Proc. R. Soc. Lond. A **390**, 283–291 (1983)
Printed in Great Britain

Wave set-up, percolation and undertow in the surf zone

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(*Received* 27 *May* 1983)

[Plates 1–4]

Waves approaching a sloping beach induce a tilt in the mean water level within the surf zone. The existence of this 'set-up' is here demonstrated by observing the mean flow in a straight tube laid parallel to the incoming waves; also by showing that the waves induce a siphon in a U-tube laid on the sloping bottom.

It is argued theoretically, and confirmed by experiment, that the set-up should help to drive an offshore bottom current (the undertow) between the shoreline and the breaker line. Seawards from the breaker line the bottom current is reversed. The consequent convergence of the bottom currents may contribute to building up the 'breaker bar'.

Further experiments show that the mean onshore pressure gradient drives a circulation of water within a porous beach. The associated pattern of streamlines also extends into the land, inshore from the run-up line. Theoretically, the injection of dye at the sediment–water interface might be used to probe the porosity of the beach material.

1. Introduction

Since the laboratory measurements by Saville (1961) of the mean bottom pressures due to waves on a sloping beach, it has been recognized that breaking waves tend to cause a change in the mean level $\bar{\zeta}$ of the water in the surf zone; this was called the wave 'set-up' by Saville. A quantitative explanation was given by Dorrestein (1961) and Lundgren (1963) in terms of the horizontal flux of momentum in the waves, or equivalently in terms of the radiation stress (Longuet-Higgins & Stewart 1962, 1963, 1964). This analysis suggests that, seawards from the breaker line, where the radiation stress is increasing towards the shore, the mean surface will tilt downwards, and that between the breaker line and the shoreline, where the stress is decreasing because of dissipation, the mean level will tilt strongly upwards, in agreement with Saville's observations. A full confirmation of this effect was provided by the careful laboratory measurements reported by Bowen *et al.* (1968), who like Saville (1961) measured the mean pressure on the bottom with a damped manometer.

Because they are masked by the oscillatory motion of the waves, these time-averaged effects, though significant, are not easy to appreciate visually. The first objective of this paper is to describe a clear demonstration of the set-up by means of the flow induced in a straight pipe laid on the sloping beach (see §2); also the flow induced in a U-tube or siphon.

In the remainder of this paper we offer some demonstrations of the effects of the wave set-up, first upon currents in the surf zone, where it can be shown to be responsible for the so-called 'undertow' (Evans 1938) and generally to influence strongly the inshore circulation (see §3).

The second effect to be demonstrated is the setting up of an internal flow or circulation of water within the beach itself, when this has a porosity characteristic of loose sand or shingle. The expected pattern of such a circulation is described in §4, and in §5 we report some simple experiments that serve as a striking confirmation (see figures 6 and 7). Further discussion of the uses and consequences of these phenomena is given in §6.

2. Demonstration of wave set-up

Experiments were done in a rectangular wave channel 60 cm wide and 14 m long; shown in figure 1. At one end of the channel an oscillating, wedge-shaped plunger generated waves of period 1 s, which progressed down the channel and broke at the far end on a plane beach with gradient 0.11 (see figures 1 and 2). The depth of water in the uniform part of the channel was 33 cm, and the wave height at the breaker line was about 14 cm. The initial break point B and the point of maximum run-up C are indicated in figure 1.

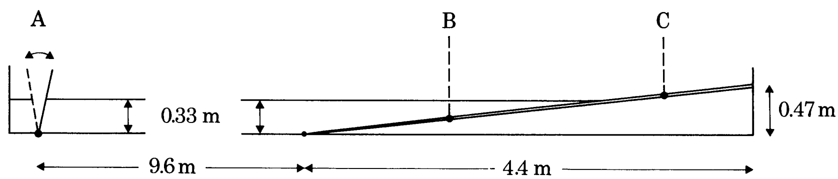

Figure 1. Sketch of wave channel, with wavemaker A, breaker line B and run-up line C.

Experiment 1

An open-ended Perspex tube of length 150 cm and internal diameter 1.3 cm was laid on the sloping beach with its lower end approximately at the break point (see figure 2*a*, plate 1). The upper end of the tube was then always submerged. The tube was held in place by a sheet of wire mesh (mesh diameter approximately 3 cm) laid across the tank, so as to cover only the middle section of the tube and not affect the flow near the two ends.

With the waves absent, a small quantity of dye (potassium permanganate solution) was injected into the upper end of the tube. This remained stationary until the wavemaker was started. As soon as the waves reached the beach, the dye began to flow strongly seawards down the tube (figure 2(*b*)), with a mean speed of about 35 cm/s. Although there was some diffusion of the dye, this was small compared

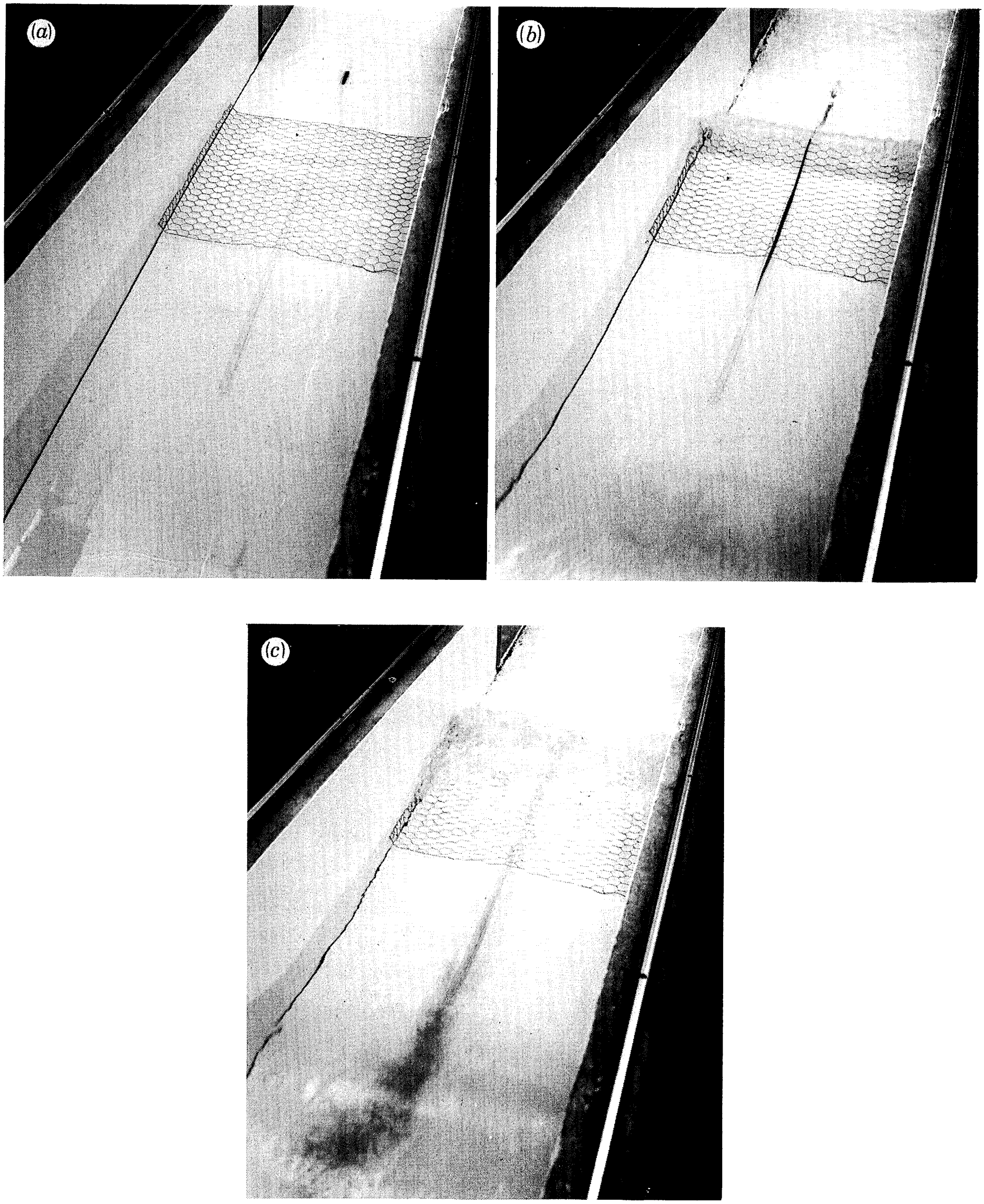

FIGURE 2. (*a*) A straight tube laid in the surf zone, with dye injected at the upper end. The water is still. (*b*) Dye moving offshore under the action of waves. (*c*) Dye emerging from the lower end of the tube.

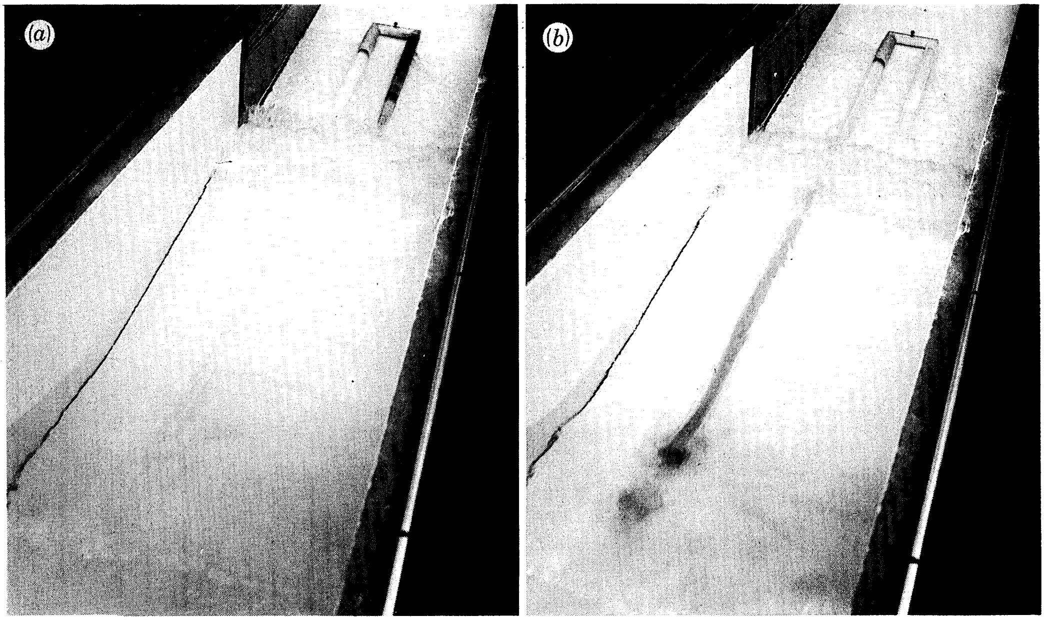

FIGURE 3. U-tube with two unequal arms laid in the surf zone. (*a*) Dye injected into the shorter arm moves towards the beach; (*b*) dye emerges from the longer arm near the breaker line.

FIGURE 4. Dye injected into the boundary layer at the bottom moves offshore.

with the mean displacement of the dye along the length of the tube. On leaving the lower end of the tube the dye dispersed rapidly into the main part of the channel (figure 2(*c*)).

The oscillatory component of the flow within the tube was small compared with the mean flow, as one would expect from the fact that the length of the tube spanned several wavelengths. We may therefore treat the flow approximately as a constant, unidirectional flow of strength U through a pipe of circular cross section and diameter $d = 1.3$ cm. The Reynolds number $R = Ud/\nu$ being about 3500 we apply the Blasius law

$$\varpi \equiv \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{0.158}{R^{\frac{1}{4}}}\frac{U^2}{d} \tag{2.1}$$

for the pressure gradient along the axis of the tube (see Prandtl 1952). Taking $U = 35$ cm/s we obtain $\varpi = 19$ dyn/cm^2†. On the other hand if we use the formula

$$\varpi' = g\frac{\partial \bar{\zeta}}{\partial x} = 0.15\beta \tag{2.2}$$

for the mean surface gradient in terms of the beach slope $\beta = -\mathrm{d}h/\mathrm{d}x$ (see Longuet-Higgins & Stewart 1963, 1964), then we get $\varpi' = 16$ dyn/cm^2, a comparable result. This suggests that the flow in the tube is due mainly to the mean pressure induced by the wave set-up.

Experiment 2

To eliminate the possibility of end effects, which involve the action of the oscillating flow on the open ends of the tube, a second experiment was performed in which each end was stopped with a cork, and access to the flow in the tube was through 16 holes, each 2 mm diameter, bored in the sides of the tube between 3 and 6 cm from each end (32 holes in all). At the start of the experiment, the tube was filled with water, the two ends were then corked, and dye was injected through one of the holes near the upper end. On activating the waves, the flow was again downwards along the tube, at a rate only slightly less than when the ends were open.

Experiment 3

A more spectacular demonstration of the set-up was achieved by inserting a U-tube having arms of unequal length, about 100 and 207 cm, into the surf zone (see figure 3, plate 2). The tube was filled with water and the upper end, or bend, of the U was drawn up on the beach. In this position the lower arm of the tube was near the breaker point B, while the upper end remained, as previously, just submerged beneath the waves. The tube was therefore ready to act as a siphon. Dye was injected into the shorter arm, and the waves were then activated, with the result that the fluid flowed easily up the shorter arm towards the bend (figure 3(*a*)) and then down the longer arm and out near the break point (see figure 3(*b*)).

† 1 dyn = 10^{-5} N.

It is clear that by placing a turbine in the tube a small amount of energy might have been extracted from the flow, and hence from the waves. However, the pressure head that is available would not be great. By (2.2) the head would be at most $0.15\,h_B$, where h_B denotes the mean depth of water at the breaker line. This is related to the breaker height $2a$ by $2a \simeq 0.75\,h_B$ (see, for example, Longuet-Higgins & Stewart 1964). Though the flow could be multiplied by laying many straight pipes side by side normal to the shoreline, it can easily be seen that the efficiency of such a device would be extremely low.

3. Currents in the surf zone

Imagine the tubes to be entirely removed. In spite of some increase in the vertical mixing, we nevertheless expect a mean flow to take place near the bottom in the same sense as when the straight tube was present. Accordingly, when the waves were running, dye was injected into the layer near the bottom; this moved strongly offshore (see figure 4). The bottom current extended only as far as the breaker line. Even before this point was reached, the dye tended to become mixed with the fluid at higher levels, and to be circulated towards the shoreline. Seawards from the breaker line, the bottom current was weaker and in the opposite direction, i.e. shorewards.

We recall that in a uniform progressive wave travelling over a smooth horizontal bottom, the water particles tend to move *forwards* near the bottom end in the direction of wave propagation (Bagnold 1946; Russell & Osorio 1956). This forwards motion was explained by Longuet-Higgins (1953, 1956) to be due to the action of viscosity, or eddy viscosity, in the boundary layer near the bottom. A similar forwards flow would be expected on a smooth sloping beach, in the absence of a mean pressure gradient. However, it appears that the horizontal pressure gradient associated with the wave set-up is so strong as to completely swamp the weak viscous forces in the laminar boundary layer, and carry the whole flow in the opposite direction.

This interpretation is strengthened by the following argument. The thickness of the Stoke oscillatory boundary layer is of order $\delta = (2\nu/\sigma)^{\frac{1}{2}}$, where ν is the kinematic viscosity and σ is the radian frequency of the waves. In the absence of a mean horizontal pressure gradient, the magnitude of the forwards drift, $\overline{U}$, just outside the boundary layer in water of uniform depth, is known to be $\frac{5}{4}q^2/c$, where q is the amplitude of the oscillating flow and c is the phase speed. On a gently sloping bed, $\overline{U}$ would still be of the same order of magnitude and of the same sign. On the other hand, the flow produced by an adverse pressure gradient $\varpi' = g\partial\overline{\zeta}/\partial x$ at a height δ above the bottom will be of order $\varpi'\delta/\nu$; that is to say it is proportional to $\nu^{-\frac{1}{2}}$. The pressure gradient will therefore clearly dominate the viscous boundary layer and determine the direction of the mean flow.

The effect of the mean pressure gradient is partly counterbalanced by a second term contributing to the momentum flux, namely $\rho\overline{u^2}$, where u is the horizontal velocity. But this term diminishes with depth, and near the bottom it will, in addition, be sharply reduced because of the decreased velocity in the Stoke layer.

By definition, the radiation stress, whose gradient balances the wave set-up, is essentially a vertically integrated momentum flux. As has often been noted, this flux is in fact a function of the vertical coordinate, being greatest near the upper boundary, and generally less at deeper levels. This unequal vertical distribution of the stress can also be expected to drive a mean circulation, which is forwards near the surface and backwards at lower levels. However, by the Kelvin circulation theorem it would clearly be impossible to generate such a circulation in the absence of any dissipation.

One further aspect of the motion is the mass flux, which also is unequally distributed in the vertical, being greatest at the upper levels. This will also tend to produce a circulation in the sense observed.

A complete theoretical model of the flow will need to balance both the mass flux, the momentum flux, and also the flux of angular momentum (see Longuet-Higgins 1980, 1983).

The reverse bottom current, or 'undertow', in the surf zone may be expected to be strongest when the beach is fairly steep, and therefore the surf zone is narrow. For very gentle beach slopes the current may not be significant. In addition, when the surf zone is relatively wide, the two-dimensional pattern of circulation seen in a narrow laboratory channel may tend to become unstable to three-dimensional perturbations, and to break up into a horizontal circulation associated with rip currents, as described in Shepard & LaFond (1939).

4. Percolation in a porous beach

Since all beaches consisting of shingle, sand or unconsolidated sediment are to some extent porous, one would expect the changes in the mean pressure associated with the set-up to produce some flow of sea water within the beach itself.

It has already been shown (Putnam 1949) that the oscillatory component of the pressure may produce some damping of the waves over a porous beach. But the mean pressure gradient, though smaller, will produce effects that, because they are cumulative in time, may be more far-reaching.

As a very simple model, consider a beach of uniform porosity, in which the flow velocity $\boldsymbol{u}$ obeys the Darcy law

$$\boldsymbol{u} = -\kappa\nabla\bar{p}, \tag{4.1}$$

where $\bar{p}$ is the pressure and κ is a constant. In other words

$$\boldsymbol{u} = \nabla\phi, \tag{4.2}$$

where $\phi = -\kappa\bar{p}$, is a potential, which by continuity must satisfy

$$\nabla^2\phi = 0. \tag{4.3}$$

In the simple theory of wave set-up outlined in §1 the pressure gradient $\partial\bar{p}/\partial x$ is uniform between the break point B and the run-up point C (see figure 1). If we agree to neglect the much smaller wave set-up (or rather 'set-down') seawards from the breaker point, and also neglect the fact that inshore from C the upper surface of the wetted porous medium will not coincide exactly with the plane surface of the beach, then the mathematical problem to be solved becomes very simple. We must find a potential function ϕ satisfying Laplace's equation (4.3) in the lower half plane below the line BC, and such that

$$\partial\phi/\partial x' = \begin{cases} K \text{ (constant)} \quad \text{between B and C,} & (4.4) \\ 0 \quad \text{elsewhere on BC.} & (4.5) \end{cases}$$

Here, x' and y' denote rectangular coordinates with the origin O at the mid-point of BC, and with Ox' in the direction of OC (see figures 1 and 5).

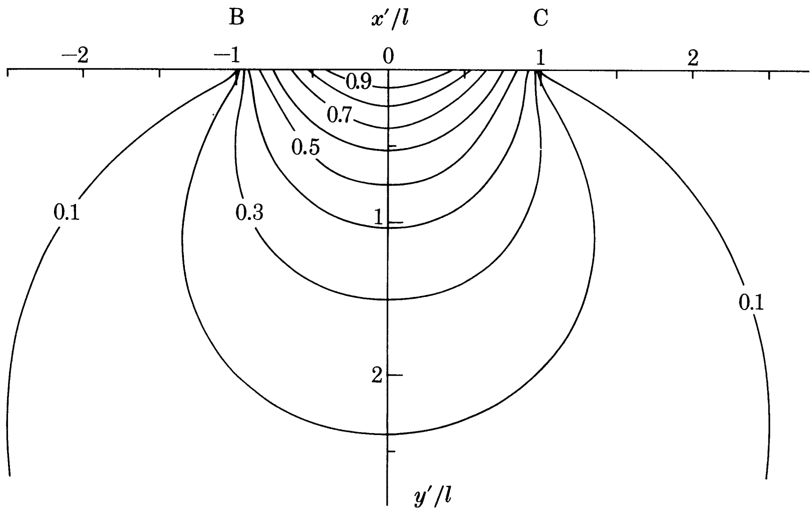

FIGURE 5. Theoretical streamlines of flow in a medium of uniform porosity bounded by $y' = 0$, when a uniform pressure gradient $\partial\phi/\partial x'$ is applied between B and C.

The solution of this problem is well known. In fact if we take elliptical coordinates (ξ, η) defined by

$$\left.\begin{aligned} x' &= l\cosh\xi\cos\eta, \\ y' &= l\sinh\xi\sin\eta, \end{aligned}\right\} \tag{4.6}$$

where $0 \leqslant \xi \leqslant \infty$, $0 \leqslant \eta \leqslant \pi$ and $l = \text{OC}$, then the function we seek is

$$\phi = Kl\,\mathrm{e}^{-\xi}\cos\eta. \tag{4.7}$$

The conjugate function

$$\psi = Kl\,\mathrm{e}^{-\xi}\sin\eta \tag{4.8}$$

is a constant along the streamlines of the flow. These are shown in figure 5, which is adapted from a paper (Longuet-Higgins 1949) on the electrical currents induced by

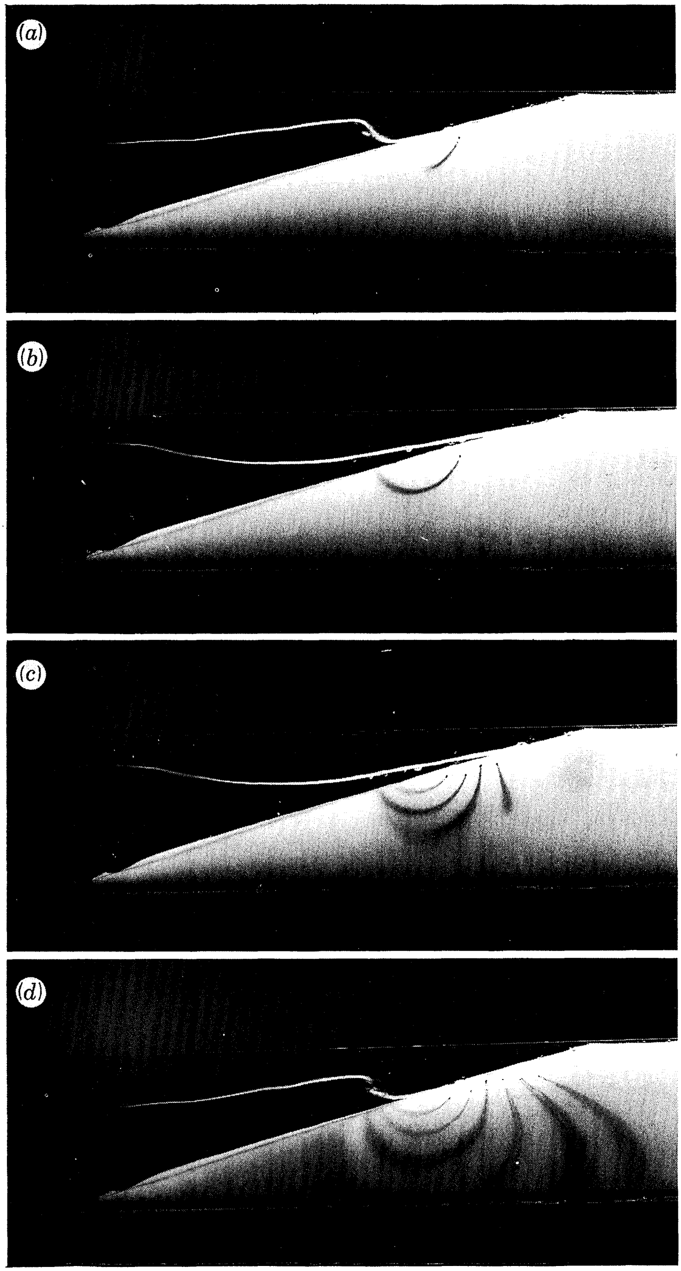

FIGURE 6. Flow pattern in a porous beach revealed by inserting particles of dye near the side wall of a wave channel. (*a*) Particle inserted near centre of surf zone, showing trajectory moving downwards; (*b*) 10 s later; (*c*) trajectories from four particles; (*d*) trajectories from six particles.

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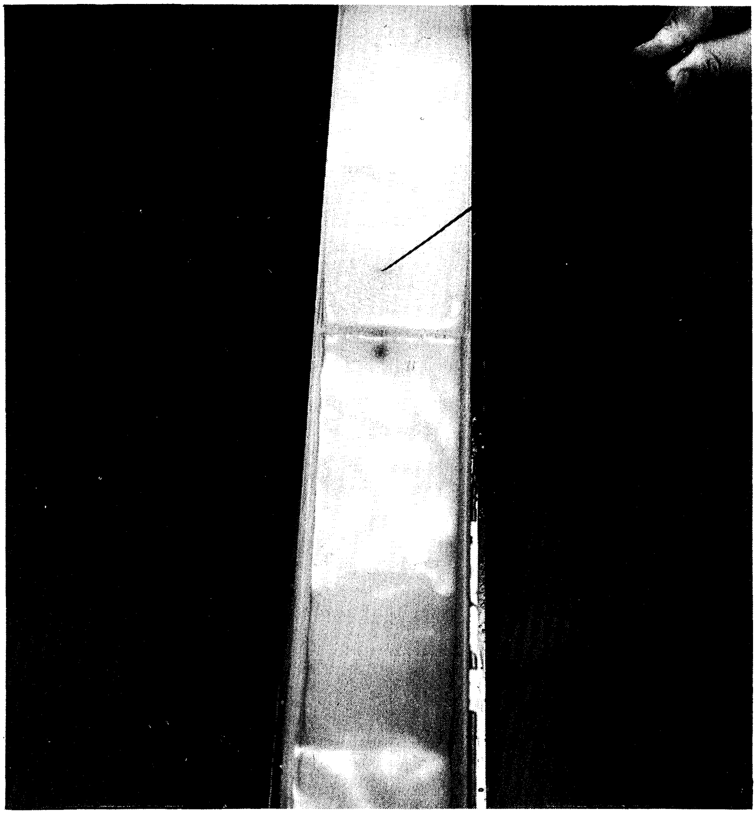

FIGURE 7. Overhead view of the smaller wave tank, which shows the injection of dye into a point near the centre line of the tank, in the upper part of the surf zone. The dye emerges later at a point lower down the beach.

tidal streams flowing in a straight channel of elliptical cross section, in the presence of the Earth's magnetic field. It can be seen that the flow extends considerably outside the segment between B and C. (It was this property that enabled the streams in the English Channel to be detected by potential measurements on one side of the channel only.)

5. Experimental vertification

Experiments were conducted in a smaller wave channel, 1.7 m long and 8.5 cm wide. Short waves of period 0.25 s were generated by the plunger at one end. A sloping beach of height 15 cm and length 60 cm (figure 6, plate 3) was placed about 50 cm from the wavemaker. For convenience, the loose beach sediment was replaced by a sponge-like packing material with a similar porosity, which pressed lightly against the glass sides of the tank. A second rectangular block, 60 cm long, consisting of the same material, was placed behind the wedge-shaped beach, and in contact with it.

Waves of period 0.25 s impinging on the beach produced a well defined breaker line and run-up, analogous to that seen in the larger wave channel.

To reveal the pattern of percolation, particles of potassium permanganate dye were inserted into the beach near one side wall, in the upper half of the surf zone. When the point of insertion was near the middle of the surf zone the dye moved at first downwards into the beach (see figure 6 (*a*)), and then returned to the surface further down the slope (figure 6 (*b*)). Repetition of the procedure with more dye particles (figures 6 (*c*, *d*)) revealed a flow pattern similar to figure 5. It can be seen that somewhat shorewards from the breaker line the flow is to the right; that is to say the waves are pumping water into the 'land'.

In the experiment, the fluid was free to return along a path below the porous beach, thus accounting for the continuous darker zone seawards from the breaker line, in figure 6 (*d*).

In a further experiment, viewed from above in figure 7, plate 4, dye was injected into the upper half of the sloping 'beach' at a point near the centre line of the channel. Again the dye disappeared down into the beach and reappeared a few seconds later at a point lower down (see figure 7).

6. Discussion

It has been demonstrated that, provided the porosity of the beach is uniform, water from the surf zone may percolate down to depths below the surface, which are comparable with the width of the surf zone. Such a circulation is likely to transport oxygen, and hence help to maintain biological activity at these levels. By preventing anaerobic conditions the circulation may also help to keep the beach mobile.

In general, the porosity will not be uniform, but is likely to diminish somewhat with distance below the surface. The time taken for the dye to travel along a given trajectory depends directly on the porosity along its path. Theoretically it should be possible to determine the distribution of porosity by an inverse method, while knowing the travel times of dye injected at different points on the beach surface. In practice, any attempt to do this might be hampered by the turbidity of the water and by other difficulties associated with making observations in the surf zone.

The presence of a small vertical component of the pore pressure gradient may have little effect on the stability of sand grains. On the other hand, the tangential movement of the sand is bound to be affected by the mean horizontal pressure gradient. The convergence of the bottom current in the neighbourhood of the breaker line suggests a partial explanation for the formation of the 'breaker bar' by horizontal convergence of the transported sand.

The experiments described in this paper were done in Cambridge, England, during the summer of 1982, and were first reported at the Coastal Engineering Workshop, University of Hawaii, in August of that year. For assistance with the photography, I am indebted to Dr J. W. Simpson and Mr N. D. Smith. The manuscript was prepared during a visit to the Department of Engineering Sciences of the University of Florida at Gainesville, Florida. I am also indebted to the Chairman, Dr K. Millsaps, and to his staff for their hospitality and assistance.

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