22nd IAHR International Symposium on Ice
Singapore, August 11 to 15, 2014

Wave Induced Iceberg Deterioration

Kashafutdinov Marat*, Raed Lubbad, Sveinung Løset
Department of Civil and Transport Engineering, Norwegian University of Science and Technology, Trondheim, Norway
Høgskoleringen 7A,7491 Trondheim, Norway
** marat.kashafutdinov@ntnu.no*

As development of oil and gas fields goes further north, there is need for more precise iceberg drift and iceberg deterioration model. Both iceberg drift model and iceberg deterioration model use wind, waves and currents as input environmental parameters. Current paper considers iceberg deterioration model which is governed by heat equation. Boundary conditions of the heat equation can be divided into three parts: upper part exposed to air, middle part exposed to wave action and lower part exposed to seawater action. Air and seawater actions are described by convective heat transfer from turbulent boundary layer to a surface while wave action is described by heat transfer from oscillating flow to a surface. Here we consider only wave action. Wave theory is used to find velocity of oscillating flow.

Proceedings of the 22nd IAHR International Symposium on Ice

ISBN: 978-981-09-0750-1
doi:10.3850/978-981-09-0750-1_1265

1. Introduction

Each year more and more licenses are given to oil and gas companies to develop offshore fields in high north. Environmental conditions are different up there from what we used to deal with when developing conventional offshore fields. Among those conditions which can restrict operations is iceberg presence. Therefore iceberg drift model is an important part in mitigating risks and defining design criteria.

The iceberg drift model consists of dynamic and deterioration parts. The dynamic part considers various forces acting on the iceberg i.e. drag forces from water and air, Coriolis force, wave radiation force, forces from surrounding sea ice and water pressure gradient force.

The deterioration part considers thermal processes and calving of overhanging slabs which cause the change in iceberg mass and shape. This change, in turn, affects the iceberg dynamics. Therefore the deterioration model is essential part of the drift model.

Job (1978) identified the eight most important mechanisms of iceberg deterioration: (1) forced convection due to difference in velocity of the iceberg and ambient environment both air and water, (2) buoyant convection, (3) wave erosion followed by calving of overhanging slab, (4) subsurface calving due to the upthrust on large underwater shelves formed by other ablation mechanisms, (5) differential melting along cracks or low-density planes in the iceberg, leading to further calving, (6) convection induced by wallowing or overturning, (7) melting due to solar radiation, and (8) failure of iceberg due to internal stresses. In this paper we consider only wave erosion as the main contributor to the iceberg deterioration (El-Tahan et al., 1987).

There are two main approaches to model iceberg deterioration processes. The first one was developed by White et al. (1980). This approach gives theoretical estimation of wave erosion and assumes the iceberg as a vertical wall in presence of incident waves. Another approach was developed by Bigg et al. (1997). Bigg et al. (1997) used sea state observations to describe mass loss of the iceberg and did not take into account its shape. Further, Gladstone et al. (2001) improved this model by including sea water temperature and sea ice concentration.

The approach developed by White et al. (1980) is used in the model described in this paper. However, the iceberg is considered to be a cylinder in finite-water depth in the presence of incident wave. In addition, the iceberg is allowed to have motion with three degrees of freedom i.e. surge, heave and pitch. These improvements give advantages to estimate more accurately the velocity potential around the iceberg, hence one can utilize more accurate iceberg deterioration model.

2. Model Description

2.1 Wave Theory

Let's consider a cylindrical iceberg with radius R and draft b in finite water depth h in the presence of incident wave of amplitude A and angular velocity σ. The iceberg is assumed to have three degrees of freedom i.e. surge, heave and pitch. Since the iceberg is in the water, there will be a diffracted wave. Assuming an irrational flow and continuity of the flow, a motion of the water can be described by potential theory with the velocity potential Φ and velocity V is a gradient of the potential. The velocity potential can be decomposed into four velocity potentials Φ_d, Φ_s, Φ_h and Φ_p, where Φ_d is velocity potential due to diffraction of incident wave on the fixed iceberg and Φ_s, Φ_h and Φ_p are velocity potentials due to the radiation due to surge, heave and pitch respectively. Total velocity potential Φ can be written as:

$$\Phi = \Phi_d + \xi_s\Phi_s + \xi_h\Phi_h + a\xi_p\Phi_p \qquad \textbf{[1]}$$

where ξ_s, ξ_h and ξ_p are displacement for surge, heave and pitch respectively.
In addition, boundary conditions are the seabed, the wall of cylinder, free water surface and the value of the scattered potential at infinity must be vanished. Then we have a following boundary value problem (BVP) in cylindrical coordinates (r,φ,z) with z-axis vertically upward from the sea water level, r measured from the z-axis and φ from the positive x-axis and g is acceleration due to gravity:

$$\nabla^2\Phi = 0 \qquad \textbf{[2]}$$
$$\sigma\Phi - g\partial\Phi/\partial z = 0 \ \text{ on } z = 0,\ r \geq a \qquad \textbf{[3]}$$
$$\partial\Phi/\partial z = 0 \ \text{ on } z = -h \qquad \textbf{[4]}$$

with boundary conditions:
for surge motion:

$$\partial\Phi/\partial z = 0 \ \text{ on } z = -b,\ 0 \leq r \leq a \qquad \textbf{[5]}$$
$$\partial\Phi/\partial r = -\mathrm{i}\sigma\xi_s\cos\varphi \ \text{ on } -b \leq z \leq 0,\ r = a \qquad \textbf{[6]}$$

for heave motion:

$$\partial\Phi/\partial z = -\mathrm{i}\sigma\xi_h \ \text{ on } z = -b,\ 0 \leq r \leq a \qquad \textbf{[7]}$$
$$\partial\Phi/\partial r = 0 \ \text{ on } -b \leq z \leq 0,\ r = a \qquad \textbf{[8]}$$

for pitch motion:

$$\partial\Phi/\partial z = -\mathrm{i}\sigma\xi_p r\cos\varphi \ \text{ on } z = -b,\ 0 \leq r \leq a \qquad \textbf{[9]}$$
$$\partial\Phi/\partial r = \mathrm{i}\sigma\xi_p z\cos\varphi \ \text{ on } -b \leq z \leq 0,\ r = a \qquad \textbf{[10]}$$

Bhatta and Rahman (2003) showed that it is possible to have analytical solution for each velocity potential Φ_d, Φ_s, Φ_h and Φ_p. The diffraction potential is found by solving BVP eqs. [2]-[4], [5], [8] with an additional condition that scattered potential at the infinity is equal to zero i.e. satisfies the radiation condition. The radiation problem for surge motion is found by solving BVP eqs. [2] -[6] with an additional condition that the surge radiation potential at the infinity is equal to zero. The radiation problem for heave motion is found by solving BVP eqs. [2]-[4],[7],[8] with an additional condition that the heave radiation potential at the infinity is equal to zero. The radiation problem for pitch motion is found by solving BVP eqs. [2]-[4],[9],[10] with an additional condition that the pitch radiation potential at the infinity is equal to zero. The solution for each velocity potential is found by dividing the domain into two parts namely interior (under the iceberg, $r \leq a$) and exterior (beyond the iceberg, $r \geq a$). Implementing the method proposed by Bhatta and Rahman (2003) one can find the velocity potential in the presence of iceberg.

2.2 Thermodynamics

Heat transfer equation in cylindrical coordinates:

$$\rho c \partial T/\partial t = 1/r\ \partial/\partial r(kr\partial T/\partial r) + 1/r^2\ \partial/\partial\varphi(k\partial T/\partial\varphi) + \partial/\partial z(k\partial T/\partial z) + \mathrm{d}M/\mathrm{d}t l_f \quad \textbf{[11]}$$

where ρ is an iceberg density, c is a heat capacity, k is thermal conductivity, dM/dt is a mass change of the iceberg and l_f is a latent heat capacity.
Let's consider the iceberg is heated by imposed flux q_{input}. Assuming an input flux in every horizontal plane is the same, implies that there is no heat flux in tangential direction:

$$\partial q/\partial\varphi = 0 \quad \textbf{[12]}$$

Due to symmetry, axis of symmetry of the cylinder is assumed to be adiabatic:

$$\partial q/\partial r = 0 \ \text{ on } r = 0 \quad \textbf{[13]}$$

Let's assume the initial temperature to be below or equal to melting temperature i.e.$T_{init} \le T_m$. Given aforementioned assumptions and the Fourier's law of heat conduction, the mathematical formulation of the iceberg melting can be written as:

$$\rho c\partial T/\partial t = 1/r\ \partial/\partial r(kr\partial T/\partial r) + \partial/\partial z(k\partial T/\partial z) + \mathrm{d}M/\mathrm{d}t l_f \quad \textbf{[14]}$$
$$T(0) = T_{init} \quad \textbf{[15]}$$
$$\partial q/\partial r = 0 \ \text{ on } r = 0 \quad \textbf{[16]}$$
$$\partial q/\partial r = q_{input} \ \text{ on } -b \le z \le 0,\ r = a \quad \textbf{[17]}$$
$$\partial q/\partial z = q_{input} \ \text{ on } z = -b,\ 0 \le r \le a \quad \textbf{[18]}$$

Problem eqs. [14]-[18] describes heat transfer in sector of cylinder with angular size $d\varphi$. In order to solve the problem eqs. [14]-[18] the enthalpy method was used (Alexiades and Solomon, 1993). Let's subdivide [0 a] into N subintervals and [0 H] into K subintervals, note that non-uniform grid spacing is allowed in both r– and z–directions. Thus the sector is subdivided into $N \times K$ control volumes. Subscripts i and j correspond to r– and z–directions, respectively. For each control volume $V_{i,j}$ the energy conservation law is applied to find a discrete heat balance. The conservation law can be written as:

$$\partial E/\partial t + \partial q/\partial x = 0 \quad \textbf{[19]}$$

where E is a thermal energy density per unit volume. Eq. [19] says that the gain of heat during the time is equal to the amount of heat entering minus the heat leaving.
Let's denote $E_{i,j}$ as thermal energy density per unit volume for control volume $V_{i,j}$. If $E_{i,j} < 0$ then $V_{i,j}$ is solid, if $E_{i,j} \ge \rho l_f$ then $V_{i,j}$ is liquid and if $0 \le E_{i,j} < \rho l_f$ then $V_{i,j}$ is partially liquid and partially solid. Let's consider that heat capacity for both solid and liquid states are independent of temperature, then E can be defined:

$$E = \rho c_S(T - T_m) \quad T < T_m \quad \textbf{[20]}$$
$$E = \rho c_L(T - T_m) + \rho l_f \quad T \ge T_m \quad \textbf{[21]}$$

or, solving for T:

$$T = T_m + E/\rho c_S \quad E \le 0 \quad \textbf{[22]}$$

$$T = T_m \quad 0 < E < \rho l_f \qquad \textbf{[23]}$$
$$T = T_m + (E - \rho l_f)/\rho c_L \quad E \geq \rho l_f \qquad \textbf{[24]}$$

Let's denote $q_{i,j-1/2}$ as inward heat flux for control volume $V_{i,j}$ from lower neighbouring cell $V_{i,j-1}$, $q_{i,j+1/2}$ as outward heat fluxes for control volume $V_{i,j}$ to upper neighbouring cell $V_{i,j+1}$, $q_{i-1/2,j}$ as inward heat flux for control volume $V_{i,j}$ from left neighbouring cell $V_{i-1,j}$ and $q_{i+1/2,j}$ as outward heat flux for control volume $V_{i,j}$ from right neighbouring cell $V_{i+1,j}$. Then the explicit scheme for eq. [19] is:

$$E_{i,j}^{n+1} = E_{i,j}^{n} + \Delta t/\Delta r_i(q_{i-1/2,j}^{n} - q_{i+1/2,j}^{n}) + \Delta t/\Delta z_i(q_{i,j-1/2}^{n} - q_{i,j+1/2}^{n}) \qquad \textbf{[25]}$$

where Δt is a time step, Δr_i and Δz_i are dimensions of the control volume $V_{i,j}$.
The initial values are:

$$T_{i,j}^{0} = T_{init\ i,j} \qquad \textbf{[26]}$$

The boundary conditions are:

$$q_{1/2,j}^{n} = 0 \qquad \textbf{[27]}$$
$$q_{i,1/2}^{n} = q_{input} \qquad \textbf{[28]}$$
$$q_{N+1/2,j}^{n} = q_{input} \qquad \textbf{[29]}$$
$$q_{i,K+1/2}^{n} = q_{input} \qquad \textbf{[30]}$$

The explicit scheme [25] with initial conditions [26] and boundary conditions [27]-[30] is the solution for the problem [14]-[18].

2.2 Coupling of Wave Theory and Thermodynamics

White et al. (1980) proposed the approach to deal with wave action. The approach is based on the two assumptions. Due to oscillating velocities from the wave near the solid surface oscillating turbulent layer is formed, resulting in periodic friction against the surface. Since water is relatively warm comparing with the iceberg surface temperature, the friction will interact with the thermal boundary layer, this process will cause periodic heat transfer to the surface, that can be averaged. The second assumption is that the wave friction can be related with the heat transfer via Reynolds analogy. White et al. (1980) proposed to use the following Reynolds analogy:

$$\mathrm{St} = 1/2\mathrm{C_f}/(1+12.8(\mathrm{Pr}^{0.68} - 1)(1/2\mathrm{C_f})^{1/2}) \qquad \textbf{[31]}$$

where St is Stanton number, $\mathrm{C_f}$ is wave friction factor defined by Jonsson (1966) as:

$$\mathrm{C_f} = 2\mathrm{Re_a}^{-1/2} \quad \mathrm{Re_a} < 31000 \qquad \textbf{[32]}$$
$$\mathrm{C_f} = 0.09\mathrm{Re_a}^{-0.2} \quad \mathrm{Re_a} > 31000 \qquad \textbf{[33]}$$

where $\mathrm{Re_a} = a_w V/v_w$ is the wave orbit Reynolds number, a_w is an amplitude of the wave, V is a wave orbital velocity and v_w is kinematic viscosity. Velocity V is obtained from eq. [1] by taking derivative in r–direction for vertical wall of the iceberg and z–direction for horizontal bottom of the iceberg.

At the same time Stanton number by definition:

$$\mathrm{St} = q_{input}/\rho V c_p \Delta T \qquad \textbf{[34]}$$

where c_p is heat capacity and ΔT is temperature difference between ambient environment and the iceberg surface. Using eqs. [31] and [34] it is possible to estimate q_{input}, hence it is possible to estimate the melting rate of the iceberg due to wave action.

3. Conclusions

A method has been presented to compute iceberg deterioration due to surge, heave and pitch motion in finite-depth water in the presence of incident wave. Firstly, the total velocity potential is obtained from analytical solution of BVP. Then the total velocity is used as input parameter for boundary condition of heat transfer problem. Thus one can estimate the mass loss of the iceberg due to wave action.

Acknowledgments

The authors would like to thank the Norwegian Research Council through the project 200618/S60-PetroRisk and the SAMCoT CRI for financial support and all the SAMCoT partners.

References

Alexiades, V., Solomon, A.D., 1993. Mathematical Modelling of Melting and Freezing Processes, Hemisphere Publishing Corporation.

Bhatta, D.D., Rahman, M., 2003. On scattering and radiation problem for a cylinder in water of finite depth. International Journal of Engineering Science 41(9): 931-967.

Bigg, G.R., Wadley, M.R., Stevens, D.P., Johnson, J.A., 1997. Modelling the dynamics and thermodynamics of icebergs. Cold Regions Science and Technology 26(2): 113-135.

El-Tahan, M., Venkatesh, S., El-Tahan, H., 1987. Validation and quantitative assessment of the deterioration mechanisms of arctic icebergs. J. Offshore Mech. Arct. Eng. 109(1): 102-108.

Gladstone, R. M., Bigg, G.R., Nicholls, K.W., 2001. Iceberg trajectory modeling and meltwater injection in the Southern Ocean. Journal of Geophysical Research: Oceans 106(C9): 19903-19915.

Job, J.G., 1978. Numerical modelling of iceberg towing for water supplies - a case study. Journal of Glaciology 20(84): 533-542.

White, F.M., Spaulding, M.L., Gominho, L., 1980. Theoretical estimates of the various mechanisms involved in iceberg deterioration the open ocean environment. Washington, D.C.: 126.