

## Directional wind wave development

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**Abstract.** Using aircraft radar instruments designed for sea surface wave measurements, we have obtained fetch-limited directional wind wave spectra under steady off-shore wind conditions. The results from these observations in different areas at different times show that, up to a fetch of 150 km, the dominant waves propagate at an angle to the wind. The angle is near to that suggested by the Phillips resonance wind wave generation condition, but with one important difference: The waves are not always symmetric to the left and right of the wind. Most of the cases show the eventual dominance of one side lobe. The asymmetry of the wave direction relative to the wind suggests that the surface wind stress vector may not always be parallel to the mean wind direction.

### Introduction

Water surface wave dynamics was an early subject of study by applied mathematicians, among them Laplace, Stokes, Airy, Helmholtz, and Lord Kelvin. Although they all made lasting contributions to wave dynamics, the solution to the wind wave generation problem remains elusive. Jeffreys (1924, 1925) proposed the "sheltering" mechanism for the wind effect on waves; later refinements of the wind flow over waves were proposed by Miles (1957, 1959a, 1959b, 1960, 1962, 1963). This was further refined by Townsend (1972), who calculated the forces on waves propagating at an angle to the wind direction, recognizing the feedback effect of waves on the turbulent boundary layer. Previously, Phillips (1957) described the initial stage of wave generation as due to a resonant interaction between surface waves and moving air pressure patterns in the boundary layer. Because of the complexity of the process of wave generation, theories can only represent radical simplifications of the physics. Extensive field and laboratory investigations have been undertaken to provide material for structuring empirical models for wind wave generation.

The Phillips theory of resonant excitation of waves by boundary layer pressure fluctuations will generate waves that travel at an angle to the wind such that the wind speed, projected on the direction of wave travel, equal the phase speed of the waves. This mechanism is merely a passive resonance, in the sense that the boundary layer pressure disturbances arise independently of the presence of surface

waves. The Miles instability mechanism provides a feedback, so that the pressure disturbances that generate waves are generated from the response of the boundary layer to the presence of waves. This is a feedback mechanism, and involves radical simplifications to the dynamics of the air boundary layer. The most unstable and fastest growing wave, according to this theory, will be travelling directly downwind, and thus this conclusion follows Squire's (1933) theorem for waves of infinitesimal amplitude. Another feature of Miles' theory is the need for the existence of a critical layer on the high wind profile curvature (where the phase speed equals the local air speed).

Townsend (1972) examined the effect of surface waves on boundary layer flow in more detail, including the case where the waves propagated at an angle to the wind. He modeled the dynamics of the response of a turbulent boundary layer to obliquely moving water waves; his analysis includes the effects of wave induced fluctuations on the turbulent energy density, the directions of momentum fluxes, the diffusion of turbulent fluctuations and viscous dissipation. Assuming the undisturbed flow in the boundary layer to be similar to that over a planar rigid surface, he managed to formulate a set of equations amenable to numerical integration and calculated the surface stress (including pressure) variations over the moving waves as functions of phase speed,  $C$ , and wave number,  $k$ . The wind field entered the equations as the mean wind velocity  $U(1/|kl|)$  at a height above the mean surface of  $z = 1/|kl|$ .

The maximum energy input to the waves by the wind was found to occur approximately when

$$\frac{C}{U(k^{-1}) \cos \theta} \approx 0.8 \quad (1)$$

where  $C$  is the phase speed of the waves, and  $\theta$  is the angle between wind and wave propagation. One should note that in a boundary layer over a rigid surface the propagation speed of pressure fluctuations is approximately 80% of the free stream velocity (Willmarth and Woolridge, 1962). The Phillips resonance would be excited by pressure fluctuations propagating at  $0.8 U_m$ , and Townsend's result shows that the waves excited by resonance will tend to grow through feedback instability.

The direction  $\theta$  of the fastest growing waves, according to Phillips' and Townsend's mechanism will have a mean or most likely frequency  $\sigma$  that satisfies

$$\sigma = \vec{k} \cdot \vec{U}_m = k U_m \cos \theta. \quad (2)$$

Using the linear dispersion relation,

$$\sigma = (gk)^{1/2} \quad (3)$$

we have

$$\theta = \cos^{-1} \frac{C}{U_m}, \quad \text{or} \quad \sigma = \frac{g}{U_m \cos \theta}. \quad (4)$$

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Here  $U_m = 0.8 U$ , approximately, where  $U$  is the wind speed,  $k$  and  $C$  are the wave number and phase speed of the dominant wind wave. Here we have concentrated only on the mean wind velocity and the most energetic wave component. If the turbulent eddies were convected rigidly, the maximum wave energy would occur at the frequency given by (4). In reality the turbulence eddies are evolving as they are convected by the mean wind. There should thus be a range of frequencies that would satisfy (4). The exact range of the resonant frequency depends on the turbulence statistics of the wind field as discussed by Phillips (1957); therefore, (4) should be treated as a mean or the most likely value of the resonant frequency.

For waves that propagate symmetrically at an angle  $\theta$  to the right and left of the wind direction, the resulting wave field

$$\begin{aligned} \eta(x,y,t) &= \frac{a}{2} \cos(kx \cos \theta + ky \sin \theta - \sigma t) \\ &+ \frac{a}{2} \cos(kx \cos \theta - ky \sin \theta - \sigma t) \\ &= a \cos(kx \cos \theta - \sigma t) \cos(ky \sin \theta) \end{aligned} \quad (5)$$

corresponds to a short-crested wave propagating in the  $x$ -direction with a cross-wind crest length of  $2\pi/(k \cos \theta)$ . As will be seen in the observational data that we present, there are situations where one side lobe seems to dominate, and the wave field becomes definitely skewed about the wind direction. Recent developments in aircraft radars, designed to resolve the directional properties of surface waves, have enabled us to obtain quantitative data of the directional spectra of wind wave to address this problem.

## Data Sources

Our data were collected primarily during MASEX and other field experiments in that general area. MASEX stands for Meso-scale Air-Sea EXperiment, a field study conducted off the east coast of the United States in January 1983. The meteorological conditions during the experiment are described by Chou (1993). The wave data used here were collected by Walsh *et al.* (1989). There is some difference of opinion on the interpretation of these data. Walsh *et al.* (1989) used slanted fetch to explain the off-wind direction of the waves. Based on this argument, any bays, bends, or dents in the coastline can give a longer fetch and cause more mature waves to radiate from such locations. We will not totally rule out such a possibility. But in the slanted fetch analysis, the key parameter, as defined by Donelan *et al.* (1985), is  $U \cos \theta / C$ , with  $\theta$  as the angle between the wind and the waves. In our case, the match of the frequency with the Phillips resonant condition gives  $U \cos \theta / C \approx 1$ , which renders the parameterization of slanted fetch meaningless.

## Analysis

We have concentrated on data taken during MASEX cold-air outbreaks with nearly steady offshore wind, so that the wave development with distance from the coast was purely spatial rather than temporal. The instruments that collected the data were the Surface Contour Radar (SCR) of Walsh (1991) and the Radar Ocean Wave Spectrometer (ROWS) of Jackson (1991).

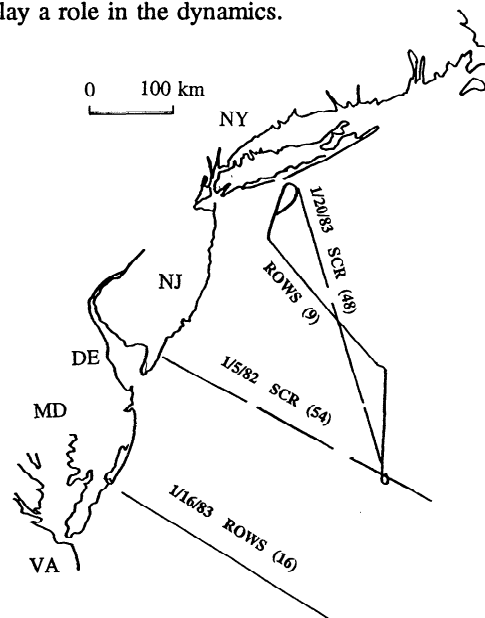
SCR is a radar with a scanning pencil beam (with half beam width of the transmitting antenna of  $1.4^\circ \times 1.4^\circ$ ), which scans across the flight direction with a range of  $\pm 15^\circ$ . This covers

approximately 400 meters of the ocean surface across-track when flying at an altitude of 400 m. The water surface elevation is measured directly to produce a digital map of sea surface topography with an absolute accuracy of  $\pm 30$  cm. Directional spectra can be calculated from these data. The accuracy of the direction is  $\pm 10^\circ$ . ROWS is a rotary scanning radar with a  $10^\circ$  to  $15^\circ$  off-nadir elliptical ( $4^\circ$  azimuth by  $10^\circ$  elevation beam width) beam rotating at a rate of 6 rpm. It obtains wave slope spectra in each complete conical scan with a direction resolution of  $\pm 15^\circ$  under typical flight conditions. More detailed discussions of the instrumental errors can be found in Walsh *et al.* (1991) and Jackson (1991).

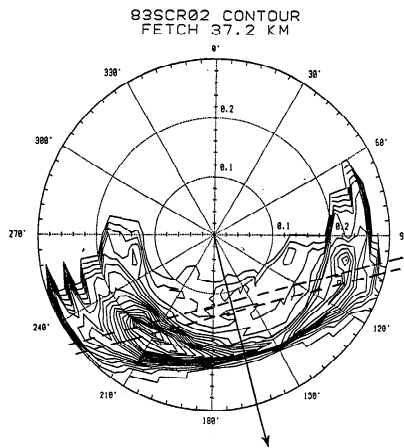
The data used in this study were collected off the Atlantic Coast between Maryland and Connecticut on January 5, 1982, and on January 16 and 20, 1983. The aircraft tracks are shown in Figure 1, identified by date and instrument.

On January 20, 1983, a cold-air outbreak covered most of the eastern seaboard of the United States, with the wind blowing persistently from a direction of  $350^\circ$  true. During the one and a half hour flight, the wind speed varied within 11 to 13 m/sec, and the directional variation was less than  $10^\circ$ . The flight covered a distance of 350 km, and the resulting directional wave spectrum obtained by the SCR at a distance of 37 km from the coast, shown in Fig. 2, is typical of the series. The arrow shows the wind direction, and the dashed lines indicate where spectral peaks should be found according to the Phillips (1957) and Townsend (1972) analyses (Eqs. 1. and 4.). Although both peaks lie along the resonant line, the peak to the right of the wind dominates.

As another example, the January 5, 1982 flight was again made during a cold-air outbreak with off-shore wind from a direction of  $300^\circ$  with a wind speed of 13 to 15 m/sec. Fig. 3 shows a typical spectrum, obtained at 35 km from the coast, in addition to a weak swell coming from the direction of Cape Hatteras. The wind waves have a clear maximum near the left resonance peak, while there is no sign of the resonant waves to the right. This indicates that the swell may play a role in the dynamics.



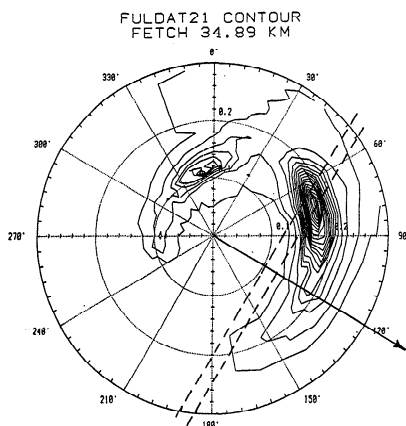
**Figure 1.** Flight tracks of the four different observational missions from which directional wave spectra cited were obtained. The numbers in parentheses beside each line were the number of directional wave spectra obtained on the flight. SCR stands for the Surface Contour Radar; ROWS, the Radar Ocean Wave Spectrometer.



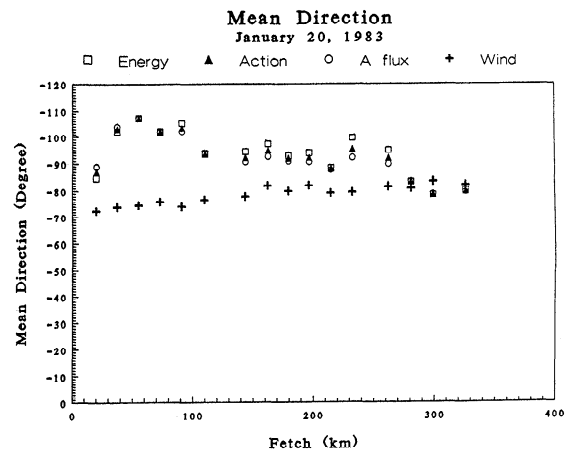
**Figure 2.** Contour plot of a typical wave directional spectrum obtained on the flight with the SCR on January 20, 1983 at a fetch of 37.2 km. The radial coordinate is wave frequency in Hz, as calculated from the linear dispersion relation. The arrow shows the wind direction; the dashed lines, the location of spectral peaks that satisfy the Phillips (1957) resonance criterion (Eq. 3) with the wind speed range of 11 to 13 m/sec.

Fig. 4 summarizes the results of sixteen spectra obtained from the flight on January 20, 1983. We have plotted the wind direction, and the mean directions of the spectra of wave energy, wave action and action flux as functions of fetch. This illustrates how the wave evolves towards the wind direction with increasing fetch and wave phase speed. At the initial stage (fetch less than 50 km, say) of wave development, the trend of the angle between wind and wave directions seems to drop down a little, that is, the angles are actually smaller. This reversion of the trend is an artifact due to our definition of the mean direction, which is

$$\bar{\theta} = \frac{\int \int \phi(\sigma, \theta) \theta \sigma d\sigma d\theta}{\int \int \phi(\sigma, \theta) \sigma d\sigma d\theta} \quad (6)$$



**Figure 3.** A spectrum as shown in Figure 2, obtained with the SCR on January 5, 1982 off the New Jersey Coast at a mean fetch of 34.89 km. Again, the spectral energy peak lies near the resonance criterion for the wind speed range of 13-15 m/sec. The fact that the spectrum is so highly asymmetric about the wind direction may be due to the presence of the swell as shown by the weak low frequency peak. The swell direction in the open ocean is more southerly. Near the coast as in this figure, its direction has been refracted towards the normal to the shoreline by the bottom topography.

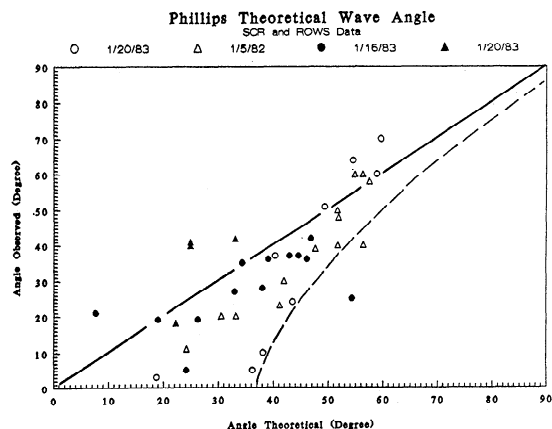


**Figure 4.** Summary of mean wave directions calculated from the data along the entire flight line of January 20, 1983. The mean direction is computed from moment integrals; therefore, a pair of nearly symmetric peaks will yield a low value for the mean deviation from the wind. At the initial stage of wave development, the two nearly symmetric peaks did gradually coalesce into a single peak on one side of the wind. This caused the initial increase in the mean deviation angle. Then, the single peak gradually shifted and aligned itself with the mean wind as the wave speed increased in conformity with the resonance condition.

The integrations cover the range in frequency of 0.1 to 0.3 Hz, which coincides with the main wind wave energy. The limits of 0 to  $2\pi$  are used because we have removed the 180° ambiguity; therefore, all the values of energy density retained are valid wind wave energy values. With this definition of the mean direction, a nearly symmetric angular distribution of energy will yield a low mean value. In our case, at the initial stage of wave generation, the two peaks in the spectrum are nearly symmetric about the wind direction, but each individual maximum is at a larger angle from the wind direction. This symmetric distribution did not persist at larger fetches: One side of the spectrum became dominant and the mean direction increased, then, as the waves became longer with still larger fetch and as their direction turned closer to the wind direction. The evolution process of direction conforms well with the predictions of the Phillips (1957) and Townsend (1972) theories.

Fig. 5 summarizes the cases from all the flights shown in Fig. 1, in which the directions of the dominant waves are plotted against the Phillips and Townsend resonance angles. Most of our data fall in between the values of the two theories. It should be pointed out that, in plotting Townsend's value, for lack of the true wind profile, we have taken the mean wind rather than the  $U(k^{-1})$  in equation (4).

Finally, we also point out that although there should be two equal side lobes according to both the Phillips (1957) and the Townsend (1972) theories, this definitely does not seem to occur. The side lobes are unequal in magnitude in all the cases we observed even when both lobes materialized: One side lobe always develops much faster than the other, and, eventually, becomes the only peak in the directional spectrum. There are also cases when only one lobe appeared. The selection between side lobes seems to depend upon the swell or the pre-existing wave field. The possible existence of swell in Fig. 2 was not well resolved because of the limited distance covered in the lateral scan of the SCR. If there were swell, logic would only allow it to come from the open ocean. Then, it would propagate against the weak side



**Figure 5.** Summary of the peak deviation angles from the mean wind direction from all four flights. The horizontal axis is the theoretical peak deviation computed from local wind and wave data according to equation (3); the vertical axis is the observed angle. The values computed according to equation (5) are given as a dashed line. Most of the data points lie in between the area predicted by Phillips' (1957) and Townsend's (1972) calculations. Open symbols represent data taken by the Surface Contour Radar (SCR), and the solid symbols represent data taken by the Radar Ocean Wave Spectrometer (ROWS).

lobe in the spectrum. The northerly propagating swell in Fig. 3 was clearly shown, which propagated nearly opposite to the absent southerly side lobe. Since the swell systems have much longer wave lengths than the locally generated wind waves, the weakly nonlinear wave-wave interactions would not be effective here; therefore, a direct effect of the swell cannot be important. The effect of waves, however, may well involve a modification of the boundary layer and the wind stress field as reported by Rieder *et al.* (1994). Such modifications of the wind field can be an indirect rather than a direct effect upon the wind wave generation processes. Our observations are also consistent with a recent study by Geerneart *et al.* (1993), who also suggested that the swell direction could have a strong influence on the stress direction.

## Conclusion

In summary, our observations indicate that the maximum wind wave growth seems to occur near the direction for the Phillips resonance, but at a smaller angle as suggested by Townsend's feedback mechanism. More work is needed to understand the reasons why one spectral side lobe is preferred, although both should be possible according to the resonance criterion. If wind waves can indeed propagate off angle to the generating wind in the short fetch coastal region, the present wind wave models may need modifications for coastal applications.

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