Optimal Power-Law Description of Oceanic Whitecap Coverage Dependence on Wind Speed

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ABSTRACT

The optimal power-law expression for the dependence of oceanic whitecap coverage fraction W on 10 m elevation wind speed U as determined by ordinary least squares fitting applied to the combined whitecap data sets of Monahan (1971) and Toba and Chaen (1973), is $W = 2.95 \times 10^{-6} U^{3.52}$. The equivalent expression, obtained by the application of the technique of robust biweight fitting, is $W = 3.84 \times 10^{-6} U^{3.41}$. These expressions fit the combined data set better than any of the previously published equations.

1. Introduction

In the interpretation of the radiances in the various short-wavelength bands measured by such remote sensing systems as the Coastal Zone Color Scanner aboard the Nimbus 7 satellite, it is necessary to account for the marked influence of whitecaps on the sea surface albedo in the various spectral bands. In at least one approach to this task, Gordon and Jacobs (1977) had recourse to the power-law description of whitecap dependence upon wind speeds given by Monahan (1971). Since the presence of whitecaps affects the apparent microwave brightness temperature of the sea surface, and this effect varies with wavelength, polarization and viewing angle (Webster *et al.*, 1976), the records from the Scanning Multichannel Microwave Radiometer on the same satellite are projected to provide a basis for determining from space the sea surface wind speed to within 2 m s^{-1} (Gloersen and Barath, 1977). With such applications in the offing, it is manifest that an accurate description of the relationship between whitecap coverage W and 10 m elevation wind speed U is needed.

Recently Wu (1979), having reanalyzed the data of Monahan (1971) and Toba and Chaen (1973), proposed a power-law expression for $W(U)$ for which he presented an essentially theoretical rationale. Wu's results, like most previously obtained expressions for $W(U)$, were obtained in the absence of a formal statistical methodology. Indeed, the only explicit published reference to a statistical treatment of whitecap data we have been able to locate appeared in Tang (1974), where he gives a $W(U)$ relationship which he credits to an unpublished report of Stogryn (1972). Tang states that Stogryn carried out a "least squares curve fit" to the whitecap data sets of Murphy (1968), Williams (1970), Rooth and Williams (1970), Monahan (1969), and Monahan (1971). From a statement in Williams (1970), and a comment in Blanchard (1971), it appears that the unpublished results of Murphy, and Williams' own conclusions, were based on a consideration of the same set of whitecap photographs (anon., 1952) that Blanchard (1963) had previously analyzed. It is not clear from Tang (1974) that Stogryn (1972) adjusted the one set of fresh-water whitecap coverage observations (Monahan, 1969) he included in his analysis in light of the shorter lifetimes of fresh-water whitecaps (Monahan and Zietlow, 1969) that result from the difference between fresh-water and salt-water whitecap bubble spectra. In light of this consideration, and the uncertainties which will be discussed shortly that surround the data base (anon., 1952) used by Murphy (1968) and Williams (1970), it is appropriate that a detailed statistical analysis of a suitable set of oceanic whitecap coverage observations be carried out to obtain an optimal expression for $W(U)$. To facilitate a comparison with the previously published results, a power-law description will in this instance be sought.

2. Previous power-law descriptions of $W(U)$

In the following paragraphs the various published expressions for $W(U)$, all in the form given by Eq. (1), are discussed:

$$W = \alpha U^{\lambda} \quad (1)$$

1) $\alpha = 4.4 \times 10^{-4}$, $\lambda = 2$, for $U > 5$ m s^{-1} ($W = 0$, for $U < 3$ m s^{-1}). This quadratic expression

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is found in Blanchard (1963), along with the remark that this implied that the wind stress was directly proportional to whitecap coverage. Blanchard obtained this result from the analysis of five photographs from a Naval Weather Squadron manual (anon., 1952) which had been prepared to aid aviators in estimating surface wind speeds. The procedure whereby these photographs were selected to represent the typical appearance of the sea surface at various wind speeds has never been published, nor has the nature of the associated sea truth wind speed measurements. In the absence of this information, questions remain about the appropriateness of this data set, as alluded to above. In this case, and the ones to follow, the α value has been set so that where the 10 m elevation wind speed U is given in m s^{-1}, the whitecap coverage W is expressed as a simple fraction of the sea surface area. The specific expression introduced in this paragraph will henceforth be referred to as expression AB.

2) $\alpha = 1.35 \times 10^{-5}$, $\lambda = 3.4$, for $4 \text{ m s}^{-1} < U < 10 \text{ m s}^{-1}$. This power-law expression, which will be referred to as EM, appears in Monahan (1971), where it is described as forming an envelope over all the data points in the designated wind range. In all, 432 photographs, taken primarily in the western North Atlantic Ocean during 70 whitecap observation intervals, were analyzed to produce these points.

3) $\alpha = 1.2 \times 10^{-5}$, $\lambda = 3.3$, for $4 \text{ m s}^{-1} < U < 10 \text{ m s}^{-1}$. Using the results from 20 fresh-water whitecap observation intervals, based on the analysis of 292 photographs (Monahan, 1969), and assuming that the fraction of the water "surface covered by whitecaps . . . is directly related to the rate of energy transfer from the air flow to the fully developed spectral components" of the waves, Cardone (1969) derived a $W(U)$ expression applicable to a fully developed fresh-water sea, where U was the wind speed at 19.5 m elevation. Taking into account the observation mentioned previously that fresh-water whitecaps have shorter lifetimes than oceanic whitecaps (Monahan and Zietlow, 1969), and making further adjustments so that the result would apply for winds measured at 10 m elevation, Monahan (1971) obtained the expression given in this paragraph, designated henceforth as CZM.

4) $\alpha = 7.75 \times 10^{-6}$, $\lambda = 3.231$, for $U < 35 \text{ m s}^{-1}$. This expression, which will be identified as AMST, is given in Tang (1974), and is the result of Stogryn's (1972) least-squares fit to a data base which was discussed in detail earlier.

5) $\alpha = 2.00 \times 10^{-6}$, $\lambda = 3.75$. This expression, designated MW, was derived by Wu (1979) from a consideration of the same data set (Monahan, 1971) that gave rise to expression EM. In this and the following two cases the λ- value, 3.75, was selected by Wu as reflecting the product of the wind dependence of the wind stress and the wind dependence of the surface drift velocity, which he equates to the wind dependence of the rate at which work is done by the wind on a unit area of the sea surface.

6) $\alpha = 1.55 \times 10^{-6}$, $\lambda = 3.75$. From the observations made by Toba and Chaen (1973) during 41 whitecap observation intervals in the East China Sea and the coastal waters of Japan, from each of which apparently four photographs were analyzed, Wu (1979) has obtained this expression which will be denoted TCW.

7) $\alpha = 1.7 \times 10^{-6}$, $\lambda = 3.75$. This expression, which will be referred to as MTCW, is the one Wu (1979) fitted to the Atlantic Ocean data of Monahan (1971), and the Pacific Ocean data of Toba and Chaen (1973), taking all the data points as one set.

3. Statistical methodology

Two alternative methods have been selected to determine optimal values of the parameters α and λ of Eq. (1), for several data sets each consisting of n wind speed measurements and the corresponding n whitecap coverage fractions, which we denote by (U_i, W_i), $i = 1, 2, \ldots, n$.

For a given λ, the standard procedure used to estimate the parameter α is ordinary least squares (OLS) fitting. The essence of this method is to find $\hat{\alpha}$, the value of α for which the quantity $R_\lambda(\alpha)$, given by Eq. (2), is a minimum. Noting that W_i is

$$R_\lambda(\alpha) = \sum_{i=1}^{n} (W_i - \alpha U_i^\lambda)^2, \qquad (2)$$

the W value determined from the analysis of photographs taken during the ith observation interval, and that αU_i^λ is the predicted value of W for this interval given by Eq. (1), it is apparent that the criterion used to determine α is the minimization of the sum of the squared errors, hence the term least squares.

For any given values of α and λ, the quantity $R_\lambda(\alpha)$ gives a measure of the goodness of fit of the particular model to the selected data set, and indeed a closely related quantity, $\text{MSE}_\lambda(\alpha)$, called

$$\text{MSE}_\lambda(\alpha) = [R_\lambda(\alpha)]/n \qquad (3)$$

the mean squared error, is often used to describe in a more intuitively apparent manner this goodness of fit. The smaller the $\text{MSE}_\lambda(\alpha)$ the better the fit of the particular model to a data set.

Since, by definition, $\hat{\alpha}$ is the value of α for which $R_\lambda(\alpha)$ is a minimum for a given λ, the OLS determined model gives the optimal fit to the data in the sense of minimizing the $\text{MSE}_\lambda(\alpha)$.

TABLE 1. MSE × 10^5 for published power-law expressions as measures of goodness-of-fit to several data sets.

$W(U)$ expression	Data set M	TC	Σ
AB	71.9	72.5	72.2
EM	88.6	105.7	95.4
CZM	29.4	34.4	31.4
AMST	4.60	6.60	5.40
MW	4.80	—	—
TCW	—	6.31	—
MTCW	—	—	4.68

It is possible that in certain situations the OLS approach may seriously misrepresent a data set, in particular where the data suggest departures from certain basic assumptions. Gaver (1979) describes some of these problems with specific reference to oceanographic data, and summarizes a technique of robust regression described by Mosteller and Tukey (1977). This technique, termed biweight fitting, provides an alternative procedure for estimating the parameter α of Eq. (1). It consists of an iterative application of OLS, with successive calculation of weights which reduce the influence of outliers in the original data set, thereby yielding an estimate $\tilde{\alpha}$ of α which better characterizes the main body of the data. The quantity MSE_λ $(\tilde{\alpha})$ can also be calculated for the $W(U)$ expressions obtained by applying the robust biweight fitting (RBF) approach to the several data sets.

It is to be noted that by the definition of the OLS method, the MSE_λ $(\hat{\alpha})$ (i.e., that corresponding to the OLS estimate $\hat{\alpha}$) for a given λ and data set will be less than the MSE_λ $(\tilde{\alpha})$ calculated for the $W(U)$ obtained by the RBF approach. But, as will be seen, in most instances the MSE_λ $(\hat{\alpha})$ is not substantially less than the MSE_λ $(\tilde{\alpha})$.

In the preceding discussion two techniques for estimating α for specific data sets were described. Now, by treating the MSE_λ (α) for each estimation procedure as a function of the parameter λ, and by applying algorithm AS 47 (O'Neill, 1971) which minimizes a function of N variables, the value of λ was determined which yielded the minimum MSE_λ (α). In this manner the λ and α values corresponding to the optimal $W(U)$ expressions for each estimation procedure, and each data set, were obtained.

It should be noted that since we are dealing in the present instance with a single explanatory variable λ a plot of MSE_λ (α) for a range of λ values would be sufficient to determine the optimal value of λ for a given estimation procedure and data set. However, in more complicated W models, e.g., those including two or more explanatory variables, the use of algorithm AS 47 (O'Neill, 1971) will greatly facilitate the determination of the optimal parameter values.

4. Results

Before presenting the optimal $W(U)$ power-law expressions determined by the application of the statistical methods described in the previous section, it is appropriate to first consider the goodness-of-fit of the seven previously published $W(U)$ expressions. To this end the MSE_λ (α) has been calculated for each of these expressions, with respect to one or more data sets. The data sets selected were the Atlantic Ocean whitecap observations of Monahan (1971) and the Pacific Ocean whitecap observations of Toba and Chaen (1973). In both of these studies the whitecap photographs were analyzed using the procedure set forth in Monahan (1969), and thus the results are directly comparable. Following the approach of Wu (1979), all observations where the mean $W(U)$ value was zero have been omitted from the statistical treatments. Thus only the 54 non-zero $W(U)$ points from Monahan (1971), henceforth referred to as data set M, and the 36 non-zero $W(U)$ points from Toba and Chaen (1973), subsequently referred to as data set TC, have been used in calculating the MSE_λ (α) values that appear in Table 1. $W(U)$ expressions AB, EM, CZM and AMST have been tested against data sets M and TC, and against the combined data set Σ. In the case of the $W(U)$ expressions given in Wu (1979), they have each been evaluated using the data set for which they were specifically intended, i.e., expression MW is measured against data set M, expression TCW against data set TC, and expression MTCW against data set Σ. The points corresponding to data sets M and TC are plotted on Fig. 1, as are expressions AB, EM, CZM, AMST and MTCW.

The α and λ values defining the optimal $W(U)$ power-law expressions determined by the OLS method, for data sets M, TC and Σ, are listed in Table 2, as are the MSE_λ (α) for each of these expressions. The same quantities, obtained by the application of the RBF procedure, are to be found in Table 3.

5. Conclusions

From the MSE_λ (α) values listed in Table 1, it is apparent that of all the previously published $W(U)$ expressions, AMST provides the best description of data set M, while expression TCW best fits data set TC, and expression MTCW is the most appropriate one for the combined data set Σ.

A comparison of the MSE_λ (α) values given in Table 2 with those listed in Table 1 shows that the

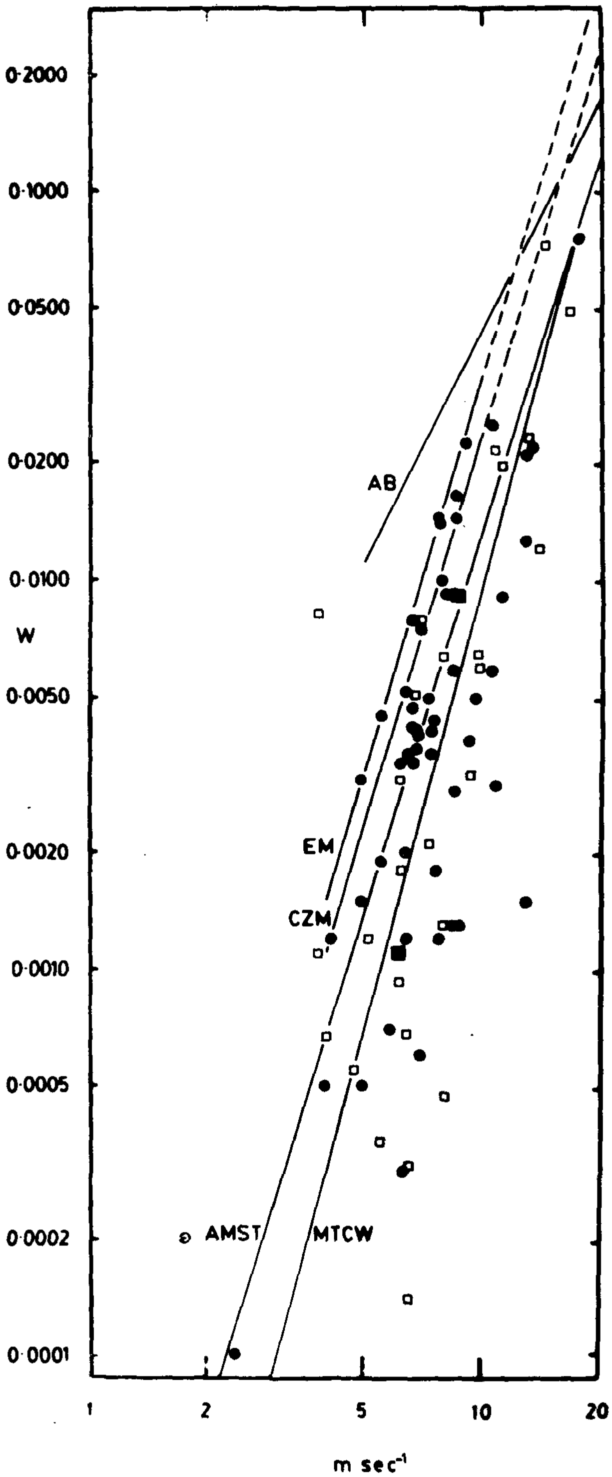

FIG. 1. The fraction W of the ocean covered by whitecaps versus the 10 m elevation wind speed U. Filled circles, mean whitecap coverage fractions for specific observation intervals from Monahan (1971); open squares, whitecap coverage values from Toba and Chaen (1973). Labeled lines represent various power-law expressions for $W(U)$ described in the text.

TABLE 2. Optimal $W(U)$ expressions from OLS fitting.

Data set	α	λ	MSE $\times 10^5$
M	1.53×10^{-6}	3.75	3.47
TC	5.56×10^{-6}	3.3	6.06
Σ	2.95×10^{-6}	3.52	4.56

$W(U)$ expressions obtained by the statistical procedure of ordinary least squares fit the several data sets better than any of the previously published expressions. While the previous best fit to data set M was afforded by expression AMST, the OLS optimal $W(U)$ expression given in Table 2 yields a 32.6% smaller $MSE_\lambda(\alpha)$. For data set TC, the $MSE_\lambda(\alpha)$ associated with TCW, the previously best fitting expression, is 4.1% larger than the $MSE_\lambda(\alpha)$ obtained for the OLS-derived expression. The optimal $W(U)$ power-law equation for the combined data set, obtained by the application of OLS, has a $MSE_\lambda(\alpha)$ that is 2.6% smaller than the $MSE_\lambda(\alpha)$ calculated for expression MTCW. Both the OLS-derived optimal $W(U)$ expression for data set Σ, and expression MTCW, are plotted on Fig. 2.

It can be seen from a comparison of the $MSE_\lambda(\alpha)$ values listed in Table 3 with those found in Tables 1 and 2 that the optimal $W(U)$ expressions obtained by the procedure of robust biweight fitting are almost as good, in the sense of minimization of $MSE_\lambda(\alpha)$, as the expressions obtained from OLS fitting, and in all cases better than those previously published.

On the basis of the results presented in the previous section, it is recommended that the $W(U)$ expression given in either

$$W_{OLS} = 2.95 \times 10^{-6}\, U^{3.52}, \quad (4)$$

or

$$W_{RBF} = 3.84 \times 10^{-6}\, U^{3.41} \quad (5)$$

be adopted for the estimation of the fraction of the ocean surface covered by whitecaps from a measurement of wind speed at 10 m elevation.

In order to assess the sensitivity of the goodness-of-fit to variations in the power-law exponent λ, the $MSE_\lambda(\alpha)$ was calculated for the optimal values of α, as determined by OLS fitting applied to the combined Σ data set as λ was varied from 2 to 4. This calculation was repeated using the optimal val-

TABLE 3. Optimal $W(U)$ expressions from robust biweight fitting.

Data set	α	λ	MSE $\times 10^5$
M	2.53×10^{-6}	3.55	3.50
TC	5.57×10^{-6}	3.3	6.06
Σ	3.84×10^{-6}	3.41	4.59

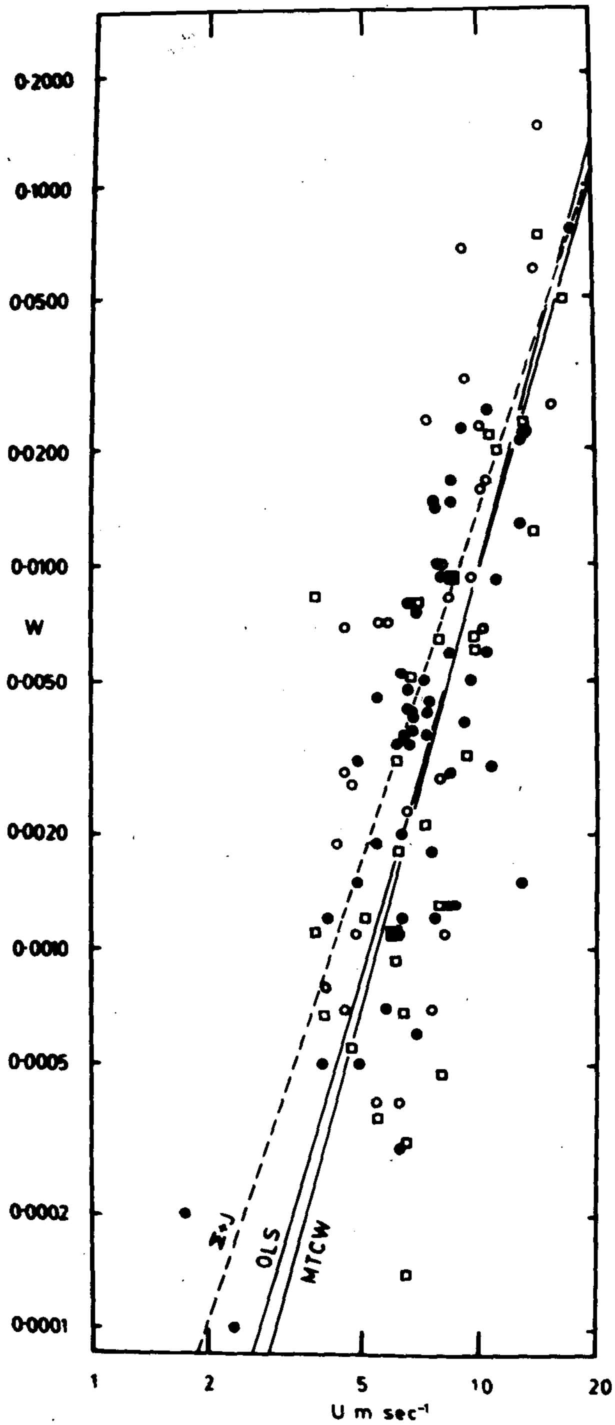

FIG. 2. The fraction W of the ocean covered by whitecaps versus the 10 m elevation wind speed U. Filled circles and open squares as defined for Fig. 1. Open circles, preliminary whitecap coverage values from 1978 JASIN experiment. Line MTCW is described in text. Line OLS is optimal power-law expression for $W(U)$ based on ordinary least-squares fitting applied to combined data of Monahan (1971) and Toba and Chaen (1973). Line $\Sigma + J$ is $W(U)$ expression obtained by applying ordinary least-squares fitting to this combined data set plus whitecap data from the 1978 JASIN experiment.

ues of α resulting from the application of RBF to the Σ data set. These results are depicted in Fig. 3. While the OLS MSE_λ (α) varies smoothly with λ, the RBF MSE_λ (α) is seen to vary irregularly with λ, reflecting the ad hoc character of this method. The less-than-optimal nature of Wu's λ of 3.75 in the context of the combined Σ data set is apparent from this figure.

The thermal stability of the lower atmosphere, as indicated by the water-air temperature difference, Δt, has been identified as a factor influencing oceanic whitecap coverage (Monahan, 1969), but the nature of this effect has recently been brought into question (Wu, 1979). The rise velocity of bubbles in the sea is influenced by the water temperature, t_w (Blanchard, personal communication), and therefore the mean lifetimes of whitecaps must alter with changes in sea water temperature. Any such factor that changes the mean lifetimes of whitecaps will affect oceanic whitecap coverage, as was pointed out by Monahan and Zeitlow (1969) in their comparison of whitecapping on lakes and oceans. In light of the above, it is evident that subsequent analyses of data sets M and TC, and new data sets,

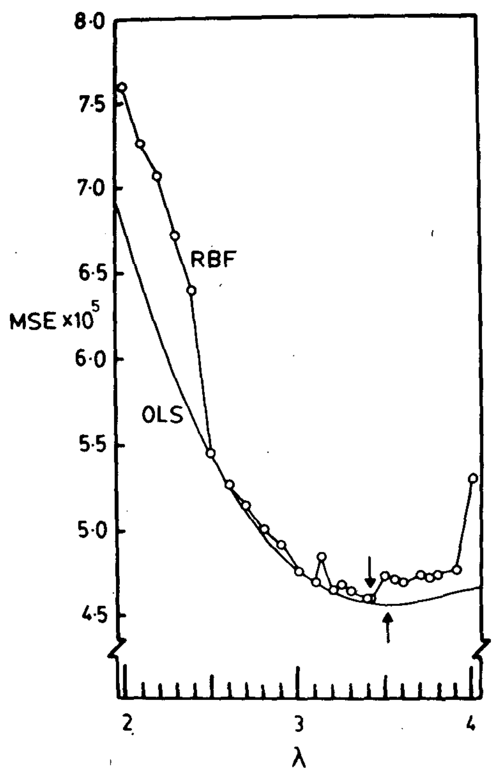

FIG. 3. The mean squared error MSE for optimal values of power-law coefficient α versus power-law exponent λ. OLS, as determined by ordinary least-squares fitting applied to combined data of Monahan (1971) and Toba and Chaen (1973); and RBF, as determined by application of robust bi-weight fitting to same combined data set. Arrows mark respective minima.

should be directed at elucidating the explicit dependence of whitecap coverage on Δt and t_w, i.e., at arriving at an optimal $W(U,\Delta t,t_w)$ expression.

It is appropriate to note that the sea water temperatures associated with 46 of the 54 points in data set M, and with all 36 points in data set TC, fell between 20° and 30°C. When the data from the initial analysis of whitecap photographs taken during the 1978 JASIN experiment, where the water temperature during all 29 non-zero whitecap coverage observation intervals was between 12° and 14°C, are combined with data sets M and TC and a new optimal $W(U)$ is obtained by the OLS method, it is found to vary markedly from the expressions of Eqs. (4) and (5) as can be seen in Fig. 2 where this $W(U)$ is included. This marked difference may be a reflection, at least in part, of the effect of sea water temperature on whitecap coverage.

It is generally recognized that whitecap coverage is nil for wind speeds less than 3 m s^{-1} (Blanchard, 1963; Gathman and Trent, 1968; Monahan, 1971). The mechanical tearing away of wave crests, with the resultant formation of spume lines, has been put forth as an additional mechanism of white water formation that becomes important at wind speeds above 9 m s^{-1} (Ross and Cardone, 1974). A consideration of these factors suggests strongly that the use of a more complex form for $W(U)$ than a simple power-law is required to describe more precisely the dependence of W upon U. The widely used statistical treatment for choice of functional form described by Box and Cox (1964) was not used in the present determination of the optimum $W(U)$ expression as such a power-law formulation does not permit the removal of an additive constant, contrary to the requirements of the Box and Cox approach (Schlesselman, 1971). In arriving at an ideal complex form of $W(U)$, one involving a finite intercept, the use of the Box and Cox approach is feasible, and should shed light on the effect of heteroskedasticity on the estimation of parameters.

In formulating more sophisticated expressions for $W(U,\Delta t,t_w)$ the dependence of apparent whitecap coverage on the limits of optical resolution of the measuring system, both in terms of size and color contrast, should be made explicit.

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REFERENCES

Anonymous, 1952: Wind estimations from aerial observations of sea conditions. (unpublished report) Weather Squadron Two (VJ-2), Jacksonville, FL.

Blanchard, D. C., 1963: The electrification of the atmosphere by particles from bubbles in the sea. *Progress in Oceanography*, Vol. 1, Pergamon Press, 71–202.

——, 1971: Whitecaps at sea. *J. Atmos. Sci.*, **28**, 645.

Box, G. E. P., and D. R. Cox, 1964: An analysis of transformations. *J. Roy. Statis. Soc.*, **B26**, 211–243.

Cardone, V. J., 1969: Specification of the wind distribution in the marine boundary layer for wave forecasting. Tech. Rep. GSL-69-1, New York University, 1–131.

Gathman, S., and E. M. Trent, 1968: Space charge over the open ocean. *J. Atmos. Sci.*, **25**, 1075–1079.

Gaver, D. P., 1979: Statistical methods of probable use for understanding remote sensing data. Tech. Rep. NPS-79-020, Naval Postgraduate School, 1–35.

Gloersen, P., and F. T. Barath, 1977: A scanning multichannel microwave radiometer for Nimbus-G and SEASAT-A. *IEEE J. Ocean. Eng.*, **OE-2**, 172–178.

Gordon, H. R., and M. M. Jacobs, 1977: Albedo of the ocean-atmosphere system: influence of sea foam. *Appl. Opt.*, **16**, 2257–2260.

Monahan, E. C., 1969: Fresh water whitecaps: *J. Atmos. Sci.*, **26**, 1026–1029.

——, 1971: Oceanic whitecaps. *J. Phys. Oceanogr.*, **1**, 139–144.

——, and C. R. Zietlow, 1969: Laboratory comparisons of fresh-water and salt-water whitecaps. *J. Geophys. Res.*, **74**, 6961–6966.

Mosteller, F., and J. W. Tukey, 1977: *Data Analysis and Regression*. Addison-Wesley, 564 pp.

Murphy, H., 1968: Percentage foam vs wind velocity. Internal report, University of Miami, Coral Gables, 27 pp.

O'Neill, R., 1971: Function minimization using a simplex procedure. Algorithm AS47. *Appl. Statis.*, **20**, 338–345.

Rooth, C., and G. Williams, 1970: Microwave radiometry of the ocean. Quarterly report, University of Miami, Coral Gables, 69 pp.

Ross, D. B., and V. Cardone, 1974: Observations of oceanic whitecaps and their relation to remote measurements of surface wind speed. *J. Geophys. Res.*, **79**, 444–452.

Schlesselman, J., 1971: Power families: A note on the Box and Cox transformation. *J. Roy. Statis. Soc.*, **B33**, 307–311.

Stogryn, A., 1972: A study of radiometric emission from a rough sea surface. NASA Contract Report CR-2088, 278 pp.

Tang, C. C. H., 1974: The effect of droplets in the air-sea transition zone on the sea brightness temperature. *J. Phys. Oceanogr.*, **4**, 579–593.

Toba, Y., and M. Chaen, 1973: Quantitative expression of the breaking of wind waves on the sea surface. *Rec. Oceanog. Works Japan*, **12**, No. 1, 1–11.

Webster, W. J., T. T. Wilheit, D. B. Ross, and P. Gloersen, 1976: Spectral characteristics of the microwave emission from a wind-driven foam-covered sea. *J. Geophys. Res.*, **81**, 3095–3099.

Williams, G. F., 1970: Comments on "Fresh water whitecaps" *J. Atmos. Sci.*, **27**, 1220.

Wu, J., 1979: Oceanic whitecaps and sea state. *J. Phys. Oceanogr.*, **9**, 1064–1068.