specular reflector the facets must be flat to within a certain fraction of a wavelength, say $\lambda/16$. Whatever the criterion selected, if it is maintained at all the microwave frequencies, the linear dimensions of the area increments will be proportional to the wavelength, but the total fraction of the surface occupied by the flat facets, ϵ, remains approximately constant at all wavelengths. If these relations are substituted in (7), σ° becomes independent of wavelength, which agrees with the measured results given in Fig. 7.

That the total fraction of the surface occupied at a given time and at a particular angle by flat reflecting facets is the same at all microwave frequencies is further substantiated by the good correlation shown at large depression angles among the instantaneous signal amplitudes obtained simultaneously at different microwave frequencies.

In these sea-scattering measurements the several possible sources of error and their magnitudes are as follows. At each microwave frequency the cross section measurement of the standard corner reflector contributes an uncertainty of ± 0.6 db; misalignment of the calibrator in the field, oscillator drift, uncertainty in the height measurements, and film shrinkage make up an additional probable error of ± 1.0 db; the calibrated attenuators contribute an uncertainty of ± 0.4 db to the calibration level; and finally, errors in measuring the signal amplitude on the microfilm reader probably do not exceed ± 0.5 db. Thus a probable error of ± 2.5 db seems reasonable for the cross section values obtained from the film averages. The signal amplitude values obtained from the electronic integration are free of some of the above uncertainties but are probably in error on the high side because the integrator responded somewhat slower to sudden decreases in signal level than to sudden increases in signal level.

As mentioned before, the film samples were only ten seconds in duration, whereas the integrator values represent nearly a two-minute sample from a run. Individual differences between values obtained by the two methods are thus partly due to the different samples taken. In general the film-read values are probably the more accurate of the two; but, since the sample for each run was short in duration, there is a possibility that the average value obtained by this method is not completely representative of the entire run.

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Radar Terrain Return at Near-Vertical Incidence*

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***Summary*—Measurements have demonstrated that, with occasional exceptions, radar return from the ground is largely due to area scatter, even at angles of incidence near the vertical.**

An expression is derived here for a power superposition integral expressing the mean pulse envelope for the pulse returned from the ground to a radar. This integral is the convolution of the transmitted pulse form in power units with a function which includes effects of antenna pattern, ground properties, and distance. This function is generalized to include the effects of specular reflection and large isolated scatterers, as well as the more prevalent area scatter.

While beam-width-limited illumination always results in inverse-square altitude variation for area scatter, it is shown that the variation with altitude for pulse-length-limited illumination varies from inverse-square to inverse-cube, and is a function of altitude as well as of ground properties and antenna pattern. Mean returned pulses are presented for various grounds and antenna orientations.

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I. INTRODUCTION

RADAR ALTIMETERS depend upon return of signals radiated from an aircraft to the ground. In order to understand the operation of such altimeters, it is necessary to understand the processes by which radar energy is returned from the ground.

A great deal of work has been done on radar return from aircraft and ship targets, and some fair amount has been done on radar return from the ground and the sea at ranges such that the angle of incidence is near grazing. Very little work has been done in the past on angles of incidence near vertical; and, so far as the authors can determine, none of the work which has been done has been published in readily available places. An early classic on the general field of radar return from the ground was that of R. E. Clapp.[1]

[1] R. E. Clapp, "A Theoretical and Experimental Study of Radar Ground Return," M.I.T. Radiation Lab. Rep. 1024; 1946.

Scattering is the principal process by which radar energy is returned from the ground to an altimeter. This process is supplemented on some occasions by specular (mirror-type) reflection. In references to the performance of radar altimeters which the authors have encountered, it has usually been assumed that the principal process was reflection. Measurements made at Sandia Corporation have indicated that these assumptions are not correct and that, in fact, reflection is a rather uncommon mode of radar return, even for vertical incidence.[2]

The usual type of scattering process is one in which large numbers of individual scatterers located on the ground contribute more or less equally to the total scattered signal. Occasionally, some scatterer is found within the pattern of illumination whose signal return stands out appreciably above all of the rest. Measurements have shown that very rarely will more than one such target be located within the area illuminated by a radar altimeter using a broad-beam antenna. It should be noted that this is contrary to the situation encountered with search radar equipment carried in aircraft where the desired information is the distinction between the strong scattering target and the general background. The principal reason for the difference is that a radar altimeter illuminates a much larger patch of ground than a properly designed search radar since it must operate when the aircraft performs various maneuvers and, therefore, must have a wide-beam antenna. This effect is compounded since the geometry of the situation near vertical incidence is such that even very short pulses spread out to cover a large area on the ground by comparison with the area which they cover at grazing angles of incidence for a reasonable beam width.

In this paper an integral is developed which is the mean scattered radar return from a collection of small scatterers located on a plane ground. It should be noted that the results here are for a mean returned pulse for a pulse-radar altimeter; this mean is taken over a large number of returned pulses. *Any individual pulse is likely to look quite different from the mean described by the integral developed in this paper.* It is interesting to note that the integral which results is a convolution of the waveform of the envelope of the transmitted pulse in *power* units with another function which includes the effects of antenna gain, distance, and scattering properties of the ground.

Examples have been calculated here to show the effect on the return from the ground of variations of different parameters of a pulse-radar system. In particular, it is pointed out for a beam-width-limited situation (one in which the leading edge of transmitted pulse passes the outer edge of antenna beam before the trailing edge of the pulse reaches the ground) that altitude signal varies inversely as the square of altitude.

Another example considered is that of a pulse-length-limited system (one in which the leading edge of the pulse has not extended to the limit of the antenna beam by the time the trailing edge reaches the ground); here the variation of the signal with altitude is inverse cube (if the variation of the scattering properties of the terrain with angle of incidence is slow). It is shown in other examples that the variation with altitude is between inverse square and inverse cube where the scattering capability of the ground drops off more rapidly with change in angle of incidence. With any scattering variation, the signal variation is inverse cube at higher altitudes and inverse square at low altitudes.

The above examples all assume that the antenna is pointed vertically. In the case where the antenna is not pointed vertically, either because the aircraft is tilted, or because the antenna's main function has to do with radar mapping or ranging on targets at a distance, the variation with altitude and the shape of the mean return pulses have been calculated for several examples.

In order that one may evaluate the integral for mean returned signal, it is necessary to know something about the nature of the ground as a scatterer. If the signal from the ground is made up of the resultant of signals from a large number of scattering elements, one may express its properties, on the average, by a quantity known as the scattering cross section per unit area, σ_0. This is a number which expresses the average amount of power radiated back from a unit area *on the ground*, provided the incident power density is known.[3] The quantity σ_0 used here is essentially that defined by Herbert Goldstein.[4]

It must be emphasized again that the expressions calculated here for returned pulse shapes for a radar altimeter are averages over many pulses. Individual pulses will look quite different from the average pulse as the returns from the various scatterers may combine as phasors in such a way as to produce a wide range of amplitudes for individual points in individual pulses. The problem of combination of large numbers of signals of similar amplitude but random phase has been treated statistically by Rayleigh and others.[5,6] This is a problem known to statisticians as the two-dimensional random-walk problem; Rayleigh's treatment is for the random walk with an infinite number of steps. At any given range, the different phasor sums for different pulses result in fading of the signal. The signals at any two ranges corresponding to different illuminated areas fade independently. For a Rayleigh distribution the range of fading from the signal level exceeded 5 per cent of the time down to that exceeded 95 per cent of the time is

[2] These experiments have not yet been reported in the literature.

[3] Note that some other writers use slightly different definitions of σ_0.

[4] D. E. Kerr (ed.), "Propagation of Short Radio Waves," McGraw-Hill Book Company, Inc., New York, N. Y., p. 483; 1951.

[5] Rayleigh, "Scientific Papers of Lord Rayleigh," Cambridge, London, vol. I, p. 491, vol. IV, p. 370; 1899–1920.

[6] J. L. Lawson and G. E. Uhlenbeck, "Threshold Signals," McGraw-Hill Book Co., Inc., New York, N. Y., p. 54; 1950.

18 db. Hence, the average pulse may be quite different from any single pulse.

In the situation where specular reflection and scatter are present together, the specular signal may be much greater than any of the individual scattered signals. If it, or one of the scattered signals, stands out above the rest so that the total power in the one large signal is comparable with that due to all small scatterers, the distribution of the returned signal amplitude is appreciably altered. This problem is the same as that of a large sinusoidal signal in noise and has been treated extensively by Rice;[7] it will not be discussed here.

II. Derivation of Expression for Scattered Return

To determine the return from a scattering ground, it will first be necessary to derive the formula for the return from a single scatterer. It will then be demonstrated that the return from two scatterers is such that the total return power is the sum of the powers in the two components, *on the average*. This will be generalized to the case of large numbers of scatterers, and it will be shown that the total returned power is the sum of the powers returned from the individual scatterers. Next we will show that, for practical purposes, the power sum may be represented as an integral over the illuminated area on the ground by use of the concept of average scattering cross section per unit area. Finally, it will be shown that this integral is actually a convolution integral involving the shape of the pulse envelope of transmitted power and a function including the effect of the ground, antenna, and distance.

A) *Return from a Single Scatterer*

Let us consider a pulse radar which at periodic or quasi-periodic intervals delivers a voltage pulse to its antenna given by

$$v_D(d) = \mathrm{Re}\{V_D(d)e^{j\omega d}\}, \tag{1}$$

where

$$V_D(d) = 0, \qquad (d < 0).$$

Lower case letters will be used to represent instantaneous voltages and powers, while upper case letters will be used to represent voltage envelopes or power averaged over an rf cycle. Hence, V_D is the envelope of the transmitted voltage pulse; d represents delay from the start of the transmitted pulse—that is, it is not assumed that phase is preserved from pulse to pulse.

The power averaged over an rf cycle is, then, for a real impedance of one ohm, given by

$$P_D(d) = \frac{V_D^2(d)}{2}\cdot \tag{2}$$

[7] S. O. Rice, "Mathematical analysis of random noise," *Bell Sys. Tech. J.*, vol. 23, p. 282; July, 1944; vol. 24, p. 46; January, 1945.

The average rf power of the signal returned by the mth scatter is

$$P_{Rm}(d) = \frac{P_D\left(d - \frac{2r_m}{c}\right)G_m}{4\pi r_m^2}\cdot\sigma_m\cdot\frac{1}{4\pi r_m^2}\cdot\frac{G_m\lambda^2}{4\pi}$$

$$= \frac{P_D\left(d - \frac{2r_m}{c}\right)G_m^2\lambda^2\sigma_m}{(4\pi)^3 r_m^4}. \tag{3}$$

New quantities introduced in this expression are:

r_m = the range to the mth scatterer from the radar,
c = the velocity of light,
σ_m = the scattering cross section of the mth scatterer,
G_m = the gain of antenna in the direction of the mth, scatterer (assuming the same antenna for transmitting and receiving),
λ = the wavelength of the carrier radiation.

The equation has been written first to show the way in which it is built up and then in a more compact form. The first factor of the first expression shows the transmitted power radiated in the appropriate direction as a power density at the receiving point (except that argument would be $d - r_m/c$). The second factor is the scattering cross section, a quantity which determines the portion of the incident energy which is reradiated toward the radar. The third factor shows the effect of the dispersion with distance on the reradiated power density, and the fourth factor shows the receiving antenna aperture.

Utilizing the results of (3), we may write for the received voltage

$$v_{Rm}(d) = \mathrm{Re}\left\{\sqrt{2P_{Rm}(d)}\,e^{j\omega_m(d-2r_m/c)}e^{j\alpha_m}\right\}$$

$$= \mathrm{Re}\left\{V_{Rm}(d)e^{j\theta_m}\right\}. \tag{4}$$

Here the phase shift has been taken into account, both that due to the travel time and the phase shift α_m on reflection. It should be realized that the frequency $\omega_m = \omega + \Delta\omega_m$ used here is not just the carrier frequency ω, but is the carrier frequency as modified by the Doppler shift. Evidently,

$$\theta_m = (\omega + \Delta\omega_m)\left(d - \frac{2r_m}{c}\right) + \alpha_m$$

and

$$V_{Rm}(d) = \sqrt{2P_{Rm}(d)}.$$

B) *Return from Two Scatterers*

The return from two scatterers is given by

$$v_R(d) = v_{R1}(d) + v_{R2}(d)$$
$$= \text{Re}\left\{V_{R1}(d)e^{j\theta_1} + V_{R2}(d)e^{j\theta_2}\right\}. \qquad (5)$$

The product of the complex voltage above with its conjugate gives the square of the envelope of $v_R(d)$:

$$V_R^2(d) = V_{R1}^2(d) + V_{R2}^2(d) + 2V_{R1}(d)V_{R2}(d)\cos(\theta_1 - \theta_2). \qquad (6)$$

This expression gives the envelope of a single return from two scatterers. The average over many returns will be designated by a bar and is

$$\overline{V_R^2(d)} = \overline{V_{R1}^2(d)} + \overline{V_{R2}^2(d)} \qquad (7)$$

provided θ_m has a uniform statistical distribution, since V_{R1}, V_{R2}, θ_1, and θ_2 are statistically independent, and $\overline{\cos(\theta_1-\theta_2)}=0$. The statistical distribution of θ_m is, of course, a result of the geometry.

In taking the average above, an average is taken at each position d in the return pulse. For that reason, the average $\overline{V_R^2(d)}$ remains a function of d. Further, in order for d to represent the same position from return to return in the signals from both scatterers, it is necessary to assume that the two scatterers remain in the same position with respect to the radar during the averaging period. Actually, of course, the range varies from pulse to pulse and it is this very variation of range which causes the angle θ to vary. However, there can be a large phase variation for a small range variation and this is the required assumption.

From another point of view the result (7) may be supposed to be due to the effect of Doppler frequencies. Since the returns from the two scatterers are sinusoids at different frequencies as indicated by (4), the application of the theorem which allows superposition of power will give the result.

Pulse-to-pulse variation may be explained in terms of sampling a pattern in space. For each point in space which may be occupied by the radar, there is a specific combination of relative phase and amplitude of the return from the two scatterers, the phase being determined by the round-trip distance and phase-shift on scattering and the amplitude by the directivity of the scatterers and antenna as well as by distance. The radar samples this pattern at the points where pulses are transmitted.

C) Return from Many Scatterers

Eq. (7) may be readily generalized to the case of many scatterers. The result is

$$\overline{V_R^2(d)} = \sum_{m=1}^{M} \overline{V_{Rm}^2(d)}$$
$$= \sum_{m=1}^{M} 2\overline{P_{Rm}(d)}. \qquad (8)$$

Then

$$\overline{P_R(d)} \triangleq \frac{1}{2}\overline{V_R^2(d)}$$
$$= \sum_{m=1}^{M} \overline{P_{Rm}(d)}$$
$$= \sum_{m=1}^{M} \overline{\left[\frac{P_D\left(d - \frac{2r_m}{c}\right)G_m^2\lambda^2\sigma_m}{(4\pi)^3 r_m^4}\right]}$$
$$= \frac{\lambda^2}{(4\pi)^3}\sum_{m=1}^{M}\left[P_D\left(d - \frac{2r_m}{c}\right)\frac{G_m^2\overline{\sigma_m}}{r_m^4}\right]. \qquad (9)$$

In the last expression above, the only quantity which must be averaged is σ_m; σ_m for a given scatterer may vary from pulse to pulse due to the changing orientation. The other quantities are all constants with the assumptions that r_m^4 and $P_D[d-(2r_m/c)]$ vary negligibly from return to return for a fixed m. Note that this can only be so if each scatterer remains *almost* fixed in position in the return pulse during the averaging; the slight change in position must provide the phase variation.

If the phase variation is not sufficient from pulse to pulse, the samples obtained are not independent. Thus, at low frequencies, it is difficult to obtain a sufficiently large sample to determine the statistical properties of the terrain. In fact, it is almost impossible to meet both assumptions of negligible change in $P_D[d-(2r_m/c)]$ during averaging and of large phase change from pulse to pulse at low frequencies.

D) Signal Returned from Scatterers on a Plane Surface

In many cases, the earth may be considered as plane for the purposes of calculating radar return. In such cases the development below may be used in its entirety. In some situations, as in flying over steep mountains or valleys, such an assumption is not justified and a summation is required over a more complex surface than a plane to obtain the mean power returned. This case will not be considered here but the approach used may be generalized to cover it.

Fig. 1 illustrates the geometry involved in this discussion. Consider a small region ΔA_m (to become dA in the limit) containing N scatterers and which is within the total illuminated area A. According to (9) the mean power returned from such a region is

$$\overline{[\Delta P_R(d)]_m} = \frac{\lambda^2}{(4\pi)^3}\sum_{n=1}^{N}\frac{P_D\left(d - \frac{2r_n}{c}\right)G_n^2\overline{\sigma_n}\Delta A_m}{r_n^4\Delta A_m} \qquad (10)$$

where the area ΔA_m has been placed in both numerator and denominator. Now if the variation of r is small enough over ΔA_m so that P_D, G, and r^4 can be considered constant over the area, the mean power can be written

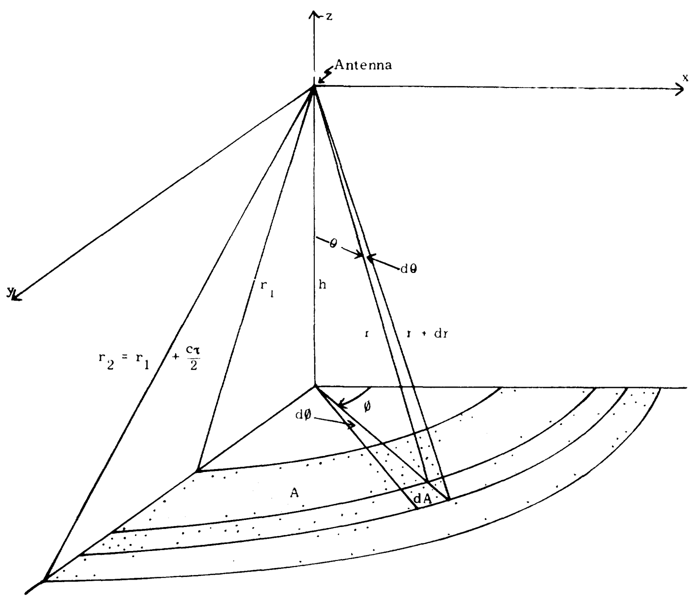

Fig. 1—Illustration of illuminated area A in one quadrant.

$$\overline{[\Delta P_R(d)]_m} \doteq \frac{\lambda^2}{(4\pi)^3} \frac{P_D\left(d - \frac{2r_m}{c}\right)}{r_m^4} G_{m2} \sum_{n=1}^{N} \frac{\overline{\sigma_n}}{\Delta A_m} \Delta A_m \qquad (11)$$

where r_m is the average distance to ΔA_m and G_m is the antenna gain in the direction of ΔA_m.

One may now *define* an average scattering cross section per unit area over this small area by

$$\sigma_{0m} \stackrel{\Delta}{=} \sum_{n=1}^{N} \frac{\overline{\sigma_n}}{\Delta A_m}. \qquad (12)$$

This is a very important quantity in the theory of radar terrain return. It is not, in general, easy to determine the characteristics of individual scatterers on the ground; in fact, it is not even easy to determine what the individual ground scatterers are. Hence, the use of some sort of average scattering cross section per unit area is almost a necessity. Of course, it is possible for the cross section to be quite different in different parts of the illuminated area simply because of the fact that homogeneous regions on the ground are hard to find. One would expect that, if the entire illuminated area were a forest, σ_0 would be essentially constant within the entire illuminated region. The same might hold true for a wheatfield, or perhaps even for a city, in which case the individual scatterers would be represented by buildings. On the other hand, one can also conceive of the situation where the average scattering cross section per unit area is quite different in different parts of the illuminated region. For example, over farm land one has different types of fields, roads, fences, small streams, and farmyards, all illuminated simultaneously. Over cities one may have industrial, residential, park, and boulevard areas illuminated simultaneously.

If we use the definition of (12), we obtain from (11) the following expression for the incremental average power:

$$\overline{[\Delta P_R(d)]_m} = \frac{\lambda^2}{(4\pi)^3} \frac{P_D\left(d - \frac{2r_m}{c}\right) G_m^2 \sigma_{0m} \Delta A_m}{r_m^4}. \qquad (13)$$

This is the power returned from the incremental area ΔA_m. The same argument which allowed us to combine the many returned signals in (10) or (8) permits us to add the average powers returned from the M different incremental areas illuminated:

$$\overline{P_R(d)} = \sum_{m=1}^{M} \overline{[\Delta P_R(d)]_m} = \frac{\lambda^2}{(4\pi)^3} \cdot \sum_{m=1}^{M} \frac{P_D\left(d - \frac{2r_m}{c}\right) G_m^2 \sigma_{0m} \Delta A_m}{r_m^4}. \qquad (14)$$

Note that the sum carried out in this case must include the transmitted pulse shape and its delay within the summation because of the fact that different delays will occur over different parts of the illuminated area; and, therefore, different incremental areas will be illuminated with different intensities. In addition, it should be noted that the gain function and the distance function are included within the summation. This is because both of these may vary appreciably over a total illuminated area. The antenna gain G_m is a function of the coordinates of the particular incremental area ΔA_m in terms of a spherical coordinate system centered on the aircraft (see Fig. 1). The scattering cross section is a function of these coordinates because of the fact that a wave incident upon the scatterer at one angle will reradiate differently from a wave incident upon the scatterer at a different angle; thus, its position relative to the altimeter is material. We write then,

$$G_m = G(\theta_m, \phi_m), \quad \sigma_{0m} = \sigma_0(\theta_m, \phi_m) \qquad (15)$$

where θ_m is angle from vertical to area element ΔA_m and ϕ_m is azimuth to ΔA_m. (See Fig. 1.)

Usually one may choose a value ΔA_m which is small enough so that all the radar parameters are reasonably constant across this area. It is then possible to pass to the limit of increasingly small incremental area and write (15) as an integral:

$$\overline{P_R(d)} \doteq \frac{\lambda^2}{(4\pi)^3} \int_{A(d)} \frac{P_D\left(d - \frac{2r}{c}\right) G^2(\theta, \phi)\sigma_0(\theta, \phi) dA}{r^4}. \qquad (16)$$

It should be noted that this is possible even though the concept of an average cross section per unit breaks down if the differential area is made small enough. This is because the only thing which varies with small changes of the size of the incremental area is the product of σ_0 and the area.

The integral of (16) is over $A(d)$, the area illuminated, which depends on d as indicated. At the beginning of this section reference was made to Fig. 1 as an illustration of the illuminated area. It should be noted that, in this coordinate system and for a plane, the area element may be taken as $rdrd\phi$. Thus, we may rewrite (16) for this particular case as

$$\overline{P_R(d)} \doteq \frac{\lambda^2}{(4\pi^3)}\int_0^{2\pi}\int_h^{cd/2}\frac{P_D\left(d-\frac{2r}{c}\right)G^2(\theta,\phi)\sigma_0(\theta,\phi)drd\phi}{r^3}. \quad (17)$$

Reference to Fig. 1 indicates that the limits in the integration on r should be r_1 to $r_2 = r_1 + c\tau/2$, where τ is the transmitted pulse width. However, the limits shown are correct since $P_D(\mu) = 0$ for $\mu < 0$ and for $\mu > \tau$, and since $r_1 \geqq h$. In other words, the exact area of illumination is taken care of by the transmitted pulse function P_D.

The integral of (16) is valid even though the surface is not a plane. If the surface is nonplanar, the area illuminated will not have a simple shape as in the planar case, but integration over this area by numerical means should be quite feasible.

E) Mean Return Signal as a Superposition Integral

It will be recognized that the integral of (17) is a superposition integral. This may be shown more readily if one converts this integral into a somewhat simpler appearing form and changes the variable of integration. Let us introduce the new variable T given by

$$T \triangleq \frac{2(r-h)}{c}. \quad (18)$$

T is the radar delay time for the difference between range r and the altitude h. Now, we define a function which includes all the effects of antenna, distance, and ground:

$$\left.\begin{aligned} B_s(T) &\triangleq \frac{c\lambda^2}{2(4\pi)^3r^3}\int_0^{2\pi}G^2(\theta,\phi)\sigma_0(\theta,\phi)d\phi, \quad (T \geqq 0)\\ &\triangleq 0, \quad (T < 0)\end{aligned}\right\}. \quad (19)$$

This integral is a function of T since r is a function of T through (18) and θ is a function of T through (18) and

$$r = h \sec \theta. \quad (20)$$

Now, (17) may be written

$$\overline{P_R(d)} = \int_0^{d-2h/c} P_D\left(d - T - \frac{2h}{c}\right)B_s(T)dT. \quad (21)$$

If we change from a function of delay time to a function of a modified delay time, *i.e.*, replace $d-(2h/c)$ by d, this may be written

$$\overline{P_R\left(d+\frac{2h}{c}\right)} = \int_0^d P_D(d-T)B_s(T)dT. \quad (22)$$

The lower limit is still zero since $B_s(T) = 0$ for $T < 0$. This is recognizable as a convolution or a superposition integral. $B_s(T)$ can be seen to represent the return signal which would be received if the transmitted pulse were an impulse or delta function. Thus, the signal received at any given time is representable as the sum of the signals received from a set of different impulses having weight given by the shape of the transmitted pulse and being transmitted at times corresponding to the times of delay of the various parts of the pulse from its leading edge.

That this is valid with power is a situation which is unique to the case in which the average signal is not dependent upon cross products of the various components. Normally, such a convolution integral can be used only with voltages or currents in linear systems.

III. Specular Reflections and Large Targets

A) Specular Reflections

As stated in the introduction, it is occasionally possible to find a surface which is smooth over a large enough area so that specular (mirror-type) reflections can take place. The mechanism of specular reflection is different from that of scattering and must be treated separately. It is not expected that any specularly reflecting surface will occur anywhere except directly beneath the radar because of the fact that its size must be so great that a surface which would be perpendicular to the incident radiation at any other point would not likely be of sufficient size. An exception may occur where corner reflector action between the ground and a wall of a large building is involved.

Specular reflection is normally treated on the image basis. In such a case, the received power is given by

$$P_R(d) = \frac{P_D\left(d-\frac{2h}{c}\right)G^2(0,\phi)\lambda^2}{(4\pi)^2(2h)^2}K \quad (23)$$

where K is the reflection coefficient of the surface and $G(0, \phi)$ the antenna gain straight down. It is convenient to express the specularly reflected wave in the same notation used for the scattered waves. In order to do this, we define a space-time function $B(T)$ in a similar manner to that defined by (19):

$$B_m(T) = \frac{G^2(\theta,\phi)\lambda^2K}{(4\pi)^2(2r)^2}\delta(T). \quad (24)$$

This is the B function for mirror-like reflection; this expression uses the Dirac delta function to indicate that the only range at which a return is expected is that corresponding to vertical incidence where $T=0$, $\theta=0$ and $r=h$. One may apply this ground function just as in the case of the ground function for scattering to obtain

$$P_R(d) = \int_0^{d-2h/c} P_D\left(d - T - \frac{2h}{c}\right) B_m(T)dT \qquad (25)$$

or

$$P_R\left(d + \frac{2h}{c}\right) = \int_0^d P_D(d - T)B_m(T)dT. \qquad (26)$$

It should be noted, of course, that (25) is directly equivalent to (23) because of the property of the delta function that

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$$

for any analytic function $f(x)$.

B) *Large Targets*

It was pointed out in Section II that there may occasionally be individual targets that stand out appreciably above the other scatterers. Such targets, while uncommon, may be present, and should be treated separately since only one is likely to be present in any given incremental area, and it is unlikely that one will be present in more than two or three of the incremental areas involved in summing up the total scattered return. Furthermore, such a target may be physically larger than an incremental area of the type described in the second section. Also, the statistics of signal returns when all the signals have roughly comparable amplitudes are the same as noise statistics, whereas the statistics for one large target and a background of smaller ones correspond to a cw signal in noise.

It was shown in Section IIA that the radar return from one target is given by

$$P_R(d) = \frac{P_D\left(d - \frac{2r}{c}\right)G^2\lambda^2}{(4\pi)^3 r^4}\sigma$$

where G is the antenna gain in the direction of the target and σ is its scattering cross section. Now if there are L large targets in the illuminated area, we define the B function for them by

$$B_l(T) \triangleq \frac{G^2(\theta, \phi)\lambda^2}{(4\pi)^3 r^4} \sum_{i=1}^{L} \sigma_i \delta(T - T_i, \phi - \phi_i) \qquad (27)$$

where σ_i is the scattering cross section of the ith large target and where the two-dimensional delta function is zero everywhere except at values $T = T_i$ and $\phi = \phi_i$ which correspond to the position of the ith large target. Then, the return due to the L large targets may be written

$$\overline{P_R(d)} = \int_0^{d-2h/c} P_D\left(d - T - \frac{2h}{c}\right) B_l(T)dT \qquad (28)$$

or

$$\overline{P_D\left(d + \frac{2h}{c}\right)} = \int_0^d P_D(d - T)B_l(T)dT. \qquad (29)$$

It should be noted that (28) yields a sum of L terms of the form of (3), one for each of the L targets.

C) *Combined Return*

In Sections IID, IIIA, and IIIB, we have shown how to set up a power impulse response function for the ground which takes into account antenna gain, scattering cross section, and distance. The function has been set forth for the three cases of area scatter, mirror-like reflection, and large individual targets. Because of the additive properties of power functions taken with random phases as described in Section II, the total return power is the sum of the powers due to the three cases. This leads to the definition of a combined B function:

$$\begin{aligned} B(T) &\triangleq B_s(T) + B_m(T) + B_l(T) \\ &= \frac{\lambda^2}{(4\pi)^2}\left\{\frac{c}{2(4\pi r^3)}\int_0^{2\pi} G^2(\theta, \phi)\sigma_0(\theta, \phi)d\phi \right. \\ &\quad + \frac{KG^2(\theta, \phi)}{(2r)^2}\delta(T) \\ &\quad \left. + \frac{G^2(\theta, \phi)}{4\pi r^4}\sum_{i=1}^{L}\sigma_i\delta(T - T_i, \phi - \phi_i)\right\}. \qquad (30) \end{aligned}$$

The total mean power due to the three types of return is, then,

$$\overline{P_R\left(d + \frac{2h}{c}\right)} = \int_0^d P_D(d - T)B(T)dT. \qquad (31)$$

It should be noted that this power is averaged over many separate pulses. The component associated with mirror-type reflection is a steady one and does not fluctuate from pulse to pulse. The component associated with a large target does not by itself fluctuate. However, if two large targets are present at the same position in the return, there is fading between them so that averaging is required for this result. In addition, the scattered components add phasorially to the other two components and cause fading of the resultant. In computing the statistical variations, it is necessary to consider the distribution function of the resulting voltage. Fortunately, because of the properties described in Section II, it is possible to utilize the power superposition integral of (31) to obtain the average pulse return. Note, however, that this average pulse is bound to be far different in appearance from any individual pulse.

IV. Examples

In this section, examples of the various types of limitation of ground illumination and their effects on the return signal will be discussed. With cw or long-pulse systems, it is possible to have the illuminated region on the ground determined by the beam width of the antenna pattern, even though the pattern may be quite broad. With narrow-beam antennas, even short-pulse systems may find their illuminated areas determined by the beam width. On the other hand, with pulse systems, and fm systems which are equivalent to pulse systems, one finds frequently that the return is limited by the pulse length rather than the beam width. The variation of the illuminated area with height and range for the beam-width-limited case is different from the variation for the pulse-length-limited case.

Of course, with nonsquare pulses and nonsymmetrical antenna patterns, the situation is somewhat more complicated, and some examples are quoted here which show this effect also. This situation becomes particularly interesting when the antenna is pointed nearly horizontally instead of vertically.

A) Beam-Width-Limited Illumination

For radars operating with long pulses or narrow beam antennas, or for certain types of cw systems, the illumination may be considered limited by the antenna beam. The simplest case to be considered here is one in which we assume the scattering cross section per unit area to be constant, the gain to be constant (antenna isotropic in region of interest, gain zero elsewhere), and the transmitted pulse to be square but of sufficient length so that it could illuminate a region bigger than the antenna will allow it to. Mathematically, these conditions are expressed as

$$\sigma_0(\theta, \phi) = \hat{\sigma}_0, \quad \text{a constant,} \tag{32}$$

$$\begin{aligned} G(\theta, \phi) &= G_0, \quad (0 \leqq \theta \leqq \theta_0), \\ &= 0, \quad (\theta_0 < \theta \leqq \pi), \end{aligned} \tag{33}$$

$$\begin{aligned} P_D(d) &= P_0, \quad (0 < d < \tau), \\ &= 0, \quad \text{(otherwise).} \end{aligned} \tag{34}$$

The pulse length is assumed to be τ and the other quantities are self-explanatory. The condition that the limitation be due to beam width is given by the inequality

$$1 + \frac{c\tau}{2h} > \sec \theta_0. \tag{35}$$

In (35) θ_0 is the angle defined by (33).

In order to determine the mean return power and consequently the mean pulse shape of the return, (17) will be used rather than those involving the convolution integral as described later in Section II. In dealing with symmetrical beams and square pulses, the original form of the integral is somewhat easier to use since it is difficult to express $B(T)$ compactly for these cases. For convenience, we repeat

$$P_R(d) \doteq \frac{\lambda^2}{(4\pi)^3} \int_0^{2\pi} \int_h^{cd/2} \frac{P_D\left(d - \frac{2r}{c}\right) G^2(\theta, \phi)\sigma_0(\theta, \phi) dr d\phi}{r^3}. \tag{17}$$

The result, when stated in full, requires three separate nonzero expressions. The first corresponds to the time before the trailing edge of the transmitted pulse reaches the ground while the illuminated circle expands on the ground. The second represents the period during which the illuminated circle remains constant; its outer boundary is determined by the antenna. The third is the period during which the trailing edge of the pulse moves outward to the edge of the antenna beam. The result is

$$\overline{P_R(d)} = 0, \qquad \left(d < \frac{2h}{c}\right) \tag{36}$$

$$\overline{P_R(d)} = \frac{\lambda^2 G_0^2 \hat{\sigma}_0 P_0}{4(4\pi)^2 h^2}\left[1 - \left(\frac{2h}{cd}\right)^2\right], \qquad \left(\frac{2h}{c} < d < \frac{2h}{c}\sec\theta_0\right) \tag{37}$$

$$\overline{P_R(d)} = \frac{\lambda^2 G_0^2 \hat{\sigma}_0 P_0}{4(4\pi)^2 h^2}\sin^2\theta_0, \qquad \left(\frac{2h}{c}\sec\theta_0 < d < \frac{2h}{c} + \tau\right) \tag{38}$$

$$\overline{P_R(d)} = \frac{\lambda^2 G_0^2 \hat{\sigma}_0 P_0}{4(4\pi)^2 h^2}\left[\left(\frac{2h}{c(d-\tau)}\right)^2 - \cos^2\theta_0\right], \qquad \left(\frac{2h}{c} + \tau < d < \frac{2h}{c}\sec\theta_0 + \tau\right) \tag{39}$$

$$\overline{P_R(d)} = 0, \qquad \left(\frac{2h}{c}\sec\theta_0 + \tau < d\right). \tag{40}$$

These results are shown for a particular example in Fig. 2.

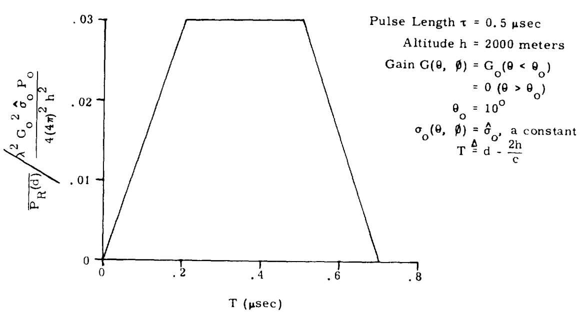

Fig. 2—Example of mean returned power for beam-limited scatter.

Note in (38) that the variation of the signal with altitude h is inverse square. (This statement applies to the peak return for the angle-limited case but does not, in general, apply to the peak return for the pulse-length-limited case.) Specular reflection also yields an inverse-square altitude variation. It is possible then, to express the *scattering* properties of the ground for this type of limitation in terms of an *equivalent reflection* coefficient. One must note, however, that in doing so the implication is that there is no fading, whereas, in point of fact, the fading is just as severe for beam-angle-limited scatter as for any other type of scatter, while a true specularly-reflected signal does not fade.

It should also be noted that the distinction between beam-angle-limited and pulse-length-limited cases is itself a function of altitude. For altitudes below those given by

$$1 + \frac{c\tau}{2h} = \sec \theta_0 \tag{41}$$

the return will be beam-angle limited. On the other hand, for altitudes above that obtained by solving this equation, the illumination will be pulse-length limited.

B) Pulse-Length-Limited Illumination, σ_0 Constant

As stated above, the condition for pulse length limitation of illumination is that

$$1 + \frac{c\tau}{2h} < \sec \theta_0. \tag{42}$$

In this first example, we shall assume that all conditions are the same as in the preceding example with beam-width limitation except for (42); that is, the conditions of (32), (33), and (34) apply here.

In this case, two separate nonzero expressions are needed to state the result. The first corresponds to the interval before the trailing edge of the transmitted pulse reaches the ground. The second is the period after the trailing edge reaches the ground and during which the illuminated annulus on the ground spreads out. The result is, for $\theta_0 = 90°$,

$$\overline{P_R(d)} = 0, \qquad \left(d < \frac{2h}{c}\right) \tag{43}$$

$$\overline{P_R(d)} = \frac{\lambda^2 G_0{}^2 \hat{\sigma}_0 P_0}{4(4\pi)^2 h^2}\left[1 - \left(\frac{2h}{cd}\right)^2\right], \qquad \left(\frac{2h}{c} < d < \frac{2h}{c} + \tau\right) \tag{44}$$

$$\widehat{\overline{P_R(d)}} = \frac{\lambda^2 G_0{}^2 \hat{\sigma}_0 P_0}{4(4\pi)^2 h^2}\left[1 - \frac{1}{\left(1 + \dfrac{c\tau}{2h}\right)^2}\right], \qquad \left(d = \frac{2h}{c} + \tau\right) \tag{45}$$

$$\overline{P_R(d)} = \frac{\lambda^2 G_0{}^2 \hat{\sigma}_0 P_0}{(4\pi)^2 c^2 d^2}\left[\frac{1}{(1 - \tau/d)^2} - 1\right], \qquad \left(\frac{2h}{c} + \tau < d\right) \tag{46}$$

$$\overline{P_R(d)} \doteq \frac{\lambda^2 G_0{}^2 \hat{\sigma}_0 P_0}{(4\pi)^2 c^2} \frac{2\tau}{d^3}, \qquad \left(\frac{2h}{c} + \tau < d,\ \tau \ll d\right). \tag{47}$$

Eq. (45) is included since this gives the peak of the mean return. Eq. (47) follows directly from (46). Eq. (47) states that at any fixed altitude h the mean returned pulse decays as $1/d^3$, *i.e.*, as the inverse cube of range. An example which behaves according to (43) through (47) is shown in Fig. 3.

We now make the assumption that the pulse length

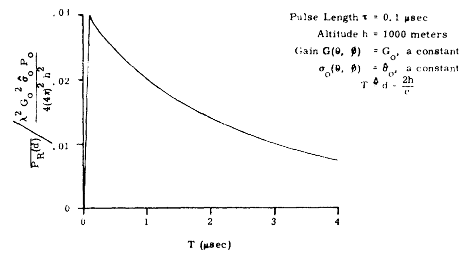

Fig. 3—Example of mean returned power for pulse-limited scatter.

in space is considerably shorter than twice the altitude, *i.e.*, that

$$c\tau \ll 2h. \tag{48}$$

Then, (45) becomes

$$\widehat{\overline{P_R(d)}} = \frac{\lambda^2 G_0{}^2 \hat{\sigma}_0 P_0 c\tau}{4(4\pi)^2 h^3} \tag{49}$$

for the peak of the mean return. This equation states that the peak of the mean return power varies as the inverse cube of altitude h. The inverse cube relationship here should be contrasted with the inverse square relationship for beam-width limitation of illumination. Again it should be pointed out that these are merely mean pulse shapes and that there will be much fading about the mean.

Because the variation of signal strength with altitude is different for the pulse-length-limited case from that for the beam-width-limited scatter or specular reflection cases, it is not possible to describe the properties of a scattering ground, insofar as a pulse-width-limited radar is concerned, by an effective reflection coefficient. Hence, measurements made at one altitude and interpreted in terms of signals at another altitude must specify the mechanism involved in the radar return so that the proper type of extrapolation may be used in going from one altitude to another. Assumption of either specular reflection or constant-σ_0, pulse-limited scatter may lead to difficulties in extrapolating measurements to different altitudes.

C) Pulse-Length-Limited Illumination, σ_0 Variable

The example of Section IVB, assumed that σ_0 was independent of depression angle. This assumption is not, in general, justified, although types of ground have been seen for which it seems fairly reasonable, particularly at the higher frequencies. Most types of ground which have been observed in the Sandia Corporation experimental program have a σ_0 which decreases as the angle with the vertical is increased. Some examples have been calculated for different types of variation of σ_0 with angle and it has been ascertained that neither an inverse square or an inverse cube altitude variation applies to all such situations.

Variations of the exponent associated with the change of signal with altitude have been determined for the cases where

$$\sigma_0 = \hat{\sigma}_0 e^{-\theta/15°}$$

and

$$\sigma_0 = \hat{\sigma}_0 e^{-\theta/5°}$$

for a rectangular transmitted pulse of duration τ and for an antenna gain which is constant over the region of interest. The results are shown in Fig. 4, where

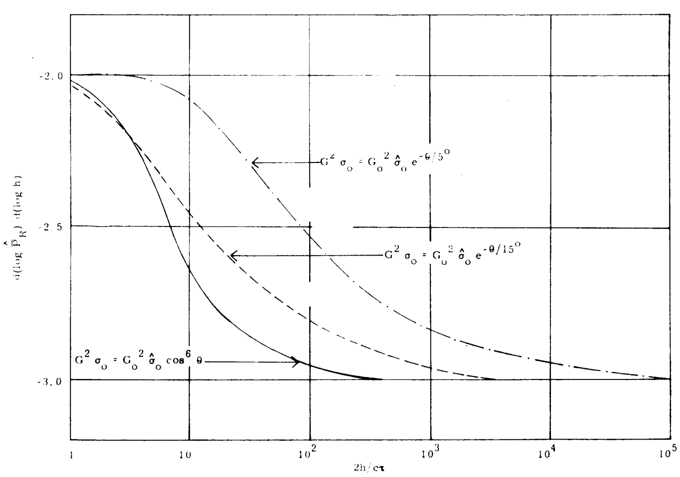

Fig. 4—Variation of peak mean return $\hat{\hat{P}}_R$ with altitude h; rectangular transmitted pulse of duration τ.

$d(\log \hat{\hat{P}}_R)/d(\log h)$ is plotted vs $2h/c\tau$. $\hat{\hat{P}}_R$ is the maximum of the mean return. This expression does give the exponent, since, if

$$\hat{\hat{P}}_R = kh^n,$$

then

$$\log \hat{\hat{P}}_R = \log k + n \log h,$$

and

$$\frac{d(\log \hat{\hat{P}}_R)}{d(\log h)} = n.$$

It is readily apparent that the inverse cube law prevails at high altitudes and the inverse square at low altitudes. This might be expected since a very small change in σ_0 across the region illuminated by the pulse approaches the case of paragraph B) of this section, where σ_0 is independent of angle. This small change occurs at high altitudes since for this case the angle that defines the illuminated region corresponding to peak return is quite small. On the other hand, for low altitudes, the area illuminated by a pulse corresponds to a very large angle and the limitation of return is due to decrease of σ_0 with angle rather than pulse length. Hence, an inverse square variation occurs.

Another example of the variation of maximum mean return with altitude has been calculated for

$$\sigma_0 = \hat{\sigma}_0 \cos^{6-n}\theta$$

and

$$G^2 = G_0^2 \cos^n \theta$$

(*i.e.*, for $\sigma_0 G^2 = \hat{\sigma}_0 G_0^2 \cos^6\theta$) and for a rectangular transmitted pulse of duration τ. The results are also shown in Fig. 4.

In this case, as in the two previous cases, the variation becomes proportional to $1/h^3$ as $2h/c\tau \to \infty$, and becomes proportional to $1/h^2$ as $2h/c\tau \to 0$. The latter case corresponds to $2h \ll c\tau$ and is of little interest for a pulse altimeter since the minimum altitude for which it is useful is $h = c\tau/2$. However, the case is of general interest.

D) Mean Return for Nonsquare Transmitted Pulse; Effect of Antenna Orientation

The pulse shown in Fig. 5 is taken as the transmitted

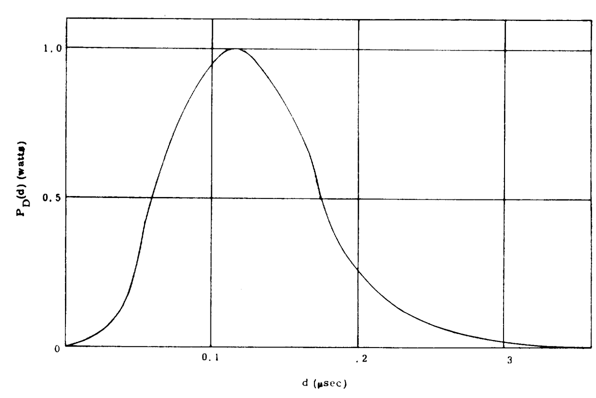

Fig. 5. Envelope of transmitted pulse in power units.

pulse. The antenna gain G and σ_0 are assumed to be

$$G = 6 \sin^2\gamma \cos^2\delta \text{ (one lobe only)}$$

where γ and δ are longitude and colatitude in a coordinate system based on the antenna, and

$$\sigma_0 = \hat{\sigma}_0 e^{-\theta/15°}.$$

The mean returned pulse (normalized) for four different

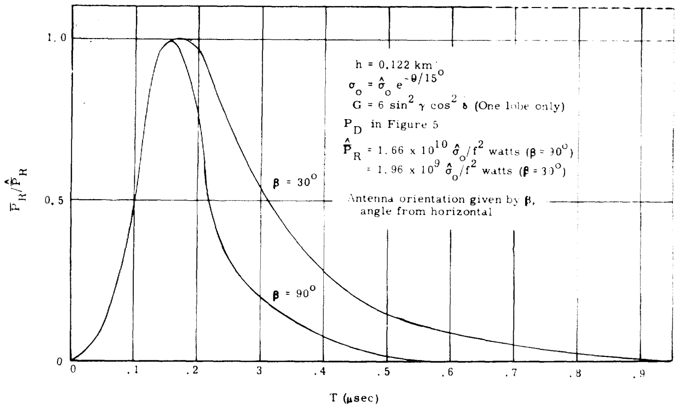

Fig. 6—Example of mean returned power.

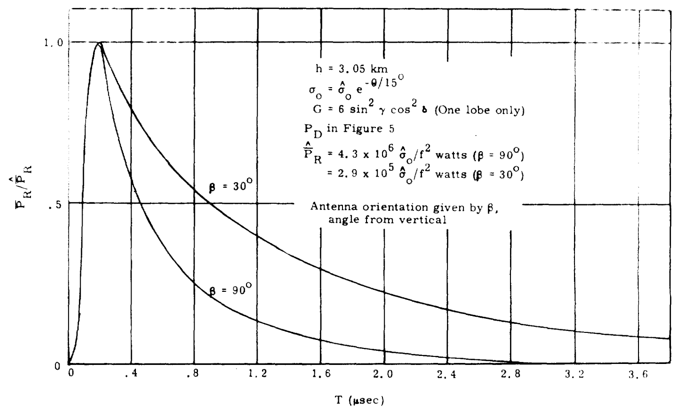

Fig. 8—Example of mean returned power.

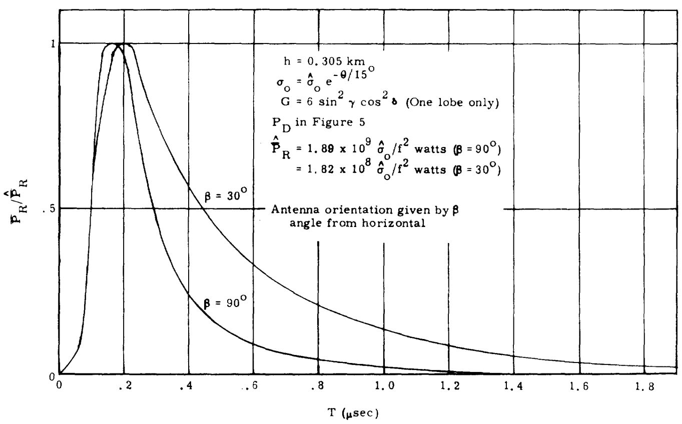

Fig. 7—Example of mean returned power.

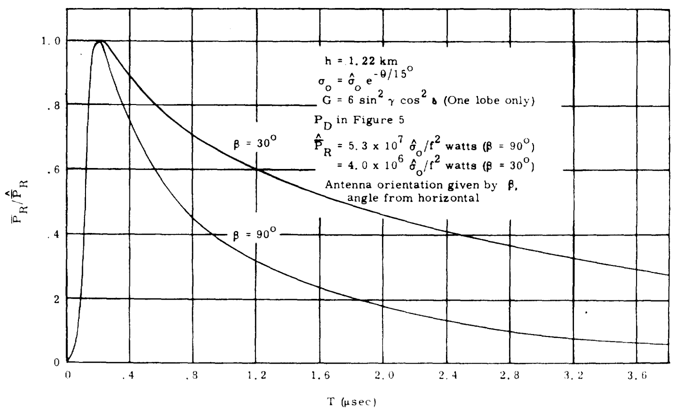

Fig. 9—Example of mean returned power.

altitudes, 0.122, 0.305, 1.22, and 3.05 km, is then given by Figs. 6, 7, 8, and 9. In each figure, two values of angle β are used. This angle represents the antenna orientation; $\beta=90°$ represents the case where the antenna is directed vertically downward and $\beta=0°$ corresponds to a horizontally directed antenna.

V. Conclusion

Radar return from the ground at near-vertical incidence is usually due to area scattering; but it may be due, at least in part, to specular reflection and scattering from individual large targets. It has been shown that the mean return to a pulse radar can be expressed by a power superposition integral involving the transmitted pulse envelope and a function including effects of ground properties, antenna gain, and distance.

With beam-width limitation of illumination, the mean peak signal varies inversely with the square of altitude. With pulse-length limitation of illumination, the mean peak signal varies inversely as the square of altitude for specular reflection, between inverse square and inverse cube for area scatter, and inversely as the fourth power for large target scatter. Curves have been presented showing altitude variation for various area scattering coefficients, and examples showing the effect of antenna tilt have been included.

VI. Acknowledgment

Original verification of the fact that the variation of peak signal with altitude changes over smoothly from inverse square to inverse cube is due to Dr. Robert A. Hessemer (now at University of Arizona), and his contribution to this and to the power superposition concept is gratefully acknowledged. Many of the original calculations which led to better understanding were performed by or under direction of Douglas M. Gragg of Sandia Corporation.

The integration of the antenna function of Section IVD), for the different orientations was done by Dr. Sheldon H. Dike of Sandia Corporation. These calculations were required in order to present the examples of Figs. 6, 7, 8, and 9.