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Toward realism in modeling ocean wave behavior in marginal ice zones

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Abstract. The model of *Meylan and Squire* [1996], which treats solitary ice floes as floating, flexible circular disks, is incorporated into the equation of transport for the propagation of waves through a scattering medium, assumed to represent open ice pack in a marginal ice zone. The time-independent form of the equation is then solved for homogeneous ice conditions allowing for dissipation due to scattering, together with extra absorption from interactions between floes, losses in the water column, and losses arising from the inelastic character of the sea ice including local brash. The spatial evolution of wave spectra as they progress through the pack is investigated with the aim of explaining the field data of *Wadhams et al.* [1986]. Specifically, the change toward directional isotropy experienced by waves as they travel into the ice interior is of interest. In accord with observations, directional spread is found to widen with penetration until eventually becoming isotropic, the process being sensitive to wave period. The effect of absorption on the solution is investigated.

Introduction

Many aspects of the marginal ice zone (MIZ), defined here as the segment of seasonal sea ice zone that is affected significantly by open ocean processes, are dominated by the presence of ocean waves. Yet several field experiments have produced data that are not explained well by contemporary theoretical models. In particular, features that relate to the evolution of directional structure as a sea or swell progresses into and through pack ice are not described well, and neither are reflections of waves at the ice margin.

Pragmatically, there is an obvious way to proceed, namely, to consider the MIZ as being composed of a large number of discrete solitary floes and then to integrate the effect of these floes. This is the preferred option and is the method of this paper, but an alternative approach, whereby the MIZ is characterized in some fashion by a continuous surface boundary condition, also has some history. The difficulty with synthesizing contributions from many floes is that the outcomes are only as good as the basic ice floe model. This has recently been improved by *Meylan* [1995] and *Meylan and Squire* [1996] (hereinafter referred to as MS) by computing scattering functions that incorporate the compliant nature of the floe and thereby allow it to respond properly to the local sea. MS show, for example, that bending of the ice floe by the waves alters its scattering significantly and that this should be taken into account when MIZ models are devised. Although MS show that modeling floes as rigid floating disks as was done by *Masson and LeBlond* [1989] is imprecise unless the wavelengths are much larger than the diameter of the floe, the qualitative agreement between observations and the theoretical conclusions of *Masson and LeBlond* [1989] is pleasing. Indeed, an observed trend toward isotropy with increased penetration is predicted. To correctly represent an ice field under a broad range of wave conditions and floe morphologies, however, requires the use of a solitary floe model that can flex as well as display the degrees of freedom associated with a completely rigid floating body.

Accordingly, in this paper we assimilate the MS model of the solitary, flexible circular ice floe into a transport equation that describes wave propagation in a scattering medium. Initially, a brief account is given of the model reported in full by MS. This is followed by the main theoretical development of the paper, where an ice field composed of many such floes is assembled with the purpose of reproducing the progression of waves

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through the MIZ. Finally, two examples of the analogue MIZ are given in the context of explaining some field observations [e.g., *Wadhams et al.,* 1986], that hitherto have not been entirely realistically modeled.

The Work of *Masson and LeBlond* [1989]

The current study is closely related to the work of *Masson and LeBlond* [1989]. In that paper a scattering model was developed using multiple scattering theory, as opposed to the approach of the current paper, which begins with the equation of transport. In many ways the differences are cosmetic, as *Masson and LeBlond* [1989] end up with a scattering equation that is almost identical to ours. Although Masson and LeBlond include extra terms such as nonlinear wave coupling and wind wave generation, they consider the temporal development of the wave spectra while we are considering the spatial development for which such terms are less significant. These terms could be included straightforwardly in the present formulation if required. An advantage of our approach over that of *Masson and LeBlond* [1989] is that the scattering equation is derived in a simpler and more transparent way. Moreover, there are terms included in the derivation of *Masson and LeBlond* [1989] that appear to make the scattering equation mathematically inconsistent. These terms do not arise in our derivation.

As was noted above, *Masson and LeBlond* [1989] also use a different model for the single floe, allowing for its draft but neglecting flexure. Accordingly, their model applies only to MIZs composed of floes of small aspect ratio. The bulk of MIZs are excluded, as they are most commonly made up of compliant floes of tens to hundreds of meters in diameter and perhaps 1–2 m in thickness.

Circular Floe Model

A crucial feature of the solitary floe solution used in this paper is that the ice floe is assumed to behave as a flexible thin plate of negligible draft, rather than as a rigid body. The circular geometry chosen simplifies the solution, as it is only for a circular disk that the free modes of vibration can be expressed in an analytic form. Solving for a more complex floe geometry would serve little purpose, since what we require is an estimate of the average floe scattering from ice floes with a distribution of geometries such as make up the MIZ. The problem considered is linear, and time dependence is removed by taking only a single frequency ω in the usual manner. In nondimensional form the boundary value problem to be solved is then

$$\nabla^2\phi = 0 \qquad -\infty < z < 0 \tag{1a}$$

$$\frac{\partial\phi}{\partial z} = 0 \qquad z \to -\infty \tag{1b}$$

$$\frac{\partial\phi}{\partial z} - \alpha\phi = 0 \qquad z = 0 \qquad 1 < r < \infty \tag{1c}$$

$$(\beta\nabla^4 + 1 - \alpha\gamma)\frac{\partial\phi}{\partial z} - \alpha\phi = 0 \qquad z = 0 \qquad 0 \le r \le 1 \tag{1d}$$

$$\frac{\partial^2}{\partial r^2}\frac{\partial\phi}{\partial z} + \nu\left(\frac{\partial}{\partial r}\frac{\partial\phi}{\partial z} + \frac{\partial^2}{\partial\theta^2}\frac{\partial\phi}{\partial z}\right) = 0 \qquad r = 1 \tag{1e}$$

$$\frac{\partial}{\partial r}\left(\nabla^2\frac{\partial\phi}{\partial z}\right) + (1-\nu)\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial^2}{\partial\theta^2}\frac{\partial\phi}{\partial z}\right) = 0 \qquad r = 1 \tag{1f}$$

where (r, θ, z) is a cylindrical-polar coordinate frame with its origin at the center of the floe, ϕ is the velocity potential in the water, ν is Poisson's ratio, and α, β, and γ are nondimensional variables defined by MS. The Sommerfield radiation condition [*Sarpkaya and Isaacson,* 1981] makes the problem well posed.

Using the Green's function for linear water waves, the boundary value problem (1) is solved by converting it to an integral equation, expanding ϕ and its normal derivative in terms of the free modes of vibration of a circular thin plate [*Itao and Crandall,* 1979], and then truncating to achieve the desired accuracy. Defining the Kochin function H [*Wehausen and Laitone,* 1960] in terms of dimensional quantities, as

$$H(\theta) = \int_\Delta (k\phi - \frac{\partial\phi}{\partial z})e^{ik(x\cos\theta + y\sin\theta)}dx\,dy \tag{2}$$

where k is the wave number, the energy radiated in unit time per unit angle is given by

$$E(\theta) = \frac{\rho\omega^3}{8\pi g}|H(\pi+\theta)|^2 \tag{3}$$

where g is the acceleration due to gravity and ρ is the density of the seawater.

Energy Scattering

In the absence of scatterers it is known that the energy intensity satisfies the following equation [*Phillips,* 1977, p. 26]

$$\frac{1}{c_g}\frac{\partial}{\partial t}I(\boldsymbol{r}, t, \theta) + \theta\cdot\nabla_r I(\boldsymbol{r}, t, \theta) = 0 \tag{4}$$

where $I(\boldsymbol{r}, t, \theta)$ denotes an intensity function that describes the rate of flow of energy traveling in a given direction, across a surface element normal to that direction, per unit of surface, per unit of solid angle of direction, and c_g is the velocity of propagation. We have neglected any wave source or dissipation terms in (4) but these could be included without difficulty as is done by *Masson and LeBlond* [1989]. Here we prefer to neglect these small terms and to focus on the effect of the scattering due to the sea ice floes. Because ice floes in the MIZ scatter energy, (4) must be rewritten according to *Howells* [1960] as

$$\frac{1}{c_g}\frac{\partial}{\partial t}I(\boldsymbol{r}, t, \theta) + \theta\cdot\nabla_r I(\boldsymbol{r}, t, \theta) = -\beta(\boldsymbol{r}, t, \theta)I + \int_0^{2\pi} S(\boldsymbol{r}, t, \theta, \theta')I(\boldsymbol{r}, t, \theta')d\theta' \tag{5}$$

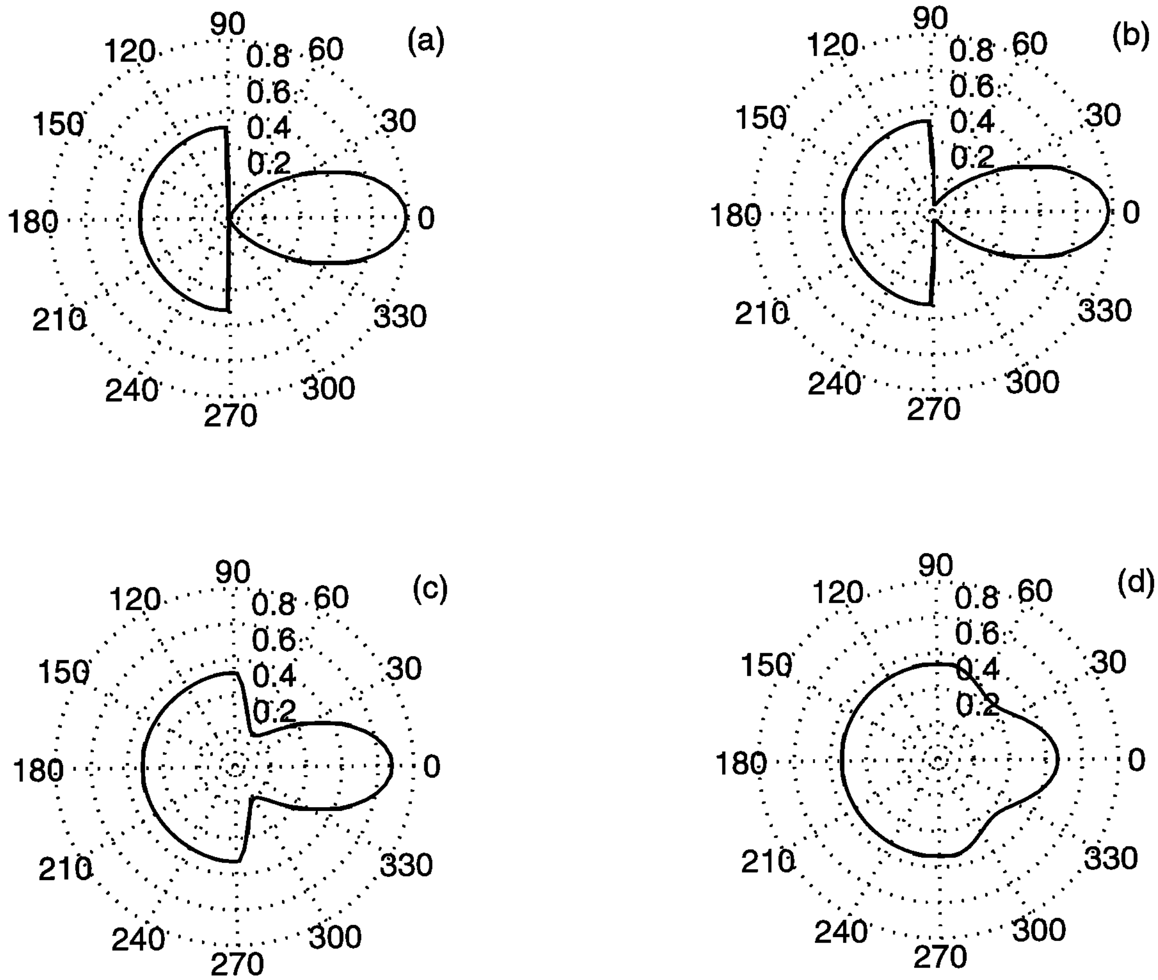

Figure 1. Polar plots showing changes in the spread of a 10-s sea traveling into a typical MIZ at penetrations (a) 0 km, (b) 1 km, (c) 10 km, and (d) 50 km. Concentration is 50%, floe thickness is 1 m, and floe diameter is 50 m. The damping coefficient $\beta' = 0$.

which is known as the Boltzmann equation. It has been assumed that each floe scatters independently of those around it and that energy from different scatterers can be added incoherently. We emphasize that this does not mean that the model omits multiply scattered waves; indeed, the fundamental aim of the Boltzmann equation is to model multiple scattering. What it means is that coherent interference between adjacent floes has been neglected. Further, because the scattering model is linear, we also do not allow an incident wave of one frequency to excite motions at a different frequency, i.e., no nonlinear coupling of different frequencies occurs.

The absorption coefficient $\beta(\boldsymbol{r}, t, \theta)$ represents the fraction of energy lost by scattering and dissipative processes from a pencil of radiation in direction θ, per unit path length traveled in the medium. As such, as well as scattering losses, it includes all irrecoverable dissipation occurring in the MIZ due to hydrodynamical turbulence and wave breaking, collisions and abrasion between ice floes, and hysteresis losses occurring in the brash between floes and through bending of the floes themselves.

The scattering function $S(\boldsymbol{r}, t, \theta, \theta')$ specifies the angular distribution of scattered energy in such a way that $SId\Omega dV d\Omega'$ is the rate at which energy is scattered from a pencil of radiation of intensity $I(\boldsymbol{r}, t, \theta')$ in a solid angle $d\Omega'$ in direction θ', by a volume dV of medium at position $\boldsymbol{r}$, into a solid angle $d\Omega$ in direction θ.

To proceed, we omit time dependence, assume that the solution is a function of the x spatial coordinate alone, and restrict our study to uniform MIZs. Then S is a function of θ and θ', β is a constant, and (5) becomes

$$\cos\theta \frac{\partial I}{\partial x} = -\beta I + \int_0^{2\pi} S(\theta, \theta') I(x, \theta') d\theta' \qquad (6)$$

At $x = 0$ a directional spectrum I_0 is presumed to be incident on the ice edge, i.e.,

$$I(0, \theta) = I_0(\theta) \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \qquad (7)$$

If the MIZ is taken to be infinite in extent, as is done

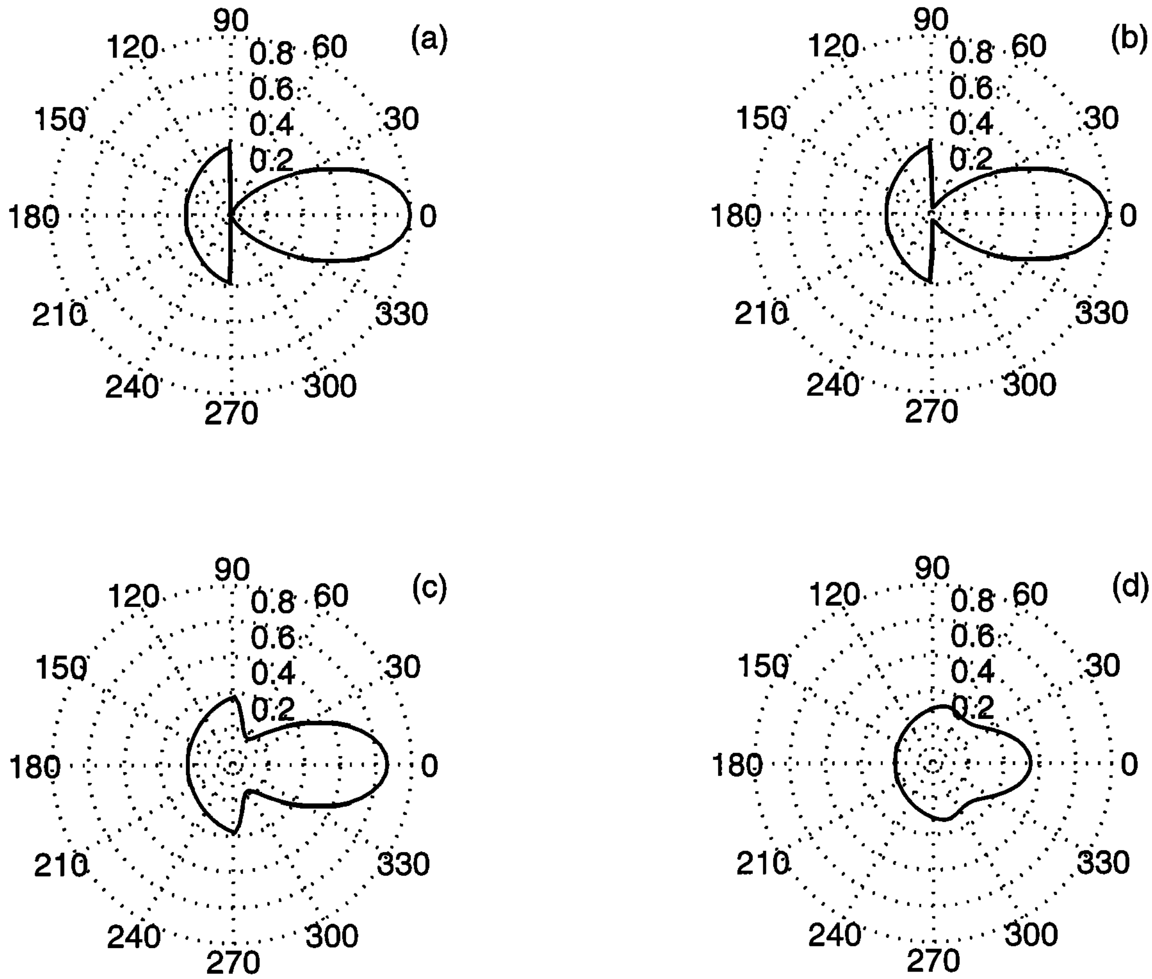

Figure 2. As Figure 1 with 5% irrecoverable damping.

in this paper, no net propagation occurs at $x = \infty$, whereas if it is finite the distant ice edge gives rise to a boundary condition that for large times will march back to affect the solution in the interior if no diffusion is applied. In either case the additional boundary condition closes the system to be solved, but the latter situation is physically implausible for a time-independent solution.

Following *Meylan and Fox* [1996], assuming incoherent scattering between floes, we can now express the scattering function in terms of E, taking the wave energy density to be $\frac{1}{2}\rho g a^2$, where a is the wave amplitude. To get the rate of energy flow per ice floe, this must be multiplied by the average area occupied by a floe, A/c, where A is the area of a floe and c is the concentration of ice floes, and by the wave phase speed ω/k, since the solution is for a single frequency. The following expression for S is obtained:

$$S(\theta, \theta') = S(\theta - \theta') = \frac{2E(\theta - \theta')ck}{A\rho g a^2 \omega} \tag{8}$$

Accordingly, the equation for $I(x, \theta)$ is

$$\frac{\partial I}{\partial x} = -\frac{\beta}{\cos\theta} I + \frac{1}{\cos\theta} \int_0^{2\pi} S(\theta - \theta') I(\theta') d\theta' \tag{9}$$

where from conservation of energy

$$\beta = \bar{\beta} + \beta' = \int_0^{2\pi} S(0, \theta') d\theta' + \beta' \tag{10}$$

The decomposition into absorption due to scattering ($\bar{\beta}$) and absorption due to irrecoverable processes (β') is made to allow us to study the effect of different degrees of (constant) dissipation. Equations (5) and (8) depend on the assumption of incoherent scattering from adjacent floes and so will become less valid as the floe concentration is increased. Even at high concentrations the scattering term will be a reasonable estimate of scattering, provided that the floe size is the same order as the wavelength. Of course, the scattering term is only an estimate, and local variations in floe size and geometry are other reasons why the scattering model will not precisely represent specific marginal ice zones.

Using the work of *Ishimaru* [1978] we convert (9) into matrix form by writing

$$\frac{\partial}{\partial x}\mathbf{I} = \mathbf{S}\mathbf{I} \tag{11}$$

with matrix coefficients

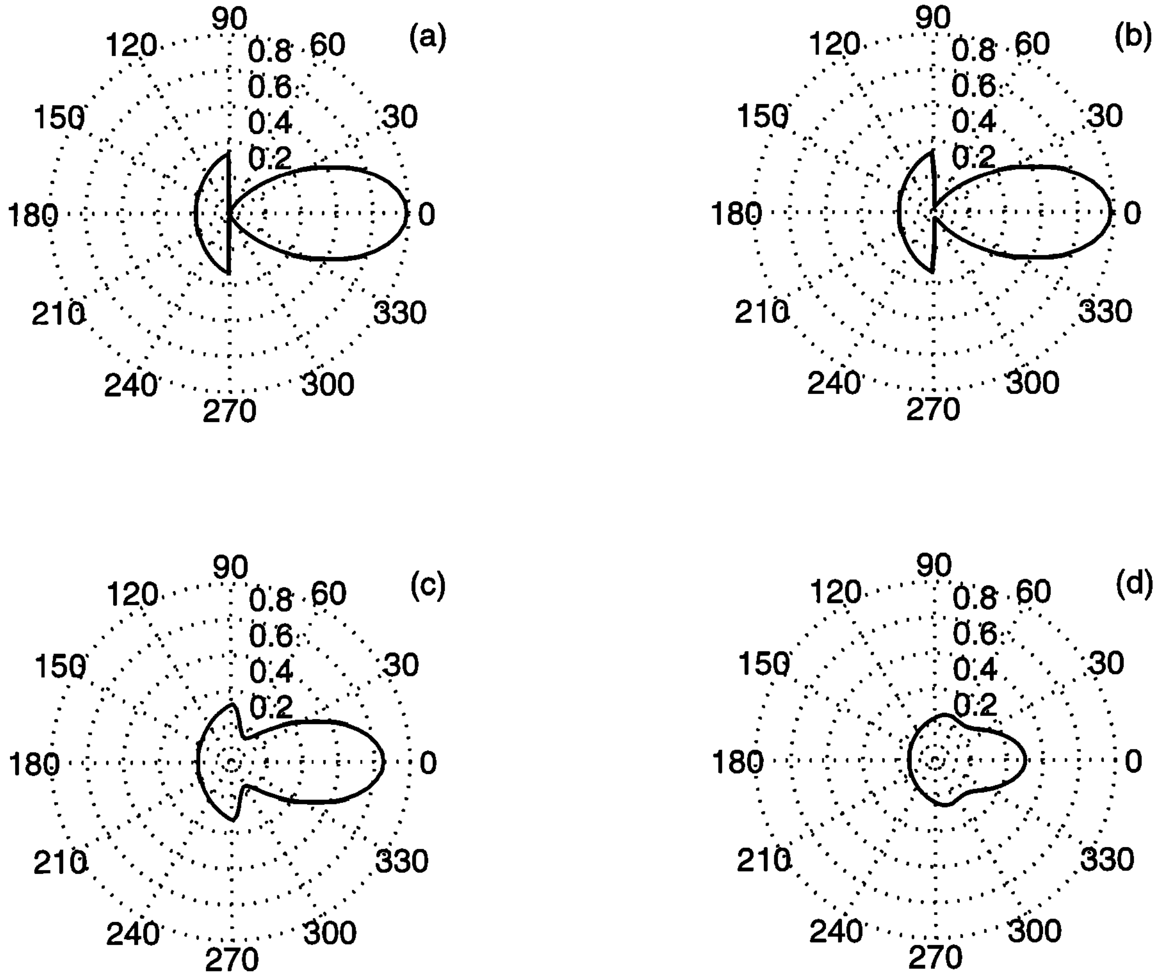

Figure 3. As Figure 1 with 10% irrecoverable damping.

$$S_{ij} = \begin{cases} \dfrac{S(0)w_i - \beta}{\cos\theta_i} & i = j \\ \dfrac{S(\theta_i - \theta_j)}{\cos\theta_i} w_j & i \neq j \end{cases} \tag{12}$$

and w_i as weight function. For $\beta' = 0$, (11) has the solution

$$\mathbf{I}(\theta_i, x) = c_0\mathbf{f}_0(\theta_i) + c_1\left(x\mathbf{f}_0(\theta_i) + \mathbf{f}_1(\theta_i)\right) + \sum_{n\geq 2} c_n \mathbf{f}_n(\theta_i)e^{\lambda_n x} \tag{13}$$

where $\mathbf{f}_n$ are the eigenvectors of the matrix $\mathbf{S}$ and $\{0, \lambda_n\}$ is its set of eigenvalues. The eigenvector $\mathbf{f}_0$ is associated with the repeated eigenvalue 0 corresponding to the isotropic solution, and because there is only one such independent eigenvector, a generalized eigenvector $\mathbf{f}_1$ must be used to obtain a second independent solution $(x\mathbf{f}_0 + \mathbf{f}_1)$ [*Boyce and DiPrima*, 1986]. The values of the coefficients c_n are determined by the boundary conditions. For an infinite MIZ, $c_n = 0$ if $\lambda_n > 0$ and $c_1 = 0$, as there is no flow as $x \to \infty$. The rest of the c_n are found from the boundary condition at $x = 0$, i.e.,

$$\sum_n c_n \mathbf{f'}_n(\theta_i) = \mathbf{I}_0(\theta_i) \qquad -\frac{\pi}{2} < \theta_i < \frac{\pi}{2} \tag{14}$$

where the set $\mathbf{f'}_n$ comprises $\mathbf{f}_0$ and the eigenvectors with negative eigenvalues.

When $\beta' \neq 0$ the zero eigenvalue vanishes, and consequently the solution becomes simply

$$\mathbf{I}(\theta_i, x) = \sum_n c_n \mathbf{f}_n(\theta_i)e^{\lambda_n x} \tag{15}$$

together with boundary conditions (14) with set $\mathbf{f'}_n$ now comprising just the negative eigenvalues.

Meylan and Fox [1996] provide equivalent equations when the MIZ is of finite extent. Because of the mathematical properties of the Boltzmann equation it does not follow that results for a semi-infinite marginal ice zone can be obtained from consideration of a suitably large but finite MIZ. We believe that a semi-infinite MIZ is a more physically realistic model for wave scattering because observations suggest that most MIZs are sufficiently broad that on their far side, where they meet the interior ice canopy, wave energy is essentially negligible.

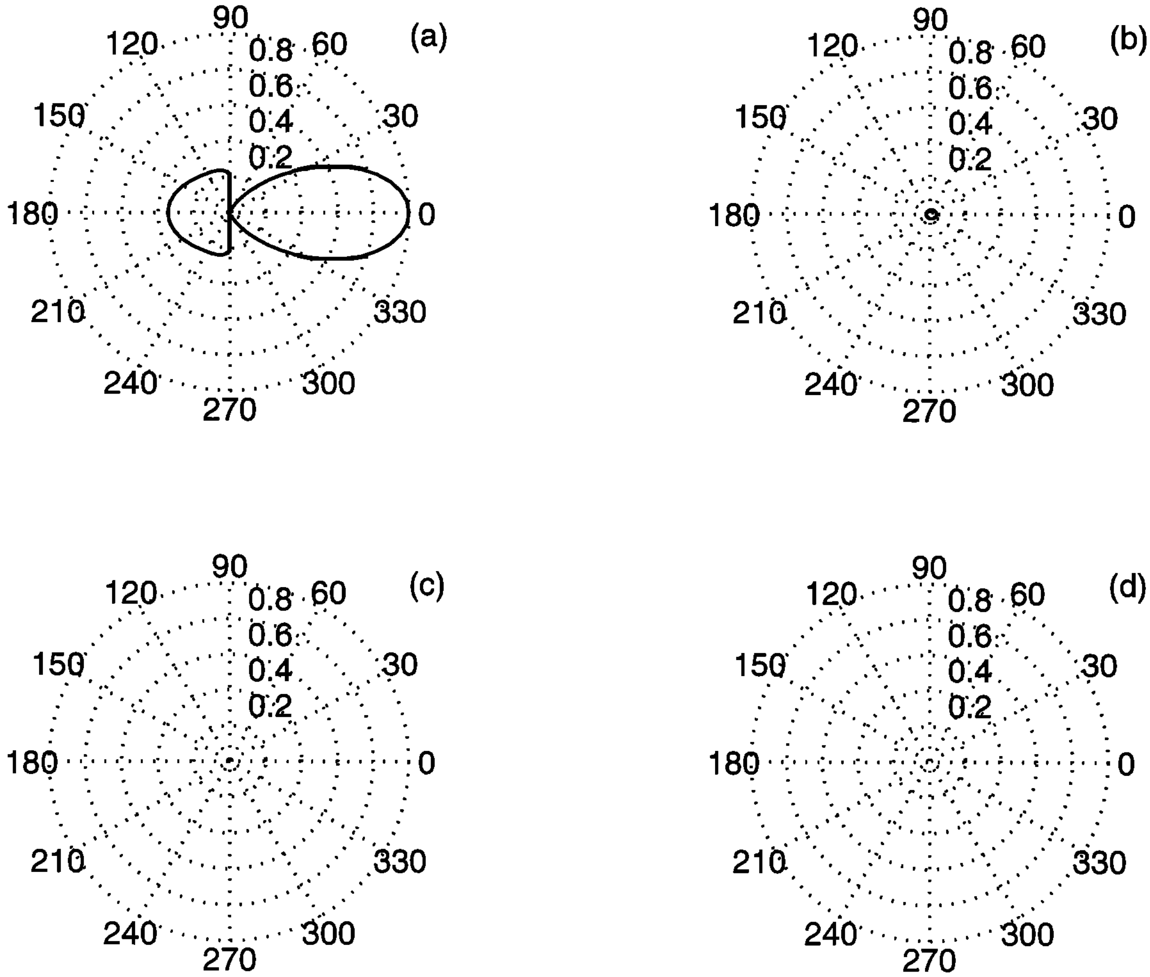

Figure 4. As Figure 2 for a 5-s sea.

Results

Throughout this section we consider the development of an open water directional spectrum $E(T, \theta)$ as it proceeds into and through an MIZ composed of 50-m ice floes. In common with other work we use [*Longuet-Higgins et al.*, 1963; *Phillips*, 1977, p. 139]

$$E(T,\theta) \propto T^5 \exp\left(-\frac{5}{4}\left(\frac{T}{T_{\mathrm{m}}}\right)^4\right)\cos^{2s}\left(\frac{\theta-\bar{\theta}}{2}\right) \quad (16)$$

where T denotes period, T_{m} is the peak period, and the principal direction $\bar{\theta} = 0°$. The exponent s is set at 10, after *Ewing and Laing* [1987]. Because we are interested only in relative change, the constant of proportionality is unimportant and we simply set $\max\{E\} = 1$.

Monochromatic Seas

In all the results that follow, unless otherwise stated we assume that Poisson's ratio for sea ice is 0.3 and Young's modulus is 6 GPa. Figures 1–3, drawn for 0, 5, and 10% irrecoverable damping, respectively, illustrate the way in which the $\cos^{2s}\theta/2$ open sea spread alters as 10-s-period ocean waves travel into an open MIZ. In each case a homogeneous distribution of 1-m-thick circular ice floes is used, and polar plots are given at the edge (0 km) and at penetrations of 1, 10, and 50 km. Each set of four polar plots is qualitatively similar. For example, Figure 1a is made up of an incident part lying to the right of the origin that is described by $\cos^{2s}\theta/2$, and a reflected isotropic part lying to the left of the origin. At 1-km penetration (Figure 1b) the appearance is similar, though a slight broadening of the forward going lobe and smearing out of the backscattered energy is evident. This effect is more noticeable 10 km from the ice edge (Figure 1c) and is very conspicuous in Figure 1d, which shows that the spread at 50 km is becoming quite isotropic. Figures 2 and 3 are similar to Figure 1 except that the nonzero β' diminishes the scattered and forward going lobes as would be anticipated. Scattered waves are particularly affected by the damping.

It is of interest to consider shorter- and longer-period seas. This is done for 5- and 20-s-period waves, respectively, in Figures 4 and 5, where the irrecoverable damping is set at 5%. In the 5-s case the spread becomes isotropic within a kilometer or so, but the waves are also strongly attenuated so that by 10 km their intensity is negligible. Figure 5, on the other hand, shows

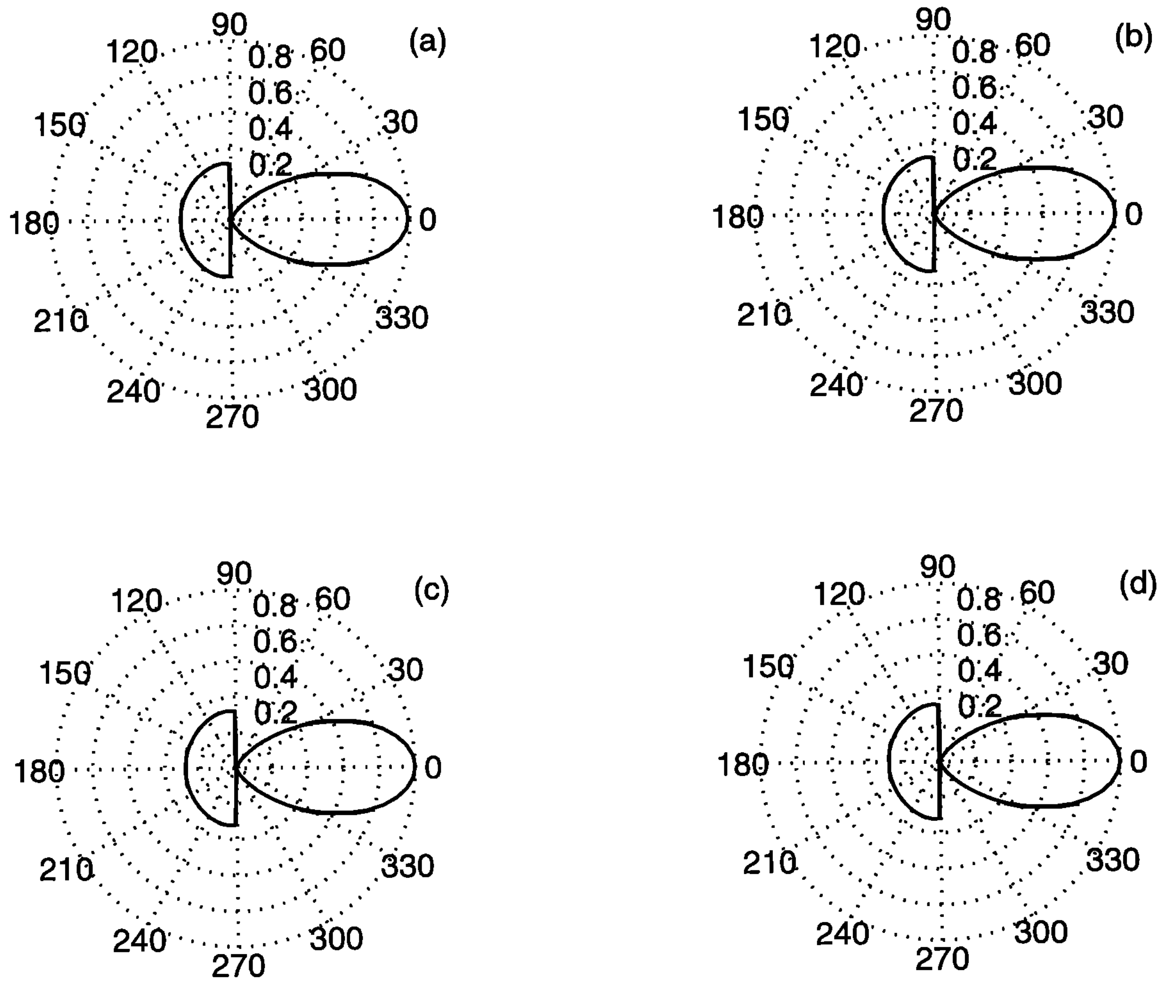

Figure 5. As Figure 2 for a 20-s sea.

that 20-s waves can travel far into the interior of the MIZ with little attenuation and next to no change in directional spread.

It is also of value to consider what effect the inclusion of floe flexure has on the calculation of the scattering operator in our results. Figure 6 is a plot of the directional spectrum for 10-s-period waves with 5% damping at the ice edge (Figures 6a and 6c) and at 500 m from the ice edge (Figures 6b and 6d). The floe thickness is 0.5 m and the floe diameter is 100 m. In Figures 6a and 6b the standard values of sea ice physical constants are used, whereas the stiffness is set to ∞ in Figures 6c and 6d; i.e., no floe flexure is allowed as in the model of *Masson and LeBlond* [1989]. While it is true that we have chosen physical values that emphasize how different the results from the two models can be, they are in no way unrepresentative of actual conditions found in typical MIZs, e.g., in the Bering Sea. The results at the ice edge are much the same, but the estimates of wave energy 500 m in are completely different. On the basis of observations collected by V. A. Squire, it is unlikely that the wave field would be modified to the extent suggested by Figure 6d so close to the ice edge. Consequently, we conclude that flexure is important in modeling the scattering process.

Summarizing, we find that the amount of directional spreading caused by ice floes in an MIZ is strongly dependent on the period of the incoming wave train. Short-period seas entering an ice field will quickly become isotropic, whereas long-period swells will retain their directionality to far greater penetrations. This is in agreement with field observations, which will be discussed later.

Evolution of Directional Spectra

Figures 7 and 8 invoke expression (16) to track the progress of a directional energy spectrum as it moves into and through an ice field composed of 50-m floes. For Figure 7, ice thickness is 1 m, and for Figure 8 it is 5 m. Note the considerable distortion of the spectral shape that occurs, both in terms of directional spread and the distribution of energy between periods. The gradual broadening of the spread seen in the earlier polar plots is evident for 1-m-thick ice floes, along with a selective filtering effect that favors the damping of shorter waves in preference to longer ones. By 50 km (Figure 7d) the spectrum is close to being isotropic for all but the longest waves, and the spectral energy is biased towards large periods. Because the scattering and consequently the damping is greater for thicker ice,

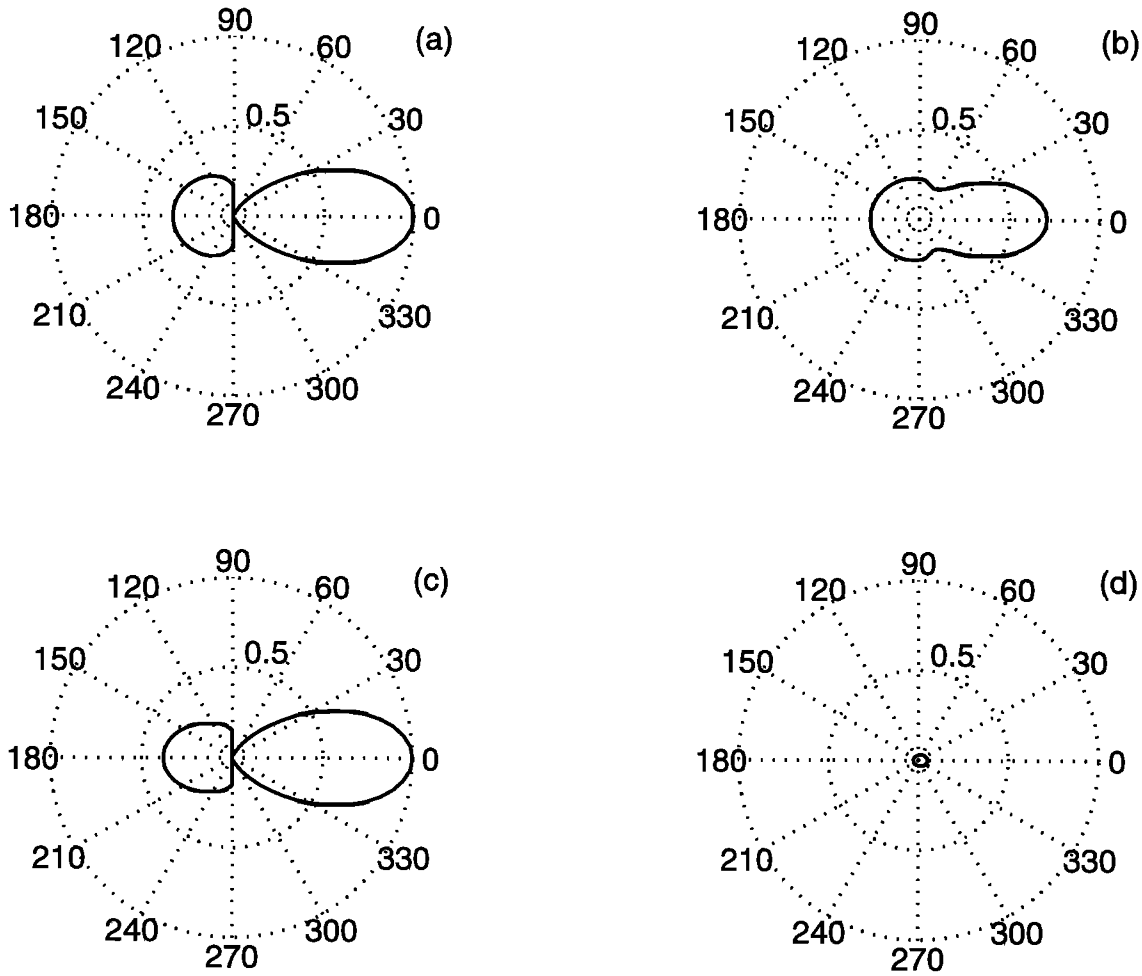

Figure 6. Polar plots showing changes in the spread of a 10-s sea traveling into a typical MIZ at penetrations of 0 km (Figures 6a and 6c) and 0.5 km (Figures 6b and 6d). Standard sea ice parameters are used in Figures 6a and 6b while no floe flexure is permitted in Figures 6c and 6d. Concentration is 50%, floe thickness is 0.5 m, and floe diameter is 100 m. The damping coefficient $\beta' = 5\%$.

Figure 8 is a more extreme version of Figure 7 that is included to illustrate the effect of thickness, even though 5-m-thick pack ice is uncommon. Here an isotropic spread is achieved within a kilometer or so of the ice edge, and by 50 km most of the energy has been removed from the spectrum; only a perfectly isotropic ring of long period energy remains with no suggestion of the original directional structure. These theoretical results are supported qualitatively by field data, as we shall now show.

Field Observations

Data reported by *Wadhams et al.* [1986] are, to the authors' knowledge, the only published field observations that follow the progress of a directional sea through pack ice. Experiments conducted in the Greenland Sea, as part of the 1984 Marginal Ice Zone Experiment, allow qualitative comparison with the theoretical predictions of the present paper. There a pitch-roll buoy was deployed in the waters off the ice edge, and an equivalent unit was placed on suitable ice floes within the ice cover. In each case, vertical acceleration and two orthogonal tilts referenced to north were measured to allow the directional wave spectrum at each location to be found using the method of *Cartright* [1963]. Because Wadhams et al. did not record the ice conditions in great detail as simultaneous aerial photography could not be done, it is not possible to perform a comprehensive quantitative comparison between the model and the data set. We must therefore rest content with replicating features in the data as convincingly as possible.

On July 12–13 the incident sea was composed of a wind sea and a swell. One open water station and three ice stations are of interest here, at penetrations of −8.2, 5.6, 17.8, and 22.5 km. The sea ice near the edge was about 8/10 in concentration, but this decreased to only 1/10 in the vicinity of the first ice station and then gradually increased again toward the interior where the ice was described as heavy pack. The ice floes at the sta-

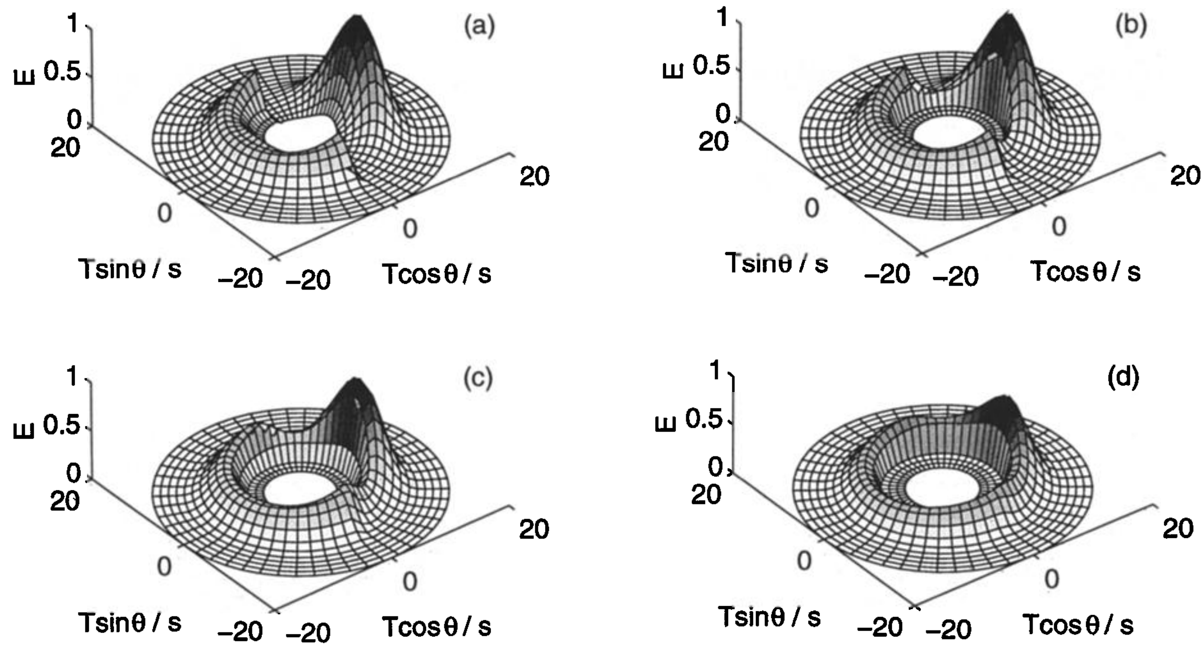

Figure 7. Three-dimensional plots showing changes in the directional spectrum traveling into an MIZ of 1-m-thick sea ice at penetrations of (a) 0 km, (b) 1 km, (c) 10 km, and (d) 50 km. Concentration is 50%, floe diameter is 50 m, and 5% damping is used.

tion locations were 0 (open water), 72, 200, and 350 m across, and the significant wave heights were found to be 150, 12.9, 2.3, and 1.4 mm, respectively. Substantial damping of the waves by the pack ice is evident, which, on examination of Figure 11 of *Wadhams et al.* [1986], occurred mainly at short periods. Of importance, the directional spread at the peak frequency changed significantly from 42° off the ice edge to 67° at 17.8 km and 76° at 22.5 km. These values suggest a very gradual change to directional isotropy with distance traveled, but the length scale over which the process occurs is large because the waves are long (14–15 s).

By taking the input spectrum at the edge of the MIZ to be identical to that at −8.2 km, the effect of these waves can be tracked theoretically as they proceed into the pack ice for comparison with the actual measurements. This is done in Figure 9 using the measured wave energy spectrum but invoking the form

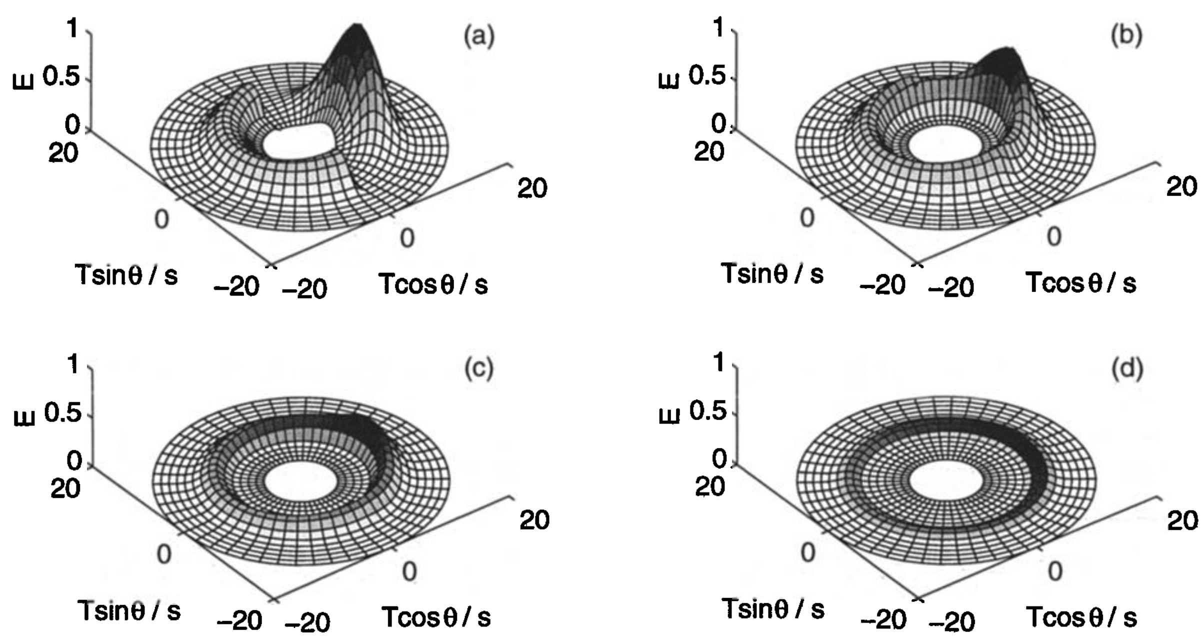

Figure 8. As Figure 7 for 5-m-thick ice floes.

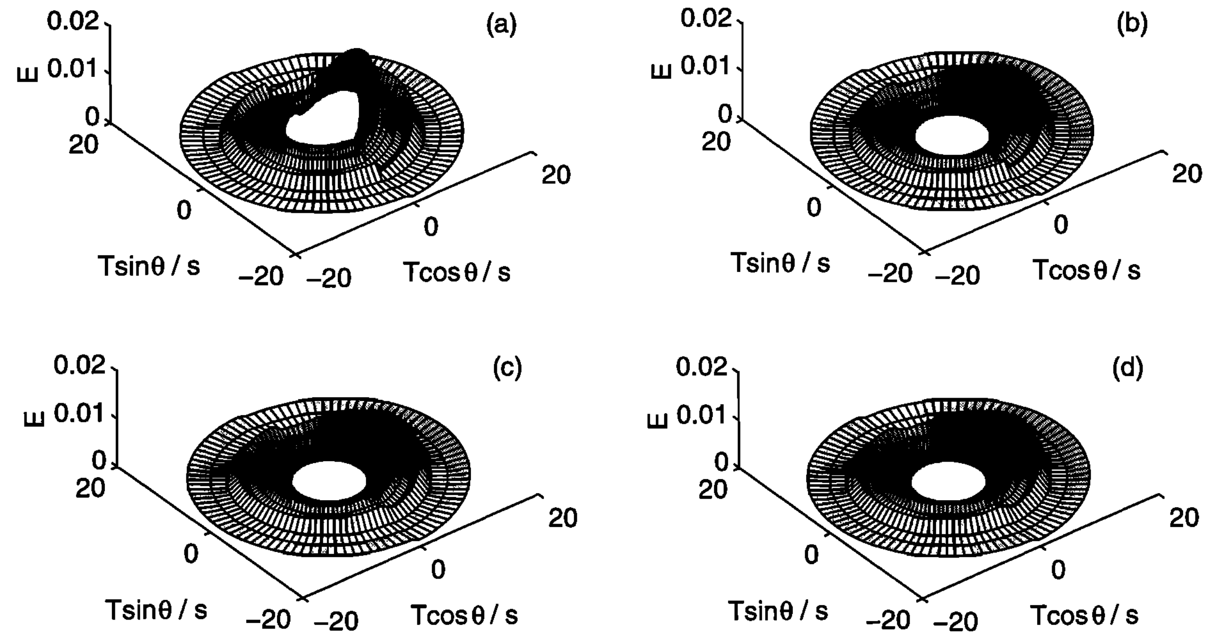

Figure 9. Three-dimensional plot of (a) the open sea spectrum measured at the ice edge, together with its computed evolution through the marginal ice zone at (b) 5.6 km, (c) 17.8 km, and (d) 22.5 km.

$\cos^{2s}(\theta - \bar{\theta})/2$ of (16) to describe the directional structure of the incoming sea, where s is found from the mean angular spread. In Figure 9 it is evident that short-period energy is present to a significant degree only in Figure 9a, and that by 5.6-km penetration (Figure 9b) the local sea contains predominantly longer period waves (8.1 s). This is in accord with Figure 11 of *Wadhams et al.* [1986], though the peak period there is somewhat greater (9.6 s). The gradual increase in the value of directional spread noted in the data is not seen in the theoretical spectra, even at the greatest penetration (Figure 9d). Indeed, the spread remains at its open sea value of 42° instead of gradually increasing to 76°. While it is certainly disappointing that slight directional broadening is not reproduced in this case, it should not come as too great a surprise, as the description of the ice cover provided by *Wadhams et al.* [1986] has insufficient detail for a precise match to the experimental conditions at the time.

In a second experiment on July 13–14, a station at 1.2-km penetration was compared with several off the ice edge. Here the change in directional spread of two distinct peaks in each spectrum could be tracked, the first at about 7 s corresponding to a short swell, and the second at about 3.3 s corresponding to a local wind sea. It was found that the wind sea's open water spread broadened from 36° on entry to the ice field, becoming undefined and isotropic by the time it reached the station at 1.2 km, where its significant wave height was only 5% of its open water value. Longer waves were less affected that close to the ice margin. Their spread increased just 3° at 1.2 km penetration from the open water value of 29°, and their significant wave height decreased in the ratio 0.72.

In Figure 10 we investigate these longer waves using the same technique as above, namely, we input a measured energy spectrum with the directional structure of (16) and then compute theoretically the spectrum at

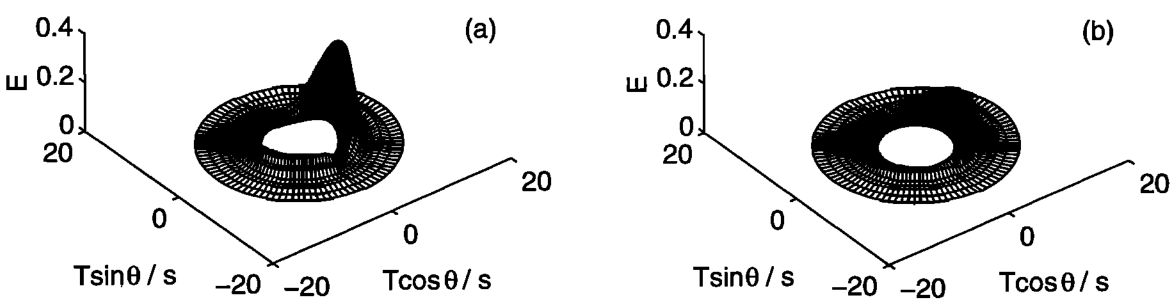

Figure 10. Three-dimensional plot of (a) open sea spectrum measured at ice edge, together with (b) its computed shape 1.2 km from the ice edge.

1.2 km. The incoming sea is modified in two ways: it is attenuated by scattering in the pack so that the predicted ratio of significant wave heights is 0.65, and the directional spread changes from 29° to 36°. This is in reasonable agreement with the data, although again uncertainties in the ice conditions during the experiment must limit the quantitative agreement possible.

Accordingly, a picture emerges in the data of a process that acts to broaden the angular spread of a penetrating sea to a degree dependent on period. Short-period waves are quickly scattered to become isotropically distributed in direction, whereas longer waves penetrate farther into the pack ice before their directional structure eventually becomes isotropic. In the long run all waves are affected, but the penetration at which isotropy is attained depends on the period in relation to the ice field's morphology. This is as predicted by the theoretical model described earlier in the paper.

Conclusion

This paper, along with that of *Masson and LeBlond* [1989], is a serious attempt to describe the behavior of ocean waves in a marginal ice zone in as authentic a manner as possible. Both papers have the same goal, but each tackles the problem in a slightly different way. We believe, supported by arguments in MS and Figure 6, that the natural compliancy of each ice floe significantly distorts the scattering function and, consequently, that it must be included when solitary floe results are synthesized over the MIZ. Our results provide compelling evidence that scattering causes the broadening in the spread of directional seas observed by *Wadhams et al.* [1986], in contrast to earlier theoretical models based on a continuous surface boundary condition that lead to collimation. The ratio of the wave's length to the floe diameter and the ice thickness are both found to be crucial in determining the penetration at which isotropy is first achieved.

The present model certainly has deficiencies. Of importance: (1) the MIZ has been assumed to be uniform, (2) the absorption coefficient β is constant, (3) no dependence on time is included, and (4) the solution depends only on the x coordinate. The first author is currently working hard to circumvent some of these assumptions.

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References

Boyce, W. E., and R.C. DiPrima, *Elementary Differential Equations and Boundary Value Problems,* 4th ed., p. 74, John Wiley, New York, 1986.

Cartright, D. E., The use of directional spectra in studying the output of a wave recorder on a moving ship, in *Ocean Wave Spectra,* p. 203, Prentice-Hall, Englewood Cliffs, N. J., 1963.

Ewing, J. A., and A. K. Laing, Directional spectra of seas near full development, *J. Phys. Oceanogr., 17,* 1696, 1987.

Howells, I. D., The multiple scattering of waves by weak random irregularities in the medium, *Proc. Roy. Soc. A, 252,* 431, 1960.

Ishimaru, A., *Wave Propagation and Scattering in Random Media,* Vol. 1, p. 202, Academic, San Diego, Calif., 1978.

Itao, K., and S. H. Crandall, Natural modes and natural frequencies of uniform, circular, free-edge plates, *J. Appl. Mech., 46,* 448, 1979.

Longuet-Higgins, M. S., D. E. Cartwright, and N. D. Smith, Observations of the directional spectrum of sea waves using the motions of a floating buoy, in *Ocean Wave Spectra,* p. 111, Prentice-Hall, Englewood Cliffs, N. J., 1963.

Masson, D., and P. H. LeBlond, Spectral evolution of wind-generated surface gravity waves in a dispersed ice field, *J. Fluid Mech., 202,* 43, 1989.

Meylan, M., The motion of a floating flexible disk under wave action, in *Proceedings of the 5th International Offshore and Polar Engineering Conference,* vol. 3, p. 450, Int. Soc. of Offshore and Polar Eng., Golden, Colo., 1995.

Meylan, M. H., and C. Fox, A model for the propagation of waves through the MIZ from a single floe solution, in *Proceedings of the 6th International Offshore and Polar Engineering Conference,* vol. 3, p. 321, Int. Soc. of Offshore and Polar Eng., Golden, Colo., 1996.

Meylan, M. H., and V. A. Squire, Response of a circular ice floe to ocean waves, *J. Geophys. Res., 101,* 8869, 1996.

Phillips, O. M., *The Dynamics of the Upper Ocean,* 2nd ed., Cambridge Univ. Press, New York, 1977.

Sarpkaya, T., and M. de St. Q. Isaacson, *Mechanics of Wave Forces on Offshore Structures,* Van Nostrand Reinhold, New York, 1981.

Wadhams, P., V. A. Squire, J. A. Ewing, and R. W. Pascal, The effect of the marginal ice zone on the directional wave spectrum of the ocean, *J. Phys. Oceanogr., 16,* 358, 1986.

Wehausen, J. V., and E. V. Laitone, Surface waves, in *Encyclopedia of Physics,* vol. IX, p. 446, Springer-Verlag, New York, 1960.

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