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Wave groups: a closer look at spectral methods

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ABSTRACT

Two well known spectral approaches to wave group analysis are critically examined: the envelope theory combined with the discrete counting correction scheme (Longuet-Higgins, 1984), and the Kimura theory as modified by Battjes and Van Vledder (1984). In both cases, the mean length of discrete wave groups, $\bar{j}$, is related, through the narrow spectral bandwidth approximation, to some characteristics of the wave spectrum: the spectral width, ν, and the spectral correlation coefficient, γ_s, respectively. Comparisons between the predictions of the models and the groupiness characteristics of both numerically simulated and field data are used to identify some deficiencies of the two methods over a wide range of ocean wave conditions. The limitations of the two methods due to the various assumptions employed are closely examined, and their effects on the models' predictions quantified. The discrete counting correction scheme of the first method is shown to use an incorrect probability distribution for the wave groups and also to neglect an important *splitting* effect. In the second approach, the spectral correlation coefficient is found to be consistently smaller than the putatively equivalent discrete wave correlation parameter, γ_h, resulting in a systematic underestimation of the mean group length by up to 12%.

1. INTRODUCTION

Sequences of high waves, known as wave groups, are evident both in visual observations of the sea and in wave records. A wave group is commonly defined as a sequence of waves with heights exceeding a certain preset level. The likelihood of encountering such a group of large waves represents an important parameter for the engineering design of moored and floating structures, as well as for various hydrodynamical problems such as surf beat, harbour resonance, and whitecaps distribution (e.g. Ouellet and Theriault, 1989). The term wave group is also associated with the extremely high waves, or *freak* waves, which have been reported to occur at sea on rare occasions and somewhat unexpectedly (e.g. Nickerson, 1986). However those giant episodic waves form a class in themselves and will not be considered here.

A standard measure of wave groupiness, the mean group length $\bar{j}$, is obtained by averaging over a record of sea surface elevation the number of consecutive waves exceeding a height threshold, H_c. Individual waves are defined here using the standard zero-upcrossing method with their height, H, taken as

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the maximum vertical excursion of the surface elevation between two zero-upcrossings, and their period, T, as the time interval separating these two events. Whereas $\bar{j}$ is readily calculated from a series of wave heights, most of the wave information collected is commonly available as the power spectrum of the surface elevation. Thus, a practical problem which has received considerable attention consists in identifying a robust relationship between some groupiness statistic computed in the time domain, such as $\bar{j}$, and a spectral parameter.

Several linear approaches have been suggested in the past to predict wave group statistics from the energy spectrum. The classical Gaussian model, first introduced by Rice (1944–1945) to analyse noise in electrical circuits, has been used to describe group behaviour in terms of characteristics of the wave spectrum. In this model, the sea surface displacement is treated as a narrow band Gaussian noise, and the group properties are derived from the envelope function of the surface displacement. Longuet-Higgins (1957, 1984), who first applied the Gaussian model to sea waves, derived an estimate of the average number of waves in a high run, $\bar{\mathscr{H}}$, in terms of a single parameter, the spectral width. Although this method has been shown to successfully predict the mean number of waves in a group defined by the wave envelope (e.g., Chandler and Masson, 1992), its usefulness in modeling the group statistics of the discrete wave heights has not yet been satisfactorily demonstrated. Vanmarcke (1975) and Goda (1976) suggested a relationship between groupiness based on continuous envelope theory ($\bar{\mathscr{H}}$) and one based on discrete counting ($\bar{j}$). However, Elgar et al. (1984) noted that this discrete correction scheme contains internal inconsistencies and yields incorrect results for narrow spectra.

An alternative approach to the analysis of wave groups was proposed by Kimura (1980). In this method, the sequence of wave heights is treated as a Markov chain, and the mean group length, $\bar{j}$, is shown to depend on one parameter, the correlation coefficient between successive discrete wave heights, γ_h. This theory has been found by several authors to adequately model the run lengths for a variety of sea states (e.g., Goda, 1983; Battjes and Van Vledder, 1984). A drawback of Kimura's formulation is that the groupiness is not conveniently defined in terms of the wave spectrum. Following Arhan and Ezraty (1978), Battjes and Van Vledder (1984) proposed to replace γ_h by a spectral correlation coefficient, γ_s, derived from Rice's envelope theory. However, there are indications that this new correlation coefficient derived from the spectrum tends to be smaller than the correlation coefficient obtained in the time domain, γ_h, (e.g. Chandler and Masson, 1992). The present uncertainty on the relation between the spectral and time-domain correlation coefficients was identified by the IAHR working group on wave generation and analysis (1992) along with the need for further research.

In view of the uncertainties associated with the known spectral approaches and the need for a practical relationship between groupiness characteristics computed in the time domain and those derived from the wave spectrum, we were motivated to examine more closely the limitations of these methods. Thus, in this study, two spectral methods of wave group analysis are applied to an extensive data set in order to determine the reliability of their predictions. The first part of this paper reviews the two commonly used spectral approaches to wave group analysis mentioned above: the envelope theory of Longuet-Higgins, and the Kimura theory as extended by Battjes and Van Vledder (1984). In the next section, the two methods are reviewed in detail. Group statistics derived from field data and numerically simulated data for a variety of sea states are then presented and compared with the predictions of the two methods. Finally, in Section 4, the limitations of the methods due to the various assumptions are examined more closely, and their influence on the results quantified. It should be noted that, in this work, the group analysis of the wave field is restricted to the application of the linear dynamics and, therefore, does not consider wave group formation as a result of nonlinear modulational instability (see e.g., Yuen and Lake, 1980). However, because of the relative success of the linear methods obtained in the past to model the observed group behaviour, the limited scope of the present study seems justified.

2. REVIEW OF SPECTRAL METHODS

2.1. Envelope theory

The sea surface elevation time history, $\eta(t)$, is treated as a random Gaussian process and can be expressed as the sum of sinusoidal components with angular frequency ω_n,

$$\eta(t) = Re\left(\sum_n c_n e^{i[\omega_n t + \epsilon_n]}\right) \tag{1}$$

where the random phases, ϵ_n, are uniformly distributed over the range $[0,2\pi]$. The fixed amplitude is determined by the frequency spectrum, $E(\omega)$, as $c_n = \sqrt{2E(\omega)\Delta\omega}$, with $\Delta\omega$ the frequency increment. By choosing a carrier wave frequency, $\bar{\omega}$, as a representative midband frequency, $\eta(t)$ may be rewritten as

$$\eta(t) = Re\left(e^{i\bar{\omega}t}\sum_n c_n e^{i[(\omega_n - \bar{\omega})t + \epsilon_n]}\right) \tag{2}$$

$$= Re(R(t)e^{i\bar{\omega}t}) \tag{3}$$

A complex wave envelope function can now be defined as

$$R(t)=\sum_{n} c_n \mathrm{e}^{i[(\omega_n-\bar{\omega})t+\epsilon_n]} \equiv a(t)\mathrm{e}^{i\phi(t)} \tag{4}$$

where $a(t)$ is the amplitude of the envelope function and $\phi(t)$ its phase. For narrow band spectra, the variation of a with time is slow compared with the carrier wave $\mathrm{e}^{i\bar{\omega}t}$, and the wave crests (troughs) closely follow the envelope function. Also, the probability density function (pdf) of the surface displacement, $\eta(t)$, and of its derivative is known to be Gaussian (e.g., Longuet-Higgins, 1957). For the wave envelope $a(t)$ of narrow spectra and its time derivative, the pdf assumes a Rayleigh and Gaussian distribution, respectively. Given those distributions, the average number of waves in a group of the envelope function, $\bar{\mathscr{H}}$, can be derived by dividing the average length of episodes for which the wave envelope exceeds the critical level, $H_c/2$, by the mean zero-upcrossing period. Longuet-Higgins (1984) conveniently expresses $\bar{\mathscr{H}}$ in terms of one single spectral parameter, the spectral width, ν,

$$\bar{\mathscr{H}}=\sqrt{\frac{2m_0\sqrt{1+\nu^2}}{\pi \quad \nu}}\,\frac{1}{H_c} \tag{5}$$

The spectral width parameter, ν, is defined as

$$\nu=\sqrt{\frac{m_2 m_0}{m_1^2}-1} \tag{6}$$

with the spectral moment $m_r=\int_0^\infty \omega^r E(\omega)\mathrm{d}\omega$. The results of this theory are asymptotically valid for narrow spectra for which the spectral bandwidth $\nu^2 \ll 1$. This method is then applicable to sufficiently narrow band processes, or to data that have been adequately filtered. Longuet-Higgins (1984) found that, for typical records of wind waves, a band-pass filter with upper and lower cut-offs at 1.5 and 0.5 times the peak frequency is the most suitable.

It is important to note that the mean group length derived using the envelope theory, $\bar{\mathscr{H}}$, is defined differently than the commonly used parameter, $\bar{j}$, obtained from the series of discrete wave heights. For example, the envelope method can identify a short episode during which the envelope exceeds the critical level but for which no discrete waves exceed the threshold (see Fig. 6a below). In other words, not all of the small $\mathscr{H}$ groups correspond to a j group, resulting generally in a smaller $\bar{\mathscr{H}}$ than $\bar{j}$ value. In fact, for relatively wide band processes or for high values of the critical levels (i.e. small $\bar{\mathscr{H}}$) the number of groups identified by the envelope method may be much larger than the one defined by the discrete method. Accordingly, the estimate of the mean group length, $\bar{\mathscr{H}}$, must be modified to account for this difference in order for the envelope theory to adequately model $\bar{j}$ in the case of realistic sea states with finite spectral bandwidth.

Vanmarcke (1975) and Goda (1976) derived an expression for the discrete parameter $\bar{j}$ in terms of the probability density function of the variable $\mathscr{H}$ derived from the wave envelope, $p(\mathscr{H})$. First, it is assumed that an envelope group with $i<\mathscr{H}<(i+1)$, where $i=0,1,2,3...$, corresponds to a run length j of i or $i+1$ discrete waves. Also, the probability that this $\mathscr{H}$ group be associated with a group of either i or $i+1$ discrete waves is assumed to be $(i+1-\mathscr{H})$ and $(\mathscr{H}-i)$, respectively. Note that for the case $i=0$, each $\mathscr{H}$ group is associated with either no j group ($j=0$) or with a j group of one wave ($j=1$). The mean number of waves per group can then be estimated as

$$\bar{j}=\frac{\sum_{i=1}^{\infty}(\alpha_{i1}+\alpha_{i2})i}{\sum_{i=1}^{\infty}(\alpha_{i1}+\alpha_{i2})} \tag{7}$$

with

$$\alpha_{i1}=\int_{i-1}^{i}(\mathscr{H}-i+1)p(\mathscr{H})\mathrm{d}(\mathscr{H})$$

and

$$\alpha_{i2}=\int_{i}^{i+1}(i+1-\mathscr{H})p(\mathscr{H})\mathrm{d}(\mathscr{H})$$

In Eq. (7), the summation over i does not include the $i=0$ case as, by definition, $j\geqslant 1$. There is difficulty in evaluating Eq. (7) because the function $p(\mathscr{H})$ is not easily obtained from the envelope theory. However, for narrow spectra, the probability that a group be larger than $\mathscr{H}$, $P(\mathscr{H})$, can be approximated by a Poisson distribution (Nolte and Hsu, 1972) with a resulting pdf of the form:

$$p(\mathscr{H})=\frac{1}{\bar{\mathscr{H}}}\mathrm{e}^{-\mathscr{H}/\bar{\mathscr{H}}} \tag{8}$$

The above distribution is obtained by assuming that successive upcrossings of the critical level by the envelope are uncorrelated. This assumption will hold whenever the time interval between the successive upcrossings is large relative to the mean period, $2\pi/\bar{\omega}$. Substituting Eq. (8) into Eq. (7) leads to

$$\bar{j}=\left(\frac{1}{1-\mathrm{e}^{-1/\bar{\mathscr{H}}}}\right) \tag{9}$$

At the limit $\bar{\mathscr{H}}\rightarrow\infty$, this expression reduces to

$$\bar{j} \approx \bar{\mathscr{H}} + 0.5 \tag{10}$$

which is the relation proposed by Longuet-Higgins (1984) for the discrete counting correction. However, as previously noted by Elgar et al. (1984), for $\bar{\mathscr{H}} \to \infty$, both discrete wave parameter and envelope parameter should be identical ($\bar{\mathscr{H}} \to \bar{j}$), in contrast with Eq. (10). The discrete counting correction will be critically examined in the following sections.

2.2. Modified Kimura theory

Kimura (1980) proposed to analyse wave groups by treating the sequence of wave heights as a Markov chain, and allowing for non-zero correlation between successive waves. The joint probability density function of successive wave heights H_1 and H_2 is taken by Kimura as a bivariate Rayleigh distribution:

$$p(H_1, H_2) = \frac{\pi^2}{4} \frac{H_1 H_2}{H_m^4 (1-\kappa^2)} \exp\left(-\frac{\pi}{4} \frac{H_1^2 + H_2^2}{H_m^2} \frac{1}{(1-\kappa^2)} \right) I_0\left(\frac{\pi}{2} \frac{\kappa}{(1-\kappa^2)} \frac{H_1 H_2}{H_m^2} \right) \tag{11}$$

where κ is a correlation parameter, H_m the mean wave height, and I_0 the modified Bessel function of zeroth order.

The correlation parameter used by Kimura, κ, is a function of the correlation coefficient between successive wave heights, γ_h:

$$\gamma_h = \frac{1}{\sigma^2(H)} \frac{1}{N-1} \sum_{i=1}^{N-1} (H_i - H_m)(H_{i+1} - H_m) \tag{12}$$

with $\sigma(H)$ the standard deviation of a large number, N, of wave heights, H_i. The coefficient, γ_h, and the correlation parameter, κ, are related through (Uhlenbeck, 1943)

$$\gamma_h = \frac{\boldsymbol{E}(\kappa) - \frac{1}{2}(1-\kappa^2)\boldsymbol{K}(\kappa) - \frac{\pi}{4}}{1 - \frac{\pi}{4}} \tag{13}$$

where $\boldsymbol{K}$ and $\boldsymbol{E}$ are complete elliptic integrals of the first and second kind, respectively.

To compute the probability of a sequence of high waves, Kimura used the conditional probability that a wave height exceeds the threshold value, H_c, given that the previous wave also exceeds H_c. This conditional probability,

$p_{22} \equiv Prob[H_{i+1} \geqslant H_c | H_i \geqslant H_c]$, is computed from the joint pdf $p(H_1,H_2)$ of Eq. (11):

$$p_{22} = \frac{\int_{H_c}^{\infty}\int_{H_c}^{\infty} p(H_1,H_2)\mathrm{d}H_1\mathrm{d}H_2}{\int_{H_c}^{\infty}\int_{0}^{\infty} p(H_1,H_2)\mathrm{d}H_1\mathrm{d}H_2} \tag{14}$$

The probability that a group is comprised of j waves can then be written as

$$p(j) = (1-p_{22})p_{22}^{(j-1)} \tag{15}$$

giving an average group length

$$\bar{j} = \frac{1}{1-p_{22}} \tag{16}$$

Through Eqs. (11)–(16), the present theory allows the mean number of waves per group, $\bar{j}$, to be estimated from one parameter, the correlation coefficient between consecutive wave heights, γ_h. Goda (1983), among others, found γ_h to control fairly well the mean group length from an analysis of long-travelled swell.

A drawback of Kimura's formulation is that the groupiness is not conveniently defined in terms of the energy spectrum but rather depends on a parameter, γ_h, computed from the series of discrete wave heights. To remedy this problem, Kimura (1980) initially suggested that, since γ_h can be derived from the wave peakedness parameter Q_p, statistical properties of wave groups can be estimated from the latter. However, Q_p has been shown to be clearly inadequate as a definite group parameter (e.g., Elgar et al., 1984). Later, Battjes and Van Vledder (1984) proposed a modification to the theory in which a new spectral wave groupiness parameter was introduced. On the basis of the work of Arhan and Ezraty (1978), they replaced the correlation parameter, κ, in Eq. (11) by a new correlation parameter, κ_s, determined by the frequency spectrum, $E(\omega)$;

$$\kappa_s = \frac{\sqrt{X^2+Y^2}}{m_0} \tag{17}$$

where

$$X = \int_0^{\infty} E(\omega)\cos(\omega T_m)\mathrm{d}\omega \tag{18a}$$

$$Y=\int_0^{\infty} E(\omega)\sin(\omega T_m)\mathrm{d}\omega \tag{18b}$$

and $T_m = 2\pi\sqrt{m_0/m_2}$ is the average period between zero-upcrossings. Using this spectral parameter, a new correlation coefficient, γ_s, analogous to γ_h, can be computed from Eq. (13). The parameter γ_s is in fact the correlation coefficient between points of the envelope wave function, $a(t)$, separated by a constant time interval equal to T_m. In this approach, the correlation coefficient between discrete waves, is replaced by the correlation coefficient between points of the wave envelope, and thus allows the wave groupiness to be determined by a single spectral parameter, γ_s. However, the parameter γ_s will be equal to γ_h given that the amplitude of the discrete waves follow the envelope function, the waves are separated by a time interval approximately constant and equal to T_m, and finally that the wave heights can be approximated by twice the value of the envelope function. These conditions will be strictly satisfied in the limit of an infinitely narrow spectra, and the non zero spectral bandwith of ocean wave spectra will make the two correlation coefficients to be different as demonstrated below.

In the next section, the various groupiness characteristics predicted by the two spectral models described above will be compared with both field data and numerically simulated data.

3. DATA VERSUS PREDICTIONS OF THE MODELS

3.1. Field data

A subset of the field data collected during the Canadian Atlantic Storms Program (CASP) in 1986 with Datawell Waverider and Wavec buoys is used in this study. The data set was collected off the coast of Nova Scotia, in water depths of 20 to 100 m (see Dobson et al., 1989 for more details). In linear theory, it is recognized that the concept of a wave group implies that most of the energy is associated with wave components of frequencies close to the peak frequency. Thus, any spectrum that has its energy distributed in two or more modes of comparable energy and widely separated in frequency is not suitable for group analysis based on linear wave dynamics. Also, it is known from both the linear theory and published data sets that such bimodal spectra with large spectral bandwidth do not exhibit any significant groupiness properties. Accordingly, we selected time series most likely to be associated with unimodal spectra only, by requiring that no more than 20% of the energy was contained in the frequency range above 1.5 times the peak frequency. It should be noted that, for the broad Pierson-Moskowitz spectrum, approximately 20% of the wave energy is contained above this cut-off frequency. Given that this spectral shape represents the limit of full wave development where the spectrum reaches a maximum spectral bandwidth, the chosen criterion ensures that most typical unimodal spectra contained in the data set are included in the analysis.

As the CASP data set consists, in large part, of bimodal spectra from the typical Atlantic conditions of a local wind sea developing on an underlying swell, the above criterion was a serious limitation to the selection of wave spectra used in our group analysis, and only about 20% of the spectra examined satisfied the selection criterion. Also, due to the particular wave climatology of the region studied, the data set does not contain any long travelled swell with very narrow spectra which have however been found in other regions (e.g. Goda, 1983). In other words, these local wave conditions are certainly not typical of all the world oceans, and the 20% success rate for the given selection criteria should by no means be considered universal. A total of 324 time series were processed and used in the group analysis described below.

A spectral analysis of the original (unfiltered) 30 minute records of surface elevation sampled at 1.28 Hz provided spectral estimates with a resolution of 0.005 Hz and 18 degrees of freedom. Each selected time series was first detrended. Then, time series containing spurious spikes (values greater than 6 times the standard deviation of the time series) were rejected. In addition, following the recommendations of Longuet-Higgins (1984), the time series were lowpass filtered with a filter cutoff frequency of 1.5 times the peak frequency prior to the group analysis by the envelope theory, but not for the analysis based on the modified Kimura approach. Also, to ensure consistency between time and spectral domains, the power spectrum was recomputed from the lowpass filtered time series. The series of discrete wave heights was then generated using the standard zero-upcrossing technique, and the wave envelope computed using the Hilbert transform.

3.2. Numerical simulations

To provide us with an additional source of data, time series of sea surface elevation were also generated numerically. The random waves were simulated using the random coefficient method in which the signal $\eta(t)$ consists of N values sampled at discrete times $t_{\rm m}$ with intervals Δt, such that

$$\eta(t_{\rm m})=\sum_{n=0}^{N/2}[a_n\cos(\omega_n t_{\rm m})+b_n\sin(\omega_n t_{\rm m})] \tag{19}$$

where $\omega_n=2\pi n/N\Delta t$. The random coefficients a_n and b_n are generated from a Gaussian distribution with variance $E(\omega_n)\Delta\omega$. This method is preferred here to the commonly used random phase method as the latter was shown to be adequate only for sufficiently large values of N (Tucker et al., 1984). Two target wave spectra, $E(\omega)$, were chosen. One is the empirical JONSWAP spectrum characteristic of growing seas, and the other a typical narrow ocean swell distribution. All spectra used in the method based on the envelope theory were truncated at $1.5f_p$. The JONSWAP spectrum has the form

$$E(\omega)=\frac{\alpha g^2}{\omega^5}\exp\left[-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)4^{-4}\right]\gamma^{\exp[-(\omega-\omega_p)^2/\omega_p^2 2\sigma^2]} \tag{20}$$

where

$$\sigma = \begin{matrix} 0.07 \text{ for } \omega\leqslant\omega_p \\ 0.09 \text{ for } \omega>\omega_p \end{matrix}$$

ω_p is the peak frequency, the Phillips constant $\alpha=0.01$, and the peak enhancement factor $\gamma=3.3$. The expression for the swell spectrum was taken from Longuet-Higgins (1984) as

$$E(\omega)=\alpha_s\omega^{-0.5}e^{-(n/2)[\beta\omega+(\beta\omega)^{-1}]} \tag{21}$$

where α_s, β, and n are constants. The spectral width is determined by the variable n through

$$\nu=\frac{(n+2)^{0.5}}{(n+1)}$$

For ocean swells, values of n typically range from 50 to 300 and, here, values $n=50$, 100, and 200 ($\nu=0.14$, 0.1, and 0.07) were used. The time series sampling and duration periods are consistent with those of the field data. For this section, a total of 600 numerically generated time series were examined for their group characteristics, with 150 for each spectral shape. For all spectra, the peak frequency was chosen as $\omega_p= 2\pi\cdot 0.1$ rad/s.

The wave height threshold, H_c, was selected as the mean wave height, $H_m=\sqrt{2\pi m_0}$. This leads to a greater number of wave groups per record than the commonly used significant wave height, which is larger than H_m, and therefore reduces the statistical variability for the estimate of the mean group length. It is important to note that the statistical variability of the mean group length also depends on the duration of the time series used in the analysis.

3.3. Group statistics

The group characteristics of the wave field are first analysed in terms of the wave envelope theory. For each time series, the predicted average number of waves in a group, $\bar{\mathscr{H}}$, was derived from the spectral width of the filtered spectrum according to Eq. (5), with $H_c=H_m$. The results are then compared with another estimate of the mean number of waves in a group, $\langle j_{en}\rangle$, where $\langle\ \rangle$

is used to indicate an ensemble average. This parameter is computed directly from the envelope by averaging the duration of the time intervals for which the amplitude of the envelope function, $a(t)$, exceeds the mean wave amplitude, $H_m/2$, and dividing this value by the mean zero-upcrossing period, T_m. Results are given in Fig. 1 and show a good agreement between the theoretical predictions and the measured wave envelope groupiness (correlation coefficient $r=0.96$). The high correlation between $\bar{\mathscr{H}}$ and $\langle j_{en} \rangle$ is seen to hold over the whole range of values presented ($0.8 \leqslant \bar{\mathscr{H}} \leqslant 5.5$) with, however, increased scatter of the data as $\bar{\mathscr{H}}$ increases because of a reduced number of wave groups detected in each record (larger statistical variability). It should be noted that the field data analysed do not cover as wide a range of $\bar{\mathscr{H}}$ as the numerical data. The spectral shape used for the narrowest swell ($n=200$ in Eq. 21) leads to higher levels of wave groupiness ($\bar{\mathscr{H}}>4$) than the ones observed in the CASP data ($\bar{\mathscr{H}}<3.5$) because of the particular wave conditions of the region mentioned earlier. However, over the range of values for which the two data sets overlap, the data presented in Fig. 1 (as well as in the following figures)

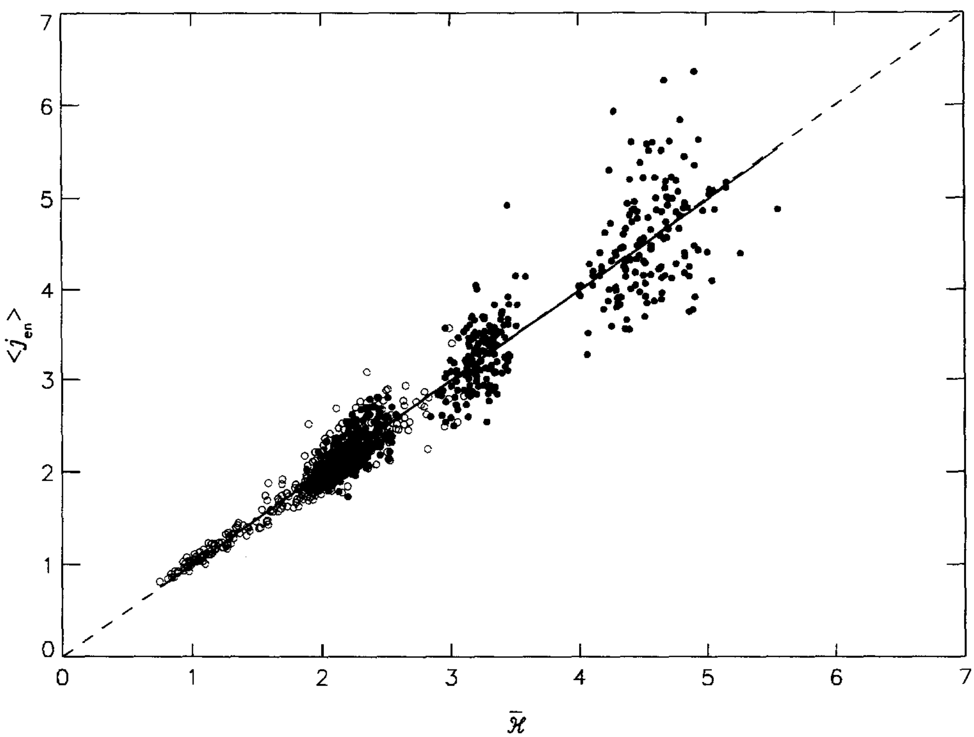


Fig. 1. The average run length derived from the amplitude of the wave envelope $\langle j_{en} \rangle$ as a function of $\bar{\mathscr{H}}$ for field (○) and simulated (●) data. The dashed line gives the equality $\langle j_{en} \rangle = \bar{\mathscr{H}}$ and the full line is the linear least squares fit.

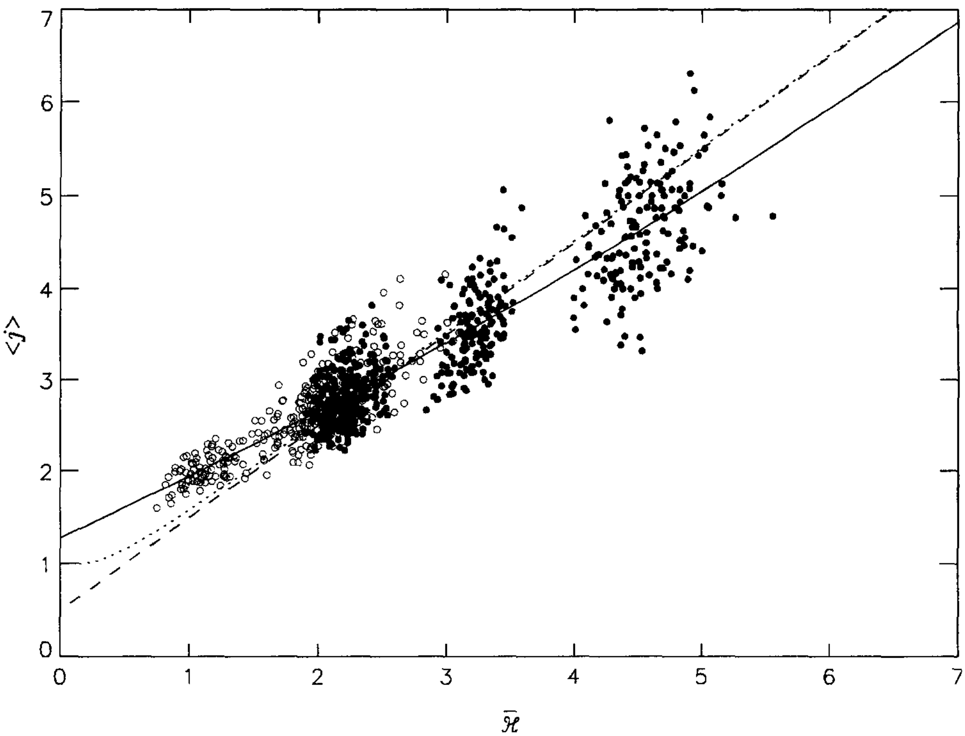
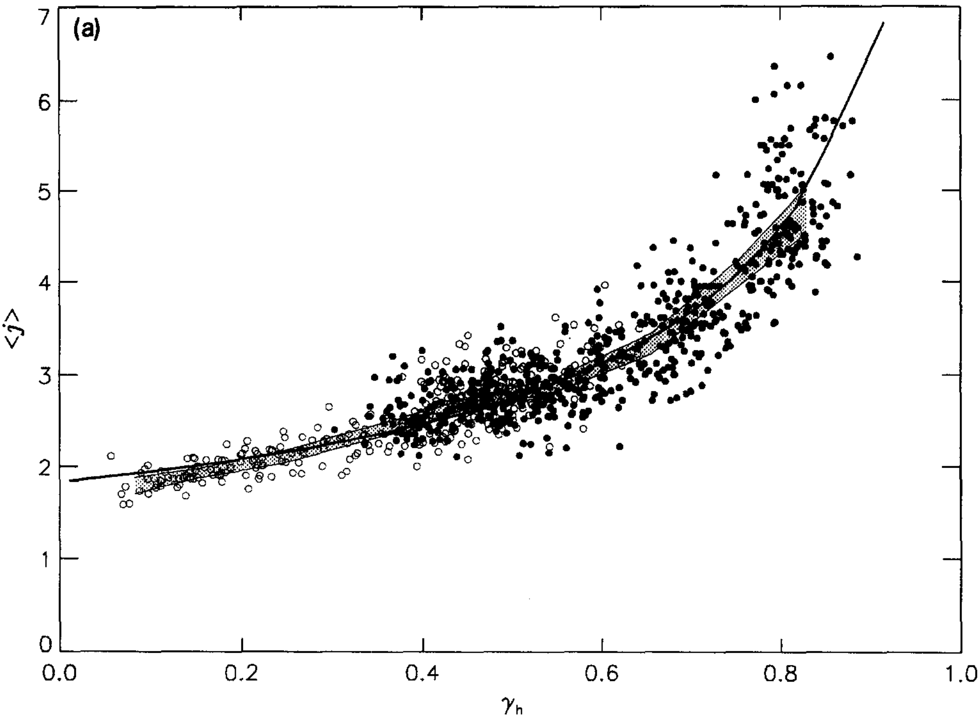


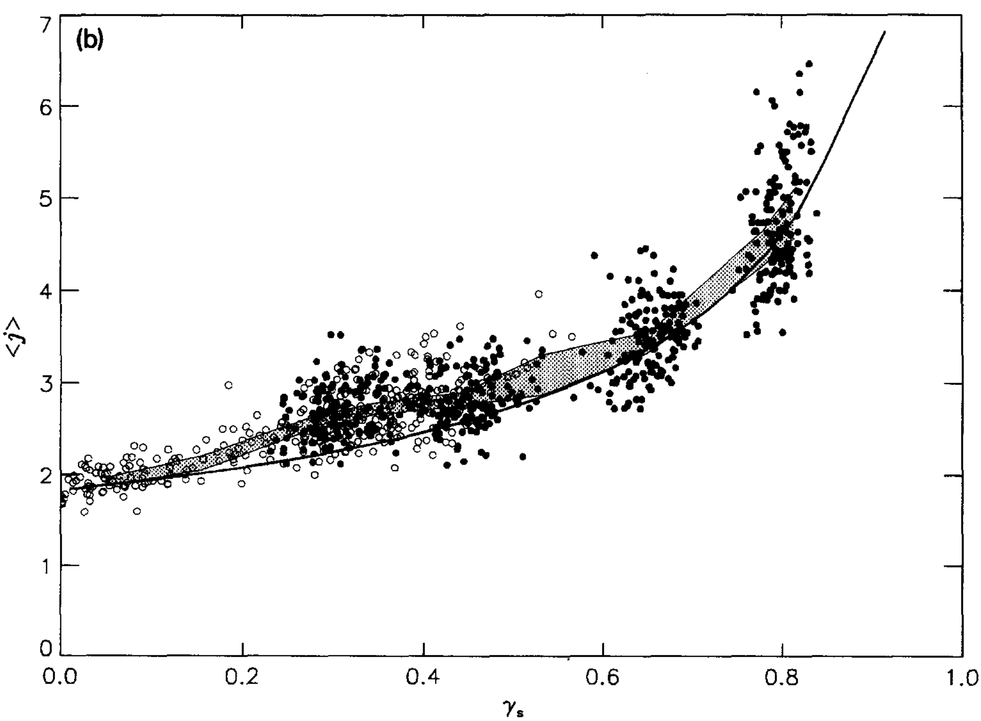
Fig. 2. The measured $\langle j \rangle$ as a function of the average run length derived from Eq. (5), $\bar{\mathcal{H}}$ for field (○) and simulated (●) data. The full line gives the second order least squares fit to the data, the dotted line the relation obtained with the discrete counting correction (Eq. 9), and the dashed line its simplified form (Eq. 10).

do not reveal any significant difference among group statistics extracted from field and simulated data, in agreement with previous works (e.g., Elgar et al., 1984; Goda, 1976). These results support the notion that the analytic linear Gaussian model for the surface elevation can adequately reproduce measured wave group characteristics.

The success of this spectral approach in predicting the mean group length of the envelope, $\langle j_{en} \rangle$, does not guarantee its usefulness in predicting wave group characteristics of the discrete wave heights. In fact, its ability to predict the parameter $\bar{j}$ also depends on the validity of the assumptions involved in the discrete counting correction expressed in Eqs. (7)–(9). In Fig. 2, the av-

Fig. 3. The average group length $\langle j \rangle$ as a function of the correlation coefficient between discrete wave heights γ_h (a) and of the spectral correlation coefficient γ_s (b) for field (○) and simulated (●) data. The solid curve represents Kimura theoretical relationship, and the shaded area covers the area bounded by the 99% confidence intervals.


(a)
<j>
γh
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5
6
7


(b)
<j>
γs
0.0
0.2
0.4
0.6
0.8
1.0
0
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2
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4
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6
7

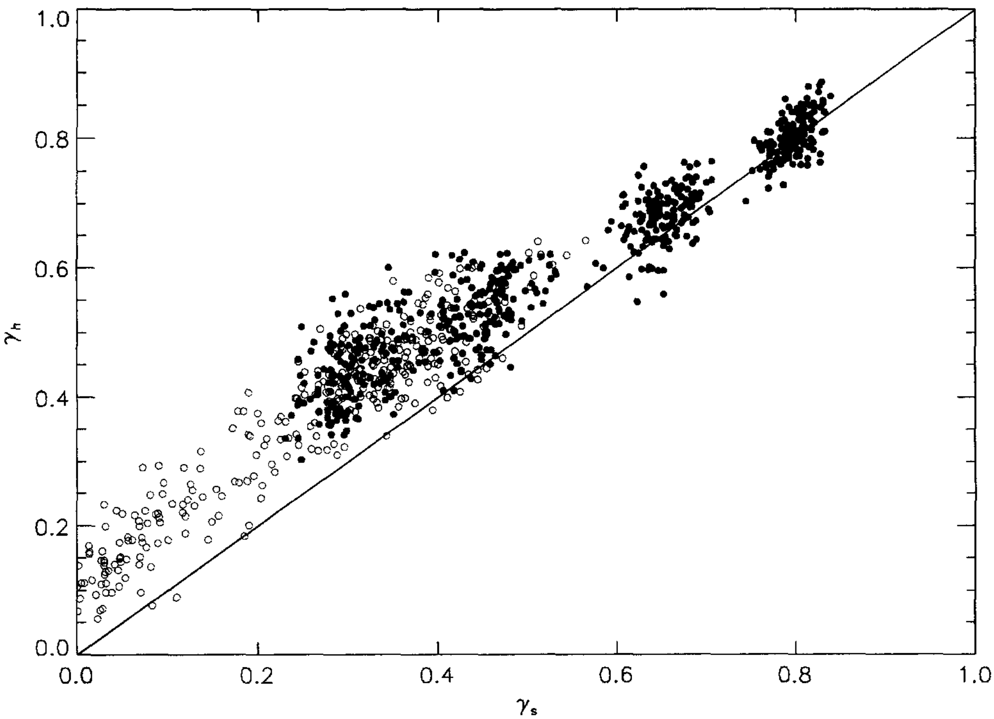

Fig. 4. The correlation coefficient between successive wave heights, γ_h as a function of the spectral parameter, γ_s for field (○) and simulated data (●).

erage number of waves in groups of measured discrete waves, $\langle j \rangle$, is compared with the average number of waves predicted by the model, $\bar{\mathscr{H}}$. Also included in the figure is the second order least squares fit to the data, and the relation between the two variables suggested by the known discrete counting correction, Eq. (9), and its simplified form Eq. (10). Although the agreement between the model and the data appears fairly good in the intermediate range of values, the present models Eq. (9) and Eq. (10) are clearly not supported by the data for both smaller ($\langle j \rangle \leqslant 2.5$) and larger ($\langle j \rangle \geqslant 4$) parameter values. For $\langle j \rangle \geqslant 4$, the present data set confirms the inconsistency of Eq. (9) or Eq. (10) noted in Section 2.1 in the case of narrow spectra for which the equality $\bar{\mathscr{H}} = \bar{j}$ prevails. On the other hand, for time series which are relatively poorly grouped ($\langle j \rangle \leqslant 2.5$), the correction for discrete counting proposed in Eq. (9) does not appear severe enough, with a measured difference between $\langle j \rangle$ and $\bar{\mathscr{H}}$ larger than predicted. These inconsistencies of the present discrete counting correction model is discussed further in the next section, where it is shown that the reasonable agreement obtained at mid range of $\langle j \rangle$ results from a fortuitous cancellation of errors due to using an inexact pdf distribution and neglecting an important *splitting* effect.

In a second attempt to model the groupiness of the present data set, the prediction of the Kimura theory for the mean length of wave groups as given by Eqs. (11)–(16) is examined. In Fig. 3a, the variations of the measured $\langle j \rangle$ with the correlation coefficient between successive wave heights, γ_h, is presented. The data points cluster around the theoretical curve which remains within the 99% confidence intervals measured for every 0.1 interval of γ_h values. Therefore, the Kimura theory models quite successfully the observed dependence of the mean group length on γ_h, as has been consistently found in the past (e.g., Goda, 1983; Thomas et al., 1986).

Battjes and Van Vledder (1984) modified Kimura's theory by relating $\bar{j}$ to a new correlation coefficent, γ_s, conveniently computed directly from the spectrum (see Eqs. 17 and 18), and presumably equivalent to γ_h. However, the modified approach is found to consistently underpredict the measured wave group length, $\bar{j}$, for all but very wide or very narrow spectral shapes (Fig. 3b), with the theoretical curve now falling below the 99% confidence intervals for all but very low or very high γ_s values. This is due to the fact that the proposed spectral correlation coefficient, γ_s, is consistently smaller than the measured coefficient, γ_h, extracted from the time series of discrete wave heights (Fig. 4). Only for very high values of correlation obtained from numerical simulations of very narrow spectra are the two correlation coefficients equal. Thus, despite the success of the Kimura theory in predicting $\bar{j}$ in terms of γ_h, its practical usefulness is limited by the lack of a valid spectral analog to γ_h. In the following section, the difference between the two correlation parameters γ_h and γ_s is examined in more detail.

4. LIMITATIONS OF THE METHODS

4.1. Envelope theory

When comparing the measured values of $\bar{j}$ with the predictions of the envelope theory combined with the discrete counting scheme, some divergence was noted. To identify a possible cause for the inability of this approach to fully model the present data set, the choice of a Poisson model for the $\mathscr{H}$ probability density distribution, resulting in the pdf (Eq. 8), is first examined. Secondly, the *splitting* of wave groups by small excursion of the envelope function below the threshold level is identified as important in the relation between the envelope and the discrete wave measure of the mean group length.

In order to obtain reliable statistical information, 600 time series were numerically generated with a truncated JONSWAP target spectrum, producing a total of 18388 envelope wave groups (j_{en}). This data set was then used to compute the probability density function $p(\mathscr{H}) \equiv p(j_{en})$. In Fig. 5, the mea-

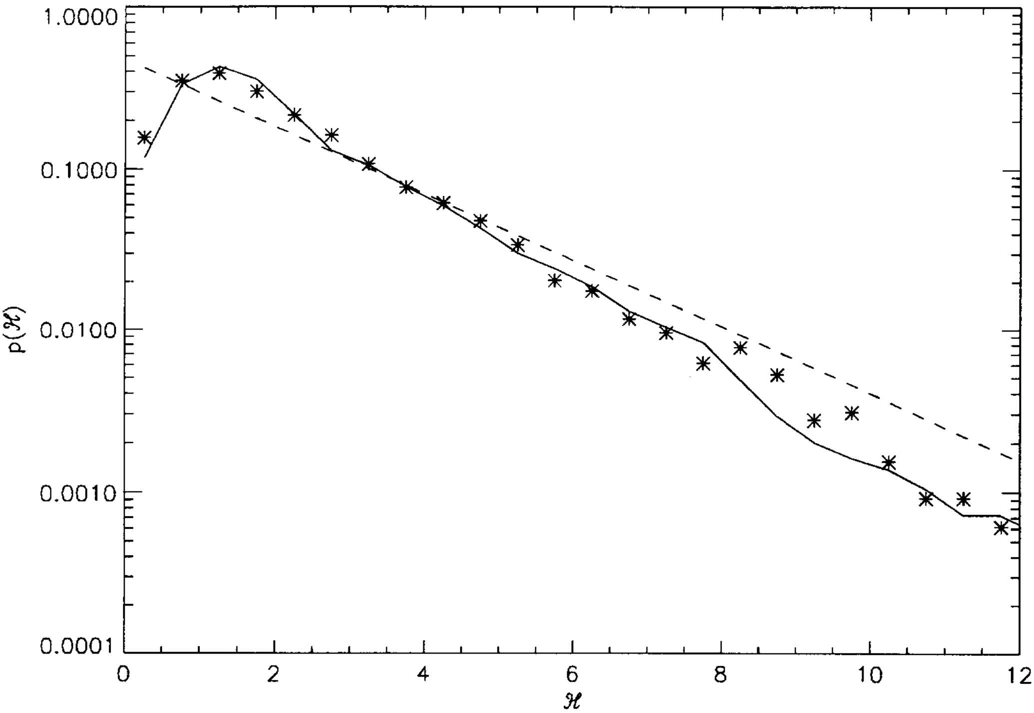

Fig. 5. The probability density function of the envelope group length, $p(\mathscr{H})$, for field (*) and simulated (——) data. The dashed line is the Poisson model distribution.

sured distribution is compared with the Poisson model (Eq. 8) for which the variable $\bar{\mathscr{H}}$ is taken as the ensemble average $\langle j_{en} \rangle = 2.09 \pm 0.03$. Here and for all the ensemble averages to follow, the 99% confidence interval is given as a measure of the statistical variability. This average envelope wave group is a very good estimate of the spectral parameter $\bar{\mathscr{H}} = 2.10$ computed as in Eq. (5) using the measured spectral width, $\nu = 0.15$, of the target spectrum. Also included in Fig. 5 is the pdf computed from 155 of the field time series for which $\bar{\mathscr{H}}$ is within the range of the values obtained with the numerical JONSWAP simulations ($1.9 \leqslant \bar{\mathscr{H}} \leqslant 2.5$). Again, the field data and the numerical data are found to have similar group characteristics, but the measured pdf is significantly different from the assumed Poisson model distribution. In particular, the data indicate a decrease in the probability density function as $\mathscr{H} \to 0$ in contrast with the assumed distribution which reaches a maximum at the small $\mathscr{H}$ limit. This deviation of the measured distribution from the Poisson model at small $\mathscr{H}$ value is not surprising given that this model's distribution is obtained by assuming that successive upcrossings are widely separated in time, which is not the case for small $\mathscr{H}$ values.

Given that the measured pdf is known to be different from the assumed

TABLE 1

Group parameters for a truncated JONSWAP target spectrum

Parameter	value
$\langle j \rangle$	2.72
$\langle j_{en} \rangle$	2.09
$\mathscr{H}$ as in Eq. (5)	2.10
$\bar{j}$ predicted by Eq. (7) (with Poisson $p(\mathscr{H})$)	2.63
$\bar{j}$ predicted by Eq. (7) (with measured $p(\mathscr{H})$)	2.29
$\langle j'_{en} \rangle$	2.47

distribution, it is interesting to examine the effect of using the measured $p(\mathscr{H})$ on the model's prediction of $\bar{j}$ for the JONSWAP target spectrum. The ensemble average of all the groups identified by the envelope and by the discrete wave method are $\langle j_{en} \rangle = 2.09 \pm 0.03$ and $\langle j \rangle = 2.72 \pm 0.04$, respectively (see Table 1). An estimate of $\bar{j}$ was computed as in Eq. (7), with both the Poisson model ($\bar{j} = 2.63$) and the measured pdf ($\bar{j} = 2.29$). Rather than improving the estimate, the use of the measured $p(\mathscr{H})$ function increases the difference between the measured and the estimated $\bar{j}$ relative to the results of the Poisson model which, in fact, are already too low. According to the discrete counting scheme, the envelope estimate is smaller than the discrete wave estimate due to all the small groups identified by the wave envelope function which do not correspond to groups of discrete waves. It is therefore understandable that the correction obtained with the measured distribution is less severe than the one obtained with the Poisson model which predicts relatively more small $\mathscr{H}$ groups.

The deterioration of the $\bar{j}$ estimate obtained with an improved pdf brought us to examine more closely the first assumption used in the discrete counting correction which leads to Eq. (7). In this scheme, it is assumed that every group identified by the envelope with $i < \mathscr{H} < (i+1)$ corresponds to a group of either i or $i+1$ discrete waves, with $i = 0,1,2,3$. For the case $i = 0$, a fraction of the small $\mathscr{H}$ groups will not be associated with any group in the wave height time series ($j = 0$). This occurs when the envelope stays over the critical level for a short time during which no wave height is identified by the zero-upcrossing method (Fig. 6a). According to the discrete counting scheme, the difference between the number of $\mathscr{H}$ groups and j groups in a time series should be accounted for by this phenomenon. It is possible to estimate the fraction, δ, of $\mathscr{H}$ groups which are not likely to be associated with any j group. If we assume that the case $j = 0$ occurs only for small groups with $\mathscr{H} \leqslant 1$, and that the probability of having a $j = 1$ group is proportional to $\mathscr{H}$, we can write

$$\delta=\int_{0}^{1}(1-\mathscr{H})p(\mathscr{H})\mathrm{d}\mathscr{H} \tag{22}$$

Using the measured pdf of Fig. 5 leads to $\delta=0.08$. Thus, the discrete counting

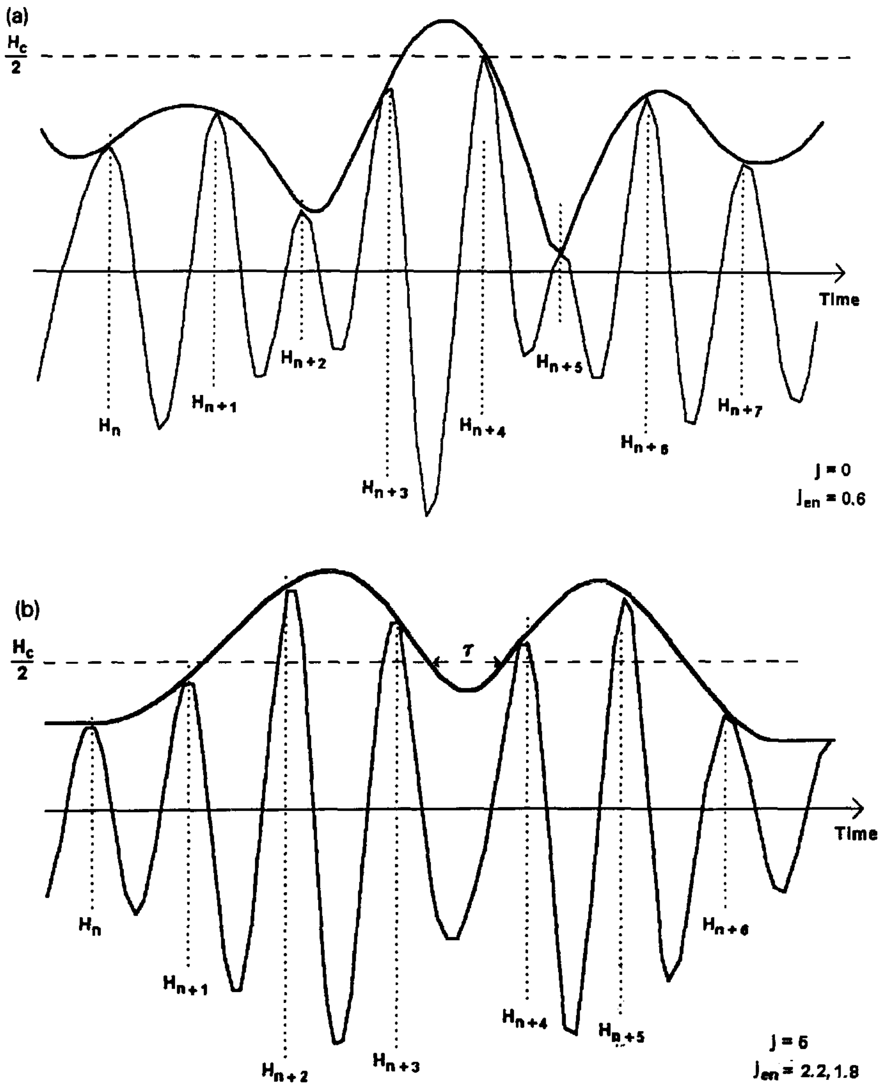

Fig. 6. Time series of water surface elevation with (a) a case $j=0$, and (b) a *split* j_{en}. The full line gives the surface elevation $\eta(t)$, the thick line the envelope function $a(t)$, and the dotted lines the discrete waves.

scheme predicts that, for the truncated JONSWAP spectrum used here, there should be 8% more $\mathscr{H}$ groups than j groups. On the other hand, in the selected (the 155 JONSWAP-like spectra) field and the numerical data there are 25% and 26%, respectively, more $\mathscr{H}$ groups than j groups.

This result clearly shows that some phenomenon not accounted for by the present discrete counting correction scheme significantly affects the $\bar{\mathscr{H}}$–$\bar{j}$ relationship. In fact, a close inspection of the data revealed a new aspect of discrete counting which was generally found in all of the time series: the *splitting* of a large $\mathscr{H}$ group by a brief excursion of the envelope below the critical level during which the series of wave heights remain above the threshold level (Fig. 6b). By dividing large $\mathscr{H}$ groups into smaller ones, the splitting effect contributes to make more $\mathscr{H}$ groups than j groups, and consequently to increase the difference between the $\bar{\mathscr{H}}$ and $\bar{j}$ values.

To understand this behaviour better, a new parameter, τ, is defined as the time interval between successive high runs, normalized by the mean period. The mean length of such an interval, $\bar{\tau}$, is simply the difference between the mean length of a total run (interval between two successive upcrossings), $\bar{G}$, and of a high run, $\bar{\mathscr{H}}$. Using Longuet-Higgins' (1984) expression for the mean length of a total run, which can be written as $\bar{G}=\bar{\mathscr{H}}\exp(H_c^2/8m_0)$, we have

$$\bar{\tau}=\bar{G}-\bar{\mathscr{H}}=\bar{\mathscr{H}}(e^{H_c^2/8m_0}-1) \tag{23}$$

For the truncated JONSWAP spectrum used here, $\bar{\tau}$ takes the value 2.5. In analogy with the δ of Eq. (22), the fraction, ϵ, of the intervals τ causing an $\mathscr{H}$ group to be divided within a j group can be estimated by

$$\epsilon=\int_0^1(1-\tau)p(\tau)\mathrm{d}\tau \tag{24}$$

A pdf for the variable τ was also computed from the 600 simulated time series, and used in Eq. (24) to estimate the fraction $\epsilon=0.07$. Note that the given values of δ and ϵ are rather crude estimates, being quite sensitive to the value of the upper limit of the integral. For example, by allowing intervals with $1\leqslant\tau\leqslant 1.25$ to have a non-zero probability to cause a *split*, the value of ϵ increases to 0.12. However, what is more significant here, is that the two ratios are approximately equal, indicating that this new effect occurs as often as the one shown in Fig. 6a, and that its neglect in the present discrete counting scheme is a serious source of error in the predicted $\bar{\mathscr{H}}$–$\bar{j}$ relationship.

To evaluate the magnitude of the *splitting* effect on the mean group length, a new parameter, $\langle j'_{en}\rangle$, was extracted from the time series. This new variable represents the average length of the groups identified by the wave envelope function modified to prevent any division of a group by a small interval

τ during which the wave height time series remained above the critical level. For the truncated JONSWAP spectrum, we obtain $\langle j'_{en} \rangle = 2.47 \pm 0.04$. A comparison with the previous results of Table 1, reveals that the splitting effect accounts for more than half of the difference between the envelope, $\langle j_{en} \rangle$, and the discrete wave, $\langle j \rangle$, measure of the mean group length.

Given that both assumptions Eq. (7) and Eq. (8) used to construct the discrete counting correction scheme Eq. (9) have been shown to be inadequate, the reasonable agreement found between mid range values of predicted $\bar{j}$ and measured $\langle j \rangle$ of Fig. 2 may be surprising. In fact, the apparent success of the model for median values of the mean group length simply results from a fortuitous combination of overpredicting the effect of the $j=0$ cases and ignoring the important *split* events. Unfortunately, the *splitting* effect is not suitable for a simple parameterization similar to Eq. (7), and the need for a solid $\bar{\mathscr{H}}$–$\bar{j}$ relationship, which would not overlook the *splitting* effect, remains, and will be explored in future work.

4.2. Modified Kimura theory

Battjes and Van Vledder (1984) proposed a spectral version of the Kimura theory in which the correlation coefficient between discrete waves, γ_h, is replaced by the spectral parameter, γ_s. However, for most of the time series analysed, the latter is found to be consistently smaller than the coefficient derived from the series of discrete waves (see Fig. 4). The spectral correlation coefficient, γ_s, is the value of the autocorrelation of the wave envelope, $a(t)$, for points separated by a time interval T_m, and can be written as

$$\gamma_s = \frac{1}{\sigma^2(a)} \int_0^{\infty}\int_0^{\infty} (a_1 - \bar{a})(a_2 - \bar{a}) p(a_1, a_2) \mathrm{d}a_1 \mathrm{d}a_2 \tag{25}$$

where $a_1 = a(t)$, $a_2 = a(t + T_m)$, and $\sigma^2(a)$ and $\bar{a}$ are the variance and the mean value the envelope function, respectively. The joint probability density, $p(a_1, a_2)$, has the bivariate Rayleigh distribution of Eq. (11) with the correlation parameter $\kappa = \kappa_s$. The coefficient γ_s will be a good estimate of γ_h assuming that: (1) the amplitudes of the discrete waves follow the envelope function, (2) the separation between zero-upcrossings is approximately constant and equal to T_m, and (3) the wave height $H(t)$ can be approximated by $2a(t)$. These three assumptions are rigorously valid for infinitely narrow spectra only, in which case one has the identity $\gamma_h = \gamma_s$. However, for most ocean spectra the finite spectral bandwidth causes the spectral estimate of the correlation coefficient to be lower than the discrete wave estimate. The contribution of each of the three assumptions to making γ_s smaller than γ_h is examined below.

In order to quantify the effect of each of the three above assumptions on the observed difference between γ_s and γ_h, we used the results derived from

300 JONSWAP numerical time series along with 163 JONSWAP-like time series of the field data set for which $0.2 \leqslant \gamma_s \leqslant 0.4$. The average value of the correlation coefficients are $\gamma_h = 0.43 \pm 0.007$ for the series of discrete waves, and $\gamma_s = 0.3 \pm 0.005$ for the spectral value (through Eqs. 17 and 13). The small 99% confidence intervals of both values clearly indicates that the two correlation coefficients are significantly different. In this case, with the Kimura theory (Eqs. 11–16), the use of the smaller spectral correlation coefficient systematically underestimates the mean group length, $\bar{j}$, by 12% compared to one obtained with γ_h.

The effect of the assumption (1) is analysed by computing the correlation coefficient of the wave envelope with a version of Eq. (12) in which each wave height is now the sum of the value of the envelope function at the time of the crest and the trough, $H_i = a(t_{\mathrm{crest},i}) + a(t_{\mathrm{trough},i})$. Throughout the analysis the envelope function was computed using the Hilbert transform. The resulting correlation coefficient is now slightly larger (0.45) than the one obtained with the actual wave heights, γ_h. Thus, the low bias of γ_s relative to the measured γ_h does not appear to be due to the fact that the wave amplitudes do not follow exactly the envelope function.

Regarding the assumption (2), we first present the work of Arhan and Ezraty (1978) who examined the effect of the scattering of periods around their mean value, T_m, on the estimate of the joint probability density function, $p(H_1,H_2)$. In the modified Kimura theory, the correlation coefficient between successive wave heights is approximated by γ_s of Eq. (25). This relation uses the joint probability, $p(a_1,a_2)$, for two points of the wave envelope separated by a constant interval, T_m, in place of $p(H_1,H_2)$. A better approximation of the function $p(H_1,H_2)$ is derived by introducing the height–period joint probability density of discrete waves, $p(H,T)$, in the model. The conditional probability density of periods, given the height, H, can be defined as:

$$p(T|H) = \frac{p(H,T)}{\int_0^\infty p(H,T)\,\mathrm{d}T} \tag{26}$$

The joint probability of two successive waves H_1 and H_2 can be estimated assuming that the lag between the crests of the two waves is the period, T, of a wave with height $(H_1+H_2)/2$, giving

$$p(H_1,H_2) \approx \frac{1}{4}\int_0^\infty p(a_1,a_2\,|\,T)\,p\!\left(T\,\Big|\,\frac{H_1+H_2}{2}\right)\mathrm{d}T \tag{27}$$

where the joint probability density function of the wave envelope is written as $p(a_1,a_2\,|\,T)$ to underline the fact that it applies to two points of the envelope separated by a time lag T. This function, $p(a_1,a_2 \mid T)$, is given by a bivariate Rayleigh distribution for which the correlation parameter, κ_s, of Eq. (17) is now computed with a varying time lag T in place of the fixed T_m. Arhan and Ezraty computed this probability density for a JONSWAP spectrum using an analytic height–period joint probability density, $p(H,T)$, and found it very close to a measured distribution.

The results of Arhan and Ezraty are used here to improve the spectral estimate of γ_h by computing a correlation coefficient as in Eq. (25) but with the improved joint probability distribution of Eq. (27). To avoid the uncertainty associated with analytical distributions, a height–period joint probability density, $p(H,T)$, was obtained by aggregating into a joint histogram a total of 96000 discrete waves identified in time series numerically simulated from a JONSWAP target spectrum. Figure 7 presents the resulting joint distribution in 0.1×0.1 dimensionless bins. The general features of this distribution, including its distinct bimodal structure, are similar to the ones observed previously by Sobey (1992), and Su and Bergin (1983). Using the measured $p(H,T)$ function leads to a larger value, $\gamma_s = 0.35$, for the correlation coeffi-

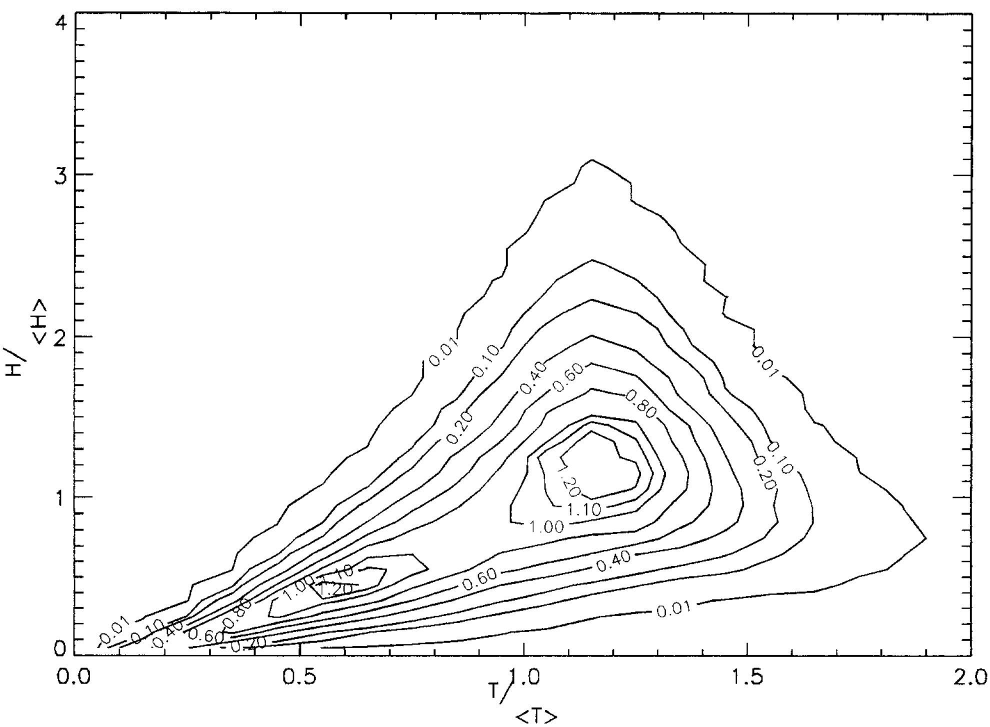

Fig. 7. Joint probability distribution of discrete wave heights and periods, $p(H,T)$, for a truncated JONSWAP target spectrum. The heights and periods are normalized by their average values, $\langle H \rangle$, and $\langle T \rangle$.

cient. Therefore, for the spectral shape studied, the discrepancy between the discrete and the original spectral estimate of the correlation coefficient is significantly reduced from its initial value by allowing the time lag between waves to vary around T_m.

The last assumption (3) used to obtain the approximation $\gamma_h = \gamma_s$ consists in estimating the height of a discrete wave as twice the value of the envelope function at the time of the crest. A better estimate of the discrete wave height, which is defined as the difference between the maximum and the minimum value of the surface elevation within a wave period, would be to use the value of the envelope function at points separated by half the wave period rather than by the whole period only. This improved estimate of H has been used by Tayfun (1990) to obtain the statistical distribution of zero-upcrossing wave heights. A new correlation coefficient can be estimated from the envelope function of a finite time series $a(t)$ of N values sampled at intervals Δt by using an analog to Eq. (12) in which the wave heights are replaced by the appropriate values of the envelope function:

$$\gamma_h \approx \frac{1}{2\sigma^2(a)\left(1+\gamma_s\left(\frac{T_m}{2}\right)\right)} \frac{1}{N-\frac{T_m}{\Delta t}} \sum_{n=1}^{N-(T_m/\Delta t)} \left[a(t_n)+a\left(t_n+\frac{T_m}{2}\right)-2\bar{a}\right]\left[a(t_n+T_m)+a\left(t_n+\frac{3T_m}{2}\right)-2\bar{a}\right] \tag{28}$$

Noting that the value of the autocorrelation function of the wave envelope for a time lag T can be expressed as

$$\gamma_s(T) = \frac{1}{\sigma^2(a)} \frac{1}{N-\frac{T}{\Delta t}} \sum_{n=1}^{N-(T\Delta t)} [a(t_n)-\bar{a}]\,[a(t_n+T)-\bar{a}]$$

Eq. (28) can be rewritten as

$$\gamma_h \approx \frac{1}{2\left(1+\gamma_s\left(\frac{t_m}{2}\right)\right)} \left[2\gamma_s(T_m)+\gamma_s\left(\frac{T_m}{2}\right)+\gamma_s\left(\frac{3T_m}{2}\right)\right] \tag{29}$$

This new estimate of γ_h has been recently presented by Van Vledder (1993). According to Eq. (29), the new spectral estimate of the correlation coefficient between successive waves depends not only on the value of the autocorrelation of the wave envelope at the mean zero-upcrossing period, but also on values of this function at 1/2 and 3/2 of this time lag (Fig. 8). For the JONSWAP spectrum, this new spectral coefficient is significantly increased to a mean value of 0.43 ± 0.005 from the initial $\gamma_s \equiv \gamma_s(T_m) = 0.3$. This improved estimate of the discrete correlation coefficient has a mean value equal to the

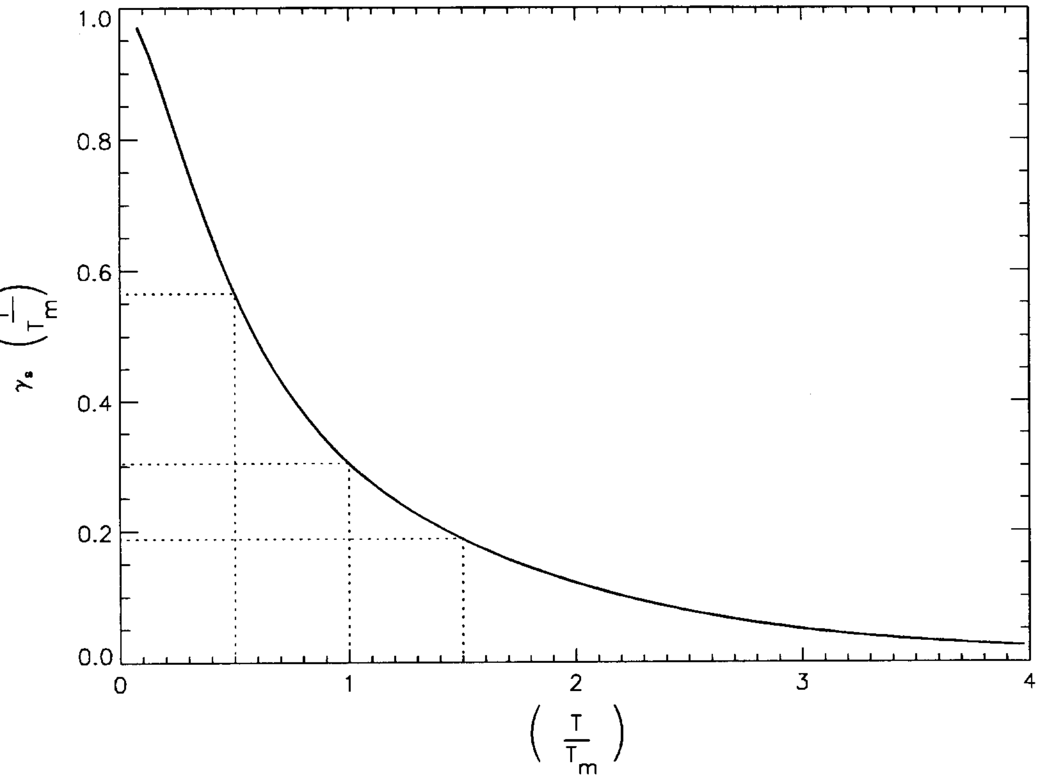

Fig. 8. Autocorrelation function of the wave envelope function for a truncated JONSWAP spectrum.

mean γ_h value. Thus, the third assumption which consists in replacing the wave heights by $2a(t)$ appears to be the main cause for γ_h to be underestimated by the spectral parameter γ_s. It is therefore also responsible for the inadequacy of this spectral approach to model the mean group length $\bar{j}$.

The spectral parameter, γ_s, is shown to be biased low because of the assumption of a fixed time interval and, to a greater extent, by assuming the wave height as twice the crest. It would be interesting to see how much of the difference between γ_s and γ_h could be explained by the combined effect of the three assumptions analysed here. However, the incorporation of the two effects into an improved spectral coefficient is not a trivial task, and is left for further study.

5. CONCLUSIONS

Two different spectral approaches to the analysis of wave grouping have been critically examined. In both cases, the suggested method is aimed at providing a reliable practical relationship between the familiar groupiness parameter, the mean length of discrete wave groups $\bar{j}$, and a spectral parameter.

The envelope theory (Longuet-Higgins, 1984) combined with a discrete counting correction scheme is based on the Gaussian noise theory, and relates $\bar{j}$ to the dimensionless bandwidth parameter, ν. To account for the difference between the group characteristics extracted from the envelope function and the ones derived from the discrete waves, a correction scheme is applied to the mean group length prediction of the envelope theory. This correction, however, is found to be inadequate because (1) it uses an incorrect pdf distribution for the envelope group length, and (2) it neglects an important *splitting* effect by which envelope groups are divided by small excursions of the envelope function below the critical level. However, the errors due to these two deficiencies of the discrete counting scheme cancel at mid range of $\bar{j}$, and lead to a reasonable agreement between measured and predicted values.

An alternative approach to wave groups analysis which treats the sequence of wave heights as a Markov chain (Kimura, 1980) is found to adequately model $\bar{j}$ in terms of the correlation coefficient between successive wave heights, γ_h. However, the extension of the Kimura theory proposed by Battjes and Van Vledder (1984), in which γ_h is replaced by the spectral correlation coefficient γ_s, is shown to systematically underestimate the mean group length $\bar{j}$ for all but very wide or very narrow spectra (12% for a typical JONSWAP spectrum). This is due to the fact that, for most of the time series examined, γ_s is consistently smaller than its discrete analog γ_h. The discrepancy between the two correlation coefficients is caused in part by the assumed fixed time lag between waves, but more importantly by assuming that the wave heights can be represented by twice the value of the envelope function at the time of the crests. Recently, Liu et al. (1993) examined the performance of this approach in relating wave spectra and groupiness. They also found an underprediction for $\bar{j}$ of roughly 15% when $\kappa \approx 0.65$ ($\gamma \approx 0.4$), leading them to conclude that the spectral correlation coefficient is a simple and unbiased group parameter for all but those relatively rare cases of very narrow spectra. However, this conclusion is clearly misleading given that a typical JONSWAP spectrum has a larger correlation coefficient value, with $\gamma_h = 0.43$.

Given the pressing need, in the fields of coastal engineering and physical oceanography, for a robust parameterization of the wave groupiness in terms of the wave spectrum, the two methods analysed here undoubtedly represent valuable steps towards a satisfactory solution. However, as this study has demonstrated, care must be taken when applying these methods because the problem of relating groupiness characteristics of the wave field derived in the frequency and time domains has not been fully solved yet. Resolution of this difficulty, through either an improved discrete counting correction scheme or a better spectral estimate of γ_h, would lead to a more reliable prediction of the groupiness parameter, $\bar{j}$.

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