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Surface-wave damping in closed basins

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The damping of surface waves in closed basins appears to be due to (a) viscous dissipation at the boundary of the surrounding basin, (b) viscous dissipation at the surface in consequence of surface contamination, and (c) capillary hysteresis associated with the meniscus surrounding the free surface. Boundary layer approximations are invoked in the treatment of (a) and (b) to reproduce and extend results that have been obtained previously by more cumbersome procedures. The surface film is assumed to act as a linear, viscoelastic surface and may be either insoluble or soluble; however, the relaxation time for the equilibrium of soluble films is neglected relative to the period of the free-surface oscillations. Capillary hysteresis is analysed on the hypothesis that both the advance and recession of a meniscus are opposed by constant forces that depend only on the material properties of the three-phase interface. The theoretical results for the logarithmic decrements of gravity waves in circular and rectangular cylinders are compared with the decay rates observed by Case & Parkinson and by Keulegan, which typically exceeded the theoretical value based on wall damping alone by factors of between two and three. It is concluded that both surface films and capillary hysteresis can account for, and are likely to have contributed to, these observed discrepancies. The theoretical effect of a surface film on wind generated gravity waves is examined briefly and is found to be consistent with the observation that the addition of detergent to water can increase the minimum wind speed (required to produce waves) by one order of magnitude.

1. INTRODUCTION

We consider the damping of surface waves owing to: (a) viscous dissipation in the neighbourhood of the rigid boundary or wall, say W, of the surrounding basin, (b) viscous dissipation at and in the neighbourhood of the free surface, say S, that is covered by a viscoelastic surface film, and (c) capillary hysteresis associated with the meniscus formed at the contact line, say L, between the free surface and the rigid boundary. We designate the logarithmic decrement of a particular mode by α and the respective contributions of (a), (b), and (c) by α_W , α_S , and α_L .

Let k and σ denote the wavenumber and angular frequency of an oscillation determined on the assumption of an inviscid liquid (see §2 below), and ν the kinematic viscosity of the actual liquid. Then 1/k is an appropriate length scale for the irrotational component of the velocity field, and

$$\delta = (2\nu/\sigma)^{\frac{1}{2}},\tag{1.1}$$

the thickness of the boundary layer on an oscillating flat plate, is an appropriate length scale for the viscous component. We base our discussion on the hypothesis that

$$\epsilon = k\delta \ll 1, \, kh, \tag{1.2}$$

where h is the mean depth of the liquid. The dispersion relation between σ and k then is $\sigma^2 = qk(1+k^2l^2) \tanh kh \left[1+O(\epsilon)\right], \quad l^2 = T'/q, \quad (1\cdot 3a, b)$

$$l^{-2} = gk(1+k^2l^2) \tanh kh [1+O(\epsilon)], \quad l^2 = T''/g,$$
 (1.3*a*, b)
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where g is the acceleration due to gravity, $T \equiv \rho T'$ is the surface tension, and l is the capillary length.

The logarithmic decrement α_W for surface waves in a cylindrical basin has been calculated by Ursell (1952; see Benjamin & Ursell 1954 for the corrected result) and is $O(\epsilon)$. We give a simpler calculation, following Stokes (1851),[†] in §§2 and 4 below. The logarithmic decrement for a *clean*, unbounded surface is (Stokes 1851; see Lamb 1932, §348)

$$\alpha_S^{(0)} = 2\pi e^2. \tag{1.4}$$

It can be shown (cf. Ursell 1952 and Case & Parkinson 1957) that this result also is valid, within a factor of 1 + O(e), for a bounded surface, but it then is negligible compared with α_{W} . (Case & Parkinson and others have retained $\alpha_{S}^{(0)}$ in the calculation of α for closed basins, but this is clearly inconsistent unless terms of $O(e^2)$ and $O(\epsilon)$ are included in the dissipation in the boundary layer on W and in the mean energy of the oscillation, respectively.)

Available measurements (Benjamin & Ursell 1954; Case & Parkinson 1957; Keulegan 1959; Van Dorn 1966) reveal that (the theoretical value of) α_{W} does not provide an adequate approximation to α for gravity waves in laboratory-size basins except under special and carefully controlled conditions and that the observed values of α/α_W may be as large as three.

Van Dorn (1966) attributed the observed discrepancies to a surface film produced by spontaneous contamination. He found that 'while the observed attenuation agreed with that computed for the solid boundaries $[\alpha_w]$ when the water was fresh, the former tended to increase with time to some higher limiting value, usually within an hour'. He also reported that 'the fully contaminated surfaces exhibited no obvious visual manifestation' and that 'the water always appear[ed] to be in every respect as clear and fresh as when uncontaminated'. He obtained an adequate approximation to the limiting value of α simply by adding to α_W the value of α calculated on the hypothesis of an inextensible film, namely (Lamb 1932, §351)

$$\alpha_S^{(1)} = \frac{1}{2}\pi\epsilon. \tag{1.5}$$

This agreement notwithstanding, we suggest that capillary hysteresis could have been significant in Van Dorn's experiments, especially since some of his measurements were made in lucite tanks (cf. Keulegan 1959).

The observation that surfactants, such as oil on water, can lead to appreciable damping of surface waves goes back to antiquity (Pliny the Elder, in the first century A.D., is cited by Levich 1962 and by Davies & Vose 1965), and Benjamin Franklin (1774) attempted a phenomenological explanation. The first satisfactory explanations appear to have been given by Reynolds (1880) and Aitken (1883), each of whom invoked the variations of surface tension with wave motion; Aitken also carried out laboratory experiments. Lamb provided the necessary mathematical analysis in support of the Reynolds-Aitken model in the second (1895) edition of his Hydrodynamics, but abbreviated the analysis to that for the inextensible

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[†] Stokes remarked that 'the effect of [viscosity] may be calculated with a very close degree of approximation by regarding each element of the [boundary] as an element of an infinite plane oscillating with the same linear velocity'. See also Landau & Lifshitz (1959, §24).

film in subsequent editions (1932, §351). Subsequent analyses, dealing with the physicochemical, as well as the hydrodynamical, problem have been given by Levich (1941, 1962), Dorrestein (1951), Goodrich (1961) and Tempel & Riet (1965).

Levich (1941, 1962) considered both insoluble and soluble surfactants, but appears to have assumed (as, implicitly, did Lamb) that damping would increase monotonically with concentration to the limiting value $\alpha_{\rm S}^{(1)}$. This tacit assumption led him to overlook the fact that his general formulation for an insoluble film yields the maximum value $\alpha_{\rm S} = 2\alpha_{\rm S}^{(1)}$ at an intermediate concentration, a result established independently by Dorrestein (1951). Levich considered various limiting cases of his general formulation, but his approximations do not appear to be entirely systematic and, in any event, do not give a clear picture of the interrelations among such parameters as frequency, bulk viscosity, surface modulus of compression, and solubility. Dorrestein (1951) considered only insoluble surfactants, but included surface viscosity in his analysis (Levich included surface diffusion of the surfactant, which is essentially similar to surface viscosity in its effect on damping). Tempel & Riet (1965) attributed differences among the results of Levich (1941), Dorrestein (1951) and Goodrich (1961) to 'the use of an incorrect boundary condition for the tangential stress'. In fact, Tempel & Riet's formulation appears to agree with those of Levich and Dorrestein when appropriate comparisons are made; detailed comparisons of their analytical results are not possible, as Tempel & Riet used an electronic computer to solve the algebraic equations implied by their boundary conditions.

None of the formulations of Levich, Dorrestein, Goodrich, and Tempel & Riet is directly applicable to closed basins (although the assumption of an unbounded surface is adequate for capillary waves in basins for which the area of the free surface is large compared with l^2). In §§ 2 to 4, a derivation of α_S is presented for either soluble or insoluble films that, by virtue of its direct appeal to boundary layer approximations, is not only simpler than its antecedents, but also is applicable to closed basins.

Davies & Vose (1965) have made careful measurements of the damping of $150 \, \text{c/s}$ capillary waves by various types of surface films. Their experiments were carried out in a 'draught-free darkroom'; their apparatus was 'cleaned with hot chromic acid followed by washings with syrupy phosphoric acid, hot tap water and distilled water' before each run. In their experiments on clean water, 'the surface was cleaned prior to the observations by spreading ignited talc on the surface and then sucking off the talc and any contamination through a fine capillary', a procedure that was repeated between observations. They monitored surface tension continuously. They found that α did approximate $\alpha_{\rm S}^{(0)}$ for clean water and also confirmed Dorrestein's prediction that α increases from $\alpha_{\rm S}^{(0)}$ to approximately $2\alpha_{\rm S}^{(1)}$ and then decreases to the limiting value $\alpha_{\rm S}^{(1)}$ with increasing concentration of an insoluble surfactant. They also obtained qualitative confirmation of Dorrestein's calculations for the effect of surface viscosity. They found that α_s was typically smaller for soluble surfactants provided that the relaxation frequency for solution was above that of the capillary waves; however, their results were not in complete agreement with the predictions of Levich (we recall that Levich did not establish, at least in any systematic way, regimes of validity for his various approximations).

Benjamin & Ursell (1954), Case & Parkinson (1957), and Keulegan (1959) all attributed the observed discrepancies between α and α_W to capillary hysteresis, but none of them attempted a direct calculation of α_L . Although Benjamin & Ursell appear to have been the first to recognize the significance of capillary hysteresis for surface-wave damping, the basic phenomenon is well known in the literature of surface chemistry (see, for example, Schwartz, Rader & Huey 1964), and Adam (1941) refers to A. A. Milne ('Waiting at the window', in *Now we are six*. London: Dutton, 1927) for an everyday observation.

Keulegan (1959) established that damping in a hydrophobic basin may be substantially larger than in a geometrically identical, hydrophilic basin. Subsequently, Keulegan & Brockman (1962, unpublished) carried out a series of measurements aimed at isolating the effect and concluded, at least tentatively, that both the advance and the recession of a meniscus are opposed by constant forces that depend only on the material properties of the three-phase interface. This conclusion agrees with that generally accepted in the literature of surface chemistry. A calculation of α_L is carried out on this hypothesis in §5 below.[†]

The results of Case & Parkinson (1957) and Keulegan (1959) are re-examined in §§ 6 and 7, where it is found probable that both α_S and α_L were significant in their experiments. It must be emphasized, however, that α_S and α_L are not likely to be entirely independent, for both the modulus of compression for the surface film and capillary hysteresis at the meniscus depend on the contamination of the free surface through mechanisms that are far from being fully understood.

Finally, in §8, the effect of an inextensible surface film on the generation of gravity waves by wind is examined. It is found that the minimum wind speed required to initiate waves of significant magnitude on a contaminated surface may be roughly an order of magnitude larger than for a clean surface. This conclusion is consistent with the observations of Keulegan (1951) and Van Dorn (1953).

2. Fluid motion

We consider a cylindrical container filled to a mean depth h with a slightly viscous liquid of density ρ and designate the upper surface of the liquid by S and the wetted boundary of the container by W. The velocity field associated with a wave motion on S will be essentially irrotational outside of viscous boundary layers at S and Wand can be derived from a velocity potential, say $\mathbf{q} = \nabla \phi$. Choosing cylindrical coordinates, with z measured vertically downward from the mean position of S, we pose this potential in the form (Lamb 1932, §257)

$$\phi(x, y, z, t) = \psi(x, y) \operatorname{sech} kh \cosh\left[k(z-h)\right] e^{i\sigma t}, \qquad (2.1)$$

where ψ is real and satisfies the two-dimensional Helmholtz equation

$$\nabla^2 \psi + k^2 \psi = 0. \tag{2.2}$$

[†] This calculation was carried out in ignorance of Keulegan & Brockman's work, but I have incorporated some of their observational results. I am indebted to Dr Keulegan for providing me with a copy of the unpublished manuscript.

The usual conventions hold in respect of complex, time-dependent variables such as ϕ . The wavenumber k is determined (as a member of an infinite, discrete set of eigenvalues) by the boundary condition

$$\mathbf{n} \cdot \nabla \psi = 0 \quad \text{on} \quad L, \tag{2.3}$$

where **n** is the inwardly directed normal to W, and L is the intersection of S and W in the plane z = 0.

The angular frequency, σ , is given by (1·3). We regard σ as real, with the implication that ψ is a slowly varying function of time for free oscillations. We then account for the time dependence of ψ only indirectly, by calculating the mean dissipation (in §4 below). The consequent errors in the kinematical description are $O(\epsilon)$.

Now let $\nabla \phi_0$ be the tangential component of $\nabla \phi$ and **u** be the actual tangential velocity at either S or W. The viscous flow in the boundary layer then must approximate that over a flat plate with the velocity

$$\mathbf{v} = \nabla \phi_0 - \mathbf{u}. \tag{2.5}$$

Evaluating $\nabla \phi_0$ at z = 0 from (2.1), and remarking that the velocity in the film must be parallel to that just outside of the boundary layer, we choose

$$\mathbf{u}_{S} = (1 - C) \nabla \psi e^{\mathbf{i}\sigma t}, \quad \mathbf{v}_{S} = C \nabla \psi e^{\mathbf{i}\sigma t}, \tag{2.6a, b}$$

where the constant C is determined (in the following section) by the equality between the viscous shearing stress in the liquid and the tangential stress in the surface film. By hypothesis, $\mathbf{u}_W \equiv 0, \quad \mathbf{v}_W = \nabla \phi_0.$ (2.7*a*, *b*)

3. SURFACE FILM

We adopt the constitutive relation (cf. Dorrestein 1951 and Goodrich 1961)

$$\boldsymbol{\tau} = \nabla T + \eta_1 \nabla \nabla \cdot \mathbf{u} + \eta_2 \nabla^2 \mathbf{u} \tag{3.1}$$

among stress, τ , surface tension, T, and (tangential) velocity, \mathbf{u} , in the surface film; ∇ and ∇^2 are the gradient and Laplacian operators[‡], and η_1 and η_2 are the dilatational and shear viscosities of the film. (Dorrestein assumes that $\eta_1 = 0$; this assumption does not simplify the present analysis, in which $\nabla \nabla \cdot \mathbf{u} = \nabla^2 \mathbf{u}$, but it appears that only η_2 has been measured directly.) Following Levich (1962, §§121, 122), we assume that the surface tension is uniquely specified by the superficial concentration of the film, say $T = T(\Gamma)$, and that the relaxation time for equilibrium between the material in the film and the dissolved material in the underlying liquid is negligible compared with $2\pi/\sigma$ (there can be little departure from this assumption for gravity waves; cf. the relaxation times given by Davies & Vose 1965). Small perturbations in T, Γ , and the bulk concentration, γ , then are related according to

$$\nabla T = (\mathrm{d}T/\mathrm{d}\Gamma)_0 \nabla (\Gamma - \Gamma_0) \tag{3.2}$$

 $(3 \cdot 3 a, b)$

and

where λ is a frequency-independent length.

† Equation (2.4) has been deleted.

 $\Gamma - \Gamma_0 = \lambda (\gamma - \gamma_0)_{z=0}, \quad \lambda = (\mathrm{d}\Gamma/\mathrm{d}\gamma)_0,$

[‡] There is no need to distinguish between the three-dimensional operators, ∇ and ∇^2 , and their two-dimensional counterparts in the plane z = 0 by virtue of the facts that **u** has no z component and that both **u** and T are independent of z.

The reduction in surface tension (from that for a clean surface) for a dilute solution is given by Szyszkowski's equation (Adam 1941, p. 116)

$$\Delta T = T(0) - T(\Gamma_0) = AT(0) \log [1 + (\gamma_0/a)], \qquad (3.4)$$

where A is a dimensionless constant (A = 0.18 for all fatty acids), and a is that value of γ_0 at which Γ_0 attains half its saturation value. Combining (3.4) with the Langmuir adsorption isotherm (Adam 1941, p. 117)

$$\Gamma = [AT(0)/R\theta] [\gamma_0/(\gamma_0 + a)], \qquad (3.5)$$

where Γ is expressed in moles/cm², R is the gas constant, and θ is the absolute temperature, we obtain

$$\chi \equiv -\Gamma_0 (\mathrm{d}T/\mathrm{d}\Gamma)_0 = AT(0) \left(\gamma_0/a\right) \tag{3.6a}$$

$$\doteq \Delta T \quad (\gamma_0/a \to 0) \tag{3.6b}$$

 $(3\cdot7a)$

(3.9)

and

$$\div (A/a) T(0) (R\theta)^{-1} \quad (\gamma_0/a \to 0). \tag{3.7b}$$

It should be noted that (3.4) to (3.7) are not applicable to insoluble films, for which $\gamma_0 \equiv 0$ and $\lambda = \infty$.

 $\lambda = AaT(0) (R\theta)^{-1} (\gamma_0 + a)^{-2}$

The distributions of Γ and γ are governed by the (linearized) conservation equation

$$(\partial \Gamma/\partial t) + \Gamma_0 \nabla \cdot \mathbf{u} = D(\partial \gamma/\partial z)_{z=0}$$
(3.8)

and the diffusion equation $\partial \gamma / \partial t = D \nabla^2 \gamma$,

where D is the bulk diffusion coefficient.

Involking the (implicit) time dependence $\exp(i\sigma t)$ and remarking that $D \ll \nu$, so that bulk diffusion is confined to a very thin (compared with both 1/k and δ) boundary layer, we pose the solution to (3·3), (3·8), and (3·9) in the form

$$(\gamma - \gamma_0) = \lambda^{-1} (\Gamma - \Gamma_0) \exp\left[-(\mathrm{i}\sigma/D)^{\frac{1}{2}}z\right] \tag{3.10}$$

and

 $\Gamma - \Gamma_{0} = -(i\sigma)^{-1} [1 + \frac{1}{2}(1-i)\eta]^{-1} \Gamma_{0} \nabla. \mathbf{u}.$ (3.11)

The dimensionless parameter

$$\eta = (2D/\sigma)^{\frac{1}{2}} (\mathrm{d}\Gamma/\mathrm{d}\gamma)^{-1}_{0} \tag{3.12}$$

is a measure of solubility and vanishes identically for an insoluble surfactant.

Substituting (3.11) into (3.1) via (3.2) and then invoking (2.6a) and (2.2), we obtain

$$\boldsymbol{\tau} = \{ (\mathrm{i}\sigma)^{-1} [1 + \frac{1}{2}(1-\mathrm{i})\eta]^{-1} (1-C) \Gamma_0 (\mathrm{d}T/\mathrm{d}\Gamma)_0 - (\eta_1 + \eta_2) \} k^2 \nabla \psi \, \mathrm{e}^{\mathrm{i}\sigma t}. \quad (3\cdot13)$$

[†] The diffusion coefficient D does not enter the subsequent analysis explicitly, and the symbol D is used for the dissipation rate in the following section. Again, the surface viscosities η_1 and η_2 enter the subsequent analysis explicitly only in (3·13) and (3·17), so that there is little danger of confusion in the use of η as a measure of diffusion.

The stress to which that of (3.13) must be matched is given by the Stokes boundary layer approximation as (Stokes 1851; cf. Landau & Lifshitz 1959, §24)

$$\mathbf{\tau} = -\left(\mathrm{i}\mu\rho\sigma\right)^{\frac{1}{2}}\mathbf{v} \tag{3.14a}$$

$$= -(i\mu\rho\sigma)^{\frac{1}{2}}C\nabla\psi\,\mathrm{e}^{\mathrm{i}\sigma t},\tag{3.14b}$$

where $\mu = \rho \nu$ denotes the dynamic viscosity, and (3.14*b*) follows from (3.14*a*) by virtue of (2.6*b*); $i^{\frac{1}{2}} \equiv 2^{-\frac{1}{2}}(1+i)$. We can show that the error in both of (3.14*a*, *b*) is $O(\epsilon)$ relative to unity.

Equating (3.13) and (3.14b) and solving for C, we place the result in the form

$$C = \frac{\xi + i\zeta + \frac{1}{2}(1+i)\eta\zeta}{\xi - 1 + i(1+\eta+\zeta) + \frac{1}{2}(1+i)\eta\zeta},$$
(3.15)

where

$$\boldsymbol{\xi} = (\frac{1}{2}\mu\rho\sigma^3)^{-\frac{1}{2}}k^2\boldsymbol{\chi} \tag{3.16}$$

 $\zeta = (\frac{1}{2}\mu\rho\sigma)^{-\frac{1}{2}k^2}(\eta_1 + \eta_2)$ (3.17)

are dimensionless measures of the surface compressional modulus and surface viscosity, respectively, and η is given by (3.12). We note the approximations

$$C \doteq (\xi + \mathrm{i}\zeta) \left[\xi - 1 + \mathrm{i}(1 + \eta + \zeta)\right]^{-1} \quad (\eta\zeta \ll 1)$$
(3.18a)

$$\ddagger \xi [\xi - 1 + i(1 + \eta)]^{-1} \quad (\zeta, \eta \zeta \ll 1).$$
 (3.18b)

It appears that, for most soluble films, the approximation (3.18a) should be valid for both capillary and gravity waves and that (3.18b) should be valid for gravity waves. Setting $\eta = 0$, we obtain

$$C = (\xi + i\zeta) [\xi - 1 + i(1 + \zeta)]^{-1} \quad (\eta = 0)$$
(3.19a)

$$\doteq \xi (\xi - 1 + i)^{-1} \quad (\eta = 0, \zeta \ll 1)$$
 (3.19b)

for insoluble films. We also note that the limiting value of C for very soluble films is given by $C = 1/(1 - i) \left(c + c \right)$ (2.20)

$$C \to \frac{1}{2}(1-i)\zeta \quad (\eta \to \infty), \tag{3.20}$$

but that this limit is not likely to be realized for typical surfactants.

4. DISSIPATION

Let E_0 be the total energy of the oscillation on the hypothesis $\epsilon = 0$, and let α be the logarithmic decrement of the damped oscillation as $\epsilon \to 0$. Invoking the hypothesis of small oscillations, which implies that the energy must be quadratic in the amplitude, we infer that the mean energy must decay like

$$\langle E \rangle = E_0 \exp\left(-\alpha \sigma t/\pi\right),\tag{4.1}$$

where $\langle \rangle$ implies an average over one cycle of the oscillation, say $t = (0, 2\pi/\sigma)$. Introducing the dissipation rate

$$D = -dE/dt, (4.2)$$

equating its mean value to the corresponding mean value implied by (4.1), and anticipating that $\alpha = O(\epsilon)$, we obtain

$$\alpha = (\pi/\sigma E_0) \langle D \rangle [1 + O(e)]. \tag{4.3}$$

The total energy of the undamped oscillation is equal to twice the mean kinetic energy and is given by

$$E_0 = \rho \iiint \langle \nabla \phi \rangle^2 \,\mathrm{d}V \tag{4.4a}$$

$$= \frac{1}{2}\rho I k^{-1} \tanh kh, \qquad (4\cdot 4b)$$

where

$$I = \int \int (\nabla \psi)^2 \,\mathrm{d}S = k^2 \int \int \psi^2 \,\mathrm{d}S. \tag{4.5}$$

The two forms of I are equivalent by Green's theorem, together with $(2\cdot 2)$ and $(2\cdot 3)$.

The rate of viscous dissipation in the liquid, say D_1 , is given by (Lamb §329)

$$D_{1} = \mu \iiint \mathbf{\omega}^{2} dV + \mu \iiint [2\mathbf{n} \cdot (\mathbf{q} \times \mathbf{\omega}) - (\partial q^{2} / \partial n)] dS, \qquad (4.6)$$
$$\mathbf{\omega} = \nabla \times \mathbf{q} \qquad (4.7)$$

where

denotes the vorticity and **n** the inwardly directed normal. The individual terms in the surface integral of (4.6), namely
$$2\mathbf{n}$$
. ($\mathbf{q} \times \boldsymbol{\omega}$) and $\partial q^2/\partial n$, each contribute terms of the same order (in ϵ) as the volume integral, but their sum is of higher order and can be

transformed to

$$2\mathbf{n} \cdot (\mathbf{q} \times \boldsymbol{\omega}) - (\partial q^2 / \partial n) = -2(\mathbf{q} \cdot \nabla) w \qquad (4 \cdot 8a)$$

$$= 2(w\nabla_2 \cdot \mathbf{u} - \mathbf{u} \cdot \nabla_2 w) \tag{4.8b}$$

(4.7)

where w is the z component of the velocity, and ∇_2 is the horizontal component of the gradient operator. Omitting the details, we find that the contribution of the surface integral of (4.8b) to $\langle D_1 \rangle$ is

$$\left\langle D_{1}^{(0)} \right\rangle = 2(1 - C_{r}) \, \sigma E_{0}[e^{2} + O(e^{3})]. \tag{4.9}$$

This is negligible compared with the remaining contributions to $\langle D_s \rangle$ if C = O(1); however, it is the only contribution as $C \rightarrow 0$, in which limit it yields the logarithmic decrement of (1.4), $\alpha_{S}^{(0)}$.

The boundary layer approximation to the volume integral in (4.6) can be reduced \mathbf{to}

$$D_1 = \left(\frac{1}{2}\mu\rho\sigma\right)^{\frac{1}{2}} \iint \mathbf{v}^2 \mathrm{d}S[1+O(\epsilon)], \qquad (4\cdot10)$$

a result that also follows directly from the flat plate analogy (cf. Landau & Lifshitz 1959, (24.14)). To this, we must add the rate at which work is done against the shearing stress in the film, namely

$$D_2 = -\int \int \boldsymbol{\tau} \cdot \mathbf{u} \, \mathrm{d}S. \tag{4.11}$$

Substituting τ from (3.14*a*) into (4.11), adding the result to (4.10), and averaging with respect to t, we obtain

$$\langle D_S \rangle = \frac{1}{2} (\frac{1}{2} \mu \rho \sigma)^{\frac{1}{2}} \mathscr{R} \int \int [|\mathbf{v}|^2 + (1+\mathbf{i}) \, \mathbf{v} \cdot \mathbf{u}^*] \, \mathrm{d}S \tag{4.12a}$$

$$= \frac{1}{2} (\frac{1}{2} \mu \rho \sigma)^{\frac{1}{2}} (C_r - C_i) I, \qquad (4.12b)$$

where \mathbf{u}^* is the complex conjugate of \mathbf{u} , and $(4 \cdot 12b)$ follows from $(4 \cdot 12a)$ by virtue of $(2 \cdot 6a, b)$ and $(4 \cdot 5)$. Substituting $(4 \cdot 4b)$ and $(4 \cdot 12b)$ into $(4 \cdot 3)$, we obtain

$$\alpha_{S} = \frac{1}{2}\pi\epsilon(C_{r} - C_{i})\coth kh + O(\epsilon^{2}).$$
(4.13)

We remark that τ and \mathbf{u} are in quadrature, and hence that $D_2 = 0$, if and only if $\eta = \zeta = 0$; more generally,

$$\langle D_2 \rangle / \langle D_1 \rangle = (C_r - C_i - |C|^2) / |C|^2 = (\xi \eta + 2\zeta) / (\xi^2 + \zeta^2) \quad (\eta \zeta \ll 1). \tag{4.14}$$

The parameter $C_r - C_i$ is a measure of the damping that owes its existence to the presence of the film; more precisely, it is the ratio of this damping to the damping



FIGURE 1. The surface-damping parameter $C_r - C_i$, as given by $(4 \cdot 15b)$.

that would be produced by an inextensible film ($\xi = \infty$, in which limit $C_r - C_i = 1$). Invoking the approximations of $(3 \cdot 18a, b)$, we obtain

$$C_r - C_i = \frac{\xi(\xi + \eta) + \zeta(\zeta + 2)}{(\xi - 1)^2 + (1 + \eta)^2 + \zeta(\zeta + 2)} \qquad (\eta \zeta \ll 1)$$
(4.15*a*)

$$= \xi(\xi+\eta) \left[(\xi-1)^2 + (1+\eta)^2 \right]^{-1} \quad (\zeta,\eta\zeta \ll 1).$$
 (4.15b)

We consider further only the approximation $(4 \cdot 15b)$, which appears to be adequate for gravity waves and is plotted against ξ for $\eta = 0, 1, 2$ in figure 1.

The parameter ξ vanishes like $\sigma^{\frac{5}{2}}$ as $\sigma \to 0$ (long gravity waves) and like $\sigma^{-\frac{1}{6}}$ as $\sigma \to \infty$ (very short capillary waves) and is typically smaller for gravity waves than for capillary waves. The parameter η varies monotonically like $\sigma^{-\frac{1}{2}}$ and therefore is more important for gravity waves. This is consistent both with the observations of Davies & Vose, who found that bulk solubility of the film material has only a slight effect on the damping of capillary waves in the absence of relaxation effects (i.e. in the frequency range for which (3.8) is valid) and with the empirical (and ancient) observation that soluble oils are less effective in the damping of gravity waves than are insoluble oils; in particular,

$$C_r - C_i \sim \xi/\eta = O(\sigma^3) \quad (\sigma \to 0).$$
 (4.16)

We emphasize, however, that the limiting behaviour of (4.16) is attained only for $\xi \ll 1$ and $\eta \gg 1$, that $\partial(C - C)$

$$\frac{\partial (C_r - C_i)}{|\partial \eta|} > 0 \quad \text{if} \quad \eta < 2^{\frac{1}{2}} |1 - \xi| - \xi, \tag{4.17}$$

and that $C_r - C_i$ may exhibit one of the partial maxima

$$C_r - C_i = \frac{1}{2} (\pm 2^{\frac{1}{2}} - 1) \xi (1 - \xi)^{-1}$$

$$\eta = \pm 2^{\frac{1}{2}} (1 - \xi) - \xi \quad (\xi \leq 2 \mp 2^{\frac{1}{2}}).$$
(4.18)

 \mathbf{at}

We also remark that

$$C_r - C_i > 1 \quad \text{for} \quad \xi > 1 + \eta (1 + \eta) (2 + \eta)^{-1} \tag{4.19}$$

and that $C_r - C_i \to 1$ as $\xi \to \infty$ for fixed η .

The maximum possible value of $C_r - C_i$ is 2 and is attained for $\xi = 2$ and $\eta = \zeta = 0$. Dr T. B. Benjamin (private communication) has pointed out that the film velocity **u** is equal in magnitude to, but in quadrature with, the irrotational velocity $\nabla \phi_0$ at this point, in consequence of which $|\mathbf{v}|^2 = 2|\nabla \psi|^2$, thereby doubling the dissipation rate for an inextensible film, for which $\mathbf{u} = 0$ and $|\mathbf{v}|^2 = |\nabla \psi|^2$.

The contribution of the boundary layer on W to α , namely α_W , can be derived by substituting $(2 \cdot 7a, b)$ into $(4 \cdot 12a)$ and integrating over W. Passing over the details, we obtain (cf. Benjamin & Ursell 1954)

$$\alpha_W = \frac{1}{2}\pi\epsilon\{\frac{1}{2}(J+K) + [2-kh(J-K)]\operatorname{cosech} 2kh\}, \qquad (4.20)$$

where

 $J = \frac{k}{I} \int_{L} \psi^2 dL \quad \text{and} \quad K = \frac{1}{kI} \int_{L} \left(\frac{\partial \psi}{\partial L}\right)^2 dL \qquad (4.21a, b)$

are essentially form factors.

5. CAPILLARY HYSTERESIS

It is an observationally established fact that the contact angle, say θ , between a liquid meniscus and a solid surface depends on the relative velocity of the three-phase (solid, liquid, and air) interface (Adam 1941; Bikerman 1958). The effect is typically much larger for hydrophopic ($\theta > 90^{\circ}$) than for hydrophilic ($\theta < 90^{\circ}$) surfaces, but its contribution to the damping of gravity waves in laboratory-size basins appears to be significant for all but carefully prepared, hydrophilic surfaces (Case & Parkinson 1957; Keulegan 1959). An adequate, deductive explanation appears to be lacking; however, the available measurements support the hypothesis, evidently due originally to Adam & Jessup (1925; see also Yarnold 1938), that the contact angles of advance and recession have unequal, but constant values, say θ_A and θ_R , for relative speeds in the range of interest for gravity waves. This implies an effective, Coulomb-like frictional force

$$F = \frac{1}{2}T\left|\cos\theta_R - \cos\theta_A\right| \tag{5.1a}$$

$$\equiv \kappa T \tag{5.1b}$$

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per unit length of meniscus. Ablett's (1923) data for water on wax yield $\kappa = 0.14$. Keulegan & Brockman's (1962) data for water on lucite and polished brass yield $\kappa = 0.31$ and $\kappa = 0.35$, respectively.

The rate dependence of θ has been measured by Ablett (1923), Rose & Heins (1962), Schwartz *et al.* (1964), and others (see papers just cited for additional references). It appears that $\cos \theta$ varies uniformly from its equilibrium value, say $\cos \theta_0$, to $\cos \theta_A(\theta_R)$ as the speed at which the meniscus advances (recedes) increases from zero to a fairly well defined critical speed, say v_c , and then remains constant for higher speeds; e.g. Ablett's results for water on wax yield $\theta_0 = 104\frac{1}{2}^{\circ}$, $\theta_A = 113^{\circ}$, $\theta_R = 96^{\circ}$, and $v_c = 0.4 \text{ mm/s}$. [The validity of Ablett's results may be accepted, at least in the present context, on the authority of Bikermann (1958, p. 349), although Rose & Heins (1962) appear to question this validity for small capillary tubes.] Unfortunately, neither the available theory nor the available measurements appear to provide an adequate basis for scaling v_c , and we can say only that v_c is typically small compared with significant gravity-wave (particle) velocities.

Calculating the vertical velocity of the meniscus from $(2\cdot 1)$ and denoting the contact line by L, we obtain

$$\langle D_L \rangle = \frac{\sigma}{2\pi} \int_0^{2\pi/\sigma} F(t) \, \mathrm{d}t \int_L |k\psi \tanh kh \cos \sigma t| \mathrm{d}L \tag{5.2}$$

as the approximate (for $e \to 0$), mean dissipation associated with capillary hysteresis. Substituting (5.1) into (5.2) and multiplying the result by $\pi/\sigma E_0$, we obtain

$$\alpha_L = \frac{4\kappa k^2 T'}{\sigma I} \int_L |\psi| \, \mathrm{d}L,\tag{5.3}$$

which varies inversely as the amplitude.

This last result is valid only if the speed of the meniscus is larger than the aforementioned speed v_c throughout most of the cycle. We consider briefly the opposite extreme, in which the meniscus speed is always less than v_c . Ablett's measurements suggest that F(t) is approximately proportional to the meniscus speed in this range, so that

$$F(t) \doteq \kappa T k \tanh k h |\psi/v_c| |\cos \sigma t|.$$
(5.4)

Substituting (5·4) into (5·2), multiplying the result through by $\pi/\sigma_0 E_0$, and invoking (4·21*a*), we obtain $\alpha_L = \pi \kappa J(T'/\sigma v_c) k^2 \tanh kh$, (5·5)

which is independent of amplitude and typically large compared with ϵ . We again emphasize, however, that (5.5) holds only for very small amplitudes—e.g. 0.03 mm maximum amplitude for water in a 6 in. diameter, wax cylinder.

6. CIRCULAR BASIN

The dominant mode in a circular cylinder of radius a is given by

$$\psi = (\sigma A/k) \coth kh \left[J_1(kr) / J_1(ka) \right] \cos \theta, \quad ka = 1.84, \tag{6.1}$$

where A is the maximum displacement of the free surface from its equilibrium position, r and θ are polar coordinates, and J_1 is a Bessel function.

Substituting (6·1) into (4·5) and (4·21), we reduce (4·20) to (cf. Case & Parkinson 1957) $\alpha_{W} = \frac{1}{2}\pi\epsilon\{1\cdot00 + 2[1-(h/a)]\operatorname{cosech}(3\cdot68\,h/a)\}.$ (6·2)

Setting k = 1.84/a in (4.13) and dividing the result through by (6.2), we find that α_s can be represented approximately by

$$\alpha_S = (C_r - C_i) \, \alpha_W \tag{6.3}$$

within a factor that tends to 1 at both h/a = 0 and ∞ and lies between 1 and 1.06 for intermediate values of h/a (the maximum error of 6% is likely to be smaller than the uncertainty in $C_r - C_i$).

Substituting (6.1) into (5.3), we obtain

$$\alpha_L = 7 \cdot 9\kappa T' k^2 \sigma^{-2} A^{-1} \tanh kh \tag{6.4a}$$

$$\doteq 14.5\kappa(l^2/aA) \quad (l \ll a). \tag{6.4b}$$

Case & Parkinson measured α for tap water in 3 and 6 in. diameter, brass cylinders and in a 20 in. diameter, steel cylinder and compared their results with the theoretical value of α_W . They found that α/α_W ranged from two to three prior to polishing, but approximated unity for the brass cylinders after careful polishing of the wetted surfaces. They concluded that the discrepancy for the rough surfaces could be 'understood on the basis of surface tension effects associated with wetting of the wall [but that] surface tension effects associated with the main part of the free surface [would be] much too small'. They also stated that the observed ratio α/α_W 'is approximately size independent'.

Neglecting α_L , we infer from (6·3), which implies $1 \leq \alpha/\alpha_W \leq 3$ $(C_r - C_i = 2$ at $\xi = 2$ and $\eta = 0$), that surface effects *could* explain the observed results *if* the contamination had been sufficiently high. Considering gravity waves in a 6 in. diameter cylinder, for which $k = 0.24 \text{ cm}^{-1}$ and $\sigma \doteq 15 \sec^{-1}(h/a \ge 1)$, we obtain $\xi \doteq 10^{-2}\chi$ from (3·16). Now, χ can achieve values in excess of 10^2 dyn/cm for condensed films. (e.g. stearic acid); moreover, Van Dorn's (1966) results suggests that contamination adequate to render a water surface inextensible ($\xi \ge 1$) tends to establish itself within an hour under ordinary laboratory conditions. It therefore appears to be entirely possible that contamination could have been responsible for the observed discrepancy between α and α_W .

The circumstantial evidence for meniscus damping is perhaps (as Case & Parkinson assert) stronger than that for surface-film damping, but the estimation of numerical bounds is even more difficult. Considering a 6 in. diameter tank and assuming T = 72 dyn/cm (although contamination would reduce this value) and $\kappa \doteq 0.35$ (for brass; Keulegan & Brockman 1962), we obtain $\alpha_L = 6.4 \times 10^{-3} (a/A)$ from (6.4*b*); the comparable value of α_W is 0.014. Case & Parkinson's published data on amplitudes is limited to their statement that the maximum value of A/a was about 0.2. Conjecturing that their average amplitudes were half this maximum values, we obtain $\alpha_L \doteq 0.06$. Recognizing the uncertainties in both κ and A/a, we can say only that this figure is at least as large as the discrepancies between their observed and calculated (α_W) logarithmic decrements for a 6 in. diameter, unpolished cylinder. We conclude that each of surface contamination and capillary hysteresis could have been responsible for a significant portion of the discrepancy between the theoretical value of α_W and the observed values of α in Case & Parkinson's unpolished cylinders, with capillary hysteresis as probably, but not certainly, the more important effect.

7. Rectangular basin

The dominant mode in a rectangular basin of breadth B and length L(L > B) is given by $\psi = (\sigma A/k) \coth kh \cos kx, \quad k = \pi/L \quad (0 < x < L),$ (7.1)

where A is the maximum displacement of the free surface.

Substituting $(7\cdot1)$ into $(4\cdot5)$ and $(4\cdot21)$, we reduce $(4\cdot20)$ to (cf. Keulegan 1959).

 $\alpha_s = \frac{1}{2}\pi\epsilon (C_r - C_i) \coth\left(\pi h/L\right).$

$$\mathbf{x}_{W} = e\{1 + (L/B) + \pi [1 - (2h/L)] \operatorname{cosech} (2\pi h/L)\}$$
(7.2)

and
$$(4.13)$$
 to

Substituting $(7 \cdot 2)$ into $(5 \cdot 3)$, we obtain

$$\mathbf{x}_{L} = 32\kappa T'(BL)^{-1} \left[1 + (\pi B/2L)\right] \tanh kh \, \sigma^{-2} A^{-1} \tag{7.4a}$$

$$= (32\kappa/\pi) \left[1 + (\pi B/2L) \right] (l^2/BA) \quad (l \ll B).$$
 (7.4b)

Keulegan (1959) measured α in a number of geometrically similar basins, for which B/L = 0.217 and h/L = 0.425, and expressed his results in the form

$$\alpha = \alpha_W + \alpha_2, \tag{7.5}$$

(7.3)

where α_W is given by (7.2) as (for B/L = 0.217 and h/L = 0.425)

$$\alpha_W = 5.67\epsilon, \tag{7.6}$$

and α_2 has the empirical form

$$\alpha_2 = KT' \, (2\pi/\sigma)^2 B^{-3} \tag{7.7}$$

Keulegan inferred from his measurements that K = 0.1 for distilled water in glass tanks and also for any of distilled water plus aerosol (a commercial wetting agent), xylene, and a mixture of xylene and marcus oil in lucite basins—all hydrophilic combinations—and K = 0.6 for distilled water in lucite basins; however, he observed that α increased with decreasing amplitudes in the lucite basins. He also found that a very slight addition of aerosol to distilled water in a glass basin caused a marked increase in the losses, which he was unable to explain, but that the addition of sufficient aerosol to reduce the surface tension of water by approximately 40 % reduced the damping in both glass and lucite basins to that observed for distilled water in glass basins.

We consider first the possible magnitude of surface-film damping in Keulegan's experiments. Setting h/L = 0.425 in (7.3) and dividing the result by (7.6) we obtain

$$\alpha_S / \alpha_W = 0.32 (C_r - C_i). \tag{7.8}$$

Keulegan's results for geometrically identical glass and lucite basins for which $B = 10 \cdot 1 \text{ cm}, L = 47 \cdot 4 \text{ cm}, h = 20 \cdot 1 \text{ cm}, \text{and } 2\pi/\sigma = 0.83 \text{ s}, \text{ are reproduced in figure 2}.$ The corresponding value of α_W for water, as given by (7.6), is 0.019. The corresponding values of $\alpha_S + \alpha_W$, as given by (7.8), for $C_r - C_i = 1$ and 2 are 0.025 and 0.031, respectively. The value of 0.025 agrees with the observed value for distilled water

(no aerosol added) in the glass basin and with the apparent asymptote, with increasing concentration of aerosol, in both the lucite and the glass basins. The value of 0.031 compares with the observed maximum (with respect to aerosol concentration) value of 0.03 for water in the glass basin. On the other hand, it does not appear that the aerosol, acting by itself, could yield the required values of our parameter ξ . We infer from Keulegan's measurements, reproduced in figure 3, that the observed variation of surface tension is well matched by Szyszkowski's equation, (3.4), with



FIGURE 2. The effect of aerosol on the logarithmic decrement for water in lucite and glass basins (from Keulegan 1959).

A = 0.29 and $a = 1.45 \,\mathrm{p/k}$ (parts per thousand). Substituting these numbers, together with the aforementioned basin parameters, into (3.6a), and (3.16), we obtain $\xi \doteq 0.044\gamma_0$, where $\gamma_0(\mathrm{p/k})$ is the aerosol concentration. Keulegan does not give adequate information to permit an estimate of η , but we infer from (4.15b) that $C_r - C_i < 0.21\xi$ for small ξ , which implies $C_r - C_i < 0.1$ for $\gamma_0 = 10 \,\mathrm{p/k}$. It appears to be possible, or even likely, that the surface was contaminated not only by the aerosol, but also by some other material, such as a condensed film (we have already referred to Van Dorn's 1966 experiments in regard to the difficult of avoiding spontaneous contamination); on the other hand, it is difficult to reconcile this conjecture with the fact that the initial (before the addition of aerosol) surface tension of the water in Keulegan's experiment was close to 72 dyn/cm (see figure 3). And, in any event, we are unable to make any direct estimate of the compressional modulus, χ , for the (conjectured) combination of prior contamination and aerosol.

Turning to meniscus damping, we set B/L = 0.217, B = 10 cm, and

where T/T_0 is the specific surface tension (figure 3). Keulegan's measurements typically were made with initial displacements of $A_0 = 0.25H \div 0.5B$ (see his

comments in regard to his figure 5). Substituting this value, $\kappa = 0.31$ (Keulegan & Brockman 1962) for distilled water on lucite, and $T/T_0 = 1$ into (7.9), we obtain $\alpha_L = 0.006 (A_0/A)$; this compares with Keulegan's measurement of $\alpha - \alpha_W \doteq 0.03$ for the 'modulus of decay from the larger amplitudes' (we also recall Keulegan's statement that α increased with decreasing amplitude in the lucite basin). If we assume that $\alpha_L = 0$ and $\alpha_S = \alpha - \alpha_W = 0.006$ for distilled water in the corresponding glass basin, we infer that $\alpha_L = 0.029$ for distilled water in the lucite basin, corresponding to $A/A_0 \doteq 0.2$ in the preceding formula; this last value is difficult to reconcile with Keulegan's reference to 'large amplitudes'.



FIGURE 3. The effect of aerosol on the specific surface tension of water (from Keulegan 1959).

We conclude from this discussion that, just as with Case & Parkinson's measurements, the most plausible explanation of Keulegan's (1959) results is that each of surface contamination and capillary hysteresis was responsible for a significant portion of the discrepancy between the theoretical values of α_W and the observed values of α in hydrophobic basins, but that capillary hysteresis was relatively more important. We also conclude that surface-film damping was the primary contributor to $\alpha - \alpha_W$ in Keulegan's hydrophilic basins. We emphasize, however, that neither of these conclusions is based on quantitive agreement between theory and experiment and that considerable uncertainty remains.

8. WIND-GENERATED WAVES

We consider briefly the effect of an inextensible surface film on the generation of gravity waves by wind. That this effect is substantial is a matter of common knowledge (see for example, Franklin 1774), and both Keulegan (1951) and Van Dorn

[†] Keulegan suggested that part of the discrepancy, $\alpha - \alpha_W$, might be charged to the reduction in effective basin width owing to the boundary layers. The resulting change in α is $O(\epsilon^2)$, which cannot be consistently retained in a boundary layer calculation and is, in any event, numerically negligible for the basins being considered (B = 10 cm). Keulegan also investigated the possibility of turbulent dissipation and concluded that it could not have been substantial in his experiments.

(1953) found that the addition of detergent prevented the formation of sensible waves by winds of at least 12 m/s. There appears to be little doubt that the primary effect of a surface film is to increase the dissipation, and hence the energy that must be supplied by the wind, although associated changes in the velocity profile of the wind also could be significant. (Dorrestein (1951) suggested that a surface film inhibits the transfer of energy from wind to waves through tangential stresses, but it is now generally accepted that this transfer takes place primarily through normal pressures.)

We first assume that the aerodynamic energy transfer can be described by the (equivalent) logarithmic decrement

$$-\alpha_a = \pi s (U_1/c)^2 \beta, \tag{8.1}$$

where $s = \rho_a/\rho_w$ is the air/water density ratio, $c = \sigma/k$ is the wave speed, U_1 is a reference wind speed, and β is a dimensionless energy-transfer coefficient that depends on c/U_1 (Miles 1957). Equating $-\alpha_a$ to α_s , as given by (1.5), and solving for U_1 as a function of c/U_1 , we obtain

$$U_1 = (g\nu/2s^2)^{\frac{1}{3}} (c/U_1)^{\frac{1}{3}} \beta^{-\frac{2}{3}}$$
(8.2*a*)

$$= 150 \, (c/U_1)^{\frac{1}{3}} \beta^{-\frac{2}{3}} \, \mathrm{cm/s}, \tag{8.2b}$$

where $(8\cdot 2b)$ follows from $(8\cdot 2a)$ after setting g = 980, $\nu = 10^{-2}$ and $s = 1\cdot 2 \times 10^{-3}$ in c.g.s. units. Invoking the theoretical model for an equivalent laminar profile of the form $U = U_1 \log y + \text{const.}$ (Miles 1957, 1959), we find that the minimum value of U_1 predicted by $(8\cdot 2b)$ is about 1 m/s at $c/U_1 \doteq 3$, corresponding to a nominal wind speed of about 12 m/s and a wavelength of about 6 m. The predicted critical wind speed of 12 m/s is at least consistent with the observations of Keulegan (1951) and Van Dorn (1953) and is roughly an order of magnitude larger than that predicted by a similar calculation based on $\alpha_S^{(0)}$ —namely $U_1 = 14$ to 15 cm/s for gravity waves of 20 to 30 cm (Miles 1959).

A rather different energy-transfer process, which depends on the existence of a viscous sublayer for the wind structure very close to the water, may be effective for $c/U_1 < 3$ (Miles 1962). Hidy (1965) has obtained qualitative confirmation of the theoretical model, but found it necessary to postulate increased dissipation, relative to that based on $\alpha_S^{(0)}$, in order to obtain approximate, quantitative agreement. He commented that, 'Although care was taken to minimize the contamination of the water surface..., small amounts of oil may have been unavoidably present [and] could easily account for the systematic deviation between [the theoretical and experimental results] shown in the data.' The original theoretical predictions (Miles 1962) were carried out for a wind profile that is linear in the viscous sublayer and logarithmic above this layer with a free-surface damping given by $\alpha_S^{(0)}$. Replacing $\alpha_S^{(0)}$ by $\alpha_S^{(1)}$ in these calculations, we find that waves of roughly 4 to 10 cm in length should be generated by a wind with a friction velocity, U_* , of 15 cm/s, corresponding to a nominal wind speed of roughly 4 to 5 m/s. This prediction is consistent with the observations of Hidy, but not with those of Keulegan (1951).

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9. Conclusions

We conclude that, in the absence of very special precautions, both surface contamination and capillary hysteresis may contribute significantly to the damping of surface waves in closed basins. The preceding analysis provides rational estimates of the corresponding contributions to the logarithmic decrement provided that the required parameters, especially the surface compressional modulus and the contact angles for advance and recession of the meniscus, are known. Further observations, including controlled experiments guided by the theoretical predictions, are necessary before we can say whether a reliable understanding and ability to predict have been achieved.

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