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A review of the analyses of ocean wave groups

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ABSTRACT

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The most common parameters and functions used to characterize wave groups in linear seas are reviewed and interrelated in a unified manner. A three-axes representation of run lengths is used to characterize wave groups using exponential and Markov chain approximations. A relationship between four parameters (Q_p, Q_e, κ^2, and ρ^2) and the correlation coefficient between consecutive wave heights [$r_{HH}(1)$] is demonstrated. The wave-height function method is reviewed in some detail in order to relate the run length theory with envelope theories. The theoretical estimates used to demonstrate the relationships between the various parameters must be considered as only first-order trends to parameter estimates computed from real wave data due to the statistical variability in these estimates when computed from real wave data.

INTRODUCTION

The tendency of ocean waves to appear in groups is receiving more attention from coastal and ocean engineers. There is a general agreement that wave grouping characteristics affect the stability of some types of maritime structures. Table 1 is a list of references in which wave groups were used to analyze various coastal phenomena and engineering problems.

In spite of the evidence that wave groups are important in a variety of coastal and ocean applications, our ability to incorporate the effects of wave groups into current design methods is limited. The difficulty of including the effects of wave groups into engineering design is due in part to the variety of theories and parameters used to characterize wave groups.

Table 2 summarizes some of the parameters and functions from various methodologies that are used to analyze and to characterize wave groups. It is obvious from Table 2 that the variety of parameters and functions from the various methodologies makes it difficult to incorporate the wave group concept into engineering design in a consistent manner. Here we attempt to identify the similarities and relationships among the various parameters, functions, and methodologies most commonly used to analyze wave groups.

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NOMENCLATURE

$A(t)$	envelope function
$AF(t)$	analytical function
$A_x(\tau)$	envelope of the function x
$a_i[=A(t_i)]$	discrete value of envelope function
B	rectangular spectrum bandwidth
C	jump parameter
C_e	constant of proportionality in Eq. 47
$E[\cdot]$	expected value
E	complete elliptical integral of the second kind
f	frequency
$f_c(=1/2\Delta T)$	cut-off frequency
f_m	frequency of the mth wave component
GF	Groupiness Factor ($SIWEH$)
$H(t)$	wave height function
H_{med}	median wave height
H_{rms}	root-mean-square wave height
H_s	significant wave height
$H_{1/3}$	average of highest one-third waves in a record
H_i	ith discrete wave height
h	threshold level
$I_0(\cdot)$	modified Bessel function of the first kind of order zero
$j(=\sqrt{-1})$	imaginary number
K	complete elliptical integral of the first kind
$k(T_{01}/\nu=T_{02}/\delta)$	parameter defined in Eq. 15
${}_1L_h(m)$	length of the mth run of high waves above threshold level, h
${}_2L_h(m)$	length of the mth total run of waves above threshold level, h
${}_1l_h\{{}_1l'_h\}[{}_1l''_h]$	length of a run of high waves computed using the envelope and normalized by $\bar{T}$ {by T_{01}} [by T_{02}]
${}_2l_h\{{}_2l'_h\}[{}_2l''_h]$	length of a total run of waves computed using the envelope and normalized by $\bar{T}$ {by T_{01}} [by T_{02}]
$LVTS(t)$	Local Variance Time Series
M_{pq}	pqth moment defined in Eq. 24
m_n	nth spectral moment
$P(\cdot)$	cumulative distribution function (c.d.f.)
$p(\cdot)$	probability density function (p.d.f.)
$p(\cdot,\cdot)$	joint probability density function
Q_e	dimensionless spectral peakedness parameter defined in Eq. 41
Q_p	dimensionless spectral peakedness parameter defined in Eq. 1
$q(\cdot)$	marginal probability density function
$R_x(\tau)$	autocorrelation function of the variable x
R_m	amplitude of the mth wave component
$r_{HH}(m)$	correlation coefficient between successive wave height with lag m
$r_{TT}(m)$	correlation coefficient between successive wave periods with lag m
$r_{HT}(m)$	correlation coefficient between successive wave heights and periods with lag m

$SIWEH(t)$	Smoothed Instantaneous Wave Energy History
$S_x(f)$	one-sided variance spectrum of the variable x
$T_{01}(=m_0/m_1)$	mean orbital period
$T_{02}(=\sqrt{m_0/m_2})$	mean zero upcrossing period
T_{SWH}	zero uprcrossing period of $SIWEH$
$T_{\mathrm{p}}(=f_{\mathrm{p}}^{-1})$	period of spectral peak frequency
T_ν	temporal spectral peakedness parameter
t	time
$\mu_{3(4)}$	cosine (sine) transform of $S_\eta(f)$
γ	peak enhancement factor for Goda-JONSWAP spectrum
$\Gamma_\eta(f)$	envelope spectral density function (unit variance)
ΔT	sampling time interval of squared wave height function
δ^2	Vanmarcke (1972) dimensionless spectral bandwidth parameter
$\eta(t)\,[\hat{\eta}(t)]$	water surface elevation [Hilbert Transform]
θ_m	random phase angle of the mth wave component
$\theta(t)+\phi$	instantaneous phase angle
κ	correlation parameter defined in Eq. (19)
ν^2	Longuet-Higgins (1957) dimensionless spectral band-width parameter
ρ	Kimura correlation parameter
$\sigma^2(\cdot)$	variance
${}_1\tau_h$	exceedance time interval of envelope above threshold level $h/2$
${}_2\tau_h$	time interval between consecutive upcrossings of envelope above threshold level $h/2$
${}_2\bar{\tau}_{H_{\mathrm{rms}}}$	mean duration of a total run of waves
$\Omega(t)$	local radian frequency function
$(\bar{\cdot})\{(\hat{\cdot})\}$	average value of $(\cdot)$ {Hilbert Transform of $(\cdot)$}

A previous review by Rye (1982) of the different wave group parameters and methodologies led to a conclusion that wave groups measured from field data compared quite well with those obtained from numerical simulations derived from linear algorithms. The validity of the linear hypothesis was also obtained by Goda (1983) and Elgar et al. (1984, 1985) from their analyses of real ocean waves measured in water depths greater than 10 meters. Field observations from Battjes and Vledder (1984) also support the linear hypothesis. Therefore, nonlinear wave–wave interaction models will not be included in this comparison of methods used to analyze wave groups in linear, random seas.

The parameters and functions listed in Table 2 that are used in the various methodologies will first be identified and interrelated where possible. Next, three methods of analyses of wave groups which incorporate most of these parameters and functions will be reviewed in some detail. These three methods of analyses are: (1) wave height function; (2) three-axes representation of run lengths; and (3) correlation coefficient between successive wave heights.

TABLE 1

Coastal phenomenon and engineering problems associated with wave groups

Coastal phenomena and engineering problems	Tucker (1950)	Hsu and Blenkarn (1970)	Ewing (1973)	Goda (1976)	Bowers (1977)	Johnson et al. (1978)	Burchart (1979)	Kimura (1980)	Rye (1982)	Sand (1982a,b,c)	Longuet-Higgins (1984)	Bruun (1985)	Mase and Iwagaki (1986)	Medina and Hudspeth (1987)	Battjes (1988)	Bowers (1988)	Agnon and Mei (1988)
Surf beat Set up/down Surf-zone dynamics	×				×				×	×			×		×	×	
Resonance Harbor resonance	×				×	×			×	×		×				×	
Oscillation and positioning of vessels Forces on mooring lines		×				×		×	×	×		×	×			×	
Damage and capsizing of ships			×	×					×		×						
Rubble mound stability						×			×			×	×				
Run-up Overtopping						×		×	×			×	×				
Design of marine structures							×			×	×						
Physical modeling										×						×	
Design wave uncertainties														×			
Internal waves																	×

REVIEW OF WAVE GROUP METHODOLOGIES

Four decades ago, Tucker (1950) identified the presence of waves in the surf zone with periods between 1 and 5 min. and suggested that these long waves were generated by wave groups. Although Tucker (1950) noted the importance of wave groups in the analysis of harbor resonance, wave groupiness did not receive much real attention until the seminal study by Goda (1970).

In this seminal study, Goda (1970) used linear numerical simulations to demonstrate that ocean waves in a random field are not completely randomly distributed. Instead, real ocean waves demonstrate a tendency to appear in groups in a manner that depends on the peakedness of the spectrum. He introduced the concepts of a length of a run of high waves and of a length of a total run of waves as well as a peakedness parameter, Q_p. The methods and parameters introduced by Goda (1970) have been the most widely used in the subsequent studies listed in Table 2.

Run lengths (Goda, 1970)

The length of a run of high waves, ${}_1L_h$, is defined as the number of consecutive wave heights that are higher than a specified threshold, h, value (e.g., $h=H_{rms}$, H_s, $\bar{H}$, H_{med},...). The length of a total run of waves, ${}_2L_h$, is the total number of wave heights that occur between the time of the first exceedance above the specified threshold value and the time of the first re-exceedance above the same specified threshold value.

Figure 1 illustrates the procedure used to determine the run lengths from a sequence of wave heights. In Fig. 1, h is the specified threshold wave height; ${}_1L_h(m)$ is the mth length of a run of high waves; and ${}_2L_h(m)$ is the mth length of a total run of waves.

Assuming that the wave heights are uncorrelated and Rayleigh distributed, Goda (1970) derived a probability density function (p.d.f.) for ${}_1L_h$ and ${}_2L_h$. However, numerical simulations based on an assumption of linear superposition demonstrated that if a random sea $\eta(t)$ was a realization from an ergodic Gaussian stochastic process, the corresponding p.d.f.'s for ${}_1L_h$ and ${}_2L_h$ were not in agreement with the Goda p.d.f.'s. Therefore, the results from his numerical simulations for a variety of spectral shapes showed that the assumption of linear superposition for random seas was not compatible with the assumption that the wave heights were uncorrelated.

High values of ${}_1L_h$ and ${}_2L_h$ are associated with larger wave groups. Therefore, parameters like the average run lengths, ${}_1\bar{L}_h$ and ${}_2\bar{L}_h$, and the sample

TABLE 2

Parameters and functions from various methodologies used to characterize wave groups

Methodologies						References
Three axes	Correlation function	Markov chains	$H^2(t)$ *LVTS* *SIWEH*	$H(t)$ Envelope	Run length	
					×	Goda, 1970
				×		Nolte and Hsu, 1972
				×	×	Ewing, 1973
					×	Rye, 1974
				×	×	Goda, 1976
				×		Arham and Ezraty, 1978
			×			Funke and Mansard, 1979
		×				Kimura, 1980
	×		×	×	×	Rye, 1982
		×	×		×	Goda, 1983
		×		×	×	Battjes and Vledder, 1984
				×	×	Elgar et al., 1984
	×			×	×	Longuet-Higgins, 1984
			×			Thompson and Seelig, 1984
		×	×		×	Mase and Iwagaki, 1986
			×	×		Medina and Hudspeth, 1987
×			×	×	×	Hudspeth and Medina, 1988
×	×	×	×	×	×	This paper, 1990

p.d.f.'s, $p({}_1L_h)$ and $p({}_2L_h)$, have been widely used to characterize wave groups in ocean wave records. Also a variety of threshold levels, h, have been used ($h = H_s, H_{med}, \bar{H}, ...$).

The numerical experiments of Goda (1970) indicated that the characteristics of wave groups in irregular wave trains were correlated with spectral peakedness. The dimensionless spectral peakedness parameter introduced by Goda (1970) was:

$$Q_p = \frac{2\int_0^\infty f S_\eta^2(f) \mathrm{d}f}{[\int_0^\infty S_\eta(f) \mathrm{d}f]^2} \tag{1}$$

Apparently related to the concept of run lengths for engineering applications is the concept of wave jumps (Bruun, 1985). A wave jump is defined as a small wave height ($H_i < C\bar{H}$, $C < 1$) followed by a big wave height ($H_{i+1} > H_s$). Burchart (1981) correlated the probability of occurrence of wave jumps with values of the wave jump parameter C using both field data from storms and laboratory data. Because linear sea states having peaked spectra produce highly correlated wave heights, the probability that a wave jump will occur decreases as the mean run lengths increases.

The p.d.f.'s for the mean run lengths defined by Goda (1970) consistently

(Table 2, continued)

Parameters and functions												
$_1\bar{T}$ $_1\bar{L}$	$_2\bar{T}$ $_2\bar{L}$	Q_e Q_p	$_1\bar{\tau}_h$	δ ν	$R_H(m\bar{T})$ $r_{HH}(m)$	$r_{TT}(m)$	r_{HT}	GF	$S_{H2}(f)$ $\epsilon(f)$	κ ρ	$\bar{T}_{SWH}$	(α_0,β_0) (α_1,β_1)
×	×	×					×					
			×	×								
×	×	×										
×	×				×							
×	×	×	×									
					×							
								×	×			
×	×	×			×					×		
		×			×	×	×	×			×	
×	×	×			×			×			×	
×					×					×		
×		×										
×	×			×	×					×		
×					×			×				
×	×				×			×			×	
		×						×	×		×	
×	×	×		×				×	×			×
×	×	×	×	×	×	×	×	×	×	×	×	×

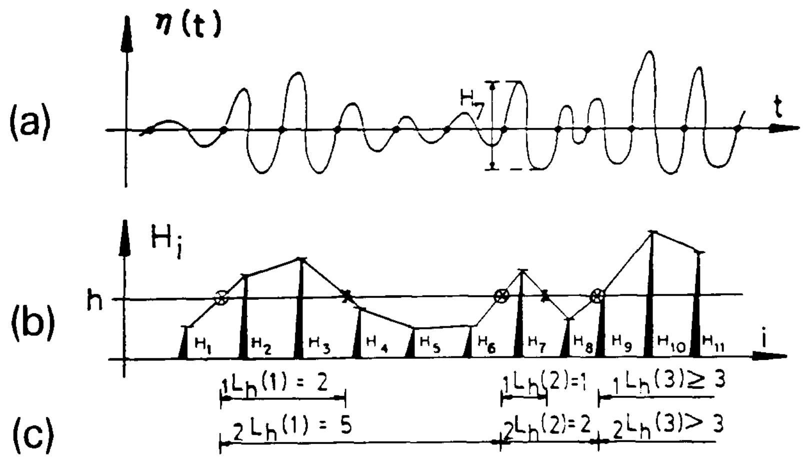

Fig. 1. Representation of (a) wave record, (b) sequence of wave heights, and (c) run lengths.

underestimated the mean run lengths computed from both field data and numerical simulations. This result lead Kimura (1980) to further refine the Goda model.

Markov chain hypothesis (Kimura, 1980)

Observing from field data that there was a positive correlation between successive wave heights, Kimura (1980) elaborated the mean run length concept for sequences of discrete wave heights that are Rayleigh distributed. To analyze the statistical properties of the run lengths, Kimura (1980) introduced a Markov chain hypothesis for a sequence of wave heights. He estimated p.d.f.'s for ${}_1L_h$ and ${}_2L_h$ as functions of a single correlation parameter, ρ. The joint p.d.f., $p(H_1,H_2)$, for two successive wave heights, $H_1=H_i$ and $H_2=H_{i+1}$, is given by:

$$p(H_1,H_2)=\frac{4H_1H_2}{(1-4\rho^2)H_{\rm rms}^4}\exp-\left[\frac{1}{(1-4\rho^2)}\left(\frac{H_1^2+H_2^2}{H_{\rm rms}^2}\right)\right]I_0\left[\frac{4H_1H_2\rho}{(1-4\rho^2)H_{\rm rms}^2}\right] \tag{2}$$

where ρ = a correlation parameter, $H_{\rm rms}$ = the root-mean-square value of wave heights, and $I_0(\cdot)$ = the modified Bessel function of the first kind of order zero. The correlation coefficient between consecutive wave heights is given by:

$$r_{\rm HH}(1)=\frac{E-[(1-4\rho^2)K/2]-(\pi/4)}{1-(\pi/4)} \tag{3}$$

where K and E are the complete elliptic integrals of the first and second kinds, respectively, with parameter (2ρ) (cf., Abramowitz and Stegun, 1968, p. 590).

The probability P_1 that H_2 does not exceed h when H_1 is below the threshold height, h; and the probability P_2 that H_2 does exceed the threshold level h when H_1 also exceeds h, may be defined as follows:

$$P_1=\frac{\int_0^h\int_0^h p(H_1,H_2)\mathrm{d}H_1\mathrm{d}H_2}{\int_0^h q(H_1)\mathrm{d}H_1} \tag{4}$$

$$P_2=\frac{\int_h^\infty\int_h^\infty p(H_1,H_2)\mathrm{d}H_1\mathrm{d}H_2}{\int_h^\infty q(H_1)\mathrm{d}H_1} \tag{5}$$

where $q(H)$ = the marginal p.d.f. for wave heights given by the Rayleigh distribution. Kimura (1980) gives the p.d.f.'s and expected values for ${}_1L_h$ and ${}_2L_h$ as:

$$p({}_1L_h)=p_2^{({}_1L_h-1)}(1-P_2) \tag{6}$$

$$E[{}_1L_h]=\frac{1}{1-P_2} \tag{7}$$

$$p({}_2L_h)=\frac{(1-P_1)(1-P_2)}{(P_1-P_2)}(P_1^{({}_2L_h-1)}-P_2^{({}_2L_h-1)}) \tag{8}$$

$$E[{}_2L_h]=\frac{1}{(1-P_1)}+\frac{1}{(1-P_2)} \tag{9}$$

The run length model of Goda (1970) is equivalent to the elaborated model of Kimura (1980) when $\rho=0$, or, equivalently, when the spectrum is extremely broad-banded. Because of the relationship between the correlation parameter, ρ and the correlation coefficient between consecutive wave heights, $r_{\mathrm{HH}}(1)$, given by Eq. 3; the p.d.f.'s $p({}_1L)$ and $p({}_2L)$ depend on only one single parameter; viz., $r_{\mathrm{HH}}(1)$. Equations 2–9 are functions of the correlation parameter ρ which may be computed either in the frequency or time domains.

Goda (1983) found $r_{\mathrm{HH}}(1)$, computed in the time domain, to be an excellent parameter to describe the run lengths from an analysis of long-travelled swell waves. He also found that the long-travelled swell data agreed quite well with the estimations from the Markov chain approximation of Kimura (1980). Battjes and Vledder (1984) observed that the distribution of lengths of runs exceeding $h=H_{1/3}$ in records from the North Sea also agreed quite well with the Markov chain approximation of Kimura (1980). They suggested using the parameter $\kappa^2=(2\rho)^2$ derived from the variance spectrum $S_\eta(f)$ to characterize the wave groups.

Earlier, Rye (1974) had identified the correlation coefficient $r_{\mathrm{HH}}(1)$ as being useful for analyzing wave groups.

Correlation coefficient for succeeding waves (Rye, 1974)

Rye (1974) identified the presence of wave groups in real ocean wave records using correlation coefficients computed from time series of wave heights and wave periods. However, only the correlation coefficient $r_{\mathrm{HH}}(1)$ appears to have received much attention (cf., Rye, 1982 for a review of correlation coefficients). The correlation coefficient for succeeding wave heights is given by:

$$r_{\mathrm{HH}}(m)=\frac{1}{r_{\mathrm{HH}}(0)}\frac{1}{(M-m)}\sum_{i=1}^{M-m}(H_i-\bar{H})(H_{i+m}-\bar{H}) \tag{10}$$

for wave periods by:

$$r_{\mathrm{TT}}(m)=\frac{1}{r_{\mathrm{TT}}(0)}\frac{1}{(M-m)}\sum_{i=1}^{M-m}(T_i-\bar{T})(T_{i+m}-\bar{T}) \tag{11}$$

and for both wave heights and wave periods by:

$$r_{\mathrm{HT}}(m)=\frac{1}{\sigma(H)\sigma(T)}\frac{1}{(M-m)}\sum_{i=1}^{M-m}(H_i-\bar{H})(T_{i+m}-\bar{T}) \tag{12}$$

where (H_i,T_i) are the wave height and the wave period, respectively, of the ith wave in a record; and M is total number of waves in the record. The variances are given by $r_{\mathrm{HH}}(0)=\sigma^2(H)$ and $r_{\mathrm{TT}}(0)=\sigma^2(T)$.

The correlation coefficient between succeeding wave heights, $r_{\mathrm{HH}}(m)$, can be related to the autocorrelation function of the envelope or the wave height function. From the relationships between H_i, $A(t)$ and $H(t)$ illustrated in Fig. 2, we may deduce that:

$$R_A(m\bar{T}) = R_H(m\bar{T}) \approx r_{HH}(m) \tag{13}$$

where $\bar{T}$ is a mean period; $R_A(\tau)$ is the autocorrelation function of the wave envelope, $A(t)$; and $R_H(\tau)$ is the autocorrelation function of the wave height function, $H(t)$. However, the estimations of the correlation coefficients based on the envelope of the autocorrelation, $R_A(m\bar{T})$, and based on the sequence of discrete waves, $r_{HH}(m)$, may show significant differences. These differences depend on the selection of mean period (T_{01}, T_{02}, Tp...); on the definition of discrete wave from continuous records (zero up crossing...); and on the spectral shape.

The envelope function, $A(t)$, had been analyzed earlier by Rice (1954) in his classic treatise on random noise.

Envelope and wave height function (Rice, 1954)

Nolte and Hsu (1972) observed that consecutive high waves in a group of waves can excite extreme forces in the mooring lines of large floating vessels. Using the analysis of groups based on the envelope concept (vide Fig. 2), they defined the average time duration for groups above a threshold level h as a basic parameter which controls the grouping characteristics.

Assuming a narrow-banded process and the results given by Rice (1954), Nolte and Hsu (1972) calculated the average duration time of excursion above a threshold level, $a = h/2$ from:

$$_1\bar{\tau}_h \approx \left(\frac{H_s/h}{2\sqrt{2\pi}}\right)k \tag{14}$$

where:

$$k^2 = \frac{1}{f_{02}^2 - \bar{f}^2} \tag{15}$$

where $f_{02}^2 = m_2/m_0$; $\bar{f} = m_1/m_0$; $m_n = n$th spectral moment; and H_s is the significant wave height.

Equation 14 may also be expressed in terms of dimensionless spectral bandwidth parameters according to:

$$_1\bar{I}_h = \frac{_1\bar{\tau}_h}{T_{01}} = \frac{H_s/h}{2\nu\sqrt{2\pi}} = \frac{(1/\nu)(\sqrt{8m_0}/h)}{2\sqrt{\pi}} \tag{16}$$

and:

$$_1\bar{I}''_h = \frac{_1\bar{\tau}_h}{T_{02}} = \frac{H_s/h}{2\delta\sqrt{2\pi}} = \frac{(1/\delta)(\sqrt{8m_0}/h)}{2\sqrt{\pi}} \tag{17}$$

where $\nu^2=(m_0m_2-m_1^2)/m_1^2$ is the dimensionless spectral bandwidth parameter defined by Longuet-Higgins (1957, 1984); $\delta^2=1-(m_1^2/m_0m_2)$ is the dimensionless spectral bandwidth parameter defined by Vanmarcke (1972, 1983); $T_{01}=m_0/m_1$; $T_{02}=\sqrt{m_0/m_2}$; and $H_s=4\sqrt{m_0}$.

Note that both ${}_1\bar{l}'_h$ and ${}_1\bar{l}''_h$ can be considered as variables analogous to the mean length of a run of high waves, ${}_1\bar{L}_h$. In addition, the parameter k, in Eq. 15, may be expressed as $k=T_{01}/\nu=T_{02}/\delta$.

Battjes and Vledder (1984) and Longuet-Higgins (1984) have identified similarities between the Markov chain approximation from the Kimura (1980) theory and the statistical properties of the envelope function given by Rice (1954). The joint p.d.f. between successive values of the envelope function, $p(a_1,a_2)$ given by Rice (1954) is:

$$p(a_1,a_2)=\frac{a_1a_2}{m_0^2(1-\kappa^2)}\exp-\left[\frac{(a_1^2+a_2^2)}{2m_0(1-\kappa^2)}\right]I_0\left[\frac{\kappa}{(1-\kappa^2)}\frac{a_1a_2}{m_0}\right] \tag{18}$$

where $a_1=A(t_1)$ and $a_2=A(t_1+\tau)$, and:

$$\kappa=\frac{\sqrt{(\mu_3^2+\mu_4^2)}}{m_0} \tag{19}$$

$$\mu_3=\int_0^\infty S_\eta(f)\cos[2\pi(f-f_{01})\tau]\mathrm{d}f \tag{20}$$

$$\mu_4=\int_0^\infty S_\eta(f)\sin[2\pi(f-f_{01})\tau]\mathrm{d}f \tag{21}$$

and $I_0(\cdot)$ is the modified Bessel function of the first kind of order zero defined by (cf., Abramowitz and Stegun, 1968, p. 376):

$$I_0(z)=\frac{1}{\pi}\int_0^\pi\exp(z\cos\theta)\mathrm{d}\theta \tag{22}$$

Longuet-Higgins (1984) assumed that $\tau=m_0/m_1=T_{01}$ and defined a correlation coefficient between successive wave heights by:

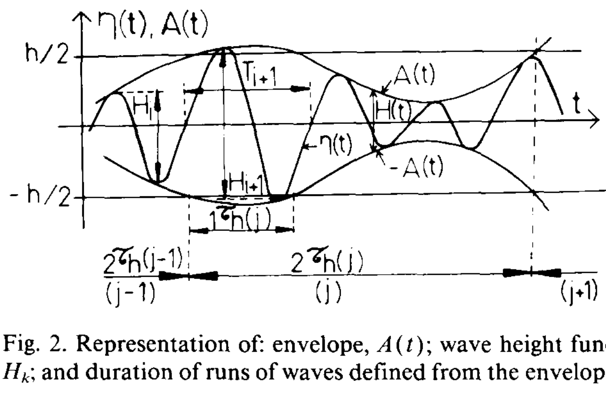

Fig. 2. Representation of: envelope, $A(t)$; wave height function, $H(t)$; discrete wave heights, H_k; and duration of runs of waves defined from the envelope; ${}_1\tau_h(i)$ and ${}_2\tau_h(i)$.

$$R_A(T_{01}) = \frac{M_{11}}{\sqrt{M_{20}M_{02}}} \tag{23}$$

where:

$$M_{pq} = \int_0^\infty \int_0^\infty (a_1 - \bar{a})^p (a_2 - \bar{a})^q p(a_1, a_2) \mathrm{d}a_1 \mathrm{d}a_2 \tag{24}$$

This correlation coefficient may be computed by:

$$R_A(T_{01}) = \frac{E - [(1-\kappa^2)K/2] - (\pi/4)}{1 - (\pi/4)} \tag{25}$$

where K and E are the complete elliptic integrals of the first and second kind, respectively, with parameter κ. Equation 25 is identical to Eq. 3 from Kimura (1980) for $\kappa = 2\rho$.

For narrow-banded processes, Longuet-Higgins (1984) found that:

$$\kappa^2 \approx 1 - 4\pi^2 \nu^2 \tag{26}$$

while Eq. 25 has the approximation:

$$R_A(T_{01}) \approx \kappa^2; \quad 0 \le \kappa^2 \le 1 \tag{27}$$

Defining a wave height function as $H(t) = 2A(t)$ and a threshold level as $h = 2a$, then a change of variables given by:

$$p(h_1, h_2)\mathrm{d}h_1 \mathrm{d}h_2 = p(a_1, a_2)\mathrm{d}a_1 \mathrm{d}a_2 \tag{28}$$

yields:

$$p(h_1, h_2) = \tfrac{1}{4} p(a_1, a_2) \tag{29}$$

$$p(h_1, h_2) = \frac{h_1 h_2}{16 m_0^2 (1-\kappa^2)} \exp - \left[\frac{(h_1^2 + h_2^2)}{8 m_0 (1-\kappa^2)} \right] I_0 \left[\frac{\kappa}{(1-\kappa^2)} \frac{h_1 h_2}{4 m_0} \right] \tag{30}$$

Equation 30 is equal to Eq. 2 derived by Kimura (1980) when $\kappa = 2\rho$, and $H_{rms} = \sqrt{8 m_0}$.

Defining ${}_1l_h$ and ${}_2l_h$ as the length of a run of high waves and the length of a total run of waves computed from the envelope function at a level $a = h/2$, Longuet-Higgins (1984) found the following estimates for $E[{}_1l_h]$ and $E[{}_2l_h]$:

$$E[{}_1l_h] = E[{}_1l_{2a}] \approx \frac{1}{\sqrt{2\pi}} \frac{\sqrt{(1+\nu^2)}}{\nu} \left(\frac{\sqrt{m_0}}{a} \right) \tag{31}$$

which can be rewritten as:

$$E[{}_1l_h] \approx \frac{1/\delta}{2\sqrt{\pi}} \left(\frac{\sqrt{8 m_0}}{h} \right) \tag{32}$$

and:

$$E[{}_2l_h]=E[{}_2l_{2a}]\approx\frac{1}{\sqrt{2\pi}}\frac{\sqrt{(1+\nu^2)}}{\nu}\left(\frac{\sqrt{m_0}}{a}\right)\exp\left(\frac{a^2}{2m_0}\right) \tag{33}$$

which reduces to:

$$E[{}_2l_h]=E[{}_1l_h]\exp\left(\frac{h^2}{8m_0}\right) \tag{34}$$

since $[(1+\nu^2)^{1/2}/\nu]=1/\delta$ and $h=2a$.

Note the similarity between $E[{}_1l_h]$ in Eq. 32 and ${}_1\bar{l}''_h$ in Eq. 17. If T_{01} is defined to be the mean wave period, then Eq. 32 may also be transformed to Eq. 16.

Rice (1954) demonstrated that the envelope and the square of the envelope have different p.d.f.'s but similar stochastic properties. Therefore, the square of the envelope or wave height function may be used to analyze wave groups.

$\eta^2(t)$ filters and $H^2(t)$

Medina and Hudspeth (1987) and Hudspeth and Medina (1988) identified the following similarities between the Smoothed Instantaneous Wave Energy History (*SIWEH*) (Funke and Mansard, 1979); the Local Variance Time Series (*LVTS*) (Thompson and Seelig, 1984); and the squared wave height function $[H^2(t)]$ defined from the envelope of the record:

$$SIWEH(t)\approx LVTS(t)\approx\frac{H^2(t)}{8}=\frac{A^2(t)}{2} \tag{35}$$

Hudspeth and Medina (1988) observed that the squared wave height function, $H^2(t)$, defined on the basis of a time series and its Hilbert transform isolates exactly the low frequency components of the squared water surface elevation, $\eta^2(t)$, for a linear stochastic model. In contrast, the *SIWEH* requires an arbitrary low-pass filter in order to isolate the low frequency contributions. They also found that for linear waves the expected value of the Grouping Factor (*GF*) as defined by Funke and Mansard should be approximately equal to unity independent of the shape of the spectrum. This implies that the Groupiness Factor is not an appropriate parameter to characterize run lengths.

Rye (1982) introduced a more appropriate parameter which was later verified by Goda (1983). This parameter was the mean zero-upcrossing period of *SIWEH*, $\bar{T}_{\rm SWH}$, defined by:

$$\bar{T}_{\rm SWH}=\left(\frac{1}{T_{\rm p}}\right)\left(\frac{1}{I}\right)\sum_{i=1}^{I}(T_{\rm SWH})_i \tag{36}$$

in which I is the total number of zero upcrossings of the mean level in the *SIWEH*; and $T_p = 1/f_p$ is the period of the peak frequency of the spectrum.

The relationship between $SIWEH(t)$ and $H^2(t)$ given in Eq. 35 implies that $\bar{T}_{SWH}$ may be related to the mean length of the total run of waves at the threshold level $h = H_{rms}$. From Fig. 2 and Eqs. 35 and 36 we find, approximately:

$$\bar{T}_{SWH} \approx \frac{{}_2\bar{\tau}_{H_{rms}}}{T_p} = {}_2\bar{l}_{H_{rms}}\left(\frac{T_{01}}{T_p}\right) \tag{37}$$

where ${}_2\bar{\tau}_{H_{rms}}$ is the mean duration of a total run of waves; and ${}_2\bar{l}_{H_{rms}}$ is the length of the total run of waves using the envelope or wave height function at level H_{rms}.

Rice (1954), Bendat and Piersol (1986), and Medina and Hudspeth (1987) give the following approximations for the spectra of $H(t)$ and $H^2(t)$;

$$S_H(f) \approx (8-2\pi)m_0\Gamma_\eta(f) \tag{38a}$$

$$S_{H^2}(f) \approx 64m_0^2\Gamma_\eta(f) \tag{38b}$$

where the envelope spectral density function (unit variance) is defined by:

$$\Gamma_\eta(f) = \frac{2}{m_0^2}\int_0^\infty S_\eta(x+f)S_\eta(x)\,dx \tag{39}$$

where $\sigma^2[H(t)] = (8-2\pi)m_0$ and $\sigma^2[H^2(t)] = 64\,m_0^2$.

Medina and Hudspeth (1987) demonstrated that the envelope spectral density function (unit variance), $\Gamma_\eta(f)$, given in Eq. 39 is usually a monotonically decreasing function with a maximum value at $f=0$. This maximum value is closely related to the grouping characteristics and to the variability of the variance of the process since:

$$\Gamma_\eta(0) = 4T_\nu = Q_e T_{01} \tag{40}$$

where T_ν is a temporal spectral peakedness parameter introduced by Medina et al. (1985); and Q_e is a dimensionless spectral peakedness parameter proposed by Medina and Hudspeth (1987). The Q_e parameter is defined as:

$$Q_e = \frac{2m_1}{m_0^3}\int_0^\infty S_\eta^2(f)\,df \tag{41}$$

This peakedness parameter is similar to the Goda peakedness parameter, Q_p, and is also related to parameters introduced by Tucker (1963) and by Blackman and Tukey (1959). Equations 38–41 were used by Medina (1990) to analyze the group-induced inshore long waves from the field data published by Nelson et al. (1988).

Finally, the envelope of the autocorrelation function has also been used to analyze wave groups. Although it is not obvious, the envelope of the autocorrelation function is mathematically related to the spectrum of the squared wave height function (see Bendat and Piersol, 1986). For this reason, it should be reasonable to use the envelope of the autocorrelation function to analyze wave groups. Rye (1982) intuitively tried to correlate wave grouping characteristics with some features of the autocorrelation function.

Envelope of the autocorrelation function (Rye, 1982)

Figure 3 illustrates the spectrum, the autocorrelation function and the corresponding envelope for a rectangular and for a Goda-JONSWAP spectra ($\gamma = 3.3$). Because the rectangular-shaped spectrum demonstrates a local maximum in the envelope of the autocorrelation function at $\tau = 1.5/B$ where B is rectangular spectral bandwidth, Rye (1982) attempted to analyze wave groups using the envelope of the autocorrelation function.

Note that in Fig. 3b the envelope of the autocorrelation function for the

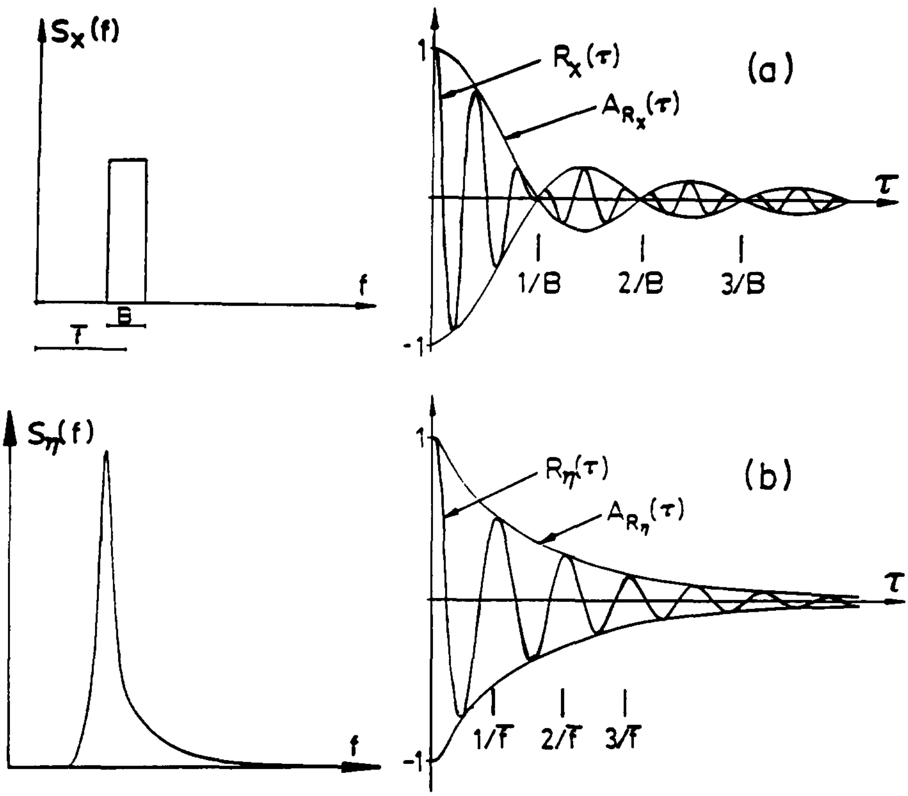

Fig. 3. Envelopes of autocorrelation functions from (a) rectangular spectrum and (b) Goda-JONSWAP spectrum.

Goda-JONSWAP spectrum decreases monotonically. Because of this monotonic behavior and because of the statistical variability in real ocean wave records, Rye was not successful in using the local maxima of envelope of the autocorrelation function to characterize wave groups.

However, it will be shown in the following that the envelope of the autocorrelation function is related to the spectrum of the wave height function. Therefore, the envelope of the autocorrelation function may be used to characterize wave groups.

WAVE HEIGHT FUNCTION ANALYSIS

Assuming that the sea surface elevation at a point is an ergodic Gaussian stochastic process having a variance spectrum $S_\eta(f)$, a realization may be approximated by:

$$\eta(t)=\sum_{m=1}^{M} R_m\cos(2\pi f_m t+\theta_m) \tag{42}$$

where M is the total number of wave components in the realization; R_m, f_m, and θ_m are the amplitude, the frequency, and a random phase angle, respectively, of the mth wave component. The random phase angle is uniformly distributed in the interval $U[0,2\pi]$. The Hilbert transform, $\hat{\eta}(t)$, of $\eta(t)$ (Bendat and Pierson, 1986), is given by:

$$\hat{\eta}(t)=\sum_{m=1}^{M} R_m\sin(2\pi f_m t+\theta_m) \tag{43}$$

and the analytical function by:

$$AF(t)=\eta(t)+j\hat{\eta}(t)=A(t)\exp\{j[\theta(t)+\phi]\} \tag{44}$$

where $j=\sqrt{-1}$; $A(t)$ is the envelope function; and $[\theta(t)+\phi]$ is the instantaneous phase angle defined by:

$$A(t)=\sqrt{\eta^2(t)+\hat{\eta}^2(t)} \tag{45a}$$

$$\theta(t)+\phi=\arctan\left[\frac{\hat{\eta}(t)}{\eta(t)}\right] \tag{45b}$$

In the complex plane, Hudspeth and Medina (1988) identified $AF(t)$ as an orbital movement consisting of a vertical displacement of a point floating in the sea surface $\eta(t)$ and a horizontal displacement $\hat{\eta}(t)$. An instantaneous wave height, $H(t)=2A(t)$, and a local radian frequency, $\Omega(t)=\mathrm{d}\theta(t)/\mathrm{d}t$, were defined. The statistical properties of these two functions were evaluated and related to characteristics of wave groups.

From the definition of $H^2(t)$ and its spectrum, $S_{H^2}(\mathrm{f})$, the inverse of the mean frequency of $S_{H^2}(\mathrm{f})$ can be interpreted as the average mean period, $_2\bar{\tau}_{H_{\mathrm{rms}}}$, of $H^2(t)$ which can be normalized by the mean wave period, T_{01}, to obtain the average length of a total run of waves at a threshold level $h=H_{\mathrm{rms}}$ (vide Eq. 37).

Figure 4 illustrates different envelope spectral density functions (unit variance) normalized by $6T_\nu$. The normalized mean period, $_2\bar{\tau}_{H_{\mathrm{rms}}}/6\mathrm{T}_\nu$, of $H^2(t)$ is equal to unity in Fig. 4 for the rectangular spectrum. Therefore, for the rectangular spectrum the average length of a total run of waves at $h=H_{\mathrm{rms}}$ may be approximated by:

$$_2\bar{l}_{H_{\mathrm{rms}}} \approx \frac{6T_\nu}{T_{01}} = \tfrac{3}{2}Q_{\mathrm{e}} \tag{46}$$

On the other hand, the normalized envelope spectral density functions for the Goda-JONSWAP spectra in Fig. 4 have normalized mean periods, $_2\bar{\tau}_{H_{\mathrm{rms}}}/6\mathrm{T}_\nu$, which are smaller than unity. As a consequence, the expected length of a total run of waves at $h=H_{\mathrm{rms}}$ is lower than for the rectangular spectrum. Therefore, the expected length of a total run of waves for real wave spectral will be proportional to the estimate for a rectangular spectrum according to:

$$_2\bar{l}_{\mathrm{H}_{\mathrm{rms}}} \doteq \tfrac{3}{2}Q_{\mathrm{e}}C_{\mathrm{e}} \tag{47}$$

where C_{e} is a constant of proportionality that is a function of the spectral shape and the cut-off frequency $f_{\mathrm{c}}=1/2\Delta T$ where ΔT is the sampling time interval of the squared-wave height nfunction.

In order to compare estimates of run lengths given by Eq. 47 with observa-

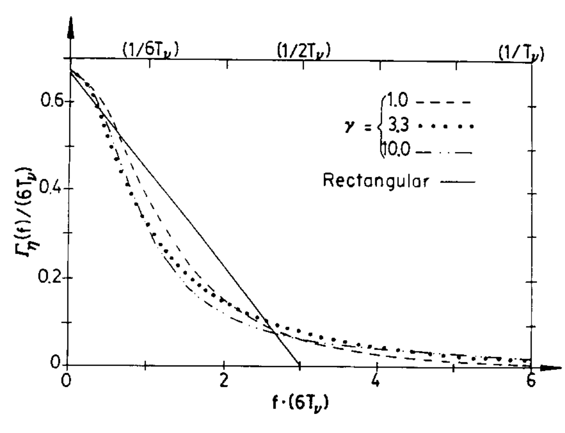

Fig. 4. Normalized envelope spectral density functions (unit variance) for rectangular and Goda-JONSWAP spectra.

tions of run lengths computed from discrete waves, the value of ΔT should be approximately equal to T_{01} because discrete wave heights should be comparable with values of $H(t)$ discretized at this time interval.

Figure 5 illustrates the effect of spectral shape ($\gamma=1.0$, 3.3 and 10.0) and of sampling time intervals ($\Delta T=T_\nu$ and $T_\nu/2$) on the constant of proportionalities, C_e, for the Goda-JONSWAP spectrum.

The similarity between the spectral peakedness parameters, Q_p and Q_e, defined by Eqs. 1 and 41 appears to be useful for comparing methods for wave group analyses. For the Goda-JONSWAP spectra, the ratio $Q_p/Q_e \approx 0.87$ with a slight tendency to increase with increasing values of γ.

The envelope spectral density function (unit variance) $\Gamma_\eta(f)$ used in Eqs. 38 and 39 suggests the possibility of using the Fourier transform of $\Gamma_\eta(f)$ to characterize wave groups. The Fourier transform of $\Gamma_\eta(f)$ is the square of the envelope of the autocorrelation function of the stochastic process (Bendat and Piersol, 1986).

Figure 6 illustrates the Wiener–Khinchine relations for (a) waves and (b) envelopes. Figure 6a shows the unit variance spectrum, $S_\eta(f)/m_0$, the corresponding autocorrelation function, $R_\eta(\tau)$, and its envelope, $A_{R_\eta}(\tau)$; and the Hilbert transform of the autocorrelation function $\hat{R}_\eta(\tau)$. Similarly, Fig. 6b shows the envelope spectral density function (unit variance), $\Gamma_\eta(f)$; its Fourier transform, $A^2_{R_\eta}(\tau)=R_{H^2}(\tau)$; as well as the square of the autocorrelation function, $R^2_\eta(\tau)$, and the square of its Hilbert transform, $\hat{R}^2_\eta(\tau)$. Note that the autocorrelation function of the squared envelope, $R_{H^2}(\tau)$, is the square of the envelope of the autocorrelation function, $A^2_{R_\eta}(\tau)$.

Even though Rye (1982) was unable to find any useful properties (i.e., no local maxima) in the envelope of the autocorrelation function, $A_{R_\eta}(\tau)$, the

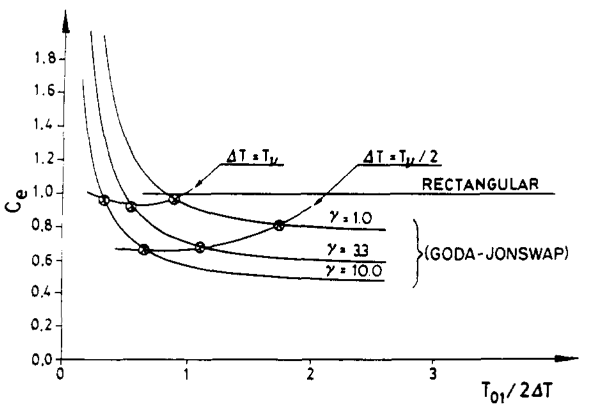

Fig. 5. Dependence of C_e on spectral shape (γ) and on sampling time interval (ΔT) for Goda-JONSWAP spectrum.

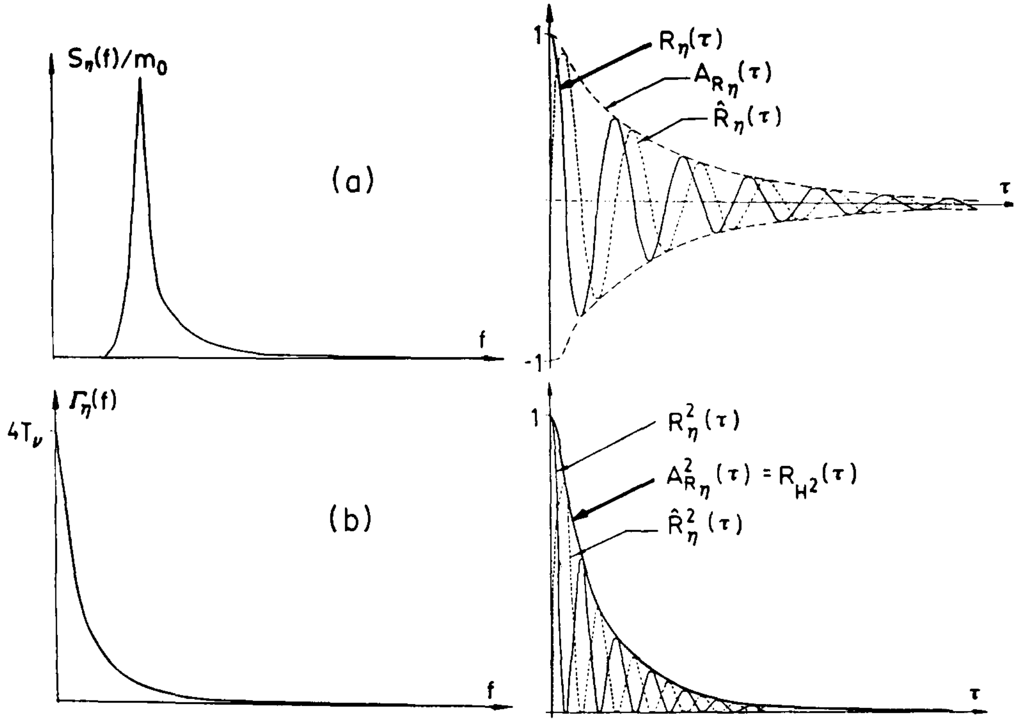

Fig. 6. Wiener–Khinchine relations for (a) waves and (b) envelopes.

following analysis appears to support the hypothesis that the square of the envelope of the autocorrelation function, $A^2_{R_\eta}(\tau)$, should contain the basic properties of the characteristics of the wave groups in a stochastic process. Note that by Eqs. 38a, b, $\Gamma_\eta(f)$ is also an approximation for the spectral density function (unit variance) of the wave height function, $H(t)$. Its Fourier transform, $A^2_{R_\eta}(\tau)$, should be an approximation for the autocorrelation function of the envelope, $R_A(\tau)$. Therefore, according to Eqs. 13, 25, and 27, we find:

$$A^2_{R_\eta}(\tau) \approx R_A(\tau) = R_H(\tau) \approx R_{H^2}(\tau) \qquad (48)$$

and:

$$A^2_{R_\eta}(T_{01}) \approx \kappa^2 = (2\rho)^2 \qquad (49)$$

where κ is the parameter defined in Eq. 19 used by Longuet-Higgins 1984 in Eq. 25; and 2ρ is the parameter used by Kimura 1980 in Eq. 3. The parameter, κ, used by Battjes and Vledder (1984) in a formula similar to Eq. 25 was defined using the envelope of the autocorrelation function, $A_{R_\eta}(\tau)$, and T_{02} instead of T_{01} as the mean period.

Figure 7 illustrates the Wiener–Khinchine relations for the envelope spec-

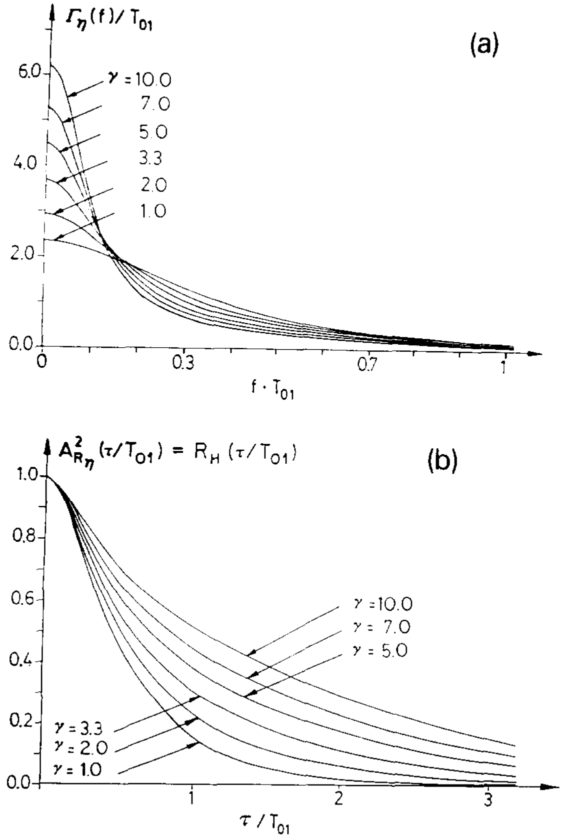

Fig. 7. Wiener–Khinchine relations for envelopes from Goda-JONSWAP spectra.

tra for different values of γ for the Goda-JONSWAP spectra. The envelope spectral density functions (unit variance) have been normalized by T_{01}. In principle, the square of the envelope of the autocorrelation function in Fig. 7b, $R_{H^2}(\tau/T_{01}) = A^2_{R_\eta}(\tau/T_{01})$, could be used in Eq. 49 to estimate the parameters κ or ρ.

STATISTICS OF RUN LENGTHS

Two simple approximations have been used to define the p.d.f.'s for the length of run of high waves and the total run of waves: viz. (1) the exponential, and (2) the Markov chain. The first approximation is based on the independence between crossings of the wave height function at a given threshold level. The second is based on a Markov chain hypothesis for the discrete wave heights as given by Eqs. 6–9.

Exponential approximation

The exponential approximation may be derived from a Poisson distribution (cf., Longuet-Higgins, 1984). Assuming that successive upcrossings of $H(t)$ at a given threshold level are uncorrelated (which is reasonable for narrow-banded processes and for large ${}_2\bar{l}_h$), then:

$$1-P({}_2l_h)=\lim_{m\to\infty}\left[1-\frac{1}{m}\right]^{[{}_2l_h/({}_2\bar{l}_h/m)]} \qquad (50)$$

where $P({}_2l_h)$ is the probability that the length of a total run of waves is less than ${}_2l_h$; ${}_2\bar{l}_h=\mathrm{E}[{}_2l_h]$ is the mean length of a total run of waves; and $({}_2\bar{l}_h/m)$ is a small subinterval of ${}_2\bar{l}_h$ which has a very low probability ($=1/m$) of detecting an upcrossing. In the limit as $m\to\infty$:

$$1-P({}_2l_h)=\exp-\left(\frac{{}_2l_h}{{}_2\bar{l}_h}\right) \qquad (51)$$

and, therefore:

$$p({}_2l_h)=\left(\frac{1}{{}_2\bar{l}_h}\right)\exp-\left(\frac{{}_2l_h}{{}_2\bar{l}_h}\right) \qquad (52)$$

in which $p({}_2l_h)$ is the probability density function of the length of a total run of waves.

With similar assumptions, analogous approximations for $P({}_1l_h)$ and $p({}_1l_h)$ are given by:

$$P({}_1l_h)=1-\exp-\left(\frac{{}_1l_h}{{}_1\bar{l}_h}\right) \qquad (53)$$

$$p({}_1l_h)=\left(\frac{1}{{}_1\bar{l}_h}\right)\exp-\left(\frac{{}_1l_h}{{}_1\bar{l}_h}\right) \qquad (54)$$

If the wave height function is Rayleigh distributed, then a ratio between the average run lengths is given by:

$$\left(\frac{{}_2\bar{l}_h}{{}_1\bar{l}_h}\right)=\frac{1}{1-P(h)}=\exp\left(\frac{h^2}{8m_0}\right) \qquad (55)$$

where ${}_2\bar{l}_h$ and ${}_1\bar{l}_h$ are the average values for the length of a total run of waves and of a run of high waves at a threshold level h, respectively; and $P(h)$ is the cumulative distribution function (c.d.f.) of the wave heights. Note that Eq. 55 is equivalent to Eq. 34.

Three-axes representation

Both the exponential (Eqs. 51–55) and the Markov Chain approximations (Eqs. 6–9) yield p.d.f.'s for ${}_1l_h$ and ${}_2l_h$ that require knowing only the average lengths of runs, ${}_1\bar{l}_h$ and ${}_2\bar{l}_h$. Therefore, estimates for the pairs of parameters $[{}_1\bar{l}_h, {}_2\bar{l}_h]$ at different threshold levels (e.g., $h=H_{med}$, $\bar{H}$, H_{rms}, H_s,...) computed from physical data provide the only information needed to compute the statistics of run lengths.

Hudspeth and Medina (1988) introduced a three-axes representation for average length of runs of waves to describe the characteristics of wave groups. Using the following change of variables:

$$u=\ln\left(\frac{h}{\sqrt{8m_0}}\right) \tag{56a}$$

$$v=\ln({}_1\bar{l}_h) \tag{56b}$$

$$w=\ln\left[\ln\left(\frac{{}_2\bar{l}_h}{{}_1\bar{l}_h}\right)\right] \tag{56c}$$

the exponential approximation will exhibit straight lines given by $w=2u$ and by $v=\alpha_0-u$ for the average length of runs of waves. High values of α_0 correspond to long runs of waves.

Figure 8 illustrates a three-axes representation for the average length of runs for the exponential approximation. Fitting straight lines given by $v(u)=\alpha_0-\beta_0 u$ and by $w(u)=\alpha_1+\beta_1 u$ to the observed pairs of parameters $[{}_1\bar{l}_h, {}_2\bar{l}_h]$ using Eqs. 52 and 54 or Eqs. 6 and 9, will yield approximate probabilistic descriptions for the run lengths at any threshold level.

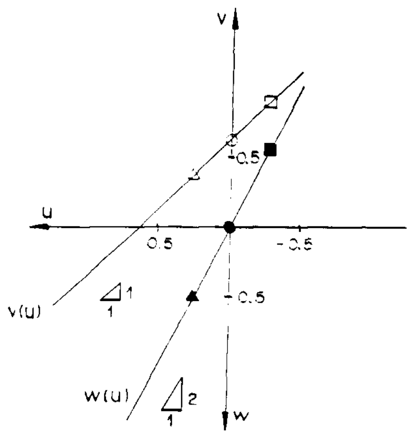

Fig. 8. Three-axes representation of run lengths from the exponential approximation.

Hudspeth and Medina (1988) and Medina and Hudspeth (1988) analyzed the three-axes representation for the average length of run of discrete waves and the envelope using both physical and numerically simulated data. They concluded that run lengths computed from discrete wave sequences did not agree very well with run lengths computed from the envelope. The envelope or wave height function produced higher run lengths when longer sampling time intervals of the envelope, ΔT, were used. Neither changes in the sampling time interval nor filtering of the data produced results that were equivalent at all threshold levels.

On the other hand, analysis of real data records using very long time series (hours) are required to sufficiently reduce the variability in the estimates of the parameters [${}_1\bar{l}_h$ and ${}_1\bar{l}_h$] (envelope), or [${}_1\bar{L}_h$ and ${}_2\bar{L}_h$] (discrete waves) in order to obtain reliable statistics for mean run lengths.

CORRELATION BETWEEN SUCCESSIVE WAVE HEIGHTS

Analyzing wave groups in long-travelled swell waves, Goda (1983) concluded that the correlation coefficient between consecutive wave heights, $r_{HH}(1)$, is a better parameter to define the length of runs of wave heights than is the spectral peakedness parameter, Q_p. On the other hand, he noted that $r_{HH}(1)$ is probably correlated with Q_p and could, therefore, be considered as an internal parameter to describe the phenomenon of wave grouping. We demonstrate this correlation later in Fig. 11.

If $\Gamma_\eta(f)$ is approximately the spectral density function of the wave height function, then its Wiener–Khinchine transform illustrated in Fig. 7b should be approximately the autocorrelation function of the wave height function. Therefore, a correlation between successive wave heights could be estimated

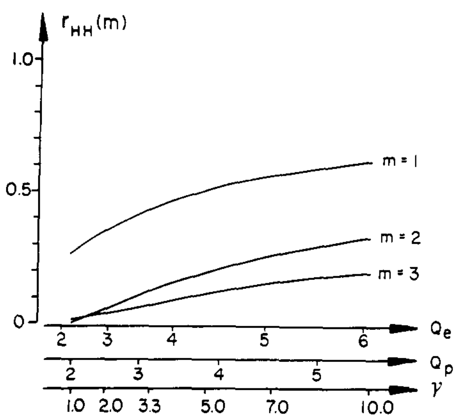

Fig. 9. Effect of spectral shape on the correlation coefficient between successive wave heights for Goda-JONSWAP spectra.

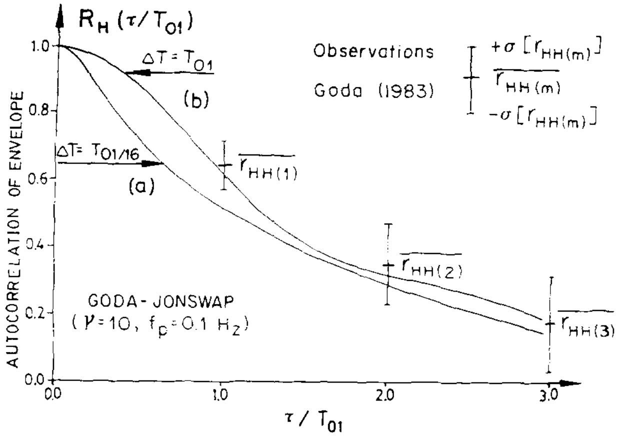

Fig. 10. Effect of sampling time interval (ΔT) on estimates of correlation between successive wave heights and comparison with swell data (Goda, 1983).

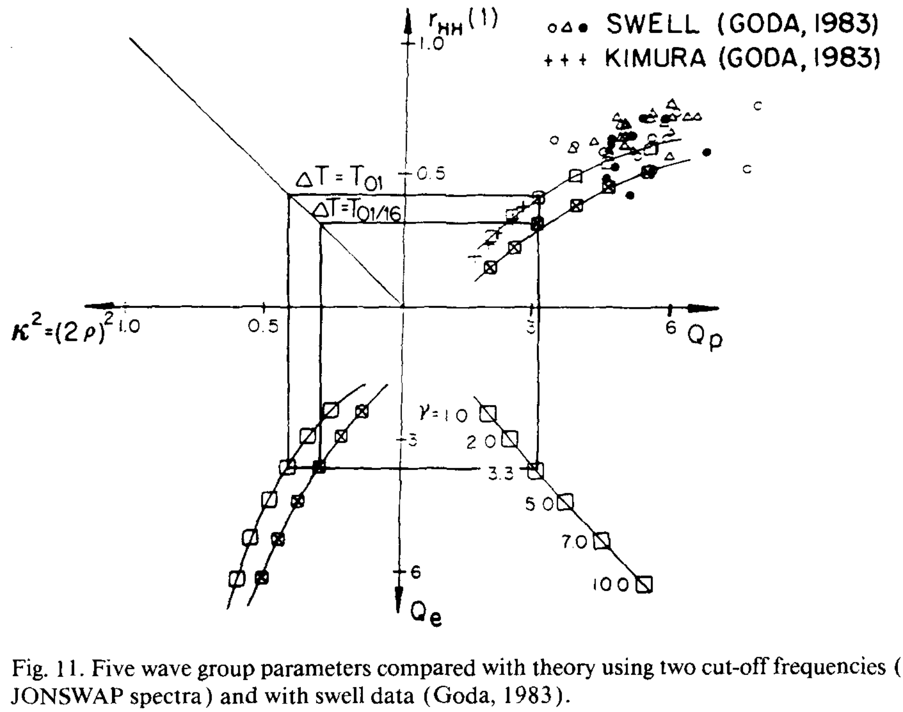

Fig. 11. Five wave group parameters compared with theory using two cut-off frequencies (Goda-JONSWAP spectra) and with swell data (Goda, 1983).

from this Wiener–Khinchine transform. Figure 9 represents estimations of $r_{HH}(m)$ from Goda-JONSWAP spectra using Eq. 48 and values from the Wiener–Khinchine transform pairs illustrated in Fig. 7.

It appears reasonable that values of $r_{HH}(1)$ computed from discrete waves should be approximately equivalent to values of $r_{HH}(1)$ computed from wave height envelopes discretized at $\Delta T = T_{01}$. Figure 10 compares three sets of values for $r_{HH}(m)$ computed from long-travelled swell waves by Goda (1983) with theoretical estimates for $R_H(\tau/T_{01})$ computed from Goda-JONSWAP spectra using two different values of ΔT (viz., $\Delta T = T_{01}/16$ and $\Delta T = T_{01}$).

Five of the parameters listed in Table 2 to characterize wave groups may be related to each other in a simple manner. Figure 11 illustrates this simple theoretical relationship using Eqs. 3, 13, and 48 and the Wiener–Khinchine pairs shown in Fig. 7. These theoretical relationships were computed from Goda-JONSWAP spectra for six values of the peak enhancement factor, γ, using two different values for the cut-off frequency given by $f_N^{-1} = 2\Delta T$ for $\Delta T = T_{01}$ and $T_{01}/16$. The first quadrant [$r_{HH}(1)$ vs. Q_p] compares the observations of long-travelled swell waves and the numerical simulations from Kimura that were reported by Goda (1983) with theoretical estimates computed using two cut-off frequencies (viz., $\Delta T = T_{01}$ and $T_{01}/16$. The second quadrant [Q_p vs. Q_e] demonstrates the ratio $Q_p/Q_e \approx 0.87$ computed from Eqs. 1 and 41, and for Goda-JONSWAP spectra. The third quadrant [Q_e vs. $\kappa^2 = (2\rho)^2$] illustrates theoretically using Eq. 49 the relationship between these two parameters as a function of the two cut-off frequencies for the envelope spectra. Finally, the fourth quadrant [$\kappa^2 = (2\rho)^2$ vs. $r_{HH}(1)$] represents the relationship defined in Eq. 3.

Of course, estimates of the five parameters that are related theoretically in Fig. 11 will be less correlated when computed from real ocean wave data due to the statistical variability of the functions used to estimate these parameters.

SUMMARY AND CONCLUSIONS

The parameters and functions derived from the most commonly used methodologies to characterize wave groups in linear waves have been reviewed and interrelated in a unified manner. The run length methodology and the exponential and Markov chain approximations have been interrelated in a three-axes representation of run lengths. A second interrelationship was made for the envelope theory; for the wave height function theory; for the $\eta^2(t)$ filter theories; and for the correlation function theories. Finally, the correlation coefficient between consecutive wave heights [$r_{HH}(1)$] was shown to be correlated with the spectral peakedness parameters (Q_p and Q_e).

The wave height function method was reviewed in some detail in order to demonstrate the relationship between the run lengths theory and the envelope theories. A three-axes representation demonstrated that run lengths computed from a discrete wave height method did not agree with run lengths computed from a continuous envelope.

Four parameters (Q_p, Q_e, κ^2, and ρ^2) used to characterize wave groups were compared with the correlation coefficient [$r_{HH}(1)$] in order to demonstrate that these parameters are interrelated. Therefore, because of this interrelationship, only one of the parameters or the correlation coefficient is required in order to evaluate wave groupiness.

Finally, run lengths and any of the wave group parameters are estimates from a linear random process, and are, therefore, subject to a statistical variability. Very long data records are required in order to reduce to an acceptable level the statistical variability of estimates of these parameters and functions. The theoretical estimates provided in this review must be interpreted as first-order trends to the estimates computed from real data.

ACKNOWLEDGEMENTS

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