# Wave Breaking and Ocean Surface Layer Thermal Response

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#### ABSTRACT

The effect of ocean surface forcing on surface temperatures and the implied layer deepening is investigated. The modification of the Mellor-Yamada turbulence closure model by Craig and Banner and others to include surface wave breaking energetics reduces summertime surface temperatures when the surface layer is relatively shallow. The effect of the Charnock constant in the relevant drag coefficient relation is also studied.

### 1. Introduction

At one time, the so-called Mellor-Yamada (1974–1982; henceforth, M-Y) turbulence closure model was thought to produce ocean surface boundary layers that were too shallow during summertime warming and consequently surface temperatures were overly warm (Martin 1985). A recent paper by Mellor (2001; henceforth M01) investigated two relevant aspects of the problem. First, it was found theoretically, and supported by numerical experiment, that one-dimensional models when forced by a realistic wind stress time series would experience an indefinite increase in surface boundary layer kinetic energy, a process that did not occur in three-dimensional models or in observations. A Raleigh drag term (Pollard 1970) in the momentum equations resulted in bounded kinetic energy but exacerbated the shoaling problem and increased summertime surface temperatures by a couple of degrees. Then M01, citing experimental

evidence, found that the problem could be "fixed" by allowing the dissipation term in the turbulence kinetic energy equation to be Richardson number dependent and by introducing an appropriate tuning constant (in a model whose constants were otherwise rather robustly related to neutral data).

Craig and Banner (1994; henceforth CB) used the M-Y model but modeled wave breaking as a surface diffusion boundary condition of the turbulence kinetic energy equation proportional to  $u_{\tau}^3$  where  $u_{\tau}$  is the surface friction velocity. We had previously thought that the constant of proportionality,  $\alpha_{CB}$ , of about 100 was rather high and that, in any event, the process would not affect boundary layer deepening. Stacey and Pond (1994), comparing wave-modified profiles with data from Knight Inlet in southwestern Canada, showed that the wave breaking model process beneficially removed sharp velocity shear gradients near the surface. Later, Stacey (1999), analyzing the same data, decided that  $\alpha_{CB} = 150$  provided the best fit to his limited data. Terray et al. (2000) similarly modified the M-Y model and favorably compared calculations with measurements of wave enhanced dissipation greater than the well known dissipation behavior in the law of the wall, near surface region. Burchard (2001) made a fairly complicated alteration to the  $k-\varepsilon$  model [which now includes equation (2) below and  $\ell \propto k^{3/2}/\varepsilon$ ]; he also obtained much reduced near surface shear gradients, but no layer deepening for a short, four day test case.

In this note we find that, contrary to our prior expectation and, understandably, contrary to the finding of Burchard (2001), the CB surface boundary condition does affect deepening and surface temperature to the extent that the modification of the dissipation term as in M01 can probably be discarded.

#### 2. The Model

The model equations for turbulence kinetic energy, length scale, momentum and salinity and temperature is the same as equations (10), (11), (12) and (13) in M01. However, we repeat the turbulence energy equation, thus,

$$\frac{\partial q^2}{\partial t} = \frac{\partial}{\partial z} \left( K_q \frac{\partial q^2}{\partial z} \right) + 2K_M S^2 - 2K_H N^2 - 2\frac{q^3}{B_1 \ell}$$
 (1)

where  $q^2/2$  is the turbulence kinetic energy, z and t are the vertical coordinate and time,  $S^2 \equiv (\partial U/\partial z)^2 + (\partial V/\partial z)^2$ ;  $N^2 \equiv -g\rho_o^{-1}\partial\widetilde{\rho}/\partial z$ ;  $B_1 = 16.6$  is a model constant,  $\ell$  is the so-called master length scale and mean velocity shear and density lapse gradients are contained in  $S^2$  and  $N^2$  respectively. A derived result of Mellor (1974, 1982) and the level  $2\frac{1}{2}$  model is that the mixing coefficients are

$$(K_M, K_H, K_a) = \ell q(S_M, S_H, S_a) \tag{2}$$

where  $S_M$  and  $S_H$  are functions of  $(\ell N/q)^2$  and we generally set  $S_q = 0.41S_H$ . Near the surface, or in neutrally stratified flow,  $(S_M, S_H, S_q) = (0.30, 0.49, 0.20)$ .

### 3. Wave breaking parameterization

Following CB, boundary conditions for (1) are that

$$K_q \frac{\partial q^2}{\partial z} = 2\alpha_{CB} u_\tau^3 , \quad z = 0$$
 (3)

Heretofore we had set  $\alpha_{CB} = 0$  (or its equivalent,  $q^3 = B_1 \ell K_M S^2 = B_1 u_{\tau}^3$  at z = 0). Then, Terray et al. (1996, 1997) found from their observations that

$$\alpha_{CB} = 15(c_p / u_*) \exp(-0.04c_p / u_*)$$
 (4)

which is a curve fit to their Fig. 8 of the 1997 paper. The parameter,  $c_p/u_*$ , is the "wave age" where  $c_p$  is the phase speed of waves at the dominant frequency and  $u_*$  is the air side friction velocity ( $u_*$ =30  $u_{\tau}$  where  $u_{\tau}$  is the water side friction velocity). For mature waves, where  $c_p/u_* \cong 30$ , one obtains  $\alpha_{CB} \cong 57$  from (4) whereas for younger waves, where, say,  $c_p/u_* \cong 10$ , one obtains  $\alpha_{CB} \cong 146$ , thus independently and convincingly bracketing the values of CB and Stacey (1999).

From the aforementioned papers, one finds that the specification of a finite value of  $\ell$  at z=0 is also equal importance to the stipulation of a non-zero  $\alpha_{\it CB}$ . In particular, Terray et al. (2000) finds that

$$\ell = \max(\kappa \ell_0, \ell_z) , \quad \ell_0 = 0.85 H_s$$
 (5a, b))

(perhaps marginally preferable to  $\ell=\ell_0+\ell_z$ ) so that  $\ell_0$  scales on the significant wave height,  $H_S$ , (equal to  $4\times$  the rms wave height). The "conventional" empirical length scale

$$\ell_z = \ell_z(z)$$

evokes many prescriptions in the literature, but generally  $\ell_z = \kappa z$  for small z where  $\kappa \cong 0.41$  is von Karman's constant. The specification of  $\ell_0$  is not simple. Donelan (1990) and Smith (1992) suggest  $H_S = 0.50(c_p/u_*)^{2.5}z_0$  (but there are many other formulas cited in Jones and Toba 2001; in the choice here, we have rounded the constants slightly) where  $z_0$  is the wind roughness parameter and  $z_0 = \alpha_{CH} u_*^2/g$  is Charnock's relation. According to Donelan (1990), Smith (1992) and Janssen (2001),  $\alpha_{CH} \cong 0.45 \, u_*/c_p$  (and there are others formulas available). Putting together these formulas with (5b) yields

$$\ell_o = \beta \frac{u_{\tau}^2}{g}, \quad \beta = 800 \left(\frac{c_p}{u_*}\right)^{1.5}$$
 (6a,b)

where we have converted to the water-side friction velocity in (6a). Stacey (1999), citing observation evidence, chose the value  $\beta = 2.0 \times 10^5$ .

Since they are uncertain, we wish now to determine the sensitivity of model simulations to the "constants",  $\alpha_{\it CB}$  and  $\beta$ . For this purpose, we have, as in past papers, used the year long Ocean Weather Station Papa data set of Martin (1985) since, we have found that, when the model performs better or worse for these data, the same holds for other data. We include an eight inertial day, Raleigh damping as in M01 where it is shown that this or something similar (for example, tacit adjustment of the Asselin filter in leap-frog temporal discretizations) must be employed in one-dimensional models when trying to simulate real data. For  $\ell_z$  we have used the differential length scale model generally associated with the M-Y model (Mellor and Yamada 1982, M01) but an algebraic length scale parameterization should also work well for surface boundary layer problems. Except for the inclusion of non-zero values of  $\alpha_{\it CB}$  and  $\beta$  and deletion of the Richardson number dependent dissipation parameterization (the last term in (1), the

dissipation term, is not altered in this paper) the model and data details are as described in M01. We have however reduced the model time step from 20 minutes to 5 minutes; this reduced the summertime surface temperature by about 1°C.

In Figs. (1a,b), we plot the surface temperatures for the Station Papa data together with calculations for the case without wave breaking,  $\alpha_{CH} = \beta = 0$ , and cases where  $\alpha_{CH} = 50$  and 100 and  $\beta = 1 \times 10^5$  and  $2 \times 10^5$ . The cooler summertime temperature imply deeper surface boundary layers. Note that the calculations are more sensitive to  $\beta$  than to  $\alpha_{CH}$ .

# 4. Drag Coefficients

While investigating the CB effect on SBL deepening, we also began to investigate the drag coefficients used in SBL simulations. Martin (1985) and, since we borrowed Martin's well designed surface forcing code, we have also been using the familiar equation of Garratt (1977)

$$C_{D10} = (0.75 + 0.067 |\mathbf{U}|) \times 10^{-3} \tag{7}$$

for the drag coefficient based on wind speed measured at 10 m above the sea surface. Another familiar expression based on the law of the wall is

$$C_{D10} = \left(\frac{\kappa}{\ln(10\text{m}/z_0)}\right)^2, \quad z_0 = \max(0.14\text{v}/u_*, \alpha_{CH}u_*^2/g)$$
 (8a, b)

Fig. 2 is a comparison of (7) and (8a,b) for  $\kappa = 0.41$  and  $\alpha_{CH} = 0.0144$ , the values suggested by Garratt. We have added the smooth wall limit, in the manner of (8b), which is well established in relatively precise laboratory measurements (Schlicting 1968). The transition from smooth to fully rough is rather abrupt, but (8b) does produce a drag curve in Fig. 2 that is more in agreement with (7) than it would be were the smooth and rough terms simply added. As cited above, Donelan (1990), Smith et al. (1992) and, recently, Janssan (2001; see also Taylor and Yelland 2001) suggest that

$$\alpha_{CH} = 0.45 \left( \frac{u_*}{c_p} \right) \tag{9}$$

all from different data sets. Considering a mix of young waves  $(c_p/u_* \cong 10)$  and mature waves  $(c_p/u_* \cong 30)$  would argue for values of  $\alpha_{CH}$  larger than 0.0144. Thus, we include drag coefficients for  $\alpha_{CH} = 0.020$  in Fig. 2 which yields values about 10% larger the other curves. It is noted that Donelan et al. (1995) displayed drag coefficient curves that increased by about 50 % for young waves relative to mature waves; his values for the mature waves are close to the Garratt values.

In Fig. 3 we compare calculations for  $\beta=2.0\times10^5$ ,  $\alpha_{\it CB}=100$  using (7) and (8a,b) for  $\alpha_{\it CH}=0.020$ . We note that the rms wind speeds in winter are about 11.5 m s<sup>-1</sup> and in summer 7.0 m s<sup>-1</sup>; the corresponding average values of  $\ell_0$  are 2.5 m and 6.0 m which seem high if (5b) is correct.

# 5. Internal wave parameterization

In Mellor (1989), there was a suggested parameterization to account for the influence of unresolved internal waves in the M-Y model by adding to  $S^2$  in (1) an rms internal wave shear gradient,  $\overline{s_{iw}^2} = 0.7N^2$ ; the constant is based on limited data and is uncertain. Calculations with this parameterization in place yield additional summertime cooling of  $0.5^{\circ}$ C or less. We have not used this parameterization in this paper, but future insight and data may persuade one to include it. In their modification to the M-Y model, we note that Kantha and Clayson (1994) have assigned considerably more importance to internal wave processes. They include additional (dimensional) diffusivity at the base of the surface boundary layer.

# 7. Summary

The main lesson to be learned is that introducing waves physics into the modeling of surface boundary layer also introduces considerable uncertainty, hardly a surprising conclusion. For users of the M-Y model and maybe other models as well, we tentatively suggest  $\beta = 2.0 \times 10^5$  following Stacey (1999),  $\alpha_{CB} = 100$  and  $\alpha_{CH} = 0.020$ . There is little doubt that the first two values should be greater than zero and their order of magnitude is deemed correct, but more precision will require more data, an interactive

wave model and, hopefully, developing confidence in equations like (4), (5), (6). There is increasing consensus that the Charnock constant should be larger than 0.015, and, we believe, the value, 0.020, is reasonable.

Whereas the need to suppress kinetic energy in one-dimensional models, certainly on climate time scales, is a robust finding in Mellor (2001), this paper may be a good excuse to expunge the Richardson number dissipation parameterization in the same paper lest the model produce too much cooling or until further wisdom is developed. Thus, the basic model is returned to its original form while attention is directed towards wind-wave forcing and parameters like wave age.

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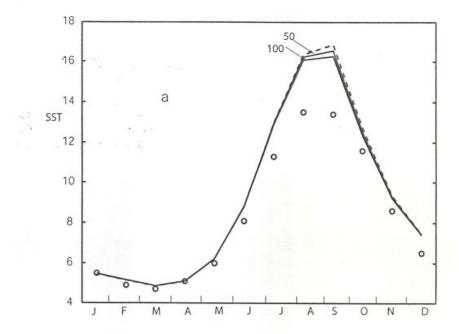
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# Figure Captions

Fig. 1. The circles are the monthly averaged sea surface temperature measured at Ocean Weather Station Papa for 1961. The dashed curve is for  $\beta = \alpha_{CB} = 0$ . The solid curves are labeled with values of  $\alpha_{CB}$  and (a)  $\beta = 1.0 \times 10^5$ , (b)  $\beta = 2.0 \times 10^5$ .

Fig. 2. Drag coefficients as functions of wind speed. The dashed curve is equation (7). The solid curves are from (8a, b) for the labeled values of  $\alpha_{CH}$ .

Fig. 3. The dashed curve is the same as in Fig. 1b where the drag coefficients were obtained from (7) The solid curve has the same values of  $\beta$  and  $\alpha_{CB}$  but the drag coefficient uses (8a, b) and  $\alpha_{CH} = 0.020$ .



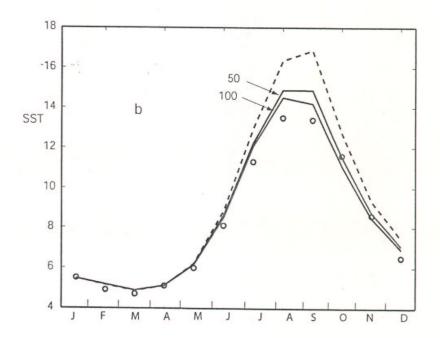


Fig. 1

