NOTES

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A note on the potential transport of scalars and organisms by surface waves

Abstract—Wave-induced transport that is not directly reflected in Eulerian current measurements, an effect known as Stokes drift, may play a significant role in the transport of organisms and solutes in the nearshore environment. Two sets of field observations are presented that illustrate the potential importance of wave-induced transport. Velocities measured near Santa Barbara, California, are used to show that the theoretical Stokes drift can be stronger than measured Eulerian currents. Dye plume measurements made at Duck, North Carolina, in both wavy and nonwavy conditions indicate that wave-induced scalar transport can be significant. To demonstrate the generality of these results, we present calculations of the Stokes drift for several depths and as functions of wave amplitude and period. Given that cross-shore velocities typical of the nearshore coastal ocean are generally comparable to or smaller than these computed values of the Stokes drift, we conclude that observations and predictions of nearshore transport should include explicit consideration of wave-induced transport.

Much attention has focused recently on transport processes on the inner shelf since these play an important role in shaping the dynamics of marine populations, e.g., through connectivity of different habitats (Cowen et al. 2000). Eulerian mean flows (the time-averaged current measured at a point in space) on the inner shelf can be driven by buoyancy, wind stresses, and tidal pressure gradients (Lentz et al. 1999). Surface waves, a prominent feature of these coastal environments, can have a profound effect both on the mean flows through the action of radiation stresses (Longuet-Higgins and Stewart 1962) and on the net movement of scalars and passive particles. This latter quantity is referred to as the Lagragian mean velocity, i.e., the time average following the motions of particles; it can be different from what would be inferred from mean currents alone (Andrews and McIntyre 1978). The purpose of this note is to review the theory that describes the difference between Eulerian and Lagrangian mean velocities, a quantity known as Stokes drift, in the context of coastal zone transport and to present several examples that highlight the potential importance of the Stokes drift.

For surface gravity waves, the Stokes drift at a point is defined as the net motion of fluid particles, generally in the direction of wave propagation, that is in excess of the time-averaged current at that point (i.e., as measured by a current meter). Why such a drift of fluid particles should occur is often explained by noting that for finite amplitude irrotational surface gravity waves, the orbital motions are not closed, an effect first described by Stokes (1847). For small-amplitude waves of any type, the Stokes drift can be calculated from the wave-induced velocity, $\mathbf{U}_w$. Assuming $\mathbf{U}_w$ has a zero Eulerian mean (the time average at a point), the Stokes drift, $\mathbf{U}_S$ can be expressed,

$$\vec{\mathbf{U}}_S = \left\langle \int^t \vec{\mathbf{U}}_w \, dt \nabla \vec{\mathbf{U}}_w \right\rangle \tag{1}$$

(see, e.g., Andrews and McIntyre 1978). Here the operator $\langle\rangle$ is the wave averaging, i.e., the time average over one wave period of a variable-like velocity. For the specific case of surface gravity waves, the wave motion, $\mathbf{U}_w$, can itself be computed from linear water wave theory in terms of the wave amplitude a, the wavenumber k, computed from the wave frequency σ (found via the dispersion relation $\sigma(k)$), the total water depth H, and elevation z (z points upward, and $z = 0$ is the mean position of the free surface). The Stokes drift can then be explicitly calculated, using Eq. 1, for the case of a wave propagating in the x direction,

$$\mathbf{U}_S = (\mathbf{U}_S, 0, 0) \tag{2a}$$

where

$$U_S = \frac{(ak)^2 C_p \cosh[2k(z + H)]}{2 \sinh^2(kH)} \tag{2b}$$

$$C_p = \frac{\sigma}{k} \tag{2c}$$

$$\sigma^2 = gk \tanh(kH) \tag{2d}$$

The details of this calculation can be found, for example, in Dean and Dalrymple (1991); we cite the results here for convenience of reference. An important aspect of (Eq. 2) is that the Stokes drift velocity is much smaller than the phase velocity of the surface waves, C_p, but increases like the square of the wave steepness, i.e., the amplitude. Both a and σ (and hence k) can be computed using bottom pressure measurements (Dean and Dalrymple 1991); although without directional wave information, the direction of Stokes drift remains unknown. Nevertheless, with recent advances in acoustic Doppler current profiler (ADCP) technology, obtaining directional wave spectra in the future should not be problematic.

Observations of Stokes drift—While the formulae cited above have been known for approximately one and a half centuries, it is worth noting that rarely do field programs and numerical models focused on transport processes include the effects of surface waves. We show here two examples drawn from field experiments on the inner shelf that demonstrate the importance of including Stokes drift effects when assessing transport.

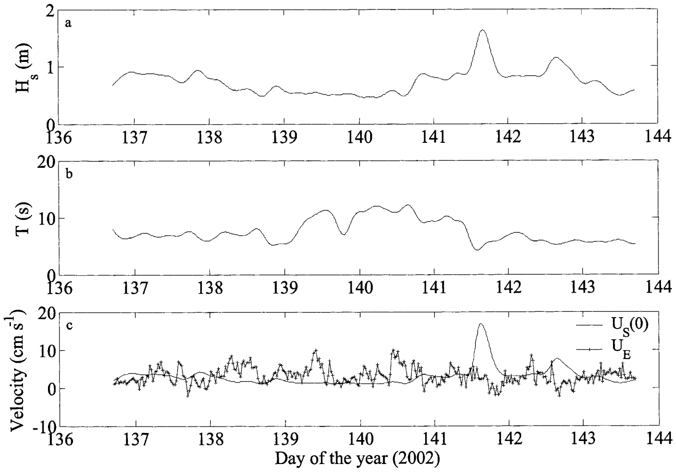

Fig. 1. Time series for May 2002 of (a) significant wave height, (b) significant wave period, and (c) Stokes drift velocity averaged over the upper 1 m of the water column and the maximum shoreward velocity measured in the upper 2 m of the water column by a 1.5 MHz ADP.

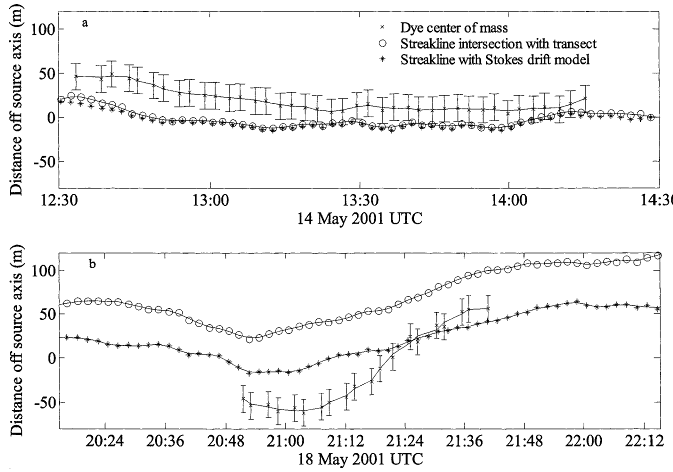

Fig. 2. Time series of plume trajectories measured at Duck, North Carolina, in May 2001. Plume center of mass 150 m downstream from source compared against streaklines, with and without Stokes drift correction, for (a) nonwavy conditions ($U_S \sim$ O(1 mm s^{-1})), and (b) wavy conditions ($U_S \sim$ O(50 mm s^{-1})). Error bars on dye center of mass reflect uncertainty in absolute AUV position while measuring dye concentrations.

The first examples are measurements taken in May 2002 as part of an experiment carried out in the environs of a kelp stand north of Santa Barbara 200 m offshore of the beach in approximately 8 m of water. During this experiment, which was aimed at understanding the effects of kelp on circulation and turbulence (to be reported elsewhere), wave measurements were made using a Seabird SBE26 Seagauge pressure logger while currents were simultaneously measured using several ADCPs (Acoustic Doppler Current Profiler) and acoustic Doppler velocimeters (ADV). As part of its suite of processing routines, the Seagauge computes the significant wave height H_s and wave period T_s from burst measurements of pressure. In this case the bursts were 240 points collected at 4 Hz every 10 min. For the sake of estimating U_S, we substituted $a = H_s/2$ and $\sigma = 2\pi/T_s$ into Eq. 2, although more refined calculations could be done from wave spectra as outlined by Kenyon (1969) and Smith (1998). Generally, the direction of wave crest propagation was perpendicular to the shore in the region of the kelp stand (J. Rosman pers. comm.) so that the direction of the Stokes drift is shoreward. Profiles of Eulerian velocities measured with a Nortek 1.5 MHz acoustic Doppler profiler (ADP, 0.25-m bins, 10-min averages) were projected on directions parallel and perpendicular to the shore, with positive velocities flowing onshore. Time series of wave elevation, period, and U_s (averaged over the upper meter of the water column) and cross-shore velocities (maximum of the upper 2 m of the water column; Fig. 1) show clear episodes during which time Stokes drift velocities near the water surface were substantial and, thus, would have a dramatic effect on the shoreward transport of materials and organisms suspended in the water column near the surface. In terms of inferring transport, neglect of the Stokes drift would have meant seriously underestimating the shoreward transport of near-surface trapped materials for several periods during the experiment.

A second case that reinforces this view and that enables us to make a direct comparison of Lagrangian transport with and without waves comes from experiments done approximately 800 m offshore of the beach at the Army Corps of Engineers Field Research Facility (FRF) in Duck, North Carolina (*see* Lentz et al. 1999). During these experiments in May 2001, carried out as part of the ONR (Office of Naval Research) program on Chemical Sensing in the Marine Environment, a continuously pumped source of Rhodamine WT dye was placed in approximately 8 m of water directly offshore of the FRF pier. The plume emanating from this source was mapped 100 to 1000 m downstream of the source using a REMUS (Remote Environment Measuring UnitS) autonomous underwater vehicle (AUV) equipped with a Seapoint dye fluorometer sampling at 9 Hz. For the same time period, velocity and wave fields were recorded. Currents were measured using a 600-kHz RD Instruments ADCP, and wave measurements were made 1 km from shore by an array of 15 pressure gauges and at the end of the FRF pier (~600 m offshore) using a pressure gauge and a Baylor staff gauge (C. Long pers. comm.). Details of the experimental protocol and hydrodynamic conditions during the experiment can be found in Fong and Stacey (2003).

Fong and Stacey (2003) focused their analysis of plume dispersion and meandering under conditions when the ambient waves were small and thus when the Stokes drift was negligible. They found, under weak wave conditions, that they were able to explain, to first order, the lateral variability in the plume's center of mass using a simple streakline model based the velocities measured by a single point ADCP. We conduct a similar analysis here for a dataset collected on 14 May 2001. Assuming that the velocity field is spatially uniform but temporally varying (in accord with the ADCP measurements near the dye source), streakline positions at a distance 150 m downstream from the source are compared with the observed center of mass of the dye concentrations measured during the same period in Fig. 2a (details of the

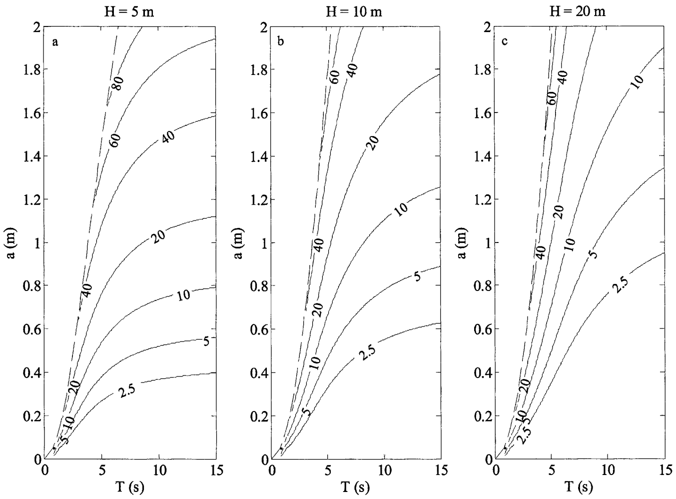

Fig. 3. Calculated values of the Stokes drift velocity (in cm s^{-1}) at the surface as a function of wave amplitude and period for three different depths: (a) $H = 5$ m; (b) $H = 10$ m; (c) $H = 20$ m. The dashed line appearing in each panel marks the approximate breaking condition $ak = 0.3$.

calculations can be found in Fong and Stacey 2003). Given the approximations of the simple model and the 15 m estimated absolute accuracy in position for the AUV (L. Sorrell pers. comm.), the streakline trajectory compares well with the observed center of mass variation.

In contrast, during conditions in which the Stokes drift was not negligible ($T = 11$ s; $a = 0.65$ m), the differences are much greater. Figure 2b shows one plume trajectory in the presence of a wave field, which exhibits significant Stokes drift (18 May 2001). The streakline model deviates significantly from the observed center of mass location. The differences between the streakline model and the center of mass position are significantly reduced, however, if one augments the measured ADCP currents with the estimated Stokes drift velocities (using measured a and σ, and wave direction inferred from the wave data; Fig. 2b). Clearly, in order to capture the shoreward migration of the plume, it is necessary to include the Stokes drift, which in fact does a reasonable job at explaining the observed translation of the plume.

Discussion—Theory and observations make clear that Stokes drift velocities can often be comparable to or larger than measured mean Eulerian velocities on the inner shelf and, because they are often directed onshore, may play an important role in, e.g., the recruitment of organisms found near or on the shore. In a like fashion, the Stokes drift associated with internal waves, which at least for linear internal waves can be computed from Eq. 1, must be taken to be a potentially important mechanism for cross-shelf transport (Shanks 1983).

The strength of the Stokes drift depends (per Eq. 2 above) on three parameters, namely, amplitude, wave period, and mean fluid depth. To help assess when Stokes drift is important, we have plotted (Fig. 3) the Stokes drift velocity at the surface ($U_S(0)$) as a function of wave amplitude and period for several depths. Note that we have excluded cases where breaking is likely to occur: $ak > 0.3$ and $a/H > 0.4$ (cf. Dean and Dalrymple 1991). For example, waves typical of the California coast off Monterey Bay have $a = 1$ m and $T = 10$ s (Gaylord et al. 2003). In 10 m of water these give $U_S(0) \approx 8$ cm s^{-1}, whereas where $H = 5$ m, i.e., closer to shore, $U_S(0) \approx 20$ cm s^{-1}. More generally, it is clear that higher frequency (and hence, for a given amplitude, steeper) waves tend to produce larger values of Stokes drift. In the preceding example, as given by the dispersion relation (Eq. 2d), the wavelength of a 10-s period wave shortens from 92 to 67 m as the wave propagates shoreward and the local depth changes from 10 to 5 m.

On the other hand, specifying a priori when the Stokes drift will be comparable to or larger than the mean Eulerian velocity is much more difficult given the wide variety of means by which nearshore flows can be forced. However, because the shore generally acts like a wall and tends to block cross-shore flows, long-shore flows tend to dominate the nearshore region in most cases. For example, in measurements of flows near the beach at Duck (e.g., Fong and Stacey 2003), several rocky coastal sites in California (Stacey et al. 2000; J. Rosman unpubl. data 2003), as well as flows over fringing coral reefs (Genin et al. 2002; R. Lowe unpubl. data) we find cross-shore flows that are generally less than 5 cm s^{-1}. More energetic, vertically sheared flows may be episodically driven by internal waves (e.g., Leichter et al. 1998) but only in stratified waters. In contrast, as surface waves approach the shore, they tend to propagate normal to the shore, and thus the Stokes drift is also directed normal to the shore. However, as shown by Andrews and McIntyre (1978), the cross-shore component of the depth-integrated mean Lagrangian flow must still be zero. Thus, if there is a shoreward Stokes drift, there must be a seaward directed compensating mean Eulerian flow that theoretically (Longuet-Higgins 1953) is distributed differently with depth than is the Stokes drift. Thus, in principle surface waves can cause vertically sheared exchange flows even in the absence of stratification.

Besides affecting the Lagrangian motions of particles, the Stokes drift also needs to be considered when formulating mass balances in wavy environments (Jay 1991). For example, consider the application of the control volume method, used by Genin et al. (2002) to study grazing on a coral reef, to the wavy environment that typifies many nearshore kelp forests. In order to properly formulate the control volume for these flows, it is necessary to account for the net advection of scalars that accompanies the wave motion. Detailed analysis given in Plumb (1979) shows that the wave-averaged local flux $\langle \mathbf{F} \rangle$ of a given scalar (e.g., carbon) can be computed as

$$\langle \mathbf{F} \rangle = (\langle \mathbf{U}_E \rangle + \mathbf{U}_S)\langle C_E \rangle \tag{3}$$

where $\langle \mathbf{U}_E \rangle$ is the wave-averaged mean Eulerian velocity (i.e., as measured by a fixed current meter), and $\langle C_E \rangle$ is the wave-averaged concentration measured at a fixed point (i.e., by a pumped sampler). As seen in both the field examples discussed above, the second term on the right-hand side of Eq. 3 can often be significant, and thus any such sampling program aimed at computing mass balances must include wave measurements.

An important facet of the analyses above is that they are based on measured currents and measured waves supplemented by simple, albeit well-tested, models of wave behavior (*see also* Smith 1998). From the standpoint of predicting transport, if waves are important (which will often be the case), circulation models like that used to model larvae transport must be supplemented by a wave model of some form. The incorporation of waves in transport predictions is made more challenging by the fact that waves and currents can act to modify each other in ways that can be complex as well as subtle. Theories have been developed to explain how waves, through the action of radiation stresses (Longuet-Higgins and Stewart 1962), can drive along-shore currents on beaches. However, many laboratory experiments show other changes in mean flows when waves are superposed (*see*, e.g., Kemp and Simons 1982). Remarkably, Nepf (1992) found reductions in mean flow that nearly matched but were opposite in direction to the Stokes drift. Extending the common ''Craik–Leibovich'' wave–current interactions to include turbulence modifications, Groeneweg and Klopman (1998) modeled these alterations with some success. In contrast, these changes in mean currents can also be explained in terms of the superposition of rotational waves, of which Gerstner waves are the best known examples, on an otherwise unchanged mean flow (Gjøsund 2000). Clearly much work remains to be done in developing accurate and complete models of wave–current interactions for the nearshore environment.

In this short paper we have focused on showing examples in which the Stokes drift of surface waves can play a significant role in scalar and particle transport. Our conclusion is that in the majority of field experiments where waves are likely important, the measurement of wave activity concurrent with Eulerian velocities will be necessary to accurately deduce dispersion and transport rates. In general, it would seem prudent when carrying out studies of transport in the nearshore coastal ocean to not assume beforehand that Stokes drift is insignificant but to make the needed wave measurements and then neglect the Stokes drift if it proves to be small in a given case. Finally, as a corollary, by the same reasoning it is clear that waves should be explicitly included in models aimed at predicting transport on the inner shelf.

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