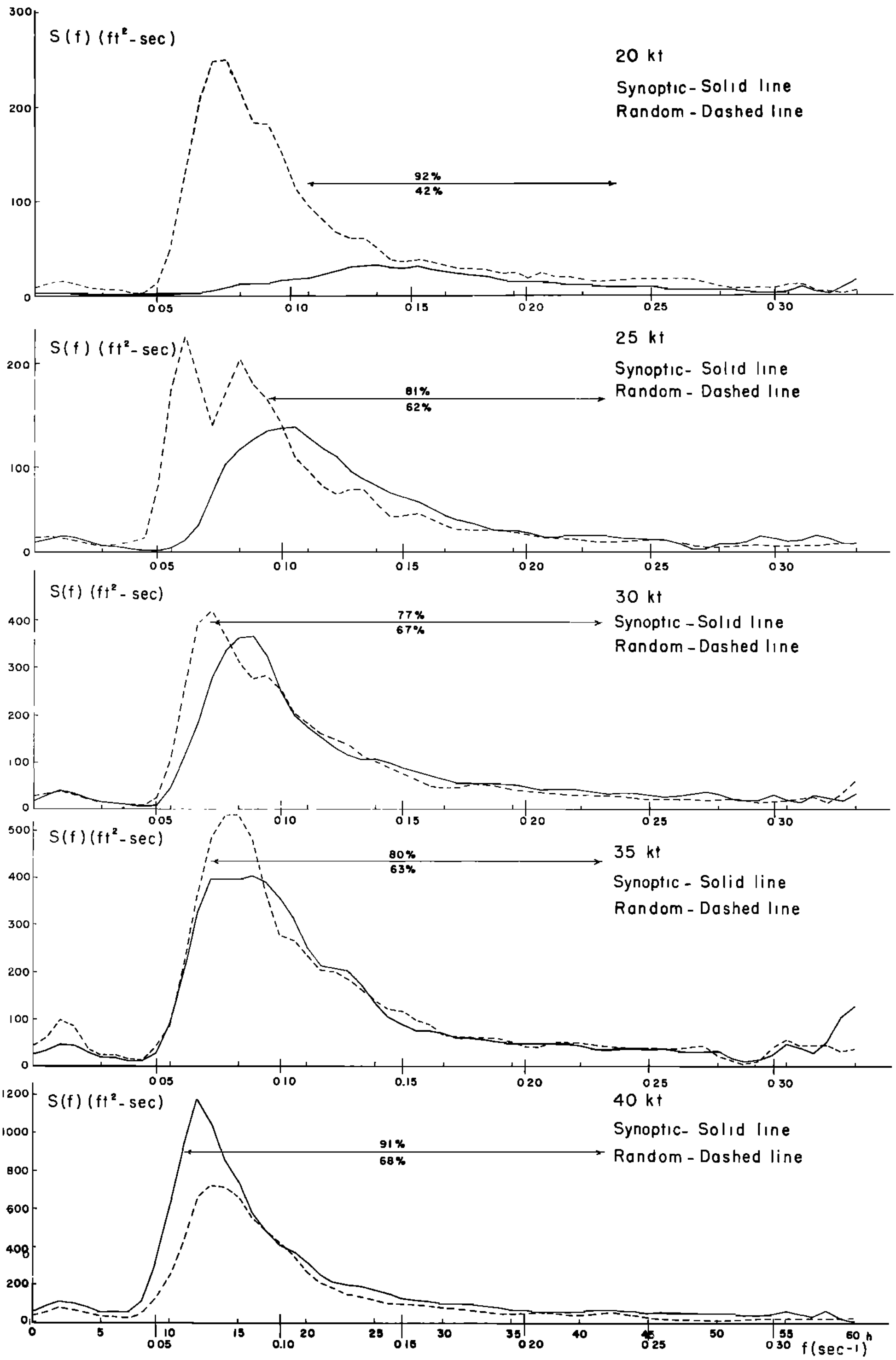

Fig. 2. Graphs of the chosen and random spectrums for each wind speed subset.

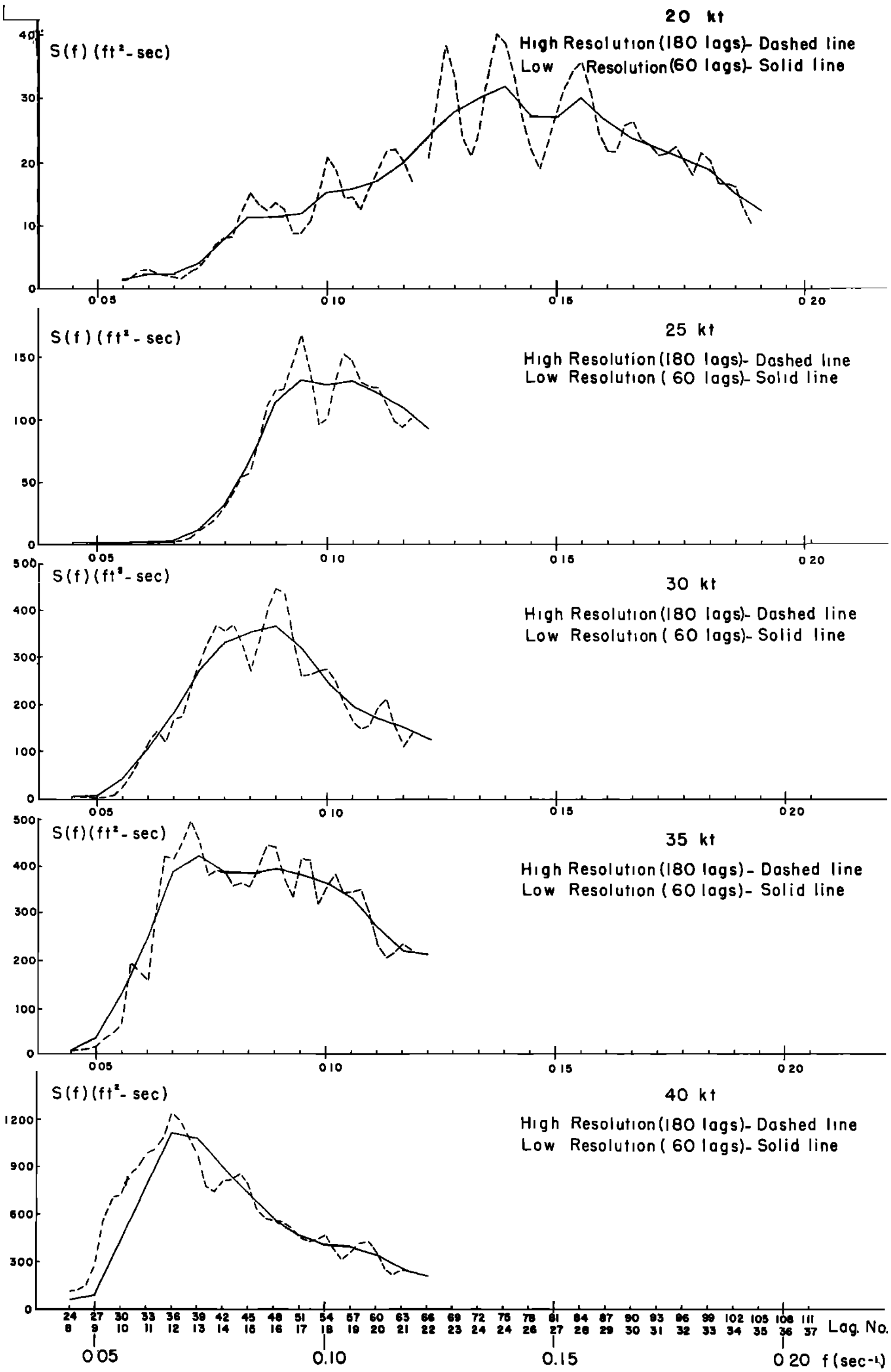

Fig. 3. Comparison of synoptically chosen spectrums using 180 and 60 lags.

of the synoptically chosen subsets were determined using the equation

$$H_{1/3} = 4(\text{Total variance})^{1/2} \tag{1}$$

which is based upon the results of *Longuet-Higgins* [1952]. All significant wave heights presented in this paper were determined in this fashion. The spectral values for the first six tabulated values were not used because they are probably not related to the wind-generated sea.

Several techniques were used in order to determine the relationship between significant wave height and wind speed. The first technique was a least-squares fit applied to the equation

$$H_{1/3} = av^2 \tag{2}$$

in order to determine the coefficient a. This was accomplished by requiring that

$$\sum [H_i - av_i^2]^2 = \text{minimum} \tag{3}$$

where H_i and v_i are the significant wave heights and average wind speeds, respectively, of each of the subsets. Second, the value of b in

$$H_{1/3} = bv^{2.5} \tag{4}$$

was obtained by using the same technique. Third, the values of d and n (n not necessarily an integer) in

$$H_{1/3} = dv^n \tag{5}$$

were found in a similar manner. In this case, the minimization yields

$$\begin{aligned} d &= \sum v_i^{n_1} H_i / \sum v_i^{2n_1} \\ &= \sum v_i^{n_1 - 1} H_i / \sum v_i^{2n_1 - 1} \end{aligned} \tag{6}$$

Various values of n from less than 2 to 2.5 were assumed, and the values of the two fractions were compared until they became equal. At this point the exponent and coefficient that provided a minimum mean square error were determined.

Similar techniques using log fits were employed to minimize the ratios of deviations from the fitted curve instead of squared absolute distances. The first of these equations is

$$m \ln a_1 + n_2 \sum \ln v_i = \sum \ln H_i \tag{7}$$

where m represents the number of subsets (5), n_2 is equal to 2, and $\ln a_1$ determines a different value for the a of (2). Equation 7 was also solved by setting $n_2 = 2.5$ and solving for $\ln b_1$. The last technique required solving (7) and (8) simultaneously for an unknown n_1 and $\ln d_1$. Equation 8 is given by

$$\left[\sum \ln v_i\right] \ln d_1 + n_1\left[\sum (\ln v_i)^2\right] = \sum (\ln v_i \ln H_i) \tag{8}$$

The results of these computations produced a v^2 law of the form

$$H_{1/3} = 0.02v^2 \tag{9}$$

where H is in feet and v in knots. The $v^{2.5}$ law did not fit the data as well as the v^2 law. Equation 5 agrees with a v^2 law but not a $v^{2.5}$ law. The use of the logarithmic fitting procedures verified the results just given and resulted in only minor changes in the constant.

These data have much less scatter than the usual data fitted to a v^2 law and, since alternative possibilities were tried and rejected, it is believed that they provide a valid observational verification of the law.

The relationship of significant wave height to the wind speed, as given in (9), still had to be modified because in its derivation the spectral values at the high frequencies were used in the determination of the significant heights. Since there is doubt as to their true values, the spectral form given by *Pierson and Moskowitz* [1964] was used to provide an equation in which an adjustment was made for the too high values in Table 3 at high frequencies. This equation is given by

$$H_{1/3} = 0.0182v^2 \tag{10}$$

and is taken to be the relationship of the significant wave height (in feet) of a fully developed sea to the wind speed (in knots). The constant is 9% lower than the one determined by the least-squares fits.

Figure 4 shows a plot of the above v^2 law and the wave height–wind speed relationships of PNJ, *Darbyshire* [1959], and Sverdrup, Munk, and Bretschneider (SMB) [as given in *Neumann and Pierson*, 1957]. The relationships for these various curves are:

PNJ	$H_{1/3} = 4.426 \times 10^{-3}\ v_{7.5}^{2.5}$
SMB	$H_{1/3} = 0.0233\ v_{10\text{–}12}^{2}$
Darbyshire	$H_{1/3} = 0.0133\ v^2$
Moskowitz	$H_{1/3} = 0.0182\ v_{19.5}^{2}$

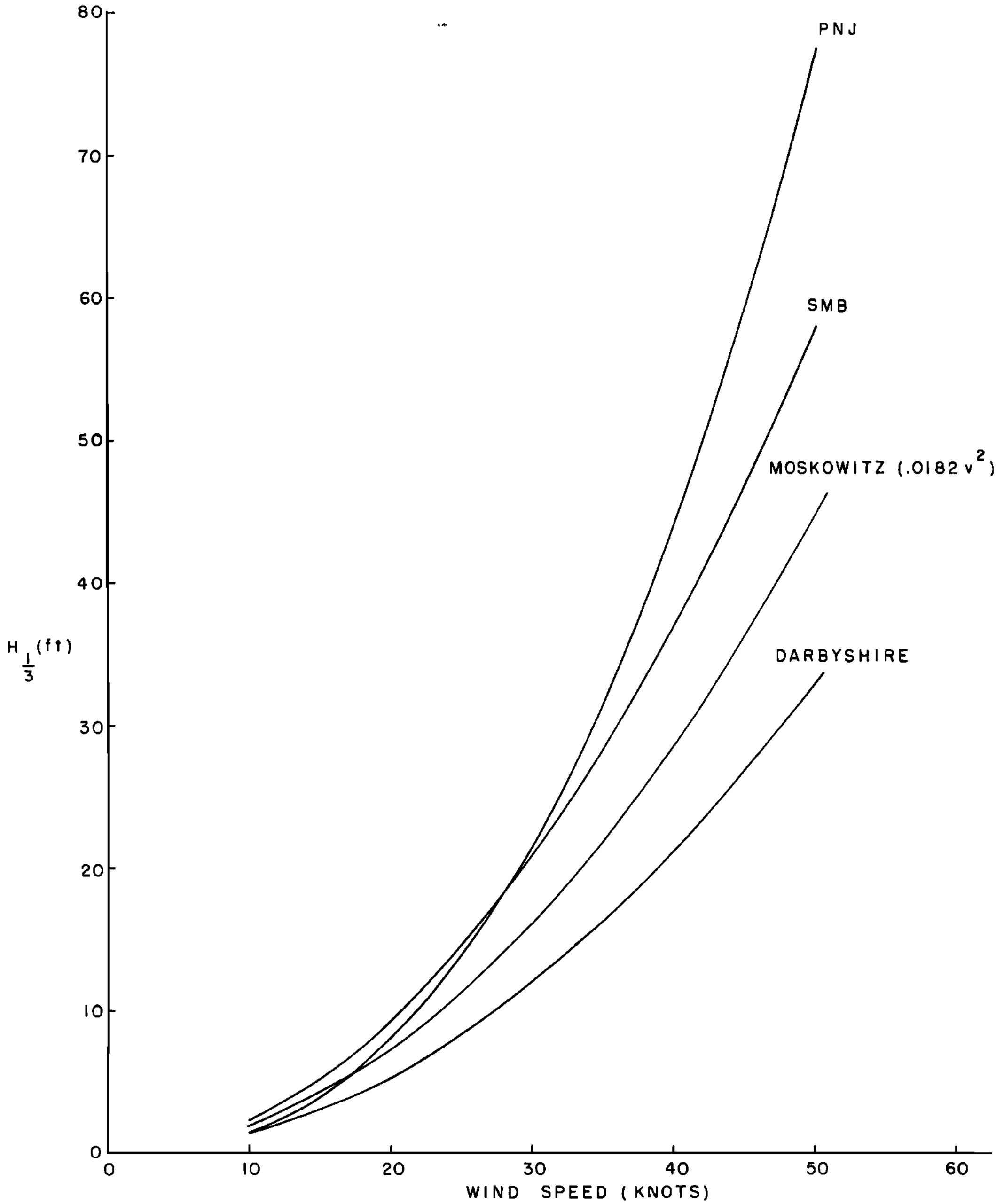

Fig. 4. Graph of significant height–wind speed relationships.

where $H_{1/3}$ is in feet and v in knots. Subscripts indicate the height in meters at which the wind is measured. As is shown by *Pierson* [1964], the PNJ, SMB, and Moskowitz curves are in substantial agreement when the variation in anemometer height is considered.

Comments on wind speed data. The results of the K-S test suggest some weaknesses in the reports of wind speeds. For those ships equipped with anemometers the reported wind speeds are 1-minute averages. For those ships not equipped with anemometers the wind speeds are estimated from the sea state by 'experienced observers.' It is known that such experienced people are able to approximate the wind speed from the sea state fairly well, but it is also possible to show many examples where an experienced observer may be in error (e.g. *Bretschneider et al.* [1962]).

Certainly a 1-minute interval for obtaining an average of the wind speed is not long enough for accurate results. The improvement of the techniques for reporting wind speed is becoming more important in many aspects of the study of air-sea boundary processes. Shipborne wave recorders on all the weather ships would eventually yield data which would help in refining these spectrums and in improving wave forecasts.

With the preceding discussion in mind, and with the aid of the derived spectral equation, it was possible to determine a modified wind speed that gave a good fit to the analytic spectrum mentioned above. These wind speeds are 19.5, 25.8, 31.6, 33.6, and 40 knots, as described by *Pierson and Moskowitz* [1964]. If these wind speeds are substituted in the spectral equation and the results plotted against the spectral values given in Table 3, the curves are very much alike.

Conclusions. The Kolmogorov-Smirnov test applied to the synoptically chosen spectrums for each mean wind speed showed that the basic assumptions used in the data reduction were valid. The results of the K-S test when applied to a randomly chosen subset for each of the mean wind speeds as compared with the synoptically chosen spectrums allow two conclusions to be drawn. First, criteria must be applied and extreme care taken in selecting synoptic situations where fully developed seas for various wind speeds might be found. Second, wind speed determinations from the observed sea state may not be accurate, especially if higher winds persisted before observation time. Stated differently, given a wind speed one cannot accurately describe the sea state at all times.

From the procedures presented in this paper, it was possible to determine an equation to describe the spectrums. This equation is not too different from equations obtained by others in previous studies. In addition a comparison was made of the various significant wave height–wind speed relationships. The resulting curve appears to represent a compromise between the PNJ, Darbyshire, and SMB methods.

In the areas of the North Atlantic where the ocean weather ships are located, it is not uncommon in the winter months to find very intense cyclones, very strong winds, and mountainous seas. It is a very rare situation, however, to find fully developed seas for wind speeds greater than 40 knots. During the 5-year period (April 1955–March 1960) only one synoptic situation for 45 knots and one for 50 knots could be found that approached the fully developed state. Perhaps, if wave records taken in the southern hemisphere, especially in the regions of the roaring forties or howling sixties, were available, more information could be obtained on the higher wind speeds. Wave statistics for light winds are also needed. However, the effects of winds of less than 20 knots are not reported here.

From the analyzed records the role that stability plays in wave generation is not obvious. No effects due to air-sea temperature differences could be detected. More interesting and higher seas are observed in winter than in summer. This is probably due to the frequency and intensity of the transient cyclones and to stability.

The general problems in reporting wind speeds at sea are a direct hindrance to the verification of wave forecasting techniques. When better techniques for wind observations are put into practice, it should be possible to analyze selected wave records precisely and to eliminate the source of variability in these results due to the lack of precision in measuring the wind speeds.

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