Effects of wave breaking on the near-surface profiles of velocity and turbulent kinetic energy.

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Abstract. A theoretical model for the near-surface velocity profile in the presence of breaking waves is presented. Momentum is accumulated by growing waves and released upon wave breaking. In effect, such a transition is a process involving a time dependent surface stress acting on the mean current. In the present study conventional theory for the Stokes drift is expanded to fourth order accuracy in wave steepness. It is shown that the higher order terms lead to an enhancement of the surface Stokes drift and a slight retardation of the Stokes volume flux. Furthermore, the results from the wave theory are used in order to obtain a bulk parameterization of momentum exchange during the process of wave breaking. The mean currents are then obtained by application of a variation of the "level 2.5" turbulence closure theory due to Mellor and Yamada. When compared to the traditional approach of a constant surface stress, the mean Eulerian current exhibits a weak enhancement in the near-surface region, compensated by a negative shift deeper in the water column. However, it is found that the results of Craig and Banner, and Craig, are not significantly affected by the present theory. Hence, this study helps to explain why the Craig and Banner model agrees well with observations when a realistic, time varying surface stress acts on the drift currents.

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1. Introduction

With the exception of coastal regions with large tidal amplitudes, the circulation of the ocean's surface layer is mainly attributed to the wind stress. This stress induces motion in the upper ocean on a variety of scales, ranging from turbulence to waves and ocean currents. The effect of the wind stress on waves can be observed as increasing wave amplitudes. This process has limitations, since waves with a steepness beyond a critical value become unstable, and wave breaking occurs. The main topics of the present paper are (1) how wave breaking affects the distribution of the momentum input to the ocean due to a stress acting on the free ocean surface, and (2) how wave breaking affects the near-surface properties of turbulence.

For a given stress at the ocean's surface, the vertically integrated momentum in the ocean is influenced only by the momentum sink at the bottom. Obviously, this is also true when momentum is distributed between wave motion and mean currents (Weber and Melsom 1993a). However, the presence of waves and wave breaking may significantly affect the vertical distribution of momentum and its temporal evolution (Melsom 1996).

In a study that has a focus on the distribution of momentum, the description of mixing processes plays an essential role. Recently, a generalization of well-known turbulence closure models (TCMs) has been developed by Umlauf and Burchard (2003). The widely used turbulence closure model by Mellor and Yamada (1982) (MY-TCM) is one of the models that can be retrieved as a special case of Umlauf and Burchard's generic model.

In their study, Craig and Banner (1994, hereafter CB) examine wave-enhanced turbulence by application of the

Mellor-Yamada model. The effect of waves is parameterized by adoption of an alternative expression for the relation between the wind stress and the turbulent kinetic energy (TKE) at the surface. Moreover, CB consider simplifications of the MY-TCM, and find that the traditional law-of-the-wall assumption of shear production of TKE being balanced by dissipation does not apply in a waveenhanced surface layer.

Numerical models for ocean circulation is one field where the presence of waves and wave breaking is largely ignored. For model results regarding features like the large scale and the mesoscale ocean circulation, the omission of effects wave breaking is not expected to be a matter of concern in the present context. However, when information in the near-surface region (on a vertical scale corresponding to the length of gravity waves or shorter) is pertinent, the presence of breaking waves may potentially have an impact on the model results. This may be the case for models which use terrain-following coordinates in the vertical, such as the Princeton Ocean Model (Mellor 1992) and the S-Coordinates Rutgers University Model (Song and Haidvogel 1994): When such models are applied to domains which span depths from a few tens of meters to thousands of meters, the uppermost layers in shallow waters may well have a thickness in the range 0.1-1 m. Traditional numerical ocean circulation models that are formulated with a z-coordinate in the vertical, e.g. Cox and Bryan (1984), may also have a resolution of a few meters (or less) in the uppermost model layer. This is also the case for a class of emerging circulation models that are formulated with a hybrid coordinate in the vertical, e.g. Bleck (2002). For a model that includes near-surface dynamics, it may be important to incorporate a realistic parameterization of wave breaking. For models with lower vertical resolution, rather than changing the model parameterization, effects of wave breaking may need to be considered if the surface velocity is derived from the results for the upper model layer. In the context of local redistribution of momentum, effects of wave breaking can safely be ignored when predictions in the deeper parts of the ocean is considered.

It is a formidable task to provide a detailed description of wave breaking and how it affects currents and turbulence. In the present paper, no efforts will be made towards such an end. Instead, effects of wave breaking will be parameterized as described in Melsom (1996), utilizing the conservation properties of mass and momentum. In that paper, it was found that wave breaking enhances the surface drift. However, vertical diffusion of momentum due to turbulence was parameterized using an eddy viscosity of constant value. In the present paper, the aim is at a realistic description of vertical momentum diffusion by invoking a version of the MY-TCM (Mellor 1973; Mellor and Yamada 1974; Mellor and Yamada 1982) with the modifications related to waveenhancement that were suggested by CB.

CB find that the Mellor-Yamada model, with their modifications, shows "agreement in detail with measured profiles of dissipation", despite the fact that the model is not parameterized or calibrated to describe drift currents in a sea-state that is influenced by breaking waves. This is particularly the case for the relatively short period from an isolated wave breaking event occurs and to the time when the wave's mass and momentum has been redistributed. One important goal in this study is to investigate how isolated wave breaking events affect the balance of the terms in the equation for the TKE, by application of a bulk parameterization of the effects of mass and momentum redistribution.

The second important goal is to obtain results that may hold valuable information to the numerical ocean circulation modeling community with respect to the need, or lack of need, for parameterization of the sub-scale process of wave breaking. The effect of wave breaking on the ocean's surface drift will be assesses in this context.

2. Mathematical Formulation

The present theory includes diverse features of the ocean circulation, namely a TCM, an investigation of wave motion to fourth order in wave steepness, and a description of mean currents. Wave motion, mean motion, and high-frequency motion are separated as described e.g. by Weber and Melsom (1993a). Assuming that distinct time scales exist, the Eulerian pressure and velocity fields are divided into three separate parts:

$$(p,\mathbf{v}) = (p',\mathbf{v}') + (\hat{p},\hat{\mathbf{v}}) + (\bar{p},\bar{\mathbf{v}})$$
(1)

Here primes, carets, and overbars denote small-scale turbulent fluctuations, wave motion, and mean motion, respectively. The governing equations for the wave motion and for the mean motion are developed by application of time-averaging. Two time-averaging processes are employed. They are denoted by $\langle \rangle$ and $\{ \}$, and correspond to integration over periods that remove linear turbulent and linear wave quantities, respectively.

The present study is confined to the upper layer of the ocean where the density ρ is taken to be constant. Furthermore, effects of molecular viscosity are omitted due to the small magnitude of the viscosity. A Cartesian frame of reference is applied, with the vertical *z* axis being positive upward. The undisturbed horizontal surface is taken to define *z* = 0. The governing Eulerian equations for wave motion and mean motion then become

$$\frac{\partial \hat{\mathbf{v}}}{\partial t} + \hat{\mathbf{v}} \cdot \nabla \hat{\mathbf{v}} - \{ \hat{\mathbf{v}} \cdot \nabla \hat{\mathbf{v}} \} =$$

$$- \nabla \frac{\hat{p}}{\rho} + \nabla \cdot [-\hat{\mathbf{v}} \bar{\mathbf{v}} - \bar{\mathbf{v}} \hat{\mathbf{v}} - \langle \mathbf{v}' \mathbf{v}' \rangle + \{ \langle \mathbf{v}' \mathbf{v}' \rangle \}]$$
(2)

and

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} + \bar{\mathbf{v}} \cdot \nabla \bar{\mathbf{v}} = -\nabla \frac{\bar{p}}{\bar{\rho}} - g\mathbf{k} + \nabla \cdot \left[-\left\{\langle \mathbf{v}' \mathbf{v}' \rangle\right\} - \left\{\langle \hat{\mathbf{v}} \hat{\mathbf{v}} \rangle\right\}\right]$$
(3)

respectively. Here g is the vertical acceleration due to gravity, and ∇ denotes the gradient operator. Details on the derivation of these equations are given by Weber and Melsom (1993a).

Note that effects of Earth's rotation have been disregarded. If such effects were to be taken into account, the wave field also becomes affected when the wave propagation direction is constant in a rotating frame of reference (Hasselmann 1970). Then, vorticity is introduced into the wave equations, introducing an additional complexity to the already fairly intricate wave theory that is presented in Appendix A. This simplification implies restriction with respect to the validity of the results beyond the first few hours of integration.

3. Turbulence and Reynolds stresses

The presence of turbulence gives rise to Reynolds stresses in (3). We shall adopt the description of these stresses as outlined by Craig (1996, hereafter C96). This description involves use of an eddy viscosity v:

$$-\{\langle u'w'\rangle\} = v\frac{\partial\bar{u}}{\partial z} \tag{4}$$

Moreover, the eddy viscosity is defined by

$$\mathbf{v} = lqS_M \tag{5}$$

where *l* is a reference length scale for turbulent fluctuations.

Introducing the MY-TCM, S_M is a parameter with a constant value in the present case of unstratified water. Moreover, we apply their "level 3" equation for the turbulent

kinetic energy (TKE), $q^2/2$. If we neglect advection of TKE, and introduce (4) and (5), we have

$$\frac{\partial}{\partial t} \left(\frac{q^2}{2}\right) = \frac{\partial}{\partial z} \left[lq S_q \frac{\partial}{\partial z} \left(\frac{q^2}{2}\right) \right] + lq S_M \left(\frac{\partial \bar{u}}{\partial z}\right)^2 - \frac{q^3}{Bl} \tag{6}$$

Here, Sq and B are additional empirical constants. Note also that in the absence of stratification, the "level 3" version for turbulence closure is identical to the popular "level 2.5" version (Mellor and Yamada 1982). Hereafter, q will be referred to as the scaling velocity for turbulence.

The reference turbulence length scale l is taken to increase linearly from the surface towards the interior of the ocean, e.g. Prandtl (1952),

$$l = \kappa(z_r - z) \tag{7}$$

where κ is von Kármán's constant and z_r is the surface roughness length.

In the presence of breaking waves, observations indicate that the TKE flux at the surface can be parametrized as

$$lqS_q\frac{\partial}{\partial z}\left(\frac{q^2}{2}\right) = \alpha u_*^3, \quad z = 0$$
(8)

(see CB), where u_* is the friction velocity due to the wind stress that acts on the ocean's surface, and α is a parameter which is generally a function of the sea state (Drennan et al. 1992).

4. Waves

4.1. Wave motion

As in Weber and Melsom (1993a), motion associated with a surface wave that travels in the *x*-direction is considered,

$$\zeta = \zeta_0 e^{\beta t} \{ \cos(kx - \omega t) + O(k\zeta_0) \}$$
(9)

where k is the wave number, ω is the frequency, β is an exponential growth rate, and ζ_0 is the initial surface amplitude of the gravest mode. The wave motion and the dispersion relation $\omega(k)$ is then found by solving (2) and the corresponding mass conservation equation with appropriate boundary conditions.

The wave motion is governed by (2) and the mass conservation equation. For the separation (1) to be meaningful, we must have $\nabla \cdot \langle \mathbf{v'v'} \rangle \sim \mathbf{A} \cos \omega t + \mathbf{B}$. Furthermore, it is assumed that there is a relatively small contribution from the Reynolds stress terms $\langle \mathbf{v'v'} \rangle$ at frequencies close to ω , i.e., $|\mathbf{A}| \ll |\mathbf{B}|$. Then, the two terms in (2) that involve the Reynolds stresses will tend to cancel each other, since $\nabla \cdot [-\langle \mathbf{v'v'} \rangle + \{\langle \mathbf{v'v'} \rangle\}] \sim -\mathbf{A} \cos \omega t$. Hence their combined effect on the wave motion will be neglected in (2). Moreover, the nonlinear terms involving $\hat{\mathbf{v}}$ and $\bar{\mathbf{v}}$ in (2) are neglected. This will restrict the accuracy of the description of the wave field close to breaking. The above approximations are the same as in Melsom (1996). Equation (2) can now be written

$$\frac{\partial \hat{\mathbf{v}}}{\partial t} + \hat{\mathbf{v}} \cdot \nabla \hat{\mathbf{v}} - \{ \hat{\mathbf{v}} \cdot \nabla \hat{\mathbf{v}} \} = -\nabla \frac{\hat{p}}{\rho}$$
(10)

If the wave field has no vorticity initially, it remains irrotational. In this case the velocity field can be represented by a potential function ϕ as $\ \hat{v} = \nabla \phi$, and the solution for exponentially growing waves is given in Appendix A. Since the results arise from inviscid theory, wave growth is produced by a surface pressure perturbation that is out of phase with the surface shape, and in phase with the vertical velocity of the ocean waves. Then, the surface pressure acts in the direction of the fluid motion, adding energy to the ocean in terms of an increased wave height.

4.2. Higher order wave drift

The particle drift may be derived in a Lagrangian manner, following the method outlined by Lamb (1932). This method is formulated as a computation of the mass transport below material interfaces that are horizontal in the absence of waves. The shape of material interfaces η may be derived e.g. by integration of the continuity equation

$$\int_{-\infty}^{\eta} \frac{\partial^2 \varphi}{\partial x^2} dz =$$

$$- \frac{\partial \varphi}{\partial z} (z = \eta) = - \frac{\partial \eta}{\partial t} - \frac{\partial \varphi}{\partial x} (z = \eta) \frac{\partial \eta}{\partial x}$$
(11)

Observing that the surface is a material interface, we find that the shape of η to $O(\epsilon^3)$ is

$$\eta = z_0 + k^{-1} \left\{ \epsilon \cos \theta \, \mathrm{e}^{k z_0} + \epsilon^2 \frac{1}{2} \cos(2\theta) \, \mathrm{e}^{2k z_0} + \right.$$

$$\left. \epsilon^3 \left[\frac{3}{8} \cos(3\theta) + \frac{5}{8} \cos \theta \right] \mathrm{e}^{3k z_0} - \right.$$

$$\left. \epsilon^3 \left[\frac{5}{8} \cos \theta - B \sin \theta \right] \mathrm{e}^{k z_0} \right\}$$
(12)

Here, $\varepsilon = k\zeta_0 e^{\beta t}$ is a non-dimensional measure of the wave steepness, $\theta = kx - \omega t$ is the phase function, and z_0 is the vertical position of the interface when at rest. The Eulerian periodic velocity profile is given by (A7) in Appendix A, and the mass transport below the interface η is

$$U(z < \eta) = \int_{-\infty}^{\eta} \frac{\partial \varphi}{\partial x} dz$$
(13)

After some rather tedious arithmetics, we find that the mean Lagrangian transport below $\eta\,$ becomes

$$\overline{U(z < z_0)} = \frac{1}{2} \varepsilon^2 c_0 k^{-1} e^{2kz_0} + \varepsilon^4 c_0 k^{-1} \left(\frac{1}{2} e^{4kz_0} - \frac{3}{8} e^{2kz_0}\right)$$
(14)

where the overbar denotes a mean quantity that results from an average over the period of the gravest wave mode, as defined in Section 2. Also, $c_0 = \sqrt{g/k}$ is the lowest order phase velocity, as defined in Appendix A.

The Stokes drift \overline{u}_S , which is the mean drift velocity of particles in a non-rotating inviscid fluid, is then given to fourth order in wave amplitude as

$$\bar{u}_{S}(z_{0}) = \frac{d}{dz}\overline{U(z < z_{0})} = \epsilon^{2}c_{0}e^{2kz_{0}} + \epsilon^{4}c_{0}\left(2e^{4kz_{0}} - \frac{3}{4}e^{2kz_{0}}\right)$$
(15)

The Stokes drift has also been calculated by an Eulerian approach, in which the particles' trajectories were derived using Taylor series expansions in time. This approach led to a number of terms that grow linearly in time, and was not suited for determination of a mean property such as the Stokes drift. However, the terms that contained periodic and constant multipliers of powers of ε were in concord with (15).

5. Parameterization of wave breaking

The process of breaking of a monochromatic wave will be parameterized by requiring conservation of mass and momentum. This approach was described by Melsom (1996), using second order accuracy in the underlying wave theory. Here, the same method is applied, and the analysis is extended to fourth order accuracy in wave steepness. This constitutes an appropriate extension of the earlier work, since wave breaking is closely associated with steep waves.

Effects of one isolated wave breaking event are examined. Let ΔM_E be the amount of Eulerian momentum that is lost from the wave motion when the amplitude is reduced by $\Delta \zeta$ to ζ_0 . Then,

$$\Delta M_E = \int_{-\infty}^{\zeta_0 + \Delta \zeta} \hat{u}(x, z, t; \varepsilon_0 + \Delta \varepsilon) dz - \int_{-\infty}^{\zeta_0} \hat{u}(x, z, t; \varepsilon_0) dz$$
(16)

$$\Rightarrow \overline{\Delta M_E} = \rho c_0 k^{-1} \Delta \varepsilon \left(\varepsilon_0 + \frac{1}{2} \Delta \varepsilon + \frac{1}{2} \varepsilon_0^3 \right) = \overline{\Delta M_1} + \overline{\Delta M_2(z < \zeta)}$$



Figure 1. Momentum conversion from periodic motion to mean motion during breaking. (a) Sketch of conversion due to shifted distribution of mass. The wave propagates to the right. (b) Sketch of mean momentum lost from the periodic motion $\hat{\mathbf{v}}$ due to reduced velocities after breaking. The magnitude of the amplitude reduction has been exaggerated. See the text for details. Figure 1 is taken from Melsom [1996].

where $\hat{u}(x, z, t; \varepsilon)$ is the velocity in the *x* direction that is associated with waves of a steepness ε , $\varepsilon_0 = \zeta_0 k$ is the wave steepness after breaking, $\Delta \varepsilon = \Delta \zeta \cdot k$ is the reduction in wave steepness due to breaking, and ΔM_1 and ΔM_2 are defined below.

As a consequence of reduction in amplitude, momentum associated with periodic motion is lost at the surface. Here, mass is redistributed by wave breaking in a process that can be observed as white-capping. In a sense, mass moving in the wave's propagation direction (dark shading in Figure 1a) is replaced by mass moving in the opposite direction (light shading in Figure 1a). We denote the momentum transfer associated with this process by ΔM_1 . Hence, this quantity represents the momentum deficit due to the redistribution of mass at the surface that is associated with white-capping, and we find

$$\Delta M_{1} = \rho \int_{\zeta_{0}}^{\zeta_{0} + \Delta \zeta} dz \qquad (17)$$

$$\Rightarrow \overline{\Delta M}_{1} = \frac{1}{2} \rho c_{0} k^{-1} \Delta \varepsilon (\varepsilon_{0} + \Delta \varepsilon + \varepsilon_{0}^{3})$$

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where ζ_0 is the shape of the surface for a monochromatic wave with steepness ε_0 , and ζ is given by (A8) in Appendix A.

Moreover, the changes in the pressure gradient due to wave breaking will give rise to an instantaneous adjustment of the wave motion in the entire water column. The amount of Eulerian momentum that is lost from the waves beneath the interface level after wave breaking becomes

$$\Delta M_{2}[z < \eta(x, z_{0}, t; \varepsilon_{0})] =$$

$$\eta(x, z_{0}, t; \varepsilon_{0})$$

$$\rho \int_{-\infty}^{0} [\hat{u}(x, z, t; \varepsilon_{0} + \Delta \varepsilon) - \hat{u}(x, z, t; \varepsilon_{0})]dz$$
(18)

$$\Rightarrow \overline{\Delta M_2(z < \eta)} = \frac{1}{2} \rho c_0 k^{-1} \varepsilon_0 \cdot \Delta \varepsilon \left[(1 - \varepsilon_0^2) e^{2kz_0} + \varepsilon_0^2 e^{4kz_0} \right]$$

where the shape of the material interface η is given by (12). The vertical distribution of ΔM_2 is indicated by the shaded area in Figure 1b.

In the present formulation, all of the momentum that is lost from the wave motion must be transferred to the mean motion. For the Stokes drift given by (15) we obtain

$$\Delta \bar{u}_{S} = -2 c_{0} \Delta \varepsilon \left[\left(\varepsilon_{0} + \frac{1}{2} \Delta \varepsilon - \frac{3}{2} \varepsilon_{0}^{3} \right) e^{2kz} + 4 \varepsilon_{0}^{3} e^{4kz} \right]$$

$$\Rightarrow \rho \int_{-\infty}^{0} \Delta \bar{u}_{S} dz' = -\overline{\Delta M}_{E}$$
(19)

so the momentum transferred to the mean Eulerian motion is exactly compensated by a reduction in the Stokes drift. Thus, the vertically integrated Lagrangian momentum is not affected by wave breaking (Weber and Melsom 1993a).

Transfer of momentum lost from the periodic motion at the surface, ΔM_1 , may be parameterized by a virtual wave stress τ_{vw} acting on the mean current at the surface. Such a parameterization was first suggested by Longuet-Higgins (1969) in a study of decaying waves. When wave breaking occurs at t = 0, we then obtain

$$\tau_{vw} = F(\omega t) \cdot \overline{\Delta M}_1 , \quad \int_0^0 F(t') dt' = 1$$
 (20)

In the case of breaking waves, the weight function F(t) must go rapidly to 0, i.e., with an exponential decay rate of one wave period or so. There is no obvious choice for the temporal development of the virtual wave stress due to breaking, τ_{vw} . However, in Melsom (1996) two extreme choices for *F* in (20) were investigated, namely the delta function and an exponential decay rate set to the wave period. It was shown that the effects of wave breaking is relatively insensitive to the choice for *F*. For this reason, only the delta function will be implemented in the present study.

The momentum transfer associated with the adjustment of the wave motion in the water column must give rise to an interior adjustment of the mean current. Hence,

$$\rho \Delta \bar{u}_i = \frac{d}{dz} \overline{\Delta M}_2(z < \eta) =$$

$$\rho c_0 \varepsilon_0 \cdot \Delta \varepsilon \left[(1 - \varepsilon_0^2) e^{2kz_0} + 2\varepsilon_0^2 e^{4kz_0} \right]$$
(21)

To the lowest order in ε , the interior adjustment Δu_i is equal to half of the change of the Stokes drift which is induced by breaking.

6. Drift currents

For the mean current, the two-dimensional problem where the horizontal current depends on the vertical coordinate and time will be considered. Moreover, in the present context, the drift of water particles (the Lagrangian drift) is different from the Eulerian velocities. The Eulerian problem given by equation (3) may be transformed to a Lagrangian problem using the relation

$$\bar{u}_L = \bar{u} + \bar{u}_S \tag{22}$$

(Phillips 1977). Then,

$$\frac{\partial \bar{u}_L}{\partial t} - \frac{\partial}{\partial z} v \frac{\partial \bar{u}_L}{\partial z} = \frac{\partial \bar{u}_S}{\partial t} - \frac{\partial}{\partial z} v \frac{\partial \bar{u}_S}{\partial z}$$
(23)

where \overline{u}_{S} is given by equation (15).

The wind stress in the atmospheric boundary layer is denoted by τ , i.e.,

$$\tau = \rho^{(a)} u_*^{(a)^2}$$
(24)

where u_* is the friction velocity and the superscripts (*a*) denote atmospheric quantities. A significant amount of the momentum that is transferred from the atmosphere to the ocean contributes to wave growth (Melville and Rapp 1985; Weber and Melsom 1993a). The stress associated with such momentum transfer is here denoted by $\tau^{(w)}$, and *f* is introduced as the fraction of $\tau^{(w)}$ to the total wind stress τ , i.e.,

$$\mathbf{t}^{(w)} = f\mathbf{\tau} \tag{25}$$

The boundary condition at the surface becomes

$$\frac{\partial \bar{u}_L}{\partial z}(z=0) = \frac{\tau^{(0)}}{\rho \nu} + \frac{\partial \bar{u}_S}{\partial z}(z=0)$$
(26)

where

$$\tau^{(o)}(t) = (1 - f) \cdot \tau + \tau_{vw}(t - t_0)$$
(27)

Here, the virtual wave stress τ_{vw} is given by equation (20), and t_0 is the time at which the latest wave breaking occurred. Note that due to wave breaking, $\tau^{(o)}$ will be a function of time even in the case of a constant wind stress τ .

Momentum associated with the wave motion will be transformed to mean motion according to equation (21). Thus, the problem must be re-initialized:

$$\bar{u}_L^+ = \bar{u}_L + \Delta \bar{u}_i + \Delta \bar{u}_S \tag{28}$$

where \bar{u}_L^+ and \bar{u}_L^- are the drift profiles after the present wave breaking event and prior to the breaking event, respectively. The instantaneous adjustments $\Delta \bar{u}_i$ and $\Delta \bar{u}_S$ are given by (21) and (19), respectively. Given a boundary condition at the bottom or in the ocean's interior, the problem given by (23)-(28) may be solved.

Furthermore, note that when T_b is the period between breaking events, we have

$$\int_{t_0}^{t_0+T_b} \tau_{vw} \, dt' + \rho \int_{-\infty}^0 \Delta \bar{u}_i \, dz' = T_b \cdot f \tau \tag{29}$$

Thus, in the present formulation all of the momentum transfer from the atmosphere to the ocean is ultimately consumed by the ocean currents.

7. Results

One aim of the present paper is to provide realistic descriptions of effects of wave breaking on drift and turbulence. Nevertheless, in Appendix B we include an analytical solution of the velocity perturbation due to a wave breaking event, based on a constant value eddy viscosity. There are two reasons for this: [1] to provide a preliminary assessment of steep waves on the drift induced by wave breaking, and [2] to provide an exact solution that may be used to test a numerical description of the problem.

The full problem is given by (6) and (23) with surface boundary conditions (8) and (26). In the numerical formulation of the problem, conditions of the Neumann type are applied in the ocean's interior:

$$\frac{\partial}{\partial z}\bar{u}_L(z=-H) = 0$$

$$\frac{\partial}{\partial z}q^2(z=-H) = 0$$
(30)

where H is a depth that is large relative to the wave length. The initial condition for the drift is

$$\bar{u}_L(t=0) = \bar{u}_S \tag{31}$$

which correspond to a state of rest prior to the onset of wind forcing and waves at t = 0. Further, (re-)initialization of \bar{u}_L after a wave breaking event is given by (28). There is no obvious choice of initial condition for the turbulent kinetic energy. For simplicity, a constant value initial background turbulence is assumed,

$$q^{2}(t=0) = q_{0}^{2}$$
(32)

which is generally in discord with the surface boundary condition (8). The initial background turbulence is set to a low value of $q_0^2 = 2 \cdot 10^{-6} \text{m}^2 \text{s}^{-2}$ (the results after 30 minutes turned out to be virtually independent of the initial value used for q_0^2).

Numerical integration is conveniently performed after invoking the transformation

$$y = \int_{0}^{z} \frac{dz'}{l(z')}$$
(33)

(see CB), which ensures that the surface boundary layer is properly resolved. The equations are solved using a standard centrally differenced form with second order accuracy in time and in the y domain. In (6) the shear production term is evaluated at the old time step. Moreover, the dissipation term is made up of two factors, $q \cdot q^2$, where the former and latter factors are evaluated at the old time step and the new time step, respectively. In (23) values for q (and v) in the diffusion term are computed from the old time step. Moreover, the temporal resolution is increased immediately after wave breaking, in order to accurately describe the sudden impact of wave breaking on the mean motion. This approach is slightly different than the numerical method applied by CB. All profiles and time series that will be presented are computed as the temporal mean values between two events of wave breaking. Results quoted as obtained with a constant surface stress (in Case 1 and Case 2 below) are analogous to the results in CB.

In order to solve the problem, a number of parameter values must be specified. For the parameters related to turbulence closure, the values quoted in Table 1 in C96 are adopted, i.e., $S_M = 0.39$, $S_q = 0.2$, B = 16.6 and $\kappa = 0.4$. Furthermore, we are faced with the "rather unpleasant problem of estimating z_r " (Madsen 1977). Here, the oceanic equivalent of the Charnock formula, i.e.,

$$z_r = \frac{au_*^2}{g} \tag{34}$$

will be used. Based on an examination of the laboratory data presented by Cheung and Street (1988), C96 determines z_r by minimizing errors in the turbulence model. The results are presented in Table 2 in C96, and yield values for a in the range 250-2000. Based on interpretation of velocity profiles, Bye (1988) proposes that $a \approx O(10^3)$. Terray et al. (1996) find good agreement between their field observations and the turbulence model by CB if $z_r \sim H_S$, then, the results in Table 1 in Terray et al. yield $a \approx O(10^4)$ and $\alpha \approx O(10^2)$. Based on observational data from Knight Inlet, Stacey (1999) finds that $a \approx O(10^3)$ when $\alpha = 150$. The data that have been quoted here clearly indicate that a universal constant value of a does not exist, and one reason for this is that the surface roughness is a function of the sea state (Donelan 1990). Nevertheless, we believe that the use of (34) is justified in this rather idealized study. A case with a friction velocity $u_* = 0.018 \text{ms}^{-1}$ will be examined here, using $a = 2 \cdot 10^4$ and $\alpha = 100$. The oceanic surface roughness length then becomes $z_r = 0.66$ m.

In order to obtain a consistent specification of the sea state and the air-sea momentum transfer, a state-of-the-art wave model, WAM (Komen et al. 1994), was used. WAM models the temporal evolution of the directional wave spectrum, for waves with lower frequencies than a specified cutoff frequency. This frequency range is usually referred to as the dynamic range of wave frequencies. For higher frequencies than the cut-off value, WAM assumes a prescribed shape of the wave spectrum, which is usually referred to as the tail of the spectrum. The energy input from the wind to the tail of the spectrum is predominantly balanced by dissipation, giving rise to a continuous flux of turbulence at the surface.

We applied a constant wind forcing of 12m/s, and inspected the results from WAM for the period 4-6 hours after the initialization. Then, the steepest waves had wave numbers in the range 0.13-0.2 m⁻¹, with the largest steepness being observed for longer waves as time progressed. The friction velocity u_* was in the range 0.18-0.19m/s, with values that declined slowly with time. The steepness of the steepest waves was approximately 0.34, and *f*, the fraction of momentum transfer from the atmosphere that was generating waves in the dynamic range of the wave spectrum, declined from 0.56 to 0.47.

Based on these results from the WAM wave model, we set $u_* = 0.18 \text{ m/s}$, $\varepsilon_0 = 0.34$, $\Delta \varepsilon = 0.0062$, $\zeta_0 = 2\text{m}$, $T_b = 600\text{s}$, and g is set to 9.8ms^{-2} . These values correspond to a wavelength of 37m, and the non-dimensional number in (A6) becomes $B_0 = 2 \cdot 10^{-4}$. Then, the reduction of wave energy due to a wave breaking event is $2\Delta \varepsilon / \varepsilon_0 = 3.6\%$, which lie within the 1%-10% range indicated by the experimental results of Melville and Rapp (1985). Also, from (29) this specification implies that the fraction of the total atmospheric stress that contributes to wave growth is f = 0.51. This value complies with the results of Mitsuyasu (1985). If we specify the density of air by $\rho = 800 \rho^{(a)}$, we have $u_*^{(a)} = 0.51 \text{ m/s}$. The phase speed of the wave then becomes $c = \sqrt{g/k} = 7.6 \text{ m/s}$, and the wave frequency is $\omega = kc = 1.3 \text{ s}^{-1}$. Furthermore, the growth rate follows from the relation

$$\varepsilon_0 + \Delta \varepsilon = \varepsilon_0 e^{\beta T_b} \tag{35}$$

We find that $\beta = 3.0 \times 10^{-5} \text{ s}^{-1}$. and the steepness at wave breaking becomes $\exp(\beta T_b) \cdot \varepsilon_0 = 0.346$. Further, we define a non-dimensional age of steep waves as $c/u_*^{(a)}$, where c is the phase speed of the waves whose momentum is partially transferred to the drift current when wave breaking occurs. Here, this non-dimensional age is 14.9.

Due to the assumption of a constant atmospheric stress in a model without a momentum sink, and where effects of Earth's rotation is disregarded, we must impose a limit on the length of the time period for which our integrations will be performed. We have chosen to set this limit to six hours, since effects of Earth's rotation will then be quite noticeable at most latitudes (see Figure 4 in Melsom (1996)).

Results from four cases will be examined. In this context, we will refer to results produced by setting f = 0 in (27) as the "constant stress" case, since the surface stress $\tau^{(o)}$ in

(27) then becomes time invariant. In this case, the wave amplitude is constant, so the virtual wave stress vanishes, and there is no re-initialization according to (28). Results from the "constant stress" case are derived identically to the CB theory, when effects of Earth's rotation are disregarded. Thus, the profiles of Eulerian velocity and the scaling velocity for turbulence will be denoted as \overline{u}_{CB} and q_{CB} , respectively.

In the realistic case, with f > 0, $\tau^{(o)}$ becomes time dependent, and this will be referred to as the "time varying stress" case. Results will be derived using both $O(\epsilon^2)$ accuracy and $O(\epsilon^4)$ accuracy in the present wave model, and the corresponding sets of profiles of Eulerian velocity, Lagrangian drift and the scaling velocity for turbulence will be denoted as $\overline{u_2}$, $\overline{u_{L;2}}$, q_2 and $\overline{u_4}$, $\overline{u_{L;4}}$, q_4 , respectively. As described above, f = 0.51 in the present specification of the sea state. Note that the TKE flux at the surface is parametrized by (8) throughout.

The cases that will be examined are defined below, and will hereafter be referred to by their case numbers:

<u>Case 1</u>: Offsets in Eulerian velocities and TKE, when results due to a time varying surface stress $\tau^{(o)}$ based on $O(\varepsilon^4)$ in the wave model (\overline{u}_4, q_4) , are compared with results that are produced in the case of a constant stress $(\overline{u}_{CB}, q_{CB})$. <u>Case 2</u>: As **Case 1**, but with $O(\varepsilon^2)$ accuracy in the wave model (\overline{u}_4, q_4) in **Case 1** is replaced by \overline{u}_2, q_2).

<u>**Case 3**</u>: Offsets in Eulerian velocities, when results based on the $O(\varepsilon^2)$ wave model (\overline{u}_2, q_2) are compared with the $O(\varepsilon^4)$ results (\overline{u}_4, q_4) .

<u>**Case 4**</u>: As **Case 3**, but for the Lagrangian drift $(\overline{u}_2, \overline{u}_4)$ in **Case 3** is replaced by $\overline{u}_{L,2}, \overline{u}_{L,4}$.

First, the temporal development of the results from **Case 1** and **Case 2** are examined. The results are depicted in Figure 2. In Figure 2a and 2b, these time series are shown for the depths of 0.4m and 4.9m, respectively. Here, the black curves are the velocity time series, and the grey curves are the time series of the scaling velocity for turbulence (q), for a period of six hours. Thick and thin lines correspond to offsets (in %) from **Case 1** and **Case 2**, respectively, so the thick and the thin black lines in Figure 2a display the temporal evolutions at z = -0.4m of $100 \cdot (\overline{u_4} - \overline{u_{CB}})/\overline{u_{CB}}$ and $100 \cdot (\overline{u_2} - \overline{u_{CB}})/\overline{u_{CB}}$, respectively.

From Figure 2, we note that the offset of q is nearly time independent for these near-surface levels in the present formulation. We also observe from Figure 2a that the surface current derived using the $O(\varepsilon^4)$ wave model is larger than for the $O(\varepsilon^2)$ model. This is due to the fact that the virtual wave stress is higher when the $O(\varepsilon^4)$ wave model is applied, see (20) and (17). The sum of surface stresses in (27) has a temporal dependence due to wave breaking, which necessarily leads to a higher level of mean kinetic energy than in the absence of wave breaking. For this reason it is expected that the surface current has a positive offset. Such an offset is displayed in Figure 2a for **Case 1** and for **Case 2**. The reduction of the wave breaking effects in time is due to the higher turbulent mixing near the surface in the presence of wave breaking.

In Figure 3a, profiles for offsets of mean current and the scaling velocity after six hours of integration are depicted.





Figure 2. Time series for offsets of \overline{u} (black curves, mean current) and q (grey curves, scaling velocity for turbulence) due to a time varying surface stress, as compared with the constant surface stress case. The offset in u is defined as $(\overline{u}_{4/2}-\overline{u}_{CB})/\overline{u}_{CB}$, and offsets in q are defined analogously. Numbers on the vertical and horizontal axes are offset values in % and time in hours, respectively. (In the upper panel, the grey lines overlap.) A box filter has been applied, with a length set to the period between two wave breaking events (presently, this period is 5 minutes). Thick and thin lines correspond to computations based on $O(\varepsilon^4)$ and $O(\varepsilon^2)$ accuracy in the wave model, respectively. Here, ε is the wave steepness. (a) Time series for a node close to the surface (at z = -0.4m) from the numerical solution. (b) Near-surface time series at z = -4.9m. See the text for further details.

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The same convention for black and grey, thick and thin lines as in Figure 2 was used. At first sight, Figure 3a seems to indicate that the integrated momentum is larger when a time varying surface stress is applied when compared to the constant stress case. However, at all depths below approximately z = -12m, the Eulerian mean current offsets are negative, in compensation for the excess momentum near the surface. Obviously, the conservative property is the Lagrangian momentum. When the differences in initial conditions are taken into account for the various numerical integrations, the results remain consistent. After application of a constant stress $\tau^{(o)} = \rho u_*^2$ (f = 0, i.e., similar to the CB theory), the vertical profiles for the Eulerian mean current and the scaling velocity for turbulence after six hours are displayed in Figure 3b.

Next, we turn our attention to **Case 3** and **Case 4**. The offsets are depicted in Figure 4, for the same domain and at the same time as in Figure 3a. The black curves are time series for offsets in the mean current, and the grey curve is the offset of the scaling velocity for turbulence (q). Thick and thin black lines correspond to offsets (in %) for the Lagrangian drift (**Case 4**) and the Eulerian current (**Case 3**), respectively. The higher order theory leads to an enhanced drift adjacent to the surface, and a somewhat



retarded drift immediately below. This is most noteworthy when the Lagrangian drift is considered in **Case 4**, as expected from (15).



Figure 4. Profiles for offsets of \overline{u} (black curves, mean current) and q (grey curve, scaling velocity for turbulence). Here, results from wave theory with accuracy $O(\varepsilon^4)$ are compared with $O(\varepsilon^2)$ results, with a time varying surface stress applied in both cases. Numbers on the vertical and horizontal axes are vertical levels in meters and offsets in %, respectively. Thick and thin black lines correspond to differences in Lagrangian and Eulerian velocities, respectively. The Lagrangian velocity offset is defined as $(\overline{u}_{L,4'}, \overline{u}_{L,2})/\overline{u}_{L;2}$, and the Eulerian offset becomes $(\overline{u}_4 - \overline{u}_2)/\overline{u}_2$. Profiles are displayed for the upper 15 meters of the water column only, after six hours of integration.



8. Discussion

First, we note that the results in this paper to a certain extent rely on the description of the wave motion. The description of the wave field was developed from irrotational theory, which follows from disregarding the stresses in (2). This is an approximation which to some degree limits the validity of the subsequent results. Nevertheless, this is the conventional approach when effects of waves on mean currents are studied, e.g. Leibovich and Radhakrishnan (1977). Moreover, this approach has been improved by expanding such theory to 4th order accuracy in wave steepness. Obviously, for a brief period around breaking the nonlinear terms cannot be neglected. This problem has been resolved here by including the bulk parameterization in Section 5 which takes advantage of the principle of momentum conservation.

As noted by Drennan et al. (1992), the scaling wave number for TKE dissipation may be related to the peak of the slope spectrum, but this spectrum is very broad. We consider steep waves of relatively long wave lengths, hence a large part of the momentum flux from the wind is initially carried by shorter gravity waves. In the previous section, we defined a non-dimensional age of the longer steep waves as $c/u_*^{(a)}$. In Figure 5, offsets of the Eulerian mean currents at three selected depths are displayed as functions of the age parameter. Here, c should be interpreted as the phase speed of the steepest waves, whose momentum feeds the drift currents upon wave breaking. Hence, this is different from the conventional wave age parameter, which results from identifying c as the phase speed of the significant wave. For moderate wave ages, around 10, the significant wave has a steepness of about $\varepsilon_0 = 0.3$ (Sverdrup and Munk, 1947). For larger wave ages, the steepness of the significant wave is smaller.

As seen from equation (8), a flux formulation has been applied as the surface boundary condition for the TKE. In



Figure 5. Drift current offset $(\overline{u}_4 - \overline{u}_{CB})/\overline{u}_{CB}$ as a function of the non-dimensional age of steep waves, after six hours of integration. From top to bottom, the curves display the results at depths of 0.3m (thick line), 1m (thin line), and 3m (thick line). With the exception of ζ_0 (and *k*), all parameter values have been retained from Section 7.

the original closure theory, the surface TKE was set proportional to the surface stress, e.g. Mellor and Yamada (1982). The rationale for the flux formulation is that the TKE diffusion is directly influenced by this condition, and the TKE profile becomes more uniform near the surface. This occurs in response to mixing generated by surface gravity waves. Note that the implementation of a virtual wave stress in (20) does not replace the flux condition for the TKE in (8): Our study is limited to steep waves of relatively long wave lengths, whereas the dissipation of wave energy from the tail of the wave spectrum provides a source for the TKE flux at the surface.

In their study, Stacey and Pond (1997) found that the best results in a numerical simulation were obtained using the TKE flux condition (8). As noted by Stacey and Pond, the main weakness of the flux condition is that the roughness length enters as the surface value for the mixing length. We also note here that in a recent study, Terray et al. (1999) found an improved model-data fit using an alternate length scale that is held constant at $-\kappa z_r$ above $-z_r$, and increases linearly below.

The surface roughness is a parameter which is frequently described with an unsatisfactory degree of perfection in ocean circulation problems. This study is no exception. In fact, we suspect that the surface roughness should be a function of time in the present formulation, with highest values shortly after wave breaking has occurred. Nevertheless, since we are unaware of observational evidence of such behavior, any prescribed temporal development of z_r would be speculative. Other phenomena that are disregarded here, but may well be important in the present context, include effects of swell (Donelan et al. 1997), and the three-dimensional response of breaking of waves with a finite crest length and non-simultaneous wave breaking events.

In order to assess the magnitude of differences due to the parameterization of the surface roughness, additional computations were performed, using a wide range of values for the constant a in the Charnock formula (34). The increase in the velocity due to wave breaking was examined, and the results after six hours of integration for the node closest to the surface are displayed in Figure 6. For a given a, the increase in velocity due to wave breaking is generally rapidly reduced with depth, as shown in Figure 3a.

In their study, CB find that in the steady state, the shear production of turbulence is balanced by dissipation beneath a "wave-enhanced surface layer". (In CB's terminology, the term "wave-enhanced surface layer" refers to the layer affected by the surface boundary condition (8).) When diffusion of turbulence is disregarded, and a time varying stress $\tau^{(o)}$ is applied with f = 0.51 in (27), the surface velocity increases by about 20% when compared to the solution obtained with the full turbulence model. For the case of a constant stress ($\tau^{(o)} = \rho u_*^2$), this value becomes approximately 24%. This difference is due to the relatively large velocity shears that arise in the vertical when waves break. Hence, the presence of a "wave-enhanced surface layer" is confirmed, though slightly offset from the results that were reported by CB.

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Figure 6. Offsets $(\overline{u}_4 - \overline{u}_{CB})/\overline{u}_{CB}$ at the surface, as a function of the constant *a* in the Charnock formula, after six hours of integration. The surface roughness z_r is proportional to *a*. With the exception of *a* (and z_r), all parameter values have been retained from Section 7.

In order to further examine the details of near-surface turbulence in the context of a time varying surface stress, the adjustment of the turbulence forcing terms in (6) was inspected. The evolution of the turbulence forcing terms is depicted in Figure 7, as a function of time since a wave



Figure 7. Forcing terms for turbulent kinetic energy as a function of time after a wave breaking event. From top to bottom, the curves in panel a show the development of the diffusion term (thick line), the shear production term (thin line), and the dissipation term (thick line), at a depth of 0.3m. In panel b, which depict the forcing terms at a depth of 5m, the diffusion term is smaller than the production term. Numbers along the horizontal and vertical axes are time in minutes after a breaking event, and magnitudes of terms in m²/s, respectively. (With the present choice of parameters, wave breaking events occur with a period of 10 minutes.)

breaking event occurred. From the results in Figure 7a, we note that while the shear production term has a significant influence immediately after breaking at a depth of 0.4m, there is a rapid adjustment to a diffusion-dissipation balance thereafter. These results help to explain the small differences between the present theory and the conventional approach, and thus shed a new light on the success of the turbulence model by CB. Figure 7b shows that at a depth of 6.9m, which is beneath the "wave-enhanced surface layer", there is mainly a balance between the shear production term and the dissipation term, which is in agreement with CB.

In Figure 8, results are displayed for the ratios of the solutions from two simplified turbulence models to results obtained with the full model. When omitting the shear production of turbulence (thick lines in Figure 8), momentum mixing is reduced, and the momentum becomes more trapped close to the surface. The current is increased by a factor of 1.35 at the surface. When the wind rotates in an non-accelerating frame of reference as in CB (e.g. leading to the steady state solution in their Figure 7), the time averaged vectorial momentum input from the wind during an inertial period, vanishes. In the latter case, CB find that omitting shear production leads to a surface current that increases by a factor of about 1.4 (see the lower panel in their Figure 6).

In the previous section, results were presented when the reduction of wave energy due to a wave breaking event was set to 3.6%. After six hours of integration, the Eulerian surface current was found to increase by 6.4% when a time varying surface stress was imposed. When the reduction of wave energy is halved to 1.8%, the impact of the time varying stress is even smaller, as the surface current enhancement becomes 3.4%.

Obviously, the impact of wave breaking on the near-surface velocity profile depends on the distribution of the airsea momentum flux between waves and drift currents. In



Figure 8. Profiles for ratios of u_4 (black curves, mean current) and q_4 (grey curve, scaling velocity for turbulence). Thick lines correspond to the ratio of the solution obtained with a dissipation-diffusion balance to the solution obtained with the full turbulence closure scheme. Thin lines show the corresponding ratios with the numerator replaced by the solution with the dissipation-shear production balance. Profiles after six hours of integration are displayed for the upper 10 meters.



Figure 9. Changes in the drift current offset $(\bar{u}_4 - \bar{u}_{CB})/\bar{u}_{CB}$ as a function of the fraction of the wind stress that induces wave growth (*f* in (25)). From top to bottom, the curves display the results at depths of 0.3m (thick line), 1m (thin line), and 3m (thick line). Various values of the fraction *f* was examined by changing the value of $\Delta \varepsilon$ (the change in wave steepness due to breaking). All other parameter values in Section 7 were retained. Results are presented after six hours of integration.

Figure 9, the changes due to the presence of breaking waves are depicted as a function of the fraction of the wind stress that induces wave growth, i.e. f in (25). We note that the changes are nearly a linear function of f, and that the results converge towards the constant stress case f = 0, i.e., the theory of CB.

The work by CB and C96 on modeling the flux of TKE as proportional to the cube of the friction velocity has later been included in other studies of turbulence in the presence of breaking waves, e.g. D'Alessio et al. (1998). This approach has recently been extended to two-equation turbulence models by Burchard (2001). Our results indicate that the assumption of a constant stress $\tau^{(o)} = \rho u_*^2$ leads to only minor errors that are largest close to the surface. In the present paper, we have applied a physically sound bulk parameterization for isolated events of wave breaking. Thus, the errors that we report here constitute an assessment of the severity of applying the modified Mellor-Yamada model in a sea state where wave breaking is abundant. Since the errors we find are small, the present theory lends support to the use of the analytical solution from C96, as it was implemented by Burchard.

One purpose of the present investigation is to quantify errors in numerical ocean circulation models due to ignoring the high frequency variation of the surface stress $\tau^{(o)}$ that arises due to wave breaking. From Figure 3a, we observe that the errors in question extend only to a depth of a few meters. Even for this narrow range in depth, the error is small (a few percent). When the near-surface currents are of interest, the results may conceivably be improved by tuning the turbulence closure parameters. As noted by C96, the effect of diffusion on the TKE profile is mainly constrained to the region immediately beneath the surface. Alternative values for S_q were assessed by comparing the shapes and offsets of the profiles in **Case 1** statistically. However, the results were inconclusive, as e.g. a reduced value for S_q in the case f = 0 led to smaller offsets between the profiles, but a deterioration in the comparison of the shapes. Hence, we do not recommend modifications of S_q based on the present study. Note also that changing the value of S_q will affect the momentum sink at the bottom when a no-slip condition is applied at this boundary.

When the parameter values from Section 7 are adopted, we find that the Lagrangian surface drift is enhanced by about 12% when effects of $O(\epsilon^4)$ accuracy in wave steepness are included in the theoretical framework (Case 4). This is the largest offset found in the present investigation, and also note the negative offsets at intermediate depths (see Figure 4). For the chosen specification of the sea state, the Stokes drift increases by 14.45% when effects of $O(\epsilon^4)$ accuracy are included (with an e-folding depth of 3.6m), and this is why the Lagrangian surface drift is more affected than the Eulerian surface current. In relation to this, we note that in an investigation where rotational effects are taken into account, Weber (1983) finds that the Stokes drift is reduced, see his equation (5.2). When the parameter values from Section 7 are used in his theoretical result, we find that this reduction is approximately 6% at the surface (with an e-folding depth of about 0.18m).

The effect of steep waves on the Stokes drift is worth notice. First, from (14) we observe that the Stokes transport is slightly enhanced as compared to traditional theory, by a factor $\varepsilon^2/4$. Moreover, close to the surface the Stokes drift is also enhanced by steep waves. At the surface, the drift is increased by a factor $1.25 \cdot \varepsilon^2$. Using the theoretical limit for steepness of gravity waves, $\varepsilon \approx 0.44$, e.g. Longuet-Higgins (1975), the limiting enhancement of the surface Stokes drift for steep waves is increased by a factor of about 0.24 when compared to the lowest order result.

9. Conclusion

The present work is an investigation of the impact of repeated wave breaking events on the near-surface profiles of velocity and TKE using an idealized, theoretical model. The model is idealized in the sense that it has been developed for monochromatic, irrotational surface waves with a uniform amplitude in a non-rotating, non-stratified fluid. The wave model is theoretical in a "pure" sense since waves are described using conservation laws throughout, and this principle is also adopted for the parameterization of effects of wave breaking via the bulk formulae in Section 5.

The TCM that is employed, is based on assumptions that are not satisfied when wave breaking is abundant. The turbulence theory has been developed for horizontally homogeneous flows, and its calibration has been performed from data that are not representative for the present sea state. Still, CB find that adoption of the present TCM yields results that are in good agreement with observations. There can be little doubt that wave breaking is the circumstance in which the assumptions of the TCM are most severely violated. Hence, we provide results when a physically sound parameterization of this circumstance is applied, in order to help bridge the gap from the turbulence theory and its intrinsic assumptions to its adaptation to a realistic sea state. When a realistic parameterization of effects of wave breaking is adopted, the near-surface profiles are only very modestly affected (by a few per cent or less) when compared to case when the wave stress is disregarded. Hence, based on a theoretical wave model that uses a bulk-formulation to parameterize wave breaking events, we are able to shed light on the success of the turbulence model as it was reported by CB.

The small differences that we report were obtained when effects of wave breaking was magnified in a number of ways, and we will mention two issues that are of relevance in this context: First, all momentum lost in the process of wave breaking was assumed to be transferred to the mean motion. This reflects the idealized notion of a monochromatic wave field, in reality, a part of this momentum would be redistributed to waves with other frequencies. Second, we have expanded the traditional wave theory to fourth order in wave steepness, and this too is a source for larger differences from the conventional theory. Other higher order effects, such as rotational theory, may give rise to adjustments that are opposite to the fourth order theory corrections.

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Appendix A: Inviscid wave motion

For two-dimensional plane waves travelling in the positive *x*-direction in an infinitely deep inviscid fluid, the motion is governed by the Laplacian equation

$$\nabla^2 \phi = 0 \tag{A1}$$

where the velocity vector has been expressed as $\hat{\mathbf{v}} = \nabla \phi$. A kinematic boundary condition is imposed at the top of the water column by assuming a material, free surface:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial \varphi}{\partial x} \frac{\partial \zeta}{\partial x} = \frac{\partial \varphi}{\partial z} \quad , \ z = \zeta$$
(A2)

Furthermore, the motion must vanish at large depths, so

$$\nabla \phi \to 0$$
 , $z \to -\infty$ (A3)

and the dynamical boundary condition at the free surface is

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2} (\nabla \varphi)^2 + g \zeta = -P_s \quad , \ z = \zeta \quad .$$
 (A4)

where $P_s = p_s / \rho$ and p_s is the pressure at the surface.

Solutions to the system (A1)-(A4) may be found by expanding the unknown variables in series after powers of the expansion parameter ε :

$$\Psi = \sum_{n} \varepsilon^{n} \Psi_{n} \tag{A5}$$

Here, ψ represents the unknown quantities φ , ζ , P_s and $c = \omega/k$, and we choose $\varepsilon = k\zeta_0 e^{\beta t}$, i.e., ε is the steepness of the gravest mode. In their discussion of observational data and analytic results, Weber and Melsom (1993b) finds that $\beta/\omega \sim 10^{-4}$ or smaller. Hence,

$$\frac{\beta}{kc_0} = \varepsilon^2 B_0 \tag{A6}$$

where c_0 is the lowest order approximation of the wave speed, and B_0 is a non-dimensional number of order unity or less.

To the lowest four orders in ε , the periodic solution of this problem of exponentially growing waves with an initial gravest mode amplitude of ζ_0 becomes

$$\varphi = k^{-1} c_0 \left\{ \epsilon \sin \theta \, \mathrm{e}^{kz} - \epsilon^3 \frac{1}{8} \sin \theta \, \mathrm{e}^{kz} + \epsilon^4 \frac{1}{2} \sin(2\theta) \, \mathrm{e}^{2kz} \right\}$$
(A7)

$$\zeta = k^{-1} \bigg\{ \varepsilon \cos \theta + \varepsilon^2 \frac{1}{2} \cos(2\theta) + \varepsilon^3 \bigg[\frac{3}{8} \cos(3\theta) + B_0 \sin \theta \bigg] + \varepsilon^4 \bigg[\frac{1}{3} \cos(4\theta) + \frac{17}{24} \cos(2\theta) + B_0 \sin(2\theta) \bigg] \bigg\}$$
(A8)

$$P_s = P_0 - \varepsilon^3 2c_0^2 B_0 \sin\theta \tag{A9}$$

$$c = c_0 \left(1 + \varepsilon^2 \frac{1}{2} \right) \tag{A10}$$

The dispersion relation may thus be written

$$\omega^2 = gk[1 + \varepsilon^2 + O(\varepsilon^4)]$$
 (A11)

e.g. Lamb (1932). Note that the solution given by (A7)-(A10) is positioned in the vertical by requiring that the surface at rest is situated at z = 0 (cfr. eq. (2) of Art. 250 in Lamb, where the mean surface is situated slightly above z = 0).

Appendix B: Analytical solution

The effect of an event of wave breaking on the near-surface drift profile may be found by analytical means in the simple case of a constant eddy viscosity v_0 . The solution to lowest order accuracy in wave steepness ε was calculated by Melsom (1996) for various specifications of the virtual wave stress τ_{vw} . Here, the virtual wave stress is assumed to act as an impulse, so in (20),

$$F(t) = \delta(\omega t) \tag{B1}$$

where δ is the Dirac delta function. This problem is described by Melsom, and constitutes a subproblem of the full formulation as given in Section 6 in the present paper:

$$\frac{\partial u^{(br)}}{\partial t} - v_0 \frac{\partial^2 u^{(br)}}{\partial z^2} = 0$$

$$\frac{\partial}{\partial z} u^{(br)} (z=0) = \frac{\tau_{vw}(t)}{\rho v_0}$$

$$u^{(br)} (t=0) = \Delta u_i + \Delta u_S$$
(B2)

The solution is obtained by Laplace transformation and subsequent inverse transformation, yielding

$$u^{(br)} = c_0 \Delta \varepsilon \Biggl[\frac{1}{2} (\varepsilon_0 + \Delta \varepsilon + \varepsilon_0^3) \frac{e^{-z^2/(4v_0 t)}}{\sqrt{\pi k^2 v_0 t}} - \frac{4}{\pi} k (\varepsilon_0 + \Delta \varepsilon - 2\varepsilon_0^3) \int_0^{\infty} \frac{\cos(z\xi) e^{-v_0 t\xi^2}}{\xi^2 + 4k^2} d\xi -$$
(B3)
$$\frac{48}{\pi} k \varepsilon_0^3 \int_0^{\infty} \frac{\cos(z\xi) e^{-v_0 t\xi^2}}{\xi^2 + 16k^2} d\xi \Biggr]$$

When only terms proportional to the lowest order in ε are kept, (B3) reduces to the solution in Melsom (1996), see equation (21) in that paper. Note that the expression above is not well defined for t = 0 due to the impulse imposed at the surface at this time. However, we have

$$\lim_{t \to 0} u^{(br)}(z < 0) = \Delta u_i + \Delta u_S$$
(B4)

in concord with the initial condition in (B2).

The temporal development and vertical profiles of $u^{(br)}$ are depicted in Melsom (1996) (to lowest order in ε ; see Figure 2a and 2b in that paper).

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