Measurement of Short-Wave Modulation Using Fine Time-Series Optical Spectra

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ABSTRACT

Photographic imagery of the ocean surface can be either optically or digitally Fourier-transformed to obtain ocean slope spectra (Monaldo and Kasevich, 1981; Stilwell, 1969). Here we outline a straightforward method of using time-series ocean photographs to determine the modulation of short-wave energy spectra by long surface waves. Specific attention is given to the isolation of actual spectral modulation of the ocean surface from spurious modulation introduced by the imaging process.

1. Introduction

Under the proper skylight illumination conditions, photographic imagery can be obtained such that the ocean surface slope is nearly linearly related to the amplitude-transmittance of the film negative (Monaldo and Kasevich, 1981; Stilwell, 1969). The Fourier transform of such an image can then be considered proportional to the ocean-slope spectrum of that patch of the ocean enclosed in the image. Although Fourier transforms can be performed digitally (Gotwols and Irani, 1980), optical transformations performed with coherent light are more convenient when dealing with large numbers of photographic images.

By evaluating rapid-time-series images of an ocean surface patch small in size compared to long ocean waves (>15 m) and large compared to short gravity-capillary waves (<40 cm), the behavior of the short-wave spectrum with respect to long waves can be determined. Our purpose here is to outline the method of accurate photographic determination of short-wave modulation.

The measurement of short-wave modulation has two areas of specific relevance. Spaceborne synthetic aperture radars (SAR's) have demonstrated the ability to monitor long ocean waves (Beal, 1980). Since the SAR is responsive to the spectrum of 30 cm waves (Elachi and Brown, 1977), the modulation of such waves by long ocean gravity waves can render such long waves detectable. A better understanding of short-wave modulation will contribute to the determination of those circumstances where SAR wave data are reliable and how such data can best be processed to yield quantitative long-wave information.

The nature of the interaction between long and short waves also is important to the understanding of wind-wave growth mechanisms. Specifically, the interaction of long and short waves provides an energy exchange mechanism applicable to the explanation of wind-wave growth.

Pursuit of the photographic method of measuring short-wave modulation is motivated by some advantages it has over other techniques. Photographic measurement of short-wave modulation is less ambiguous than point measurements *in situ*. Since the photographic measurement is inherently spatial, there is no Doppler modulation of the short waves riding on the varying horizontal orbital velocity of the long waves to be corrected for.

As opposed to radar measurements, photographic modulation measurements can be made at a variety of short wavenumbers simultaneously. Moreover, the photographic technique allows for the measurement of the modulation of short waves not traveling parallel to the long waves.

2. Model of short-wave modulation by long waves

The parameter that has been used to characterize the nature of short-wave modulation is the modulation transfer function, $M(\omega, \kappa, U^*, \theta, \psi)$ (Keller and Wright, 1975). M is a complex number having both magnitude and phase. In general, M is a function of long-wave frequency (ω), the wavenumber associated with the spectral region being modulated (κ), the frictional wind speed (U^*), the angle between the propagation directions of the long and short waves (θ), and the angle between the short-wave propagation direction and the wind (ψ). When a particular modulation measurement is made it will be at a fixed U^*, θ, ψ, so we will suppress the explicit dependence of M on such terms for convenience.

Consider a two-scale model of the ocean: a short-wave spectrum, generated primarily by the wind, superimposed on the long gravity waves. If $S_\kappa(t)$ is the time variation in short-wave spectral energy at wavenumber κ in a small patch of ocean, then

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$$S_\kappa(t) = \langle S_\kappa \rangle \left\{ 1 + \int M(\omega)\Phi(\omega)e^{i\omega t}d\omega \right\}, \quad (1)$$

where $\langle S_\kappa \rangle$ is the time-averaged $S_\kappa(t)$ and $\Phi(\omega)$ is the long-wave slope magnitude and phase at frequency ω.

If we designate $\phi_L(t)$ as the time-series variation in long-wave slope, then $M(\omega)$ is given by

$$\hat{M}(\omega) = \hat{\gamma}_{\phi,s}\left(\frac{\hat{G}_s(\omega)}{\hat{G}_\phi(\omega)}\right)^{1/2}, \quad (2)$$

where the hat above the symbols represents the estimated values from the respective series. The estimate coherence between $S_\kappa(t)$ and $\phi_L(t)$ is $\hat{\gamma}_{\phi,s}$ and $\hat{G}_s(\omega)$ and $\hat{G}_\phi(\omega)$ are the power spectral densities of $S_\kappa(t)$ and $\phi_L(t)$, respectively (Bendat and Piersol, 1971).

3. Time-series short-wave spectra and wave slope

In order to evaluate (2) for $M(\omega)$ we require only two things: the time series variation in short-wave spectral energy and long-wave slope. These both can be determined from time-series photographic imagery.

Consider the squared Fourier transform of a single ocean image. Slice this spectrum along any radial direction. For simplicity, we will hereafter assume this slice has been taken to consider waves traveling either towards or away from the camera. From this slice select a section associated with a particular wavenumber (see Fig. 1). This section represents the short-wave spectral energy traveling in a known direction at wavenumber κ at a single instant of time. A time series of such sections of squared Fourier transforms gives $S_\kappa(t)$.

The energy at the center of the square of the Fourier transform of an image we call the DC spot energy. This energy is proportional to the square of the average slope in the photograph. We can therefore relate the square root of the DC spot to the long-wave slope in a single image. The time series variation in the square root of the DC spot gives $\phi_L(t)$.

Taking the temporal Fourier transform of $S_\kappa(t)$ and $\phi_L(t)$ and squaring, we obtain $\hat{G}_s(\omega)$ and $\hat{G}_\phi(\omega)$. Using (2) we then calculate the modulation transfer function versus ω for specific values of U^*, θ, ψ and κ.

4. Corrections to estimated modulation transfer function

Ideally, $\hat{M}(\omega)$ as calculated above would represent the extent of modulation of short-wave spectral energy density by long waves caused solely by oceanographic processes. However, there are imaging effects which cause apparent modulation that does not actually exist on the sea surface. The $\hat{M}(\omega)$ is to first order a summation of $M(\omega)_h$ which reflects

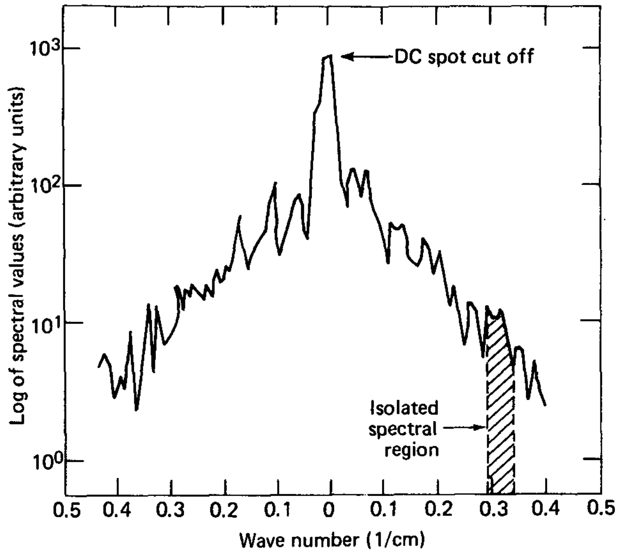

FIG. 1. A slice from a two-dimensional optical slope spectrum, where each side is a mirror image (plus noise) of the other. The center peak represents the DC spot energy. This DC spot is arbitrarily cut off because on this scale it would exceed the limits of the graph. The hatched region represents the slope spectral energy at wavenumber κ.

oceanographic conditions, geometric modulation, $M(\omega)_g$, and nonlinear modulation, $M(\omega)_n$. Hence the actual hydrodynamic modulation is given by

$$M(\omega)_h = \hat{M}(\omega) - M(\omega)_g - M(\omega)_n. \quad (3)$$

Note that all these quantities have a phase as well as magnitude. To determine the hydrodynamic interaction on the surface, it is necessary to specify $M(\omega)_g$ and $M(\omega)_n$.

Geometric modulation results from effects of photographing a small wave on the tilted surface of a long gravity wave. The Fourier-analysis process performed on the film negatives separates waves not by actual surface wavenumber (or wavelength), but by the projection of the real surface wavelength onto a plane perpendicular to the optical axis of the camera. As small waves ride on the long surface waves, their apparent wavelength as measured by the camera varies. Spectral energy will appear to be modulated by the long wave because the various tilts at various parts of the long wave will cause the measurement of different parts of the short-wave spectrum.

If κ is the actual wavenumber on the surface, then κ', the photographically measured wavenumber, equals $\kappa/\sin(\phi_0 - \phi_L)$ where ϕ_0 is the camera depression angle. The spectral energy would then be a function of measured wavenumber and long-wave slope. The spectral energy is proportional to $\kappa^{-2} = [\kappa' \sin(\phi_0 - \phi_L)]^{-2}$. We designate this quantity $S(\kappa', \phi_L)$. The geometric modulation transfer value, $M(\omega)_g$, is independent of long-wave frequency and is given by

$$M_g = \lim_{\phi_L \to 0} \left\{ \left[\frac{S(\kappa', \phi_L)}{S(\kappa', \phi_L = 0)} - 1 \right] \phi_L^{-1} \right\}, \quad (4)$$

which can be reduced to

$$M_g = \lim_{\phi_L \to 0} \left\{ \left[\left(\frac{\sin\phi_0}{\sin(\phi_0 - \phi_L)} \right)^2 - 1 \right] \phi_L^{-1} \right\}$$
$$= 2.86 e^{i0} \text{ slope}^{-1}. \quad (5)$$

The effect of nonlinear modulation is another source of spurious modulation. For surface waves traveling along any particular direction, there is a transformation between wave slope and amplitude transmittance resulting on the film. The transformation curve is not perfectly linear. There are different gain factors associated with different parts of the curve. For example, on one part of the curve a wave-slope variation of, say, 1° is transformed into an amplitude-transmittance variation of 0.05, whereas on another part of the curve the same wave-slope variation causes a transmittance variation of 0.06. A long wave passing through the patch being photographed changes the average slope in the photograph, the operating point of the transfer curve, and therefore, the multiplicative factor mapping wave slope to transmittance.

Allow ϕ_s to be the surface slope of small waves and ϕ_L to be the long-wave slope. At any point on the surface being photographed, the total slope ϕ for waves traveling along the camera azimuth, is given by

$$\phi = \phi_s + \phi_L. \quad (6)$$

The patch on the ocean enclosed by a single image is small enough that it can be assumed that within any particular photograph, ϕ_L is constant. This value of long-wave slope determines the operating point of the transfer curve. This curve can be expressed as a quadratic polynomial of the form

$$t(\phi) = t_0 + t_1\phi + t_2\phi^2. \quad (7)$$

Eq. (7) can be rewritten as

$$t(\phi) = t_0 + t_1'(\phi)\phi, \quad (8)$$

where $t'(\phi)$ equals $t_1 + t_2\phi$. For most circumstances (8) can be rewritten as

$$t(\phi) = t_0 + t_1'(\phi_L)\phi_s, \quad (9)$$

where ϕ_L, the long-wave slope, determines the gain factor $t_1'(\phi_L)$.

For short waves, a wave slope variation results in a transmittance variation of $t_1'(\phi_L)\phi_s$. The spectral energy density representing a variation of ϕ_s as measured optically is proportional to $[t_1(\phi_L)\phi_s]^2$. Modulation of spectral energy due to the nonlinear transfer curve and the long-wave slope is given by

$$M_n = \lim_{\phi_L \to 0} \left\{ \left[\left(\frac{t_1'(\phi_L)\phi_s}{t_1'(\phi_L = 0)\phi_s} \right)^2 - 1 \right] \phi_L^{-1} \right\}. \quad (10)$$

Explicit expression of terms yields

$$M_n = \lim_{\phi_L \to 0} \left\{ \left[\frac{t_1^2 + 2t_1t_2\phi_L + t_2^2\phi_L^2}{t_1^2} - 1 \right] \phi_L^{-1} \right\}. \quad (11)$$

If $(t_2^2\phi_L)$ is significantly smaller than $2t_1t_2$, i.e., the transfer curve is not strongly linear, then

$$M_n = \frac{2t_2}{t_1}. \quad (12)$$

Eq. (12) agrees with intuition. If t_2 equals zero and the transfer curve can be considered linear, there is no modulation due to nonlinear imaging effects.

By subtracting M_g and M_n from $\hat{M}$ we obtain $\hat{M}_h$, the modulation transfer function due to oceanographic process. This technique, of course, must be applied in a variety of wind conditions at different short-wave wavenumbers and propagation directions before a complete knowledge of the behavior of $M(\omega, \kappa, U^*, \theta, \psi)$ can be determined.

5. Conclusion

A method of quantifying short-wave modulation by long ocean waves with time-series photographic images is outlined. Careful attention is paid to estimating and correcting for modulation caused by geometric and nonlinear effects.

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