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Why is a water wave like a grandfather clock?

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The orbital motion of particles at the surface of a steep gravity wave consists very nearly of a uniform translation superposed on the backward swing of a pendulum.

In steep, irrotational gravity waves in deep water, the paths of the individual particles are not closed but have a net forward drift known as the mass-transport velocity.[1] For low waves this is a second-order quantity, but to calculate it accurately for steep waves requires a numerical integration.[2]

It appears, however, that for a wave of maximum height, in which the interior angle at the crest is 120° (see Ref. 3), there is a remarkably close approximation to the orbital motion based on a simple physical analog, as follows.

In Fig. 1 a wave of length L travels to the right with speed c. We can approximate the free surface between two adjacent crests A and B by the arc of a circle center C. To agree with the slope angle of 30° at the crests[3] the arc must correspond to a 60° sector. Hence, the triangle ABC is equilateral, and the radius of the arc equals the wavelength L. This gives a crest-to-trough wave height $L(1-\cos 30°)$ or $0.1340L$, which differs from the accurate[4,5] value $0.1412L$ by only $0.0072L$.

Now consider the dynamics of the motion. At the free surface the pressure is constant, so that the pressure gradient is always normal to the surface. A particle at the surface therefore behaves like the bob of a free pendulum of length L and radian frequency $\omega \sim (g/L)^{1/2}$. The particle travels from B to A in one half-period

$$\tau \sim \pi (L/g)^{1/2}. \tag{1}$$

At A it comes to rest relative to the profile, and assuming that it does not surf-ride, it then transfers, like a trapeze artist, onto a similar pendulum at the left of A. In a single swing the particle has advanced relative to deep water, through a distance $(c\tau - L)$. Its mean speed of advance is therefore

$$U = (c\tau - L)/\tau = c(1 - L/c\tau). \tag{2}$$

Using the expression $c \sim (gL/2\pi)^{1/2}$ for deep-water waves of low amplitude, we obtain

$$U/c = 1 - L/c\tau \sim 1 - (2/\pi)^{1/2}. \tag{3}$$

The more accurate values $c = 1.0923(gL/2\pi)^{1/2}$ and $\tau = 1.0174\pi(L/g)^{1/2}$ for waves of maximum height[4,5] and for a pendulum of swing ±30° (Ref. 6) gives $U/c = 0.282$. Thus, the average speed of advance is about 0.28 times the phase-speed.

The particle orbits are given by

$$x = ct - L\sin\theta, \quad y = L(1 - \cos\theta), \tag{4}$$

where θ denotes the angle between the pendulum and the vertical. θ varies almost sinusoidally with the time t, but more exactly[6] is given by

$$\sin\tfrac{1}{2}\theta = k\,\mathrm{sn}[k, (g/L)^{1/2}t], \quad k = \sin 15° \tag{5}$$

(see Fig. 2). After describing each orbit the particle has advanced a distance $[x]$, where

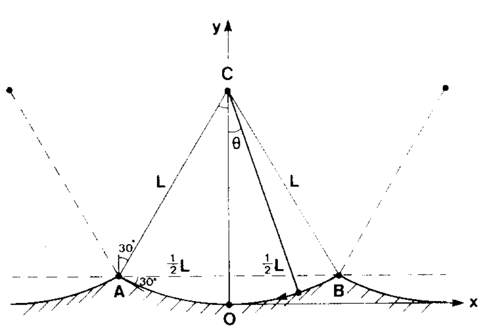

FIG. 1. Approximation of the profile of a deep-water gravity wave by the arc of a circle.

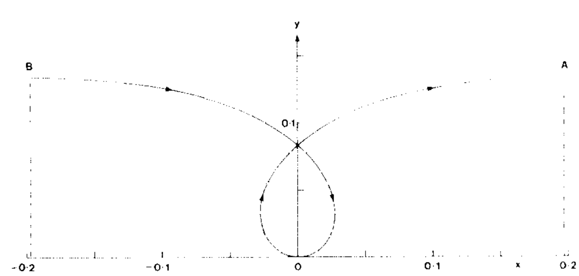

FIG. 2. The orbit of a particle at the surface of a deepwater gravity wave, as given by Eqs. (4) and (5).

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$$\frac{[x]}{L} = \frac{c\tau - L}{L} = \frac{c\tau}{L} - 1 = 0.393\,. \qquad (6)$$

Precise numerical calculations[2] based on the profile of the highest deep-water wave as given numerically by Yamada[4] yield, respectively, $U/c = 0.274$ and $[x]/L = 0.377$. The "clock" model is thus quite accurate.

The application of these results to real water waves, in the ocean and in the laboratory, is discussed in Ref. 2.

[1]H. Lamb, *Hydrodynamics* (Cambridge University Press, Cambridge, England, 1932), p. 419.
[2]M. S. Longuet-Higgins, J. Fluid Mech. (to be published).
[3]G. G. Stokes, Math. Phys. Papers **1**, 326 (1880).
[4]H. Yamada, Rep. Res. Inst. Appl. Mech. Kyushu Univ. **5**, 37 (1957).
[5]L. W. Schwartz, J. Fluid Mech. **62**, 553 (1974).
[6]F. Bowman, *Elliptic Fluctions* (Wiley, New York, 1953), p. 28.

"*Ad hoc*" liquid spray vacuum leak detection method

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The common method of spreading volatile liquid on vacuum systems to check for leaks is reexamined. Simple experiments are made to determine the parameter that characterizes the effectiveness of candidate liquids. It is found that the most important properties are density and viscosity, not vapor pressure.

The simplest method often used in laboratories to detect leaks in vacuum installations is to sprinkle the suspected leak regions with a "volatile" fluid and observe the rate of increase of pressure inside the vacuum chamber. The efficiency of this method is proved by its common use in laboratories. It seemed of some interest to have a closer look at the flow problem to see what parameters of the test liquid give the best results, e.g., a high vapor pressure or a low viscosity. An estimate of the dimensions of the leak channels involved can be obtained from the acceptable leak rates, in high vacuum installations, which are of the order of 10^{-6} Torr liters/sec.[1] Assuming the flow through the leak to be sonic, due to the high pressure ratio between atmosphere and vacuum, a vacuum volume of 10 liters and a circular shape of the leak channel, the corresponding diameter will be about 2.5×10^{-5} cm. Even for small wall thicknesses of the vacuum chamber, say 0.05 cm, the length-to-diameter ratio of the leak conduit is of the order of 10^3 and therefore, the flow through a fine capillary could be used as a possible leak model. Of course, this model may not be adequate for very different geometries, say leaks through narrow slits.

A series of very simple experiments were carried out in an existing vacuum installation used for liquid-helium experiments[2] with an equivalent vacuum chamber capacity of about half a liter. Observations were made of the steady rate of increase of pressure, dp/dt, in the vacuum chamber, due to the fluid flow through a capillary which was connected to atmospheric pressure and was fed by different liquids from a beaker. The leak flow rates were measured with the vacuum chamber valved off from the pumping system. The starting pressure of the vacuum chamber was below 1 Torr. The leak rate through the capillary was always much larger than the natural leak rate of the vacuum chamber. Glass and metal capillaries of 0.003–0.015 cm i.d. and 12–35 cm length were used for these tests. Unfortunately, the inner diameter of the glass capillaries available varied over a range of about ±25% of its nominal value and, although the metal capillaries were much more uniform, their surface roughness was larger. These qualities of the capillaries and the difficulties connected with insuring the cleanliness of the capillaries greatly influenced the scatter of the experimental results, although no doubt these conditions were closer to the real laboratory leak test conditions.

Twelve organic liquids (acetone, benzene, butyl alcohol, carbon tetrachloride, ethyl ether, ethyl alcohol, hexane, isopropyl alcohol, methyl alcohol, pentane, toluene, and xylene) covering a wide range of physical parameters were used in the tests. The measurements in different capillaries were normalized relative to the

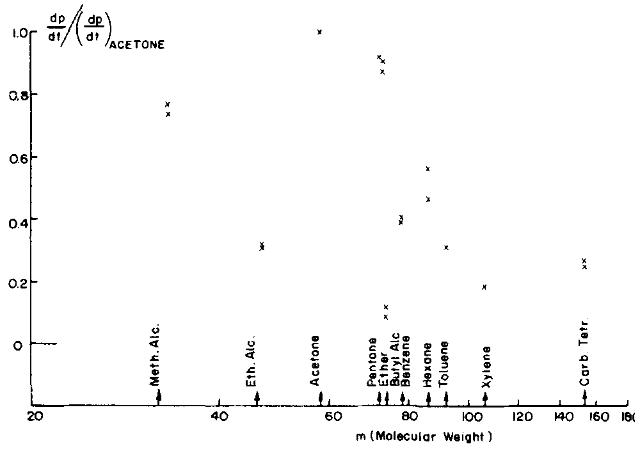

FIG. 1. Rates of increase in pressure.