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A direct comparison of a depth-dependent Radiation stress formulation and a Vortex force formulation within a three-dimensional coastal ocean model

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ABSTRACT

In this study a model system consisting of the three-dimensional General Estuarine Transport Model (GETM) and the third generation wind wave model SWAN was developed. Both models were coupled in two-way mode. The effects of waves were included into the ocean model by implementing the depth-dependent Radiation stress formulation (RS) of Mellor (2011a) and the Vortex force formulation (VF) presented by Bennis et al. (2011). Thus, the developed model system offers a direct comparison of these two formulations. The enhancement of the vertical eddy viscosity due to the energy transfer by white capping and breaking waves was taken into account by means of injecting turbulent kinetic energy at the surface. Wave-current interaction inside the bottom boundary layer was considered as well.

The implementation of both wave-averaged formulations was validated against three flume experiments. One of these experiments with long period surface waves (swell), had not been evaluated before. The validation showed the capability of the model system to reproduce the three-dimensional interaction of waves and currents. For the flume test cases the wave-induced water level changes (wave set-up and set-down) and the corresponding depth-integrated wave-averaged velocities were similar for RS and VF. Both formulations produced comparable velocity profiles for short period waves. However, for large period waves, VF overestimated the wave set-down near the main breaking points and RS showed artificial offshore-directed transport at the surface where wave shoaling was taking place. Finally the validated model system was applied to a realistic barred beach scenario. For RS and VF the resulting velocity profiles were similar after being significantly improved by a roller evolution method.

Both wave-averaged formulations generally provided similar results, but some shortcomings were revealed. Although VF partly showed significant deviations from the measurements, its results were still physically reasonable. In contrast, RS showed unrealistic offshore-directed transport in the wave-shoaling regions and close to steep bathymetry.

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1. Introduction

The interaction of surface wind waves and slowly varying currents in shallow coastal oceans has been the focus of many studies. Since the fundamental paper of Longuet-Higgins and Stewart (1962), various aspects of these interactions have been studied Bowen, 1969; Craik and Leibovich, 1976; Garrett, 1976; Hasselmann, 1971; Phillips, 1977. Different observations supported the importance of the surface wave effects in shallow waters and near-shore regions. Wolf and Prandle (1999) concluded that the maximum effects of waves with periods longer than 6 s take place in depths shallower than 20 m. Furthermore, the necessity of considering surface waves with a significant wave height

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greater than 1 m was emphasized by Prandle et al. (2000). However, within the framework of practical ocean modeling applications surface waves are not resolved. Therefore, their effect on the resolved wave-averaged flow must be included by additional forcing terms. These terms depend on wave properties that can be provided by a wave model coupled to the ocean model.

For the depth-integrated Navier–Stokes equations the additional forcing can be derived as a net wave-induced momentum flux, represented by the divergence of a 2D Radiation stress tensor (Longuet-Higgins and Stewart, 1964). Mellor (2003) extended this concept to a depth-dependent Radiation stress tensor and presented a closed set of equations for the description of 3D wave-current interaction. However, due to a mistake in the transformation of the horizontal pressure gradient to sigma-coordinates, the Radiation stress tensor derived by Mellor (2003) was wrong. Mellor (2008) derived another depth-dependent Radiation stress tensor including a singular surface term. This term was necessary for the consistency with the depth-integrated Radiation stress tensor





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of Longuet-Higgins and Stewart (1964), but it originates in an inconsistent treatment of the pressure (Bennis and Ardhuin, 2011). Furthermore, within the momentum equations of Mellor (2008) the horizontal derivatives of the elements of the Radiation stress tensor were taken in *z*-coordinates, resulting in depth-integrated equations inconsistent with Phillips (1977). Finally, based on the correct transformation of the horizontal pressure gradient in the equations of Mellor (2003) and with the Radiation stress tensor of Mellor (2008), Mellor (2011a) presented a closed set of equations in sigma-coordinates consistent with Longuet-Higgins and Stewart (1964) and Phillips (1977).

A general framework for the derivation of 3D wave-averaged equations is provided by the Generalised Lagrangian Mean (GLM) theory of Andrews and McIntyre (1978b). Within the GLM theory a Lagrangian average (obtained along a Lagrangian trajectory) is referenced to the corresponding averaged position to provide a description within a Eulerian framework. The difference between the Lagrangian and Eulerian average (with the latter taken at a fixed position) defines the corresponding Stokes correction. Andrews and McIntyre (1978b) derived two equivalent sets of exact equations for both the Lagrangian and the quasi-Eulerian averaged velocity ($\overline{\mathbf{u}}^{L}$ and $\overline{\mathbf{u}}^{QE}$ respectively). The latter was named by Jenkins (1989) and is defined as the difference between the Lagrangian averaged velocity and the (specific) pseudomomentum of the waves (Andrews and McIntyre, 1978a). To lowest order the pseudomomentum of the waves differs from the Stokes drift $\overline{\mathbf{u}}^{\text{Stokes}} = \overline{\mathbf{u}}^{\text{L}} - \overline{\mathbf{u}}^{\text{E}}$ only by the vertical shear of the Eulerian averaged velocity $\overline{\mathbf{u}}^{\mathrm{E}}$ (Ardhuin et al., 2008).

As outlined by Andrews and McIntyre (1978b), the GLM equations for the Lagrangian averaged velocity are forced by the divergence of a tensor that can indeed be identified with a depthdependent Radiation stress tensor under certain conditions. On the other hand, based on the GLM equations for the quasi-Eulerian averaged velocity Leibovich (1980) could rederive the equations of Craik and Leibovich (1976) containing the Vortex force $\overline{\mathbf{u}}^{\text{Stokes}} \times (\nabla \times \overline{\mathbf{u}}^{\text{E}})$ for the description of Langmuir circulations. In this context the Radiation stress and Vortex force concepts seem to be formally equivalent. However, for practical use the different forcing terms within each set of GLM equations must be consistently closed up to a given order. As argued by Ardhuin et al. (2008) and Bennis et al. (2011), a consistent closure of the GLM equations for the Lagrangian averaged velocity is rather impractical. Furthermore, Lane et al. (2007) showed that the lowest order Radiation stress equations are asymptotically inconsistent, because they do not capture all dynamics of the corresponding Vortex force equations.

Based on the GLM equations for the quasi-Eulerian averaged velocity Ardhuin et al. (2008) derived a closed set of equations that is consistent with the one of McWilliams et al. (2004) derived earlier by multiple asymptotic scale analysis. The latter was validated with the Regional Ocean Modeling System (ROMS; Uchiyama et al., 2010; Kumar et al., 2012). Recently, Michaud et al. (2012) validated the formulation of Ardhuin et al. (2008) with the ocean model SYMPHONIE. They also confirmed, that for weak vertical shear, the simplified forcing proposed by Bennis et al. (2011) is sufficient. Despite the issues mentioned above regarding the Radiation stress concept, the different sets of equations proposed by Mellor are widely used (CH3D: Sheng and Liu, 2011: FVCOM: Wang and Shen, 2010). Kumar et al. (2011) implemented the Radiation stress formulation of Mellor (2011a) into ROMS and confirmed that the equations captured the dynamics in the surfzone. However, they also stressed the occurrence of spurious flows in shoaling regions and due to the singular surface term in the Radiation stress tensor.

In the present study the validity of the Radiation stress formulation of Mellor (2011a) and the Vortex force formulation of Bennis et al. (2011) was investigated. Therefore, both formulations were implemented into the General Estuarine Transport Model (GETM; Burchard and Bolding, 2002). Coupled to the third generation wind wave model SWAN, several simulations were performed to validate the model system and to directly compare both formulations. The validation was carried out against measurements obtained in different flume experiments, representing wave regimes ranging from short period waves to swell, and on a realistic barred beach.

The outline of the paper is as follows: the model system is described in Section 2. In Section 3 the test cases are presented. Conclusions are given in Section 4.

2. The model system

The model system developed in this study consists of the 3D coastal ocean model GETM (Burchard and Bolding, 2002) and the third generation wind wave model SWAN (Booij et al., 1999; Booij et al., 2004). The data exchange between both models is realized by the Model Coupling Toolkit (MCT; Jacob et al., 2005). Providing the water level and the ambient current to SWAN, GETM receives mean wave properties (obtained by spectral integration) like significant wave height H_s , wave length λ , relative wave period T, wave direction $\overline{\theta}$, orbital velocity at the bottom as well as the dissipation rates due to bottom friction $S^{ds,b}$, surface wave breaking $S^{ds,br}$ and white capping $S^{ds,w}$ (Booij et al., 2004).

The inclusion of wave effects into GETM requires the modification of its governing equations. The transformation to a general vertical coordinate *s* (Kasahara, 1974), representing the bottom topography with depth -D(x, y) and the free surface with elevation $\eta(x, y, t)$ by coordinate lines, is carried out. The vertical space is discretised into n_{\max} arbitrary layers with interfaces at $z_{n+\frac{1}{2}}(x, y, t)$ for $n \in [0, n_{\max}]$ and with $z_{\frac{1}{2}} = -D$ and $z_{n_{\max}+\frac{1}{2}} = \eta$. With layer heights $h_n = z_{n+\frac{1}{2}} - z_{n-\frac{1}{2}}$ and center positions $z_n = \frac{1}{2} \left(z_{n-\frac{1}{2}} + z_{n+\frac{1}{2}} \right)$ for $n \in [1, n_{\max}]$ layer-integrated equations can be derived (Burchard and Petersen, 1997). Note that, unless marked by $()_z$, all partial derivatives refer to the same layer (with a constant value of the general vertical coordinate *s*) instead of to constant *z* due to the vertical coordinate transformation.

The continuity equation is given by:

$$\mathbf{0} = \frac{\partial h_n}{\partial t} + \frac{\partial}{\partial x} \left\{ h_n u_n^{\text{mass}} \right\} + \frac{\partial}{\partial y} \left\{ h_n v_n^{\text{mass}} \right\} + \left(w_{n+\frac{1}{2}}^s - w_{n-\frac{1}{2}}^s \right), \tag{1}$$

In (1) u_n^{mass} and v_n^{mass} are the layer-averaged horizontal mass transport velocities. Within the wave-averaged equations these are always given by the Lagrangian wave-averaged velocities $(u_n^{\text{mass}}, v_n^{\text{mass}}) = (\overline{u}_n^{\text{L}}, \overline{v}_n^{\text{L}})$. The grid-related vertical velocity w^s obeys the kinematic boundary conditions $w_{\frac{1}{2}}^s = w_{k_{\text{max}}+\frac{1}{2}}^s = 0$. Under the Boussinesq and hydrostatic pressure approximation the momentum equations can be written as:

$$\frac{\partial}{\partial t} \{h_n u_n\} + \frac{\partial}{\partial x} \{h_n u_n^{\text{mass}} u_n\} + \frac{\partial}{\partial y} \{h_n v_n^{\text{mass}} u_n\}
+ \left(w_{n+\frac{1}{2}}^s u_{n+\frac{1}{2}} - w_{n-\frac{1}{2}}^s u_{n-\frac{1}{2}}\right)
= fh_n v_n^{\text{mass}} - h_n g \frac{\partial \eta}{\partial x} + F_{x,n}^{\text{ip}} + F_{x,n}^{\text{fric}} + F_{x,n}^{\text{wave}},$$
(2a)

$$\frac{\partial}{\partial t} \{h_n v_n\} + \frac{\partial}{\partial x} \{h_n u_n^{\text{mass}} v_n\} + \frac{\partial}{\partial y} \{h_n v_n^{\text{mass}} v_n\}
+ \left(w_{n+\frac{1}{2}}^s v_{n+\frac{1}{2}} - w_{n-\frac{1}{2}}^s v_{n-\frac{1}{2}}\right)
= -fh_n u_n^{\text{mass}} - h_n g \frac{\partial \eta}{\partial y} + F_{y,n}^{\text{ip}} + F_{y,n}^{\text{fric}} + F_{y,n}^{\text{wave}}.$$
(2b)

In Eqs. (2a) and (2b) f and g are the inertial frequency and the gravitational acceleration. $(F_{x,n}^{i,p}, F_{y,n}^{i,p})$ represent internal pressure gradients caused by density differences related to a reference den-

sity ρ_0 . Friction is incorporated by $(F_{x,n}^{\text{fric}}, F_{y,n}^{\text{fric}})$. Additional forcing induced due to nonresolved surface waves is included by $(F_{x,k}^{\text{wave}}, F_{y,k}^{\text{wave}})$.

As shown in Table 1, the prognostic horizontal velocities (u_n, v_n) and the corresponding forcing terms depend on the choice of formulation. Within the Radiation stress formulation the prognostic velocities are the Lagrangian wave-averaged velocities $(\overline{u}_n^L, \overline{v}_n^L)$. For comparison with measurements and with results from the Vortex force formulation, the corresponding Eulerian wave-averaged velocities $(\overline{u}_n^E, \overline{v}_n^E)$ can be calculated based on the Stokes drift $(\overline{u}_n^{\text{Stokes}}, \overline{v}_n^{\text{Stokes}})$. For a monochromatic wave the Stokes drift is given by:

$$\overline{u}_{n}^{\text{Stokes}} = \overline{u}_{n}^{\text{L}} - \overline{u}_{n}^{\text{E}} \approx 2\frac{k_{x}E}{c} \frac{\cosh\left(2\|\mathbf{k}\|(z_{n}+H)\right)}{\sinh\left(2\|\mathbf{k}\|(\eta+H)\right)},\tag{3a}$$

$$\overline{\nu}_{n}^{\text{Stokes}} = \overline{\nu}_{n}^{\text{L}} - \overline{\nu}_{n}^{\text{E}} \approx 2 \frac{k_{y}E}{c} \frac{\cosh\left(2\|\mathbf{k}\|(z_{n}+H)\right)}{\sinh\left(2\|\mathbf{k}\|(\eta+H)\right)}.$$
(3b)

In (3a) and (3b) $E = \frac{1}{16}gH_s^2$, $c = \frac{\lambda}{T}$, $\|\mathbf{k}\| = \frac{2\pi}{\lambda}$ and $(k_x, k_y) = \|\mathbf{k}\|(\cos \overline{\theta}, \sin \overline{\theta})$ are the (specific) wave-averaged wave energy, the wave celerity, the wave number and the corresponding directional components.

The prognostic velocities within the Vortex force formulation are given by the quasi-Eulerian wave-averaged velocities $(\overline{u}_n^{\rm QE}, \overline{v}_n^{\rm QE})$. Below the troughs the quasi-Eulerian wave-averaged velocities differ from the Eulerian wave-averaged velocities to lowest order only due to the vertical shear of the Eulerian wave-averaged velocity. Thus, for weak vertical shear the comparison of the quasi-Eulerian wave-averaged velocities integrated within the Vortex force formulation with the Eulerian wave-averaged velocities diagnosed from the Radiation stress formulation and obtained by fix-point measurements is reasonable.

2.1. Details of the Radiation stress formulation

Within the Radiation stress formulation of Mellor (2011a) the prognostic velocities can be interpreted as Lagrangian wave-averaged velocities (related to Eulerian wave-averaged velocities within a wave-following vertical coordinate). The forcing term given in Table 1 is obtained as the divergence of a depth-dependent Radiation stress tensor (Mellor, 2008, 2011b) with:

$$(f^{cc}f^{cs})_n = \frac{1}{h_n} \int_{z_{n-\frac{1}{2}}}^{z_{n+\frac{1}{2}}} f^{cc}(z+D) f^{cs}(z+D) dz,$$
(4a)

$$(f^{ss}f^{sc})_n = \frac{1}{h_n} \int_{z_{n-\frac{1}{2}}}^{z_{n+\frac{1}{2}}} f^{ss}(z+D) f^{sc}(z+D) dz$$
(4b)

and:

$$f^{\rm cc}(\zeta) = \frac{\cosh\left(\|\mathbf{k}\|\zeta\right)}{\cosh\left(\|\mathbf{k}\|(\eta+D)\right)},\tag{5a}$$

$$f^{\rm cs}(\zeta) = \frac{\cosh\left(\|\mathbf{k}\|\zeta\right)}{\sinh\left(\|\mathbf{k}\|(\eta+D)\right)},\tag{5b}$$

$$f^{\rm ss}(\zeta) = \frac{\sinh\left(\|\mathbf{k}\|\zeta\right)}{\sinh\left(\|\mathbf{k}\|(\eta+D)\right)},\tag{5c}$$

$$f^{\rm sc}(\zeta) = \frac{\sinh\left(\|\mathbf{k}\|\zeta\right)}{\cosh\left(\|\mathbf{k}\|(\eta+D)\right)}.$$
(5d)

The function $f_n^{\text{RS}}(\zeta)$ appearing in the last term of $F_{\alpha,n}^{\text{wave}}$ for the radition stress method (see Table 1) is still subject to discussion. Mellor (2008) proposed $f_n^{\text{RS}} = \delta_{n,n_{\text{max}}}$. In contrast, Kumar et al. (2011) applied a smooth distribution in the vertical to decrease spurious flow in shoaling regions.

2.2. Details of the Vortex force formulation

The Vortex force formulation of Ardhuin et al. (2008) is based on the GLM equations for the quasi-Eulerian wave-averaged velocities. The Vortex force does not appear explicitly, because its contributions are incorporated into the advection term and the dynamic pressure. For weak vertical shear the forcing was simplified according to Bennis et al. (2011). The terms $F_{\alpha,k}^{ds}$ represent sources of momentum transferred from the waves due to dissipation by bottom friction $S^{ds,b}$, surface breaking $S^{ds,br}$ and white capping $S^{ds,w}$:

$$F_{\alpha,n}^{\mathrm{ds}} = \frac{k_{\alpha}g}{\|\mathbf{k}\|c} \left(f_n^{\mathrm{ds},\mathrm{b}}(z_n + D)S^{\mathrm{ds},\mathrm{b}} + f_n^{\mathrm{ds},\mathrm{s}}(\eta - z_n)S^{\mathrm{ds},\mathrm{s}} \right),\tag{6}$$

with:

$$S^{ds,s} = S^{ds,br} + S^{ds,w}.$$
 (7)

The empirical functions $f_n^{ds,b}(\zeta)$ and $f_n^{ds,s}(\zeta)$ distribute these forces in the vertical Uchiyama et al. (2010).

2.3. Wave-enhanced bottom friction

To account for the generated turbulence at the bottom due to the nonresolved oscillating wave motions, an enhanced bottom roughness length z_0^0 is obtained as a function of the base roughness z_0 and wave properties (e.g. the bottom orbital velocity of the waves) according to Styles and Glenn (2000).

2.4. Wave-enhanced turbulence

For both wave-averaged formulations the terms $(F_{x,n}^{\text{fric}}, F_{y,n}^{\text{fric}})$ in Eqs. (2a) and (2b) represent the diffusion of the Eulerian waveaveraged velocities. These terms are given in Burchard and Bolding (2002) depending on horizontal and vertical viscosities A_{m} and v_{t} . The vertical eddy viscosity v_{t} is obtained via an interface from the General Ocean Turbulence Model (GOTM; Umlauf and Burchard, 2005).

Umlauf et al. (2003) introduced a generic length scale twoequation turbulence closure model including an application to shear-free cases e.g. wave turbulence injection. They showed a similar behavior of the $k-\omega$ and the generic length scale model for this kind of application. Jones and Monismith (2008) also successfully applied the $k-\omega$ two-equation turbulence model in

Table 1

Juxtaposition of formulations. Summation is carried out over repeated indices with $\alpha, \beta \in \{x, y\}$ and $x_x = x, x_y = y, u_x = u, u_y = v$. Please see Sections 2.1 and 2.2 for the description of specific terms.

	$u_{lpha,n}$	$u_{\alpha,n}^{\mathrm{mass}}$	$F_{\alpha,n}^{wave}$
Standard GETM	$= u_{\alpha,n}$	$= u_{\alpha,n}$	= 0
Radiation stress	$=\overline{u}_{\alpha,n}^{L}$	$=\overline{u}_{\alpha,n}^{L}$	$= -\frac{\partial}{\partial x_{\beta}} \left\{ h_{n} \ \mathbf{k} \ \mathbb{E} \left[\left(f^{cc} f^{cs} \right)_{n} \frac{k_{\alpha} k_{\beta}}{\ \mathbf{k} \ ^{2}} - \delta_{\alpha\beta} \left(\left(f^{ss} f^{sc} \right)_{n} - \frac{I_{n}^{sc} (\eta - 2n)}{2 \ \mathbf{k} \ h_{n}} \right) \right] \right\}$
Vortex force	$=\overline{u}_{lpha,n}^{ extsf{E}}$	$=\overline{u}_{lpha,n}^{ m L}$	$=h_n\overline{u}_{\beta,n}^{\text{Stokes}}\left(\frac{\partial\overline{u}_{\beta,n}^{\text{E}}}{\partial x_{\alpha}}\right)_{z}-h_n\left(\frac{\partial}{\partial x_{\alpha}}\right)_{z}\left\{\frac{\ \mathbf{k}\ _{\text{E}}}{\sinh\left(2\ \mathbf{k}\ (\eta+D)\right)}\right\}+F_{\alpha,n}^{\text{ds}}$

shallow tidal environments. In addition, we performed a comprehensive sensitivity analysis for different surface roughness and a variety of two-equation turbulence closure models (e.g. $k-\epsilon$, $k-\omega$) and generic length scale). Our results showed that with the same surface roughness for the $k-\omega$ and the generic length scale model (setting according to Umlauf et al. (2003)) both models represent a similar wave injected TKE profile inside the water column. Umlauf et al. (2003) could actually show that for scenarios with turbulence injection at the surface due to surface wave breaking the $k-\omega$ model performed far better than the $k-\epsilon$ model. However, the $k-\epsilon$ model showed less depth of penetration. For instance to get the same TKE vertical distribution for $z_s^0=0.3H_s$ in $k-\omega$ one needs to use about $z_c^0=0.7H_s$ for the $k-\epsilon$. Furthermore, the $k-\omega$ turbulence closure model produces numerically more stable solutions when using roughness lengths comparable to those used by Newberger and Allen (2007b). Therefore for the present study a $k-\omega$ turbulence closure model was chosen.

The calculation of the vertical eddy viscosity requires boundary conditions for the turbulent kinetic energy (TKE) and the vorticity (ω) . Therefore GETM has to provide friction velocities and roughness lengths at the bottom and at the surface to GOTM. The direct availability of wave data from SWAN offers the description of proper boundary conditions to simulate the injection of TKE due to breaking waves (Craig and Banner, 1994; Burchard, 2001; Umlauf et al., 2003). This process can modify the profiles of the turbulence quantities down to several wave heights.

In this context the surface roughness z_0^s represents a length scale for the height of the wave-turbulent sublayer (Terray et al., 1999). A relatively wide range of values for z_0^s has been published. According to Stips et al. (2005) the magnitude of z_0^s depends on the method of observation. For example $z_0^s = H_s$ was reported from a fixed tower measurement but $z_0^s = 0.2$ m was calculated with a floating instrument for $H_s = 5$ m. A sensitivity analysis for different values of z_0^s is presented in Appendix A. According to Newberger and Allen (2007b) the surface friction velocity provided to the turbulence model is given by:

$$u_{*s} = \sqrt{\frac{g}{c}} S^{\mathrm{ds},s} + \tau_{w},\tag{8}$$

with $S^{ds,s}$ defined in (5f) and τ_w as the wind stress.

2.5. Vertical distribution of wave-induced forcing

In the previous subsections the empirical functions $f_n^{\text{RS}}(\zeta)$, $f_n^{\text{ds,b}}(\zeta)$ and $f_n^{\text{ds,c}}(\zeta)$ were introduced without a closed specification. Following the vertical distribution of the Stokes drift in (3a) and (3b), Uchiyama et al. (2010) and Kumar et al. (2011) suggested

$$f_n^X(\zeta) \propto \cosh\left(\frac{\eta + D - \zeta}{\mathcal{L}^X}\right),$$
(9)

with $X \in \{(RS), (ds, b), (ds, s)\}$. The corresponding length scales can be given in terms of the significant wave height H_s and the thickness of the wave-induced BBL δ_w (see Uchiyama et al., 2010 for the definition of δ_w):

$$\mathcal{L}^{\text{RS}} = \alpha^{\text{RS}} H_{\text{s}}, \tag{10a}$$

$$\mathcal{L}^{\mathrm{ds},\mathrm{b}} = \alpha^{\mathrm{ds},\mathrm{b}} \delta_{\mathrm{w}},\tag{10b}$$

 $\mathcal{L}^{ds,s} = \alpha^{ds,s} H_s. \tag{10c}$

The parameters α^{X} were determined by a sensitivity analysis presented in Appendix A.

3. Numerical experiments

The developed model system was verified by several numerical experiments. These also facilitate the validation and direct comparison of the Radiation stress and Vortex force formulations. In the first set of test cases different idealised wave regimes were simulated and the results were compared with measurements of corresponding flume experiments. Only for the comparison of the Radiation stress and Vortex force formulations the model system was operated in one-way mode for these experiments. This guaranteed the identical forcing due to a stationary wave field and offered the investigation of the corresponding wave-induced currents for both formulations. Finally the validated model system was applied to a realistic barred beach. For this test case the model system was operated in two-way mode to investigate the response of the wave and currents on the wind forcing. The common model parameters for the simulations are given in Table 2. Some of them are based on the sensitivity analysis presented in Appendix A.

3.1. Validation against barred beach flume experiments

The LIP-11D experiments were performed in the Delta flume of Delft Hydraulics for different idealized wave conditions. This flume has a length of 200 m and a width of 5 m (Arcilla and Roelvink, 1994; Roelvink et al., 1995). The validation was done against three flume experiments presented by Arcilla and Roelvink (1994), Roelvink et al. (1995) and Boer (1995). These range from short period waves with different wave heights (case 1A and 1B) to long period waves (case 1C). Table 3 shows the hydraulic conditions for the different experiments.

During these experiments physical quantities such as wave height, wave setup, profiles of across-shore velocity, sediment transport and bottom topography were measured.

For all experiments the bathymetry decreases from 4.1 m at the wavemaker towards an idealized shoreline. Within the shoaling region the bathymetry is different for each experiment (see Fig. 1). The model domain is discretised with a grid spacing of 1 m in across-shore direction, 10 m in alongshore direction and 50 equi-

Fable 2					
Settings	for	GETM	and	SWAN.	

GETM settings	
Am	0.05 m ² /s
z ^b ₀	Styles and Glenn (2000) with $z_0 = 0.001$ m
z_0^{s}	0.3 <i>H</i> s
α ^{RS}	0.2
$\alpha^{ds,s}$	0.2
$\alpha^{ds,b}$	3
SWAN settings	
Third generation mode	Komen et al. (1994)
Depth-induced breaking	Constant breaker index
	$(\alpha = 1.0, \gamma = 0.73)$
Whitecapping	Komen et al. (1994)
Bottom friction	JONSWAP
	(Hasselmann et al., 1973)

Table 3

LIP-11D experiments hydraulics specifications. H_{rms} is the root mean square of the wave height at the wave generator, T_p is the wave peak period and h_0 is the still water level at the wave generator.

Test	H _{rms} (m)	T_p (s)	<i>h</i> ₀ (m)
1A	0.6	5.0	4.1
1B	1.0	5.0	4.1
1C	0.4	8.0	4.1



Fig. 1. Comparison of across-shore profiles of different parameters of the LIP-11D experiment for case 1A, 1B and 1C by VF and RS methods. (a1)-(a3) Root mean square wave heights, (b1)-(b3) wave dissipation at the surface including white capping and depth induced breaking, (c1)-(c3) water level variation, (d1)-(d3) across-shore depth-integrated quasi-Eulerian wave-averaged velocity for VF and Eulerian wave-averaged velocity for RS and (e1)-(e3) bottom topography for 1A, 1B and 1C respectively. The circles are the flume measurements.

distant vertical layers ($183 \times 20 \times 50$). In alongshore direction the domain size is several times larger than the actual flume width. The intention was to prevent wave shadowing close to the corners of the domain and try to have uniform wave and hydrodynamic condition in alongshore direction. The external and internal time step of GETM was set to 0.1 and 1.0 s respectively. The time step of SWAN was set to 1.0 s, which is identical to the exchange time step of the models. In spectral dimensions, SWAN used 24 directional and 27 frequency bins.

3.1.1. Wave forcing and depth averaged results

In order to compare RS and VF, identical wave information from SWAN was passed to GETM to ensure identical forcing for both methods. The root mean square wave height, water level variation (wave setup) and depth-integrated across-shore velocity for RS and VF as well as across-shore bottom topography are presented in Fig. 1.

For all cases, two main breaking points are visible. For case 1A, they occur at 140 m and 165 m where sudden changes in the bottom profiles are present. For cases 1B and 1C, the main wave breaking points are slightly shifted offshore. The wave height provided by SWAN shows reasonable agreement with the measurements. In case 1C, indication of wave shoaling with slight increase in wave height around the breaking points is visible. A substantial amount of energy is injected into the water column due to the breaking

events (Fig. 1(b1)–(b3)). The amount of wave dissipation at the surface is varying among the cases. For case 1B dissipation starts far before the main breaking points and reaches to values of $0.054 \text{ m}^3/\text{s}^3$ and $0.041 \text{ m}^3/\text{s}^3$ at the first and second breaking point (Fig. 1(b2)). In contrast, in case 1C, most of the dissipation is taking place directly at the breaking points (Fig. 1(b3)).

Both, RS and VF produce comparable water surface elevation, in agreement with the measurements. For case 1A, both methods show identical water elevations offshore and close to the beach. However, VF is closer to the measurement at the main breaking points. The results for the case 1B show a slight overestimation for both methods. VF reproduces more detailed features in terms of the water surface elevation. For case 1C, VF clearly overestimates the set-down at the breaking points. The responsible mechanism for the rapid changes in the water level close to the breaking points is the remaining VF term (i.e. the first term of $F_{\alpha,n}^{\text{wave}}$ in VF related equation mentioned in Table 1). This term in case of a simple one-dimensional case in steady-state condition would be $D\overline{U}^{\text{Stokes}}\left(\frac{\partial\overline{U^{\text{E}}}}{\partial x}\right)$. From the continuity equation it follows that $\overline{U}^{\text{E}}=-\overline{U}^{\text{Stokes}}$ such that the forcing term is proportional to $-\overline{U}^{\text{Stokes}} \xrightarrow{\partial \overline{U}^{\text{Stokes}}}$. Moving towards the shallow water breaking point in the wave propagation direction, the Stokes velocity increases rapidly and decreases afterwards. Overall, the Stokes velocity is always positive but the sign of its slope is positive before the breaking point and negative after that. Therefore, a pair of forces pointing in opposite directions is formed around each breaking point. It seems that this forcing term, which changes its sign at the breaking point is the responsible mechanism for the extreme set down. Our numerical experiments also supports this argument.

A feature similar to the set-down of the water level shown for VF for the 1C case is also visible in Fig. 1(c) of Uchiyama et al. (2010).

In agreement with the depth-integrated continuity equation, the depth-integrated Stokes drift was compensated by the depth-integrated quasi-Eulerian wave-averaged velocity for VF and the Eulerian wave-averaged velocity for RS (Fig. 1(d1)–(d3)). With the same wave mass flux for both RS and VF methods, producing less water depth by VF, results in higher undertow velocity at both main breaking points. This is more obvious in case 1C.

3.1.2. Three-dimensional structure of hydrodynamical results

A comparison of the TKE and the eddy viscosity for case 1A with and without the injection of the TKE at the surface is given in Fig. 2. Only the results for VF are depicted, since RS and VF show nearly identical results. Without the wave injected TKE, the shear of the quasi-Eulerian wave-averaged velocity for VF and of the Eulerian wave-averaged velocity for RS are the dominant sources of TKE production. If injection of TKE due to breaking waves is considered, the TKE profile is modified throughout the water column down to about one significant wave height (Figs. 2(a,c) and 3(a)). Carniel et al. (2009) showed a comparable results in terms of the vertical distribution of TKE (in their Fig. 2) by applying of Generic length scale turbulence model (Umlauf et al., 2003; Warner et al., 2005). Also the injection of TKE causes a deviation from the typical parabolic profile of the vertical eddy viscosity throughout the whole water column (Figs. 2(b,d) and 3(b)). The same feature was presented in Fig. 14 of Jones and Monismith (2008) using $k-\omega$ twoequation turbulence model. The enhanced eddy viscosity improved the vertical distribution of the guasi-Eulerian wave-averaged velocity (Fig. 3(c)). For the case 1C, the wave dissipation and thus the injection of TKE is concentrated at the two main breaking points (not shown).

For both methods the profiles of the across-shore quasi-Eulerian wave-averaged velocity for VF and of the Eulerian wave-averaged velocity for RS show a general agreement with the observations for 1A and 1B (Fig. 4). Although the depth-integrated velocities are nearly the same for both methods (Fig. 1(d1) and (d2)), the VF results showed a more pronounced vertical velocity gradient. VF captures the vertical separation of the velocities better, towards the shore at the surface and an offshore-directed undertow close to the bottom. RS did not show the shore-directed surface current. Although the velocity profile is reproduced by RS and VF are similar, but both show an underestimation of the velocities at the top and between the two under water bars (Fig. 4).

A different situation for RS results of case 1C was observed. In the shoaling region off-shore directed Eulerian velocities at the surface were present (Fig. 4). Due to continuity equation the Eulerian wave-averaged velocity shapes the anti-Stokes velocity profile. The additional wave forcing terms (i.e. $F_{\alpha,n}^{\text{wave}}$) can be interpreted as an improvement to the constructed anti-Stokes velocity profile due to wave released momentum in the direction of wave propagation. As an example a section at x = 125 m where in the case 1C wave shoaling occurred is shown in Fig. 6. Vertical distribution of TKE (Fig. 6(a)), indicates that the wave breaking was already start. Therefore, a shoreward wave related momentum flux close to the surface is expected. However, RS predicts an offshore directed momentum flux (Fig. 6(b)). As a direct consequence, RS shows powerful offshore-directed velocities which even could exceed the anti-Stokes velocity (Fig. 6(c)). The last term in the Radiation stress divergence is the most important cause for creation of this erroneous velocities. In the shoaling region with increasing wave height and E towards shore the last term in RS forcing mentioned in Table 1, as:

$$-\frac{\partial}{\partial x_{\beta}} \left(\frac{E}{2} f_n^{\rm RS}(\eta - z_k) \right) \tag{11}$$

becomes negative. Therefore a relatively significant momentum flux close to the surface towards offshore (i.e. opposite to wave propagation direction) will be generated. In this case even suggested vertical smoothing by Kumar et al. (2011) did not remove or significantly reduced the incorrect off-shore directed velocities. This implies that even with such a treatment the application of RS for large period waves (e.g. swell) in shoaling regions could not make reasonable results.

Fig. 5(c) and (f) present these differences between RS and VF for 1C. Close to the first breaking point (between x = 120 m and x = 140 m), offshore-directed across-shore velocity close to water



Fig. 2. TKE and eddy viscosity computed by VF for the 1A case. (a) TKE and (b) eddy viscosity without the effects of TKE injection due to breaking waves. (c) TKE and (d) eddy viscosity with the effects of TKE injection due to breaking waves.



Fig. 3. Turbulent kinetic energy (a), eddy viscosity (b) and across-shore quasi-Eulerian wave-averaged velocity for VF (c) for 1A at *x* = 138 m. Profiles obtained with (red) and without (blue) the inclusion of TKE injection at the surface due to breaking waves are shown. The circles depicts velocity measurements. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)



Fig. 4. Profiles of the across-shore quasi-Eulerian wave-averaged velocity for VF (blue) and of the across-shore Eulerian wave-averaged velocity for RS (red) for cases 1A, 1B and 1C. The circles are velocity measurements. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

surface can be seen. The same feature is also visible at x = 160 m. This erroneous behavior is discussed by Ardhuin et al. (2008) and Bennis and Ardhuin (2011). Additionally they argued that the use of the Lagrangian wave-averaged velocity as the prognostic variable can lead to large errors over sloping bottoms and can cause

significant spurious acceleration during wave shoaling. They strongly insisted that solving for quasi-Eulerian flow, as a prognostic variable is the only correct way for introducing wave effects in a 3D ocean model. As one can see, VF generally produces physically sound results for all cases (Fig. 5).



Fig. 5. Quasi-Eulerian across-shore velocity for VF (a)-(c) and Eulerian across-shore velocity for RS (d-f) for 1A (a,d), 1B (b,e) and 1C (c,f).



Fig. 6. Across-shore Eulerian wave-averaged velocity profiles by the RS method for flume experiments case 1C at x = 125 m. The circles are velocity measurements. (a) TKE, (b) forces (i.e. summation of the Radiation stress terms in the right hand side of momentum equation) and (c) Across-shore Eulerian wave-averaged velocity. The blue line is the RS method results. The green line indicates the anti-Stokes velocity profile. The circles represent the measurements. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

3.2. Application to a realistic barred beach

The Sandy Duck94 campaign was conducted during October 1994 at Duck, North Carolina. Vertical velocity profiles at 0.41, 0.68, 1.01, 1.46, 1.79, 2.24 and 2.57 m above the bed were measured by a mobile sled. The measurements for each profile took 1 h and seven profiles were measured during one day. Directional wave spectra and additional physical parameters were measured as well (Faria et al., 1998, 2000; Feddersen et al., 1996, 1998; Newberger and Allen, 2007b). The available data from 12 October 1994 in a stormy condition were used for comparison with the modeling results. For this experiment, all forcing information such as wind, wave and tidal parameters were averaged over the measurement period and kept constant for the simulation. The mean tidal eleva-

Table 4

Forcing parameters for the Duck94 simulation. All wind and wave values are averaged over the measurement period and kept constant in the simulation. T_p is the wave peak period, θ_0 is the incident wave direction at the offshore boundary (Uchiyama et al., 2010).

Variable	Value
Across-shore wind stress (Pa) Alongshore wind stress (Pa) Offshore tidal elevation (m) Lateral momentum diffusion (m ² /s) Offshore wave height H_{rms} (m) Offshore wave period T_p (s)	-0.2532 -0.1456 0.7 0.05 1.6 6.0
Offshore wave angle θ_0 (°)	193.0

tion of 0.7 m was added to the surface elevation. The forcing information is given in Table 4.

The model settings are given in Table 2. The model domain has a size of 768 m in cross-shore direction and 3000 m in alongshore with 2 m and 100 m grid resolution respectively. The model domain was uniform in alongshore direction and periodic boundary condition at the alongshore boundaries were applied. 40 vertical levels with zooming towards surface and bottom were employed. The coupling time step was identical with the wave model time step of 2 s. Internal and external time step of GETM were 1.0 s and 0.1 s respectively. At the open boundary a Flather-type condi-

tion (Flather, 1976) with zero surface elevation (representing the mean tidal elevation of 0.7 m) and zero depth-integrated Lagrangian wave-averaged normal velocity was applied. The JOWNSWAP wave spectrum with wave height and peak period specified in Table 4 were used as open boundary forcing for SWAN.

In Fig. 7 vertical profiles of TKE and eddy viscosity are shown. High values of TKE at the main breaking regions are visible. Considerable effects of the injected energy on the eddy viscosity structure could be seen as well. Distribution patterns of both the TKE and the eddy viscosity are in agreement with the results of Newberger and Allen (2007b), although they applied a constant surface roughness



Fig. 7. Across-shore sections for VF of TKE (a) and eddy viscosity (b) for the Duck94 case. Applied is a wave dependent surface roughness formulation as $z_5^n = 0.3H_s$.



Fig. 8. Comparison of RS and VF methods results for Duck94 case with observations for across-shore velocity (a) and alongshore velocity (b). Blue: VF, green: RS and circles are the observations. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)



Fig. 9. Application of different energy transfer rates (0 %, 50 % and 75%) for VF and RS. (a) The across-shore quasi-Eulerian wave-averaged velocity for the VF, (b) the Eulerian wave-averaged velocity for the RS and (b,d) alongshore velocity. The circles denotes the measurements.

of $z_0^s = 0.3$ m using the two-equation turbulence model of Mellor and Yamada (1982). Our simulations confirmed that an increase of the surface roughness decreased the vertical gradient of the across-shore velocities. This resulted in a better performance offshore of the breaker line but lead to a degradation in the vicinity of the bars.

In Fig. 8, the numerical results of RS and VF for across-shore and alongshore velocities are compared to available observations. VF and RS produced similar alongshore currents. In general, the vertical shear of the alongshore current is less sensitive to the change of surface roughness (not shown here). However, an underestimation of alongshore current between underwater bar and shoreline is visible (Fig. 8(b)). In terms of the across-shore quasi-Eulerian wave-averaged velocity, VF showed higher vertical shear off-shore the breaker line. Between the bar and shoreline also at the first breaking point (x = 130 m), VF was more successful to reproduce the shoreward directed surface velocity.

It is observed that part of the dissipated energy from the broken waves forms the surface rollers. The energy content of the rollers will be slowly dissipated into the water column while they are moving towards the shoreline with the wave phase velocity (Tajima, 2004). Newberger and Allen (2007b) and Uchiyama et al. (2010) show that the inclusion of a roller model for transferring part of wave dissipated energy and momentum further towards the shoreline can improve the simulation result tremendously. This was tested using an implementation of a roller evolution model following Reniers et al. (2004) (detail formulations and implementation procedure can be found in Uchiyama et al. (2010) for VF and Kumar et al. (2011) for RS).

In the following, three scenarios are tested (Fig. 9). As a base experiment, the effects of roller are neglected. The second one assumes that 50% of the broken waves turned into rollers. In the third scenario a transfer rate of 75% is assumed. In the base experiment the center of alongshore current is situated at the bar crest. The inclusions of the roller pushed the location of the maximum alongshore current significantly towards the shoreline. Additionally, the formation of the undertow could be improved. Also the acrossshore quasi-Eulerian wave-averaged velocity for the VF and the Eulerian wave-averaged velocity for the RS is increased in this region. Surface mass flux of the roller causes segregation of the surface on-shore velocity and undertow at the bottom more pronounced and formed an apparent circulation cell between the two under water bars for both VF and RS methods. VF showed better performance in capturing onshore directed surface currents and alongshore currents between bar and shoreline.

The final results and the demonstration of importance of the implementation of surface rollers in this kind of applications were in agreement with recent studies (Newberger and Allen, 2007b; Uchiyama et al., 2010; Kumar et al., 2012). The main difference among these studies and the results presented here is the treatment of the TKE injection, the choice of turbulence closure and in choosing its sensitive parameters (e.g. surface roughness). This has direct effects on the vertical distribution of eddy viscosity and across-shore velocities. In terms of alongshore current profile the difference between the different implementation is not significant.

4. Conclusion

A two-way coupled model system between a full three-dimensional coastal ocean model (GETM) and a third generation spectral wind wave model (SWAN) has been developed. In order to take into account effects of waves two different approaches for including the 3D momentum transfer from waves into the water column have been implemented. It should be noted that for both formulations the underlying theories are based on assumptions limiting their practical applicability. The aim of the present study was to investigate these limitations from the practical side. If possible, findings in the results of the performed simulations were linked to the corresponding theoretical limitation. For a detailed discussion of all underlying assumptions on the theoretical side the reader is referred to the original publications.

The first method was proposed by Mellor (2011a) as a depthdependent Radiation stress formulation (RS method). In the second method, the set of equations developed by Ardhuin et al. (2008) and Bennis et al. (2011) based on the Vortex force were implemented (VF method). In the momentum equation of VF, wave related conservative terms and non-conservative terms are clearly distinguished which is not the case for RS. For both methods wave enhanced eddy viscosity due to energy transfer from dissipated surface waves (i.e. broken waves) was taken into account. Additionally, both RS and VF methods could benefit from the inclusion of surface rollers.

Three laboratory flume test cases (LIP-11D experiments) to validate the model system and a realistic application (Duck94) to control its performance in realistic situation, were simulated. Comparison of the depth-integrated parameters for the flume cases showed that RS and VF are in general agreement regarding wave induced water level changes (wave set-up and set-down) and Eulerian undertow velocity. Only for long period wave, VF showed a weak performance regarding the wave set-down before both main breaking points.

It should be noted that considering the contribution of the wave rollers and their modification in terms of wave mass flux and forcing could make the VF results for the wave set-down at the breaking points even slightly worse. This could also increase the water surface elevation between two breaking points which is not considered as an improvement as well.

In general, VF reproduced the higher vertical gradient for across-shore quasi-Eulerian wave-averaged velocity. Inclusion of the mixing effects of waves resulted in a reduction of this vertical gradient especially close to the surface. Increasing the surface roughness lead to an intensification of these effects. In terms of longer period waves, RS showed an artificial offshore directed transport at the surface where the wave shoaling was taking place. This feature was not present in the VF results.

As final test, the model system was applied to the Duck94 test case (Faria et al., 1998, 2000; Feddersen et al., 1996, 1998; Newberger and Allen, 2007a). The results of RS and VF for alongshore profiles were generally the same. The across-shore quasi-Eulerian wave-averaged velocity from VF method showed larger shear in comparison to the Eulerian wave-averaged velocity for the RS. Taking into account momentum transfer due to surface rollers leads to a better representation of the position and magnitude of the alongshore current and some improvement in the across-shore quasi-Eulerian wave-averaged velocity for the VF and the Eulerian wave-averaged velocity for the RS.

Due to unrealistic offshore-directed transport for RS method in wave-shoaling regions situated at rather steep slopes, this method seems to be limited to short period waves on mild slopes like locally generated waves in lakes or tidal flats. Doubtful results in general application of this method specially for coastal oceans with wave open boundaries is expected. Although VF showed some shortcomings, it generally produced physically reasonable results in a wider range of applications, which makes VF a more reliable base for prospective research in this field.

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Appendix A. Sensitivity analysis of the physical parameters

In order to investigate the sensitivity of the wave-current interaction to different parameters, a number of simulations for the three test cases (1A)–(1C) of LIP-11D experiments were carried out. Two statistical quantities, the normalized root mean square error (NRMSE) and BIAS were calculated and form the basis of the performance measure:

$$NRMSE = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N} \left(X_{obs,i} - X_{model,i}\right)^2}}{X_{obs,max} - X_{obs,min}},$$
(A.1a)

$$BIAS = \frac{1}{N} \sum_{i=1}^{N} (X_{model,i} - X_{obs,i}), \qquad (A.1b)$$

where $X_{obs,i}$ are observations and $X_{model,i}$ are model results. *i* is used as summation index over *N* points of available observations.

In the first set of experiments the parametrization of the surface roughness z_0^s due to the presence of waves were investigated. Additionally, the coefficients α^{RS} for RS and $\alpha^{ds,s}$ for VF and the effects of the wave-current bottom boundary layer z_0^b were studied. Effects of inclusion of bottom streaming term in VF method has also been tested.

A.1. Sensitivity to surface roughness

A spectrum of different surface roughness parametrization is investigated, with constant values and wave height dependent formulations (Table A.1). For all simulations, the $k-\omega$ turbulence closure model was used. As a base run, simulations without wave injected TKE were preformed.

Starting from no surface flux of TKE due to breaking waves (No injection), the inclusion of the wave related surface flux of TKE with a surface roughness of $z_0^s = 0.1$ m, results in an increase of both BIAS and NRMSE for all cases for VF (Table A.1). A further increase of z_0^s (i.e. $z_0^s > 0.1$ m) lead to a systematic decrease of both error measures. The same holds for a wave height dependent formulation. Our numerical experiments showed that the level of the errors reached to a constant level by further increase of the surface roughness (e.g. for $z_0^s > H_s$). For values of $z_0^s > 0.5$ m or $z_0^s > H_s$,

the near surface vertical gradient of the across-shore quasi-Eulerian wave-averaged velocity for the VF and the Eulerian wave-averaged velocity for the RS around the main wave breaking regions disappeared due to a large eddy viscosity throughout whole water column (not shown). Our findings are in agreement with Newberger and Allen (2007b) which showed that a larger value of z_0^s will reduce the vertical gradient of the across shore the quasi-Eulerian wave-averaged velocity for the VF and the Eulerian wave-averaged velocity for the RS, which results in a better performance far from breaking points. However, close to the breaking points, where one expect a larger vertical gradient, the performance is deteriorated. Our experiments revealed a high sensitivity of the effects of TKE injection due to wave breaking onto the vertical velocity profile. This effect was most pronounced for the across-shore velocities and not for the alongshore ones. Newberger and Allen (2007b) proposed to use a constant value for surface roughness. However, since a wave depended formulation as $z_0^s = \alpha H_s$, with α a constant, seems physically more plausible, such a formulation is favored. Taking into account both error measures for all test runs we concluded to use α = 0.3 as an optimal value.

A.2. Sensitivity to α^{RS} and $\alpha^{ds,s}$

For VF and RS, a range of different α^{RS} and $\alpha^{ds.s}$ (0.1–0.5) were tested. Relatively little sensitivity to the changes of this parameter were detected (not shown). The overall changes for all cases were in the range of 0.01 m/s and –0.004 m/s for BIAS and 0.233 and 0.239 for NRMSE for RS and VF respectively. However, it should be noted that the bulk of the measurements were taken from the middle and the lower parts of the water column. This is related to the fact that the velocity sensors need to be in water during a whole wave period to compute a proper phase averaged velocity. In order to give a precise comparison of processes close to the water surface, high-resolution measurements through out the whole water column are essential. To preserve the vertical shear close to the surface we chose 0.2 for both RS and VF related coefficient (see also Uchiyama et al., 2010).

A.3. Sensitivity to bottom roughness

Different bottom roughness such as $z_0^b = 0.001$ m as base run, calculated values using wave-current bottom boundary layer (BBL) model of Styles and Glenn (2000) and average increased value of bottom roughness to $z_0^b = 0.004$ m suggested by Boer (1995), were tested for all cases and both methods.

Boer (1995) presented an evaluation of the bottom roughness based on available measurements. For instance for 1A case they showed that the bottom roughness varied from 0.001 m at the wave maker to a maximum values of 0.01 m around the breaking point. The bottom roughness calculated by using Styles and Glenn

Table A.1

LIP-11D experiments sensitivity analysis for the effects of the surface roughness (z_0^s) in surface flux of TKE due to surface	wave breaking.
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			No injection	0.1	0.2	0.3	0.2 <i>H</i> s	0.3 <i>H</i> s	0.4Hs
RS	1A	BIAS (m/s)	0.022	-0.006	0.006	0.012	-0.001	0.007	0.011
		NRMSE	0.244	0.156	0.182	0.197	0.147	0.169	0.185
	1B	BIAS (m/s)	0.011	-0.041	-0.022	-0.011	-0.028	-0.015	-0.008
		NRMSE	0.285	0.385	0.335	0.304	0.308	0.273	0.262
	1C	BIAS (m/s)	0.035	0.037	0.043	0.045	0.037	0.041	0.043
		NRMSE	0.283	0.253	0.275	0.286	0.251	0.263	0.274
VF	1A	BIAS (m/s)	-0.026	-0.033	-0.014	-0.004	-0.026	-0.014	-0.005
		NRMSE	0.210	0.262	0.194	0.180	0.219	0.172	0.160
	1B	BIAS (m/s)	-0.054	-0.066	-0.042	-0.027	-0.052	-0.034	-0.022
		NRMSE	0.424	0.517	0.420	0.359	0.424	0.341	0.298
	1C	BIAS (m/s)	0.022	0.023	0.029	0.033	0.024	0.029	0.031
		NRMSE	0.243	0.232	0.222	0.237	0.222	0.219	0.228

Table A.2
LIP-11D experiments sensitivity analysis wave-current bottom boundary layer effects

			0.001	0.004	Styles and Glenn (2000)
RS	1A	BIAS (m/s)	0.002	0.005	0.007
		NRMSE	0.162	0.166	0.166
	1B	BIAS (m/s)	-0.016	-0.015	-0.015
		NRMSE	0.286	0.277	0.273
	1C	BIAS (m/s)	0.043	0.042	0.042
		NRMSE	0.371	0.351	0.341
VF	1A	BIAS (m/s)	-0.017	-0.014	-0.013
		NRMSE	0.188	0.174	0.168
	1B	BIAS (m/s)	-0.042	-0.038	-0.036
		NRMSE	0.371	0.351	0.341
	1C	BIAS (m/s)	0.019	0.025	0.024
		NRMSE	0.216	0.207	0.207

Table A.3						
LIP-11D experiments	sensitivity	analysis	effects of	inclusion	of bottom	streaming

			No	As bottom stress	$\alpha^{\mathrm{ds},\mathrm{b}}=3$
VF	1A	BIAS (m/s)	-0.017	-0.015	-0.011
		NRMSE	0.188	0.179	0.165
	1B	BIAS (m/s)	0.042	-0.047	-0.039
		NRMSE	0.371	0.361	0.353
	1C	BIAS (m/s)	0.019	0.024	0.034
		NRMSE	0.216	0.260	0.275

(2000) starts from 0.002 m off-shore and increases to a value of 0.018 m in the main breaking area.

A slight improvement in the results in terms of NRMSE and BIAS in case of employing BBL model or average increasing of bottom roughness for VF method only for 1A and 1B could be seen (Table A.2). But take into account wave-current BBL showed negative effects for 1C case. The effects of different BBL options on statistical measures for the results of the RS method are typically very small. We decided to employ the BBL method of Styles and Glenn (2000) to be able to have physical description of this phenomenon in the bottom boundary layer.

A.4. Sensitivity of VF to bottom streaming

The wave induced mass flux inside the wave bottom boundary layer is called bottom streaming (by Longuet-Higgins, 1953). However inside surf-zone undertow directed opposite to the wave direction and bottom streaming, is the dominant flow. We assess the effects of bottom streaming for VF method by inclusion of related momentum at the bottom similar to bottom stress or specifying it with a vertical distribution as a body force inside water column with $\alpha^{ds,b} = 3$ (Table A.3). This forcing could slightly improve the VF results for 1A and 1B. But it may make 1C results worse. We included bottom streaming in the VF methods numerical experiments with vertical distribution using $\alpha^{ds,b} = 3$ for all simulations (Reniers et al., 2004).

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