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A new predictive formula for inception of regular wave breaking

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ABSTRACT

Efforts are made to enhance the predictive formula for the inception of wave breaking. To achieve success, the existing formulas are extensively reviewed. They are categorized into four types, i.e., the McCowan type, the Miche type, the Goda type and the Munk type. The inherent relations among the different types are then exploited. The differences among each formula within a group are also discussed. Four representative formulas from the different types are chosen to compare with the measured data for a total number of 1193 cases reported in literatures. It is shown that Goda's and Ostendorf and Madsen's formulas are advantageous in general among the selected ones. Goda's formula, however, is found to be inaccurate as the beach slope becomes steeper than 1/10. Ostendorf and Madsen's formula is fairly good even for cases of very steep slopes, but its accuracy for the cases of ordinary slopes is not as good as Goda's. A new predictive formula for the inception of wave breaking is proposed. The unique index, defined by $\psi'_b=(1.21-3.30\lambda_b)(1.48-0.54\gamma_b)\psi_b$, where $\psi_b=gH_b/C_b^2$, H_b is the breaking wave height, C_b is the breaking wave celerity, λ_b is the breaking wave steepness, γ_b is the relative breaking wave height, and g is the gravity acceleration, is introduced. The incipient condition of wave breaking is then given by $\psi'_b=0.69$. This formula is a significant improvement to the existing ones in terms of the accuracy. In addition, it is a local relation. Further verification shows that the proposed formula performs similarly well when applied to the field and to the waves over permeable bed.

1. Introduction

Wave breaking is an amazing phenomenon to various people. The artists are amazed at its dynamic beauty and the scientists are amazed at its beautiful dynamics. The feeling of the coastal engineers on wave breaking, however, is often mixed. Breaking wave dynamics is a theoretically unsolved topic, and the breaking processes are so complicated that an observation can provide only limited information with generality even by means of the most recent high technology. On the other hand, the practicing engineers often have to provide solutions anyhow to the phenomenon because it is so closely related to the important problems such as the wave force on structures and the wave induced sediment transport in the surf zone.

It is believed that a wave break as an intrinsic relation is satisfied among the breaker height, the local water depth, the local wavelength, the bottom slope, and probably also some other parameters. To find this relation in a general form, however, is not easy. Efforts have been continued for more than a century. The initial contributions now are usually attributed to Michell (1893) and McCowan (1894). Based on the assumption that a solitary wave breaks as its crest angle approaches a limiting value or the fluid velocity at the crest surpasses the celerity of the profile, McCowan (1894) derived the following relation:

$$\frac{H_b}{h_b}=0.78 \tag{1}$$

where H_b is the breaker height, h_b is the water depth at the breaking point. Michell (1893) found the limiting steepness of deepwater waves or the breaking condition of deepwater waves:

$$\frac{H_b}{L_b}=0.142 \tag{2}$$

where L_b is the breaking wavelength. Miche (1944) then generalized Michell's (1893) condition and obtained the distinguished formula for periodic waves over arbitrary water depth:

$$\frac{H_b}{L_b}=0.142\tanh\frac{2\pi h_b}{L_b} \tag{3}$$

In engineering applications, Goda's (1970, 1975) formula seems to have gained the highest reputation. After a slight modification to enhance its performance for steep slopes (Goda, 2010), Goda's formula can be written as

$$\frac{H_b}{L_0}=0.17\left\{1-\exp\left[-1.5\frac{\pi h_b}{L_0}\left(1+11s^{4/3}\right)\right]\right\} \tag{4}$$

where s is the beach slope.

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Table 1
McCowan type formulas for inception of wave breaking.

Functional form	Sources
0.73	Boussinesq (1871)
	Laitone (1960)
0.78	McCowan (1894)
0.83	Gwyther (1900)
	Davies (1952)
	Yamada et al. (1968)
	Yamada (1957)
	Lenau (1966)
	Longuet-Higgins and Fenton (1974)
	Witting (1975)
	Longuet-Higgins and Fox (1977)
0.87	Chappelear (1959)
1.03	Packham (1952)
$[1.40-\max(s,\ 0.07)]^{-1}$	Galvin (1969)
$0.72(1+6.4s)$	Madsen (1976)
$1.062+0.137\log(s\lambda_0^{-1/2})$	Battjes (1974)
$1.1s^{1/6}\lambda_0^{-1/12}$	Sunamura (1980)
$0.937s^{0.155}\lambda_0^{-0.13}$	Singamsetti and Wind (1980)
$1.14s^{0.21}\lambda_0^{-0.105}$	Larson and Kraus (1989)
$1.12(1+e^{-60s})^{-1}-5.0(1-e^{-43s})\lambda_0$	Smith and Kraus (1990)
$0.284\lambda_0^{-1/2}\tanh[\pi\lambda_0^{1/2}]$	Camenen and Larson (2007)

It is of interest to note that a tremendous number of formulas have been proposed to describe the incipient condition of wave breaking up to now. The earlier studies were mainly focusing on the solitary waves and the waves under deep water conditions. The periodic waves at the breaking point were usually approximated as solitary waves at that time. After the 1940s, the breaking of periodic waves on sloping beaches has been emphasized, and rather general formulas for the inception of breaking then became available. Detailed reviews of the existing researches have been made by Galvin (1972), Sawaragi (1973), Rattanapitikon et al. (2003), Camenen and Larson (2007), and Goda (2010). Difficulties in the establishment of a universal formula are probably that too many factors affect wave breaking. The inherent variability of the phenomena (Goda, 2010) further complicates the problem. In spite of this fact, the efforts to obtain a "better" formula have never stopped during the past decades, probably because a generally valid formula is of too much keen interest to scientists and engineers.

A reliable formula or diagram for the inception of wave breaking may have to be fitted or verified by a significantly large number of data covering a wide range of beach topography and wave conditions. Weggel's (1972) formula as well as Komar and Gaughan's (1972) formula, both recommended by Coastal Engineering Manual authorized by Coastal and Hydraulics Laboratory, US Army Corps of Engineers, are certainly based on a reasonably large number of laboratory data. Goda's (1970, 1975) formula, which has been considered as the standard in the Asian coastal engineering community and also highly appreciated by the coastal engineers from other countries, fitted 215 sets of data from 8 different sources and covered a wide range of the bottom slope from 1/100 to 1/9. Other achievements that have been frequently mentioned in literatures or otherwise are equivalently reliable, including those of Iversen (1951), Ostendorf and Madsen (1979), Singamsetti and Wind (1980), Larson and Kraus (1989), Smith and Kraus (1990), Rattanapitikon and Shibayama (2000), Camenen and Larson (2007), are all based on a large number of data measured by the authors themselves, or by collecting from various sources, or by a combination of both ways.

Table 2
Miche type formulas for inception of wave breaking.

α	ξ	Sources
0.142	1.0	Miche (1944)
0.14	0.9	Battjes and Janssen (1978)
0.14	$0.8+5.0\min(s,0.1)$	Ostendorf and Madsen (1979)
0.14	$0.57+0.45\tanh(33\lambda_0)$	Battjes and Stive (1985)
$0.127e^{4s}$	1.0	Kamphuis (1991)
0.14	$-11.21s^2+5.01s+0.91$	Rattanapitikon and Shibayama (2000)

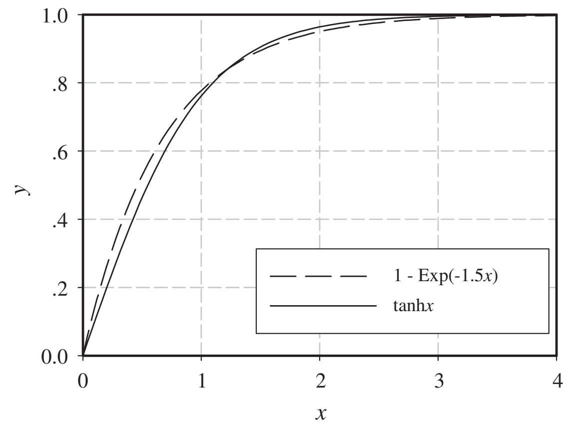


Fig. 1. Comparison of tanh x and $1-\mathrm{Exp}(-1.5x)$.

Critical investigations into the accuracy of the existing formulas for the inception of wave breaking by comparing them with a large number of measured data have also been carried out during the past two decades. Kamphuis (1991) used 225 sets of data obtained by himself to investigate the accuracy of 11 formulas; Rattanapitikon and Shibayama (2000) collected 574 sets of data from 24 sources to examine 24 formulas; Rattanapitikon et al. (2003) further solidified the conclusions of Rattanapitikon and Shibayama (2000) by adding another 121 sets of data measured in large scale wave flumes; Camenen and Larson (2007) also collected more than 500 sets of data from 22 published sources covering a wide range of beach slopes and wave conditions to compare the accuracy of 6 existing formulas.

The present study is trying to make an even more critical comparison of the formulas that are most widely preferred by coastal engineers and are considered as accurate enough within the inherent variability of the phenomena, by increasing the number of data for verification. At the same time, we propose a new formula with an expectation that it can fit the experimental data with evidently smaller errors, and then verify the formula under some critical conditions.

2. Inherence of existing formulas

The breaking condition for the solitary waves over a constant water depth can be expressed by Eq. (1) as many authors pointed out. Different authors, however, obtained different values for the constant. If the formula is generalized to represent the breaking condition for periodic waves on beaches, the constant may then have to be replaced by a function of the beach slope and probably also the deep water wave conditions. Thus, Eq. (1) has a general form:

$$\frac{H_b}{h_b}=\gamma(s,\lambda_0) \tag{5}$$

where $\lambda_0=H_0/L_0$ is the deep water wave steepness with H_0 being the incident wave height. Eq. (5) is called the McCowan (1894) type formula for the inception of wave breaking in the present study. The

Table 3
Goda type formulas for inception of wave breaking.

α'	ξ'	Sources
0.17	$0.5+7.5s^{4/3}$	Goda (1970)
0.17	$0.52+2.36s-5.40s^2$	Rattanapitikon and Shibayama (2000)
0.17	$0.5+5.5s^{4/3}$	Goda (2010)

Table 4
Munk type formulas for inception of wave breaking.

m	β	Sources
$-1/3$	0.3	Munk (1949)
$-1/4$	$0.76s^{1/7}$	LeMehaute and Koh (1967)
$-1/5$	0.56	Komar and Gaughan (1972)
$-1/4$	$s^{1/5}$	Sunamura and Horikawa (1974)
-0.254	$0.575s^{0.031}$	Singamsetti and Wind (1980)
$-1/4$	$0.68s^{0.09}$	Ogawa and Shuto (1984)
-0.24	0.53	Larson and Kraus (1989)
$-0.30+0.88s$	$0.34+2.47s$	Smith and Kraus (1990)
-0.28	0.478	Gourlay (1992)
$-1/5$	$0.55+1.32s-7.46s^2+10.02s^3$	Rattanapitikon and Shibayama (2000)

functional forms of γ previously obtained by different authors are summarized in Table 1.

The general form of the formula for the inception of breaking of periodic waves can also be obtained by modifying Eq. (3):

$$\frac{H_b}{L_b}=\alpha(s,\lambda_0)\tanh\left[\xi(s,\lambda_0)\frac{2\pi h_b}{L_b}\right] \tag{6}$$

where $\alpha(s,\lambda_0)$ and $\xi(s,\lambda_0)$ take different forms according to different authors, as summarized in Table 2. Eq. (6) is called the Miche (1944) type formula in the present study. Since Camenen and Larson (2007) showed that h_b/L_b depends on λ_0 nearly uniquely, Eq. (6) is formally consistent with Eq. (5). A slightly altered form of Eq. (6) can be written as

$$\frac{H_b}{L_0}=\alpha'(s,\lambda_0)\tanh\left[\xi'(s,\lambda_0)\frac{2\pi h_b}{L_0}\right]. \tag{7}$$

Eq. (7) is of merit because the breaking wavelength is not involved. If we note that tanh x can be approximated by $1-\exp(-1.5x)$ with rather good accuracy as shown in Fig. 1, Eq. (7) can be approximated by

$$\frac{H_b}{L_0}=\alpha'(s,\lambda_0)\left\{1-\exp\left[-1.5\xi'(s,\lambda_0)\frac{2\pi h_b}{L_0}\right]\right\}. \tag{8}$$

Eq. (8) is called the Goda (1975) type formula for breaking waves. The relevant parameters determined by Goda (1970, 2010) and modified by others are summarized in Table 3.

When considering shoaling of the normally incident wave train on a sloping beach, the shoaling coefficient that resulted from the conservation of energy flux is known to be almost uniquely determined by the local value of the relative water depth, which implies that

$$\frac{H_b}{H_0}=f\left(\frac{h_b}{L_0}\right) \tag{9}$$

where f is a definite function. Substituting Eq. (9) into Eq. (7) we have

$$\frac{H_b}{L_0}=f\left(s,\lambda_0,\frac{H_b}{H_0}\right) \tag{10}$$

where f is another known function. Eq. (10) provided a basis for another popular type of formulas for the inception of wave breaking:

$$\frac{H_b}{H_0}=\beta(s)\left(\frac{H_0}{L_0}\right)^m \tag{11}$$

called the Munk (1949) type in the present study. The parameters suggested by different authors are summarized in Table 4.

The McCowan type formulas for the inception of wave breaking is expressed as the relative wave height (i.e., the ratio of the wave height to the water depth) being limited by a critical value. For solitary waves over a constant water depth, the critical value equals to 0.83 according to the relatively recent researches (Longuet-Higgins and Fenton, 1974; Longuet-Higgins and Fox, 1977; Witting, 1975; Yamada et al., 1968), although it was reported to vary from 0.73 to 1.03 in the earlier literatures. In general, the relative wave height at the breaking point depends on the beach slope and the incident wave steepness. No consensus, however, has been reached on the functional form of the relationship, as shown in Table 1. Fig. 2 further indicates that there are significant differences among suggestions by different authors. Even so, a majority of the authors who studied the problem still realized a slight increase of the relative wave height at the breaking point with the beach slope. A decreasing tendency of the relative wave height at the breaking point with the incident wave steepness is also clear. The critical value of the relative wave height is usually less than 0.83 for waves over gentle slopes ($s\leq1/100$) and its range of variation tends to increase as either the beach slope or the incident wave steepness increases.

Different from the McCowan type, the Miche type of formulas for the inception of wave breaking proposed by various authors does not vary in a

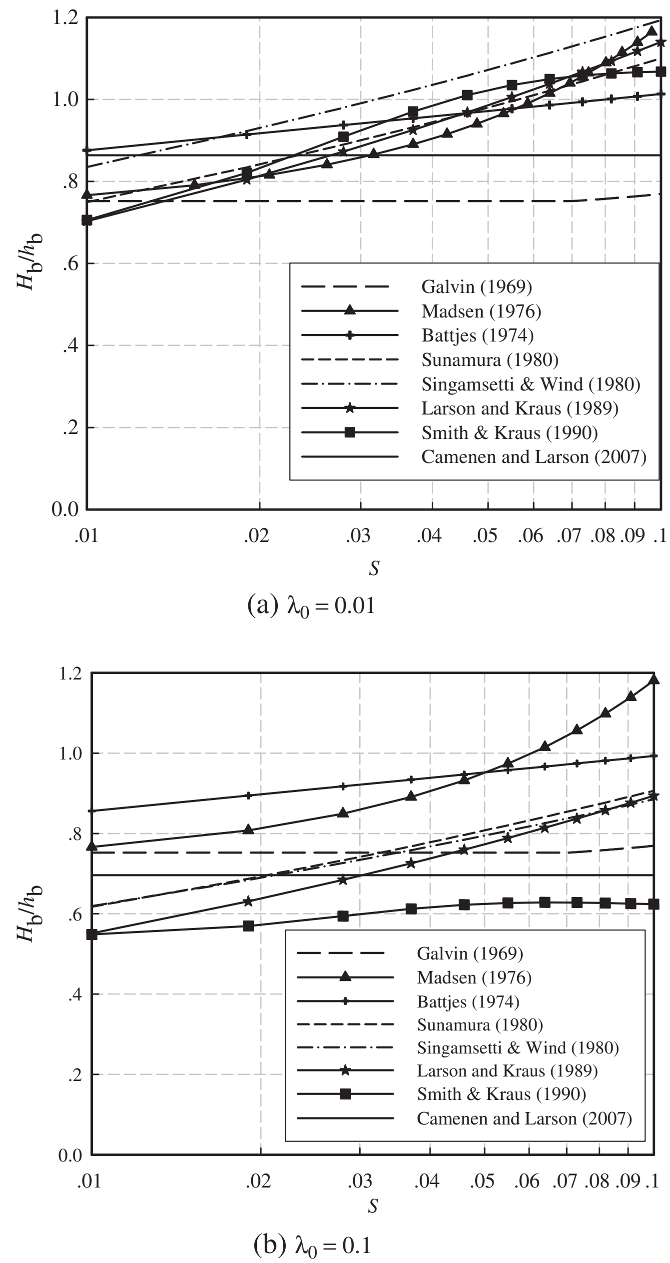
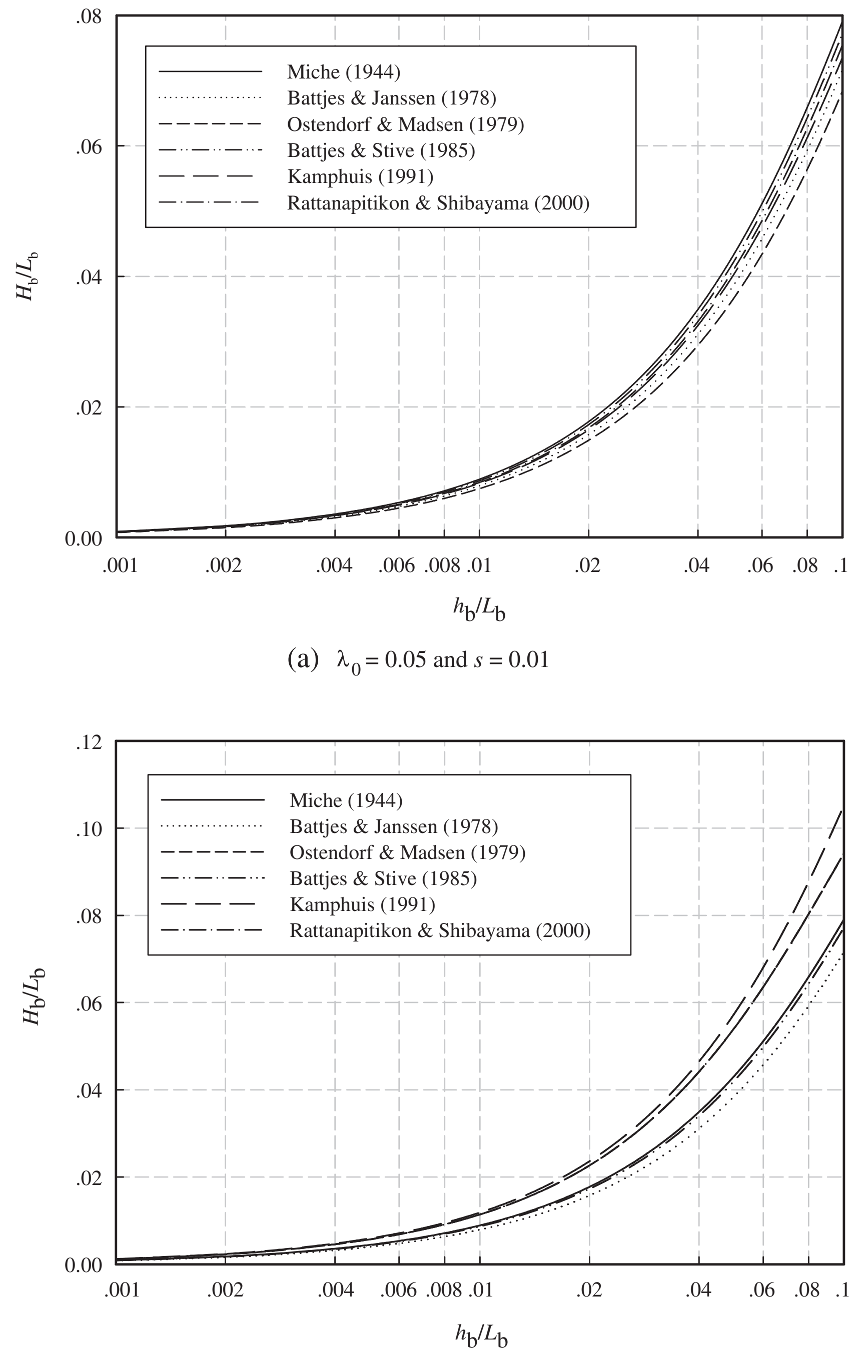


Fig. 2. Comparison of McCowan type formulas for wave breaking.



(a) $\lambda_0 = 0.05$ and $s = 0.01$

(b) $\lambda_0 = 0.05$ and $s = 0.1$

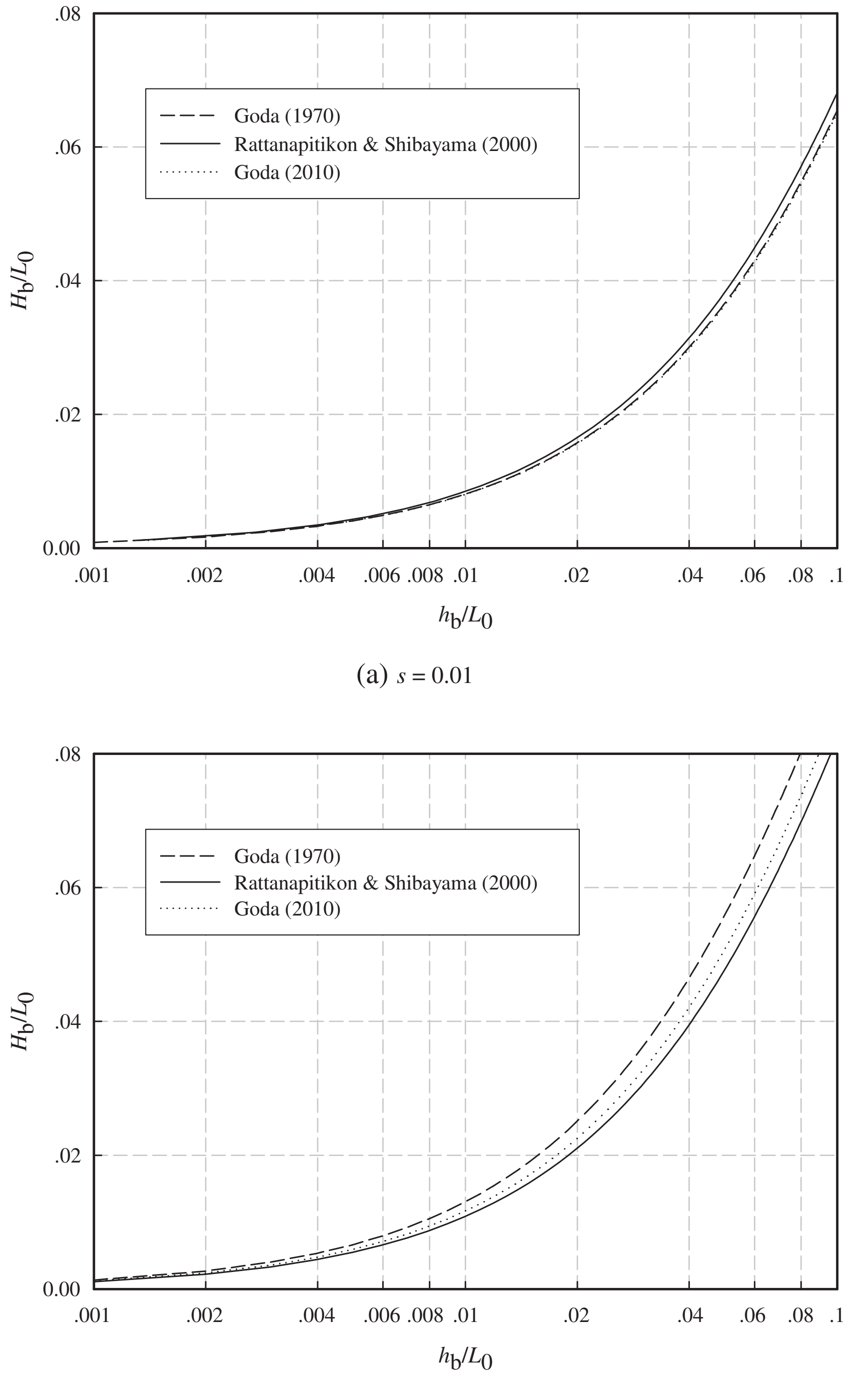
Fig. 3. Comparison of Miche type formulas for wave breaking.

wide range as shown in Fig. 3. The Goda type formulas are very similar to the Miche type, as demonstrated in Fig. 4. In this sense, the Miche or Goda type formulas are probably the most reliable representation of the intrinsic relation among the breaking wave parameters. In fact, the McCowan type formulas can be treated as a special case of Miche type under the long wave condition. As $2\pi h_b/L_b \rightarrow 0$, Eq. (6) reduces to

$$\frac{H_b}{L_b} \rightarrow \alpha(s,\lambda_0)\xi(s,\lambda_0)\frac{2\pi h_b}{L_b} \tag{12}$$

which is exactly Eq. (5).

The Munk type formulas for the inception of wave breaking are actually a little controversial. As demonstrated in Fig. 5, the formulas proposed by many authors have almost the same tendency and, in fact, all of them agree with the experimental data obtained in wave flumes with a reasonable accuracy, considering the variability of the measured data. However, as mentioned above, the basic form of the formulas depends on the unique relation between the local wave height and incident wave height governed by the conservation of energy flux, so it is definitely not valid for the waves undergoing a significant deformation in the horizontal plane without a modification. In addition, the formulas give only the breaking wave height, and their accuracy when used to determine the breaking point can hardly be ensured.



(a) $s = 0.01$

(b) $s = 0.1$

Fig. 4. Comparison of Goda type formulas for wave breaking.

3. Accuracy of representative formulas

To demonstrate the accuracy of the existing formulas for the inception of wave breaking, four representatives are selected to compare with the experimental data collected from literatures. The four formulas are:

(1) Singamsetti and Wind's (1980)

$$\frac{H_b}{h_b} = 0.937 s^{0.155} \lambda_0^{-0.13} \tag{13}$$

(3) Rattanapitikon and Shibayama's (2000)

$$\frac{H_b}{H_0} = \left(0.55 + 1.32s - 7.46s^2 + 10.02s^3\right)\left(\frac{H_0}{L_0}\right)^{-1/5} \tag{14}$$

(3) Ostendorf and Madsen's (1979)

$$\frac{H_b}{L_b} = 0.14\tanh\left\{[0.8 + 5\min(s, 0.1)]\frac{2\pi h_b}{L_b}\right\} \tag{15}$$

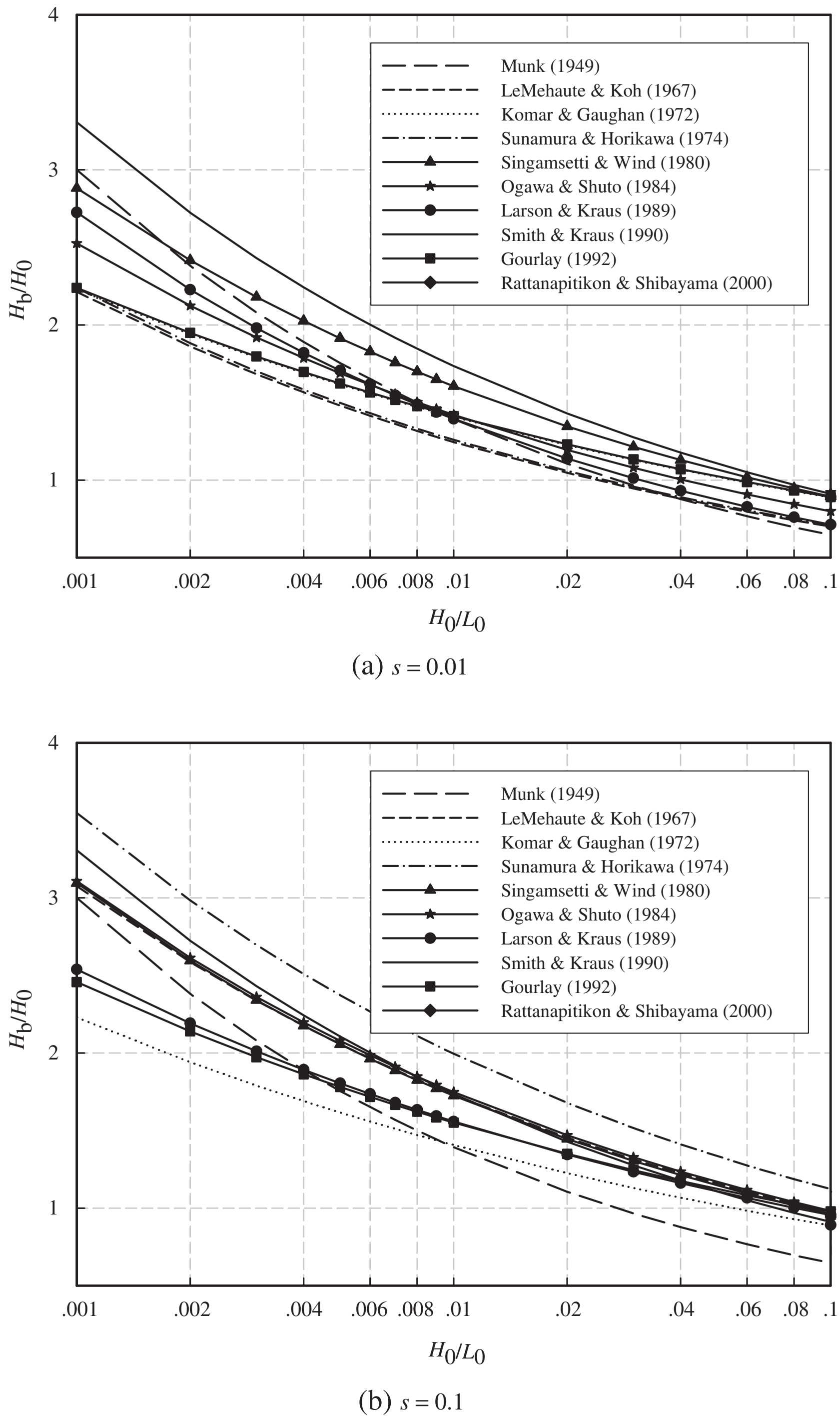


Fig. 5. Comparison of Munk type formulas for wave breaking.

(4) Goda's (2010)

$$\frac{H_b}{L_0} = 0.17\left\{1-\exp\left[-1.5\frac{\pi h_b}{L_0}\left(1 + 11s^{4/3}\right)\right]\right\} \tag{16}$$

Singamsetti and Wind's (1980) formula belongs to the McCowan type, Rattanapitikon and Shibayama's (2000) formula is the Munk type, Ostendorf and Madsen's (1979) formula is the Miche type, while Goda's (2010) formula was recently modified by the original author himself. The data collected from the different sources are listed in Table 5. A total number of 1193 experimental cases, covering a wide range of the beach slope from 1/100 to about 1/3, are included. It may be necessary to point out that totally 55 cases in the data set do not have information on H_0. Therefore, only 1138 cases are available when verifying Eqs. (13) and (14).

For the cases in which the beach slope is steeper than 0.1, we compared their deepwater wave steepness with Miche's (1944) limiting value below which waves are fully reflected before breaking, and found that no case is reflection-dominated in the data sets.

For a meaningful comparison, we rewrite Eqs. (14), (15) and (16) as

$$\frac{H_b}{h_b} = \left(0.55 + 1.32s - 7.46s^2 + 10.02s^3\right)\left(\frac{H_0}{h_b}\right)\left(\frac{H_0}{L_0}\right)^{-1/5} \tag{17}$$

Table 5
Summary of data on breaking waves.

Source	No. of cases	Beach condition	Beach slope
Iversen (1951)	63	Plane	0.02–0.1
Hamada et al. (1956)	4	Plane	0.1
Kishi and Ihara (1956)	27	Plane	0.059–0.11
Goda (1964)	33	Plane	0.01
Horikawa and Kuo (1966)	60	Step	–
Horikawa and Kuo (1966)	97	Plane	0.01–0.05
Goda et al. (1966)	6	Plane	0.1
Nakamura et al. (1966)	374	Plane	0.01–0.1
Galvin (1968)	4	Plane	0.05–0.2
Toyoshima et al. (1968)	66	Plane	0.033–0.05
Bowen et al. (1968)	11	Plane	0.083
Galvin (1969)	18	Plane	0.05–0.2
Sugie and Kawaguchi (1972)	21	Plane	0.067
Saeki and Sasaki (1973)	2	Plane	0.02
Iwagaki et al. (1974)	23	Plane	0.01–0.05
Walker (1974)	15	Plane	0.03
Singamsetti and Wind (1980)	95	Plane	0.025–0.2
Mizuguchi (1980)	1	Plane	0.01
Ishida and Yamaguchi (1983)	6	Plane	0.1
Visser (1982)	7	Plane	0.05–0.1
Maruyama et al. (1983)	7	Plane	0.03
Iwata et al. (1983)	3	Plane	0.1111
Stive and Battjes (1984)	2	Plane	0.03
Battjes and Stive (1985)	20	Plane	0.025–0.07
Sakai et al. (1986)	19	Plane	0.02–0.033
Maruyama and Shimizu (1986)	10	Plane	0.02–0.05
Aono and Hattori (1988)	7	Plane	0.05
Smith and Kraus (1990)	5	Plane	0.03–0.044
Smith and Kraus (1990)	75	Barred	0.033–0.38
Chanson and Lee (1995)	39	Plane	0.0836
Kakuno et al. (1996)	55	Plane	0.033–0.1
Xiao (2005)	6	Plane	0.005
Lara et al. (2006)	12	Plane	0.05
Total	1193		0.01–0.38

$$\frac{H_b}{h_b} = 0.14\left(\frac{h_b}{L_b}\right)^{-1}\tanh\left\{[0.8 + 5\min(s, 0.1)]\frac{2\pi h_b}{L_b}\right\} \tag{18}$$

$$\frac{H_b}{h_b} = 0.17\left(\frac{h_b}{L_0}\right)^{-1}\left\{1 - exp\left[-1.5\frac{\pi h_b}{L_0}\left(1 + 11s^{4/3}\right)\right]\right\}. \tag{19}$$

Denoting $\gamma_b \equiv H_b/h_b$, we can thus compare the values of γ_b calculated from Eqs. (13), (17), (18), and (19) with those measured, as shown in Fig. 6. The dash lines in the figures are the 20% error lines. The comparison, however, reveals that none of the selected formulas can avoid a significant scattering. To quantify the error, we define the bias E and the standard deviation ν by

$$E = \frac{1}{N}\sum_1^N \frac{\gamma_{b,\text{calculated}} - \gamma_{b,\text{measured}}}{\left(\gamma_{b,\text{calculated}} + \gamma_{b,\text{measured}}\right)/2} \tag{20}$$

$$\nu = \sqrt{\frac{1}{N}\sum_1^N \left[\frac{\gamma_{b,\text{calculated}} - \gamma_{b,\text{measured}}}{\left(\gamma_{b,\text{calculated}} + \gamma_{b,\text{measured}}\right)/2}\right]^2} \tag{21}$$

where $\gamma_{b,\text{ calculated}}$ and $\gamma_{b,\text{ measured}}$ are the calculated and the measured values of γ_b, respectively, and N represents the total number of data. We also introduce the error index P_{10} and P_{20}, defined by the percentage of data of which the relative errors are less than 10% and 20%, respectively. The results of the error analysis for the four formulas are given in Table 6. It is noted that, in general, Singamsetti and Wind's (1980), Rattanapitikon and Shibayama's (2000), and Goda's (2010) formulas overestimate while Ostendorf and Madsen's (1979) formula underestimates γ_b. The standard deviation of Singamsetti and Wind's (1980), Ostendorf and Madsen's (1979), and Goda's (2010) formulas are of the same level, while Rattanapitikon and Shibayama's (2000) formula, which belongs to the Munk type, is obviously less reliable. If we pay attention to P_{10} and P_{20}, it is not difficult to find that Ostendorf and Madsen's (1979) and Goda's (2010) formulas are advantageous.

For some further information, we rewrite Eqs. (13) and (14) as follows

$$\frac{H_b}{L_b} = 0.937s^{0.155}\lambda_0^{-0.13}\left(\frac{h_b}{L_b}\right) \tag{22}$$

$$\frac{H_b}{L_0} = \left(0.55 + 1.32s - 7.46s^2 + 10.02s^3\right)\left(\frac{H_0}{L_0}\right)^{4/5}. \tag{23}$$

Denoting $\lambda_b = H_b/L_b$ and $\lambda'_b = H_b/L_0$, we can thus compare the values of λ_b or λ'_b calculated from Eqs. (13), (17), (18) and (19) with those measured, as shown in Fig. 7. In Fig. 7, different symbols are used to distinguish the data corresponding to the different range of the beach slope. The dash lines again are the 20% error lines. Now it becomes clear that Goda's (2010) formula is the best among the selected four if data for the beach slopes larger than 1/10 are excluded. Considering that the natural beach with a slope steeper than 1/10 is rare, Goda's (2010) formula has no critical problem in practice. Ostendorf and Madsen's (1979) formula is also very well developed. It is applicable to cases with extremely steep slopes, but it is a little less accurate than Goda's (2010) formula for some ordinary cases.

4. A new formula for inception of wave breaking

The celerity of a small amplitude wave is known to be governed by

$$C = \sqrt{\frac{gL}{2\pi}\tanh\frac{2\pi h}{L}} \tag{24}$$

If Eq. (24) is used to approximate the celerity of a breaking wave, Miche's (1944) formula (3) can then be transformed into

$$\left(\frac{gH}{C^2}\right)_b = 0.89 \tag{25}$$

At the shallow water condition, we have $C \rightarrow \sqrt{gh}$. Thus, Eq. (25) reduces to

$$\left(\frac{H}{h}\right)_b = 0.89 \tag{26}$$

This is not very different from the result of Yamada et al. (1968), Longuet-Higgins and Fenton (1974), Witting (1975), and, Longuet-Higgins and Fox (1977). Now, we can argue that the breaking index defined by

$$\psi = \frac{gH}{C^2} \tag{27}$$

may be very close to a universal constant at the breaking point.

As ψ is considered to be a combination of the local wave parameters, evaluation of the breaking wave celerity in terms of the water depth, the wave period, and the wave height becomes a problem that must be solved. Many coastal engineers believe that Eq. (24) is a reasonable choice for this purpose. But as Goda (1970) pointed out, the rate of celerity is 19.3% larger than that of a small amplitude wave in deepwater and 28.5% larger than that of a solitary wave, based on Yamada and Shiotani's (1968) study. A detailed study by Catalan and Haller (2008) found that, among quite a number of proposed formulas, Kirby and Dalrymple's (1986) and Booij's (1981) are more reliable. Since Booij's (1981) formula has a simpler form than Kirby and Dalrymple's (1986), it is employed in the present study. Thus, we have

$$C_b = \sqrt{\frac{gL_b}{2\pi}\tanh\frac{2\pi}{L_b}\left(h_b + \frac{H_b}{2}\right)}. \tag{28}$$

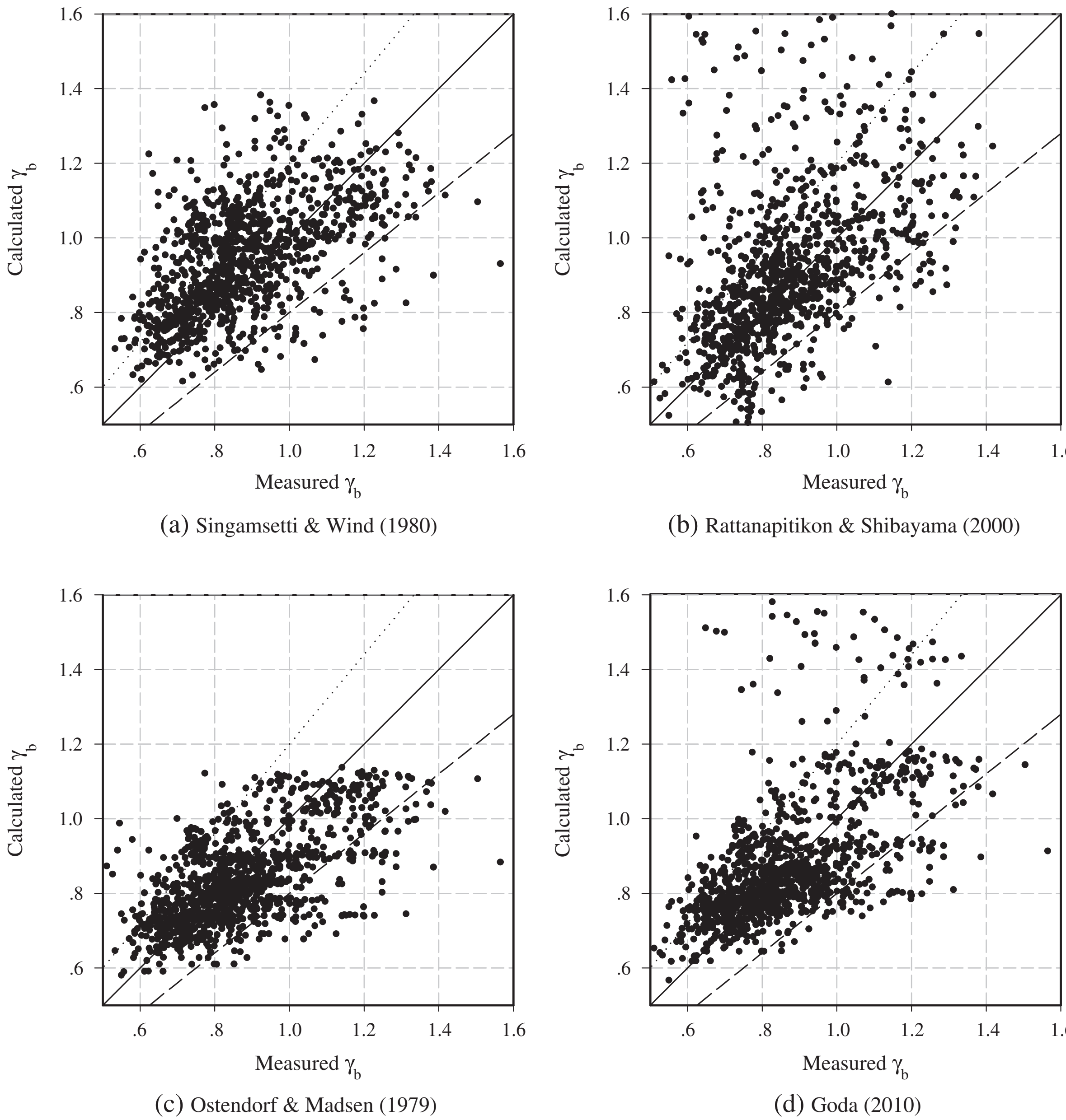


Fig. 6. Comparison of formulas with data.

By Eq. (28), the wave celerity C_b can be calculated from the water depth h_b, the wave height H_b, and the wave period T through an iteration procedure, considering $L_b=C_bT$. It can be shown that the wave celerity given by Eq. (28) is to some extent larger than that given by Eq. (24). Assuming that $H_b/h_b=0.8$, the celerity determined by Eq. (29) is 19% larger than the celerity of small amplitude waves at $h_b/L_b=0.01$ and is 13% larger at $h_b/L_b=0.1$, which are not qualitatively different from the results mentioned in Goda (1970).

Now we calculate ψ_b using the data set we collected (i.e., Table 5) and plot the results against the beach slope s, as shown in Fig. 8. It then becomes evident that the incipient condition of wave breaking in terms of ψ should be expressed, instead of Eq. (25), as

$$\psi_b \equiv \frac{gH_b}{C_b^2} = f(s) = 0.60 + 1.92s - 4.40s^2 \tag{29}$$

The parabolic function on the right hand side is obtained by curve fitting. The dash lines are the 20% error lines. It is also evidently clear that there is no case beyond $\psi_b>1.0$ and $\psi_b<0.4$, indicating that a wave should have broken if $\psi_b>1.0$ and is not breaking if $\psi_b<0.4$.

To make a comparison of the newly established formula with the existing ones, i.e., Eqs. (15), (16), (22) and (23), we rewrite Eq. (29) in the following form:

$$\frac{H_b}{L_b} = 2\pi f(s) \tanh \frac{2\pi}{L_b}\left(h_b + \frac{H_b}{2}\right) \tag{30}$$

and compare the values of λ_b calculated from Eq. (30) with those measured, as shown in Fig. 9. It is not difficult to find that the newly

Table 6
Results of error analysis.

Formula	E	v	P_{10}	P_{20}
Singamsetti and Wind (1980)	7.3%	17.9%	44.4%	74.3%
Rattanapitikon and Shibayama (2000)	6.2%	25.7%	45.5%	71.7%
Ostendorf and Madsen (1979)	−4.4%	15.0%	50.3%	82.9%
Goda (2010)	2.2%	17.5%	50.3%	80.1%

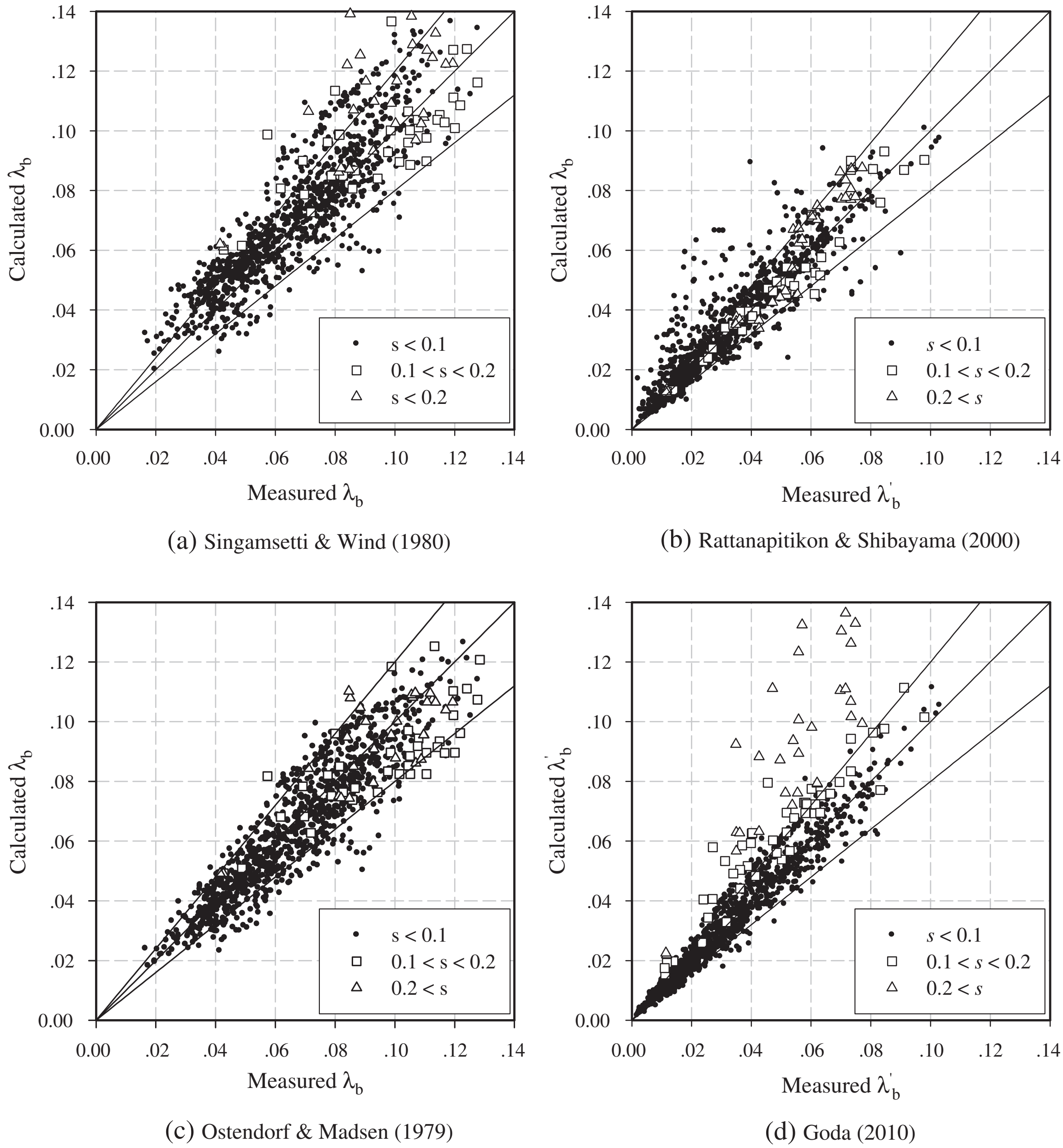


Fig. 7. Comparison of formulas with data.

established Eq. (29) is more accurate than the existing formulas by comparing Fig. 9 with Fig. 7. In fact, P_{20} and P_{10} of the new formulas are 95.6% and 68.8%, respectively, and are much larger than other formulas that we examined in the previous section. When compared with Goda's (2010) formulas, the new formula performs well also for waves on very steep slopes.

A detailed study shows that the relative error of Eq. (29) with the measured data we collected has some identifiable relation with the local values of the relative wave height and the relative wave steepness at the breaking point. This is probably because some minor factors which certainly affect the breaking process are not included in the breaking index ψ, or the breaking wave celerity evaluated by Eq. (28) is not accurate enough. Anyhow, this fact allows us to further improve the breaking index by letting:

$$\psi'_b = f_1(\lambda_b)f_2(\gamma_b)\psi_b. \tag{31}$$

After an iteration to minimize the relative error of ψ'_b, we obtain

$$\psi'_b = (1.21-3.30\lambda_b)(1.48-0.54\gamma_b)\psi_b. \tag{32}$$

Now we calculate ψ'_b using the data we collected and plot them against the beach slope s, as shown in Fig. 10. Then, we obtain a modified formula for the incipient condition of wave breaking in terms of ψ'_b:

$$\psi'_b = 0.69 \tag{33}$$

The modified formula is simple and has an excellent accuracy. The relative errors of the 1193 sets of data we collected are now all within ±20%, 99.3% of them are within ±10%, and 93% are within 5%. This is the accuracy never achieved by any existing formula. Considering the variability of the phenomenon, this accuracy can be regarded as satisfactory.

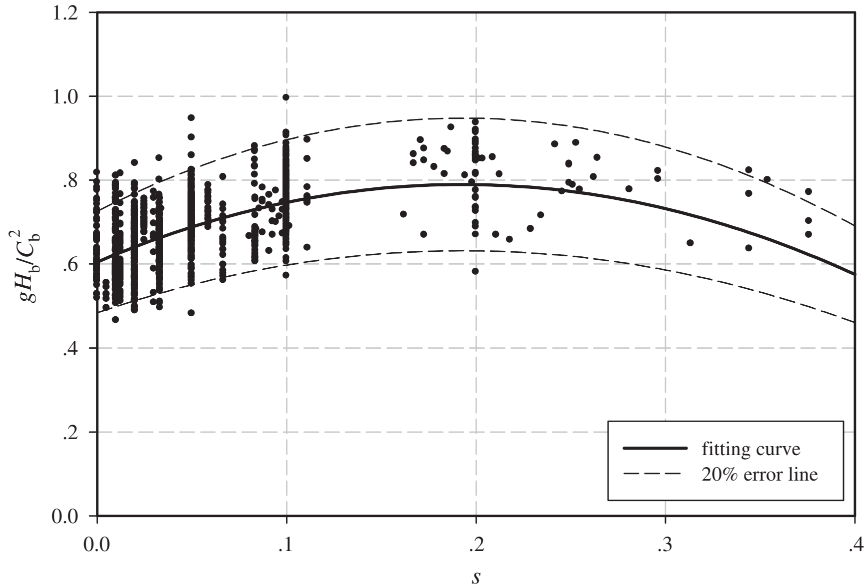


Fig. 8. New breaking index.

Application of Eq. (33) is as follows. At the given wave period T, wave height H and water depth h, we calculate the wavelength L as well as the wave celerity C by Eq. (28). Then, we obtain a value of ψ by $\psi = gH/C^2$. We can be sure that the wave should have broken if $\psi > 1.0$. Otherwise, we compute ψ' based on $\psi' = (1.21 - 3.30H/L)(1.48 - 0.54H/h)\psi$. If $\psi' > 0.69$, the wave should also have broken. An iteration is necessary when solving Eq. (28).

5. Further verification of new formula

To show the reliability of the newly proposed formula for the inception of wave breaking, some further verification by data that were not used to fit Eq. (33) may be expected. Here we first refer to Nakamura et al.'s (1968) field data. The data were observed at Watari coast in Miyagi Prefecture, Japan, from September to October, 1967. The field beach slope was approximately 1/30–1/10. The deep water wave height was estimated to be within a range of 0.5–2.0 m. The agreement between the present formula and the observations is very good as shown in Fig. 11. P_{20} of the observed data is 100%, P_{10} is 87.5%.

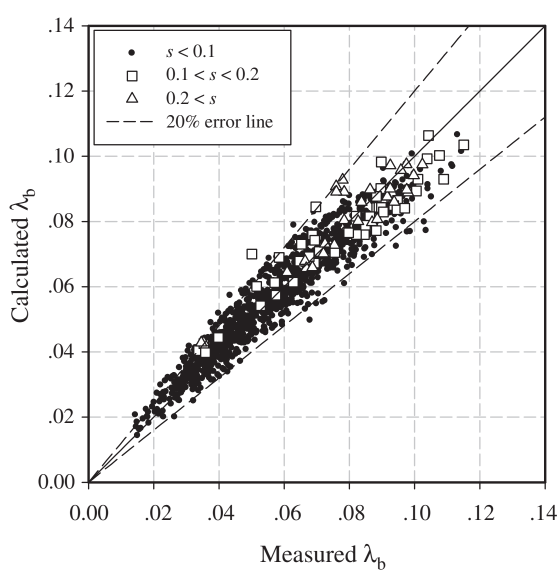


Fig. 9. Comparison of formula with data.

If Goda's (2010) formula is employed, no matter how we adjust the value of the bottom slope, P_{20} can be about 50% and P_{10} about 15% at best.

Another data set that we use to verify the new formula for the inception of wave breaking was recently published by Lee and Mizutani (2010). Lee and Mizutani's (2010) data were obtained in their laboratory as they were interested in wave motion over a permeable slope. The bottom in their experiments was made of uniform gravel sand (the median grain diameter $d_{50} = 5$ mm), and the bottom slopes were 1/7, 1/10 and 1/12, respectively. Lee and Mizutani (2010) verified Goda's (2010) formula using their experimental data and concluded that the relative error is minimum when the constant in Goda's formula is set to $A = 0.12$ for the bottom slope of 1/7 and to $A = 0.14$ for the bottom slopes of 1/10 and 1/12, both are different from the standard value $A = 0.17$ that Goda (2010) recommended. The excellent agreement of Lee and Mizutani's (2010) data with the modified formula proposed in the present study is shown in Fig. 12. An error analysis indicates that 100% of Lee and Mizutani's data are within the 20% error line, 99.7% are within the 10% error line, and 97.6% are within the 5% error line.

6. Conclusions

The objective of the present study is to improve our knowledge on the incipient condition of wave breaking. For this purpose, the existing empirical formulas proposed by different researchers have been classified into four categories, i.e., the McCowan type, the Miche type, the Goda type and the Munk type. The inherent relations between different types of formulas are exploited. The differences among each formula within the same group are also discussed. Four representative formulas of different types are chosen to compare with a total number of 1193 cases of the laboratory experiments reported in literatures. It is shown that Goda's (2010) formula, of Goda type, and Ostendorf and Madsen's (1979) formula, of Miche type, are advantageous in general. Goda's (2010) formula agrees very well with the experimental data when the beach slope is milder than 1/10, but it loses its accuracy as the beach slope becomes steeper than 1/10. Ostendorf and Madsen's (1979) formula is fairly good even for very steep slopes, but its accuracy for cases with ordinary beach slope is not as good as Goda's (2010).

A new predictive formula for the inception of wave breaking is proposed. A unique index, defined by $\psi'_b = (1.21 - 3.30\lambda_b)(1.48 - 0.54\gamma_b)\psi_b$, where $\psi_b = gH_b/C_b^2$, H_b is the breaking wave height, C_b is the

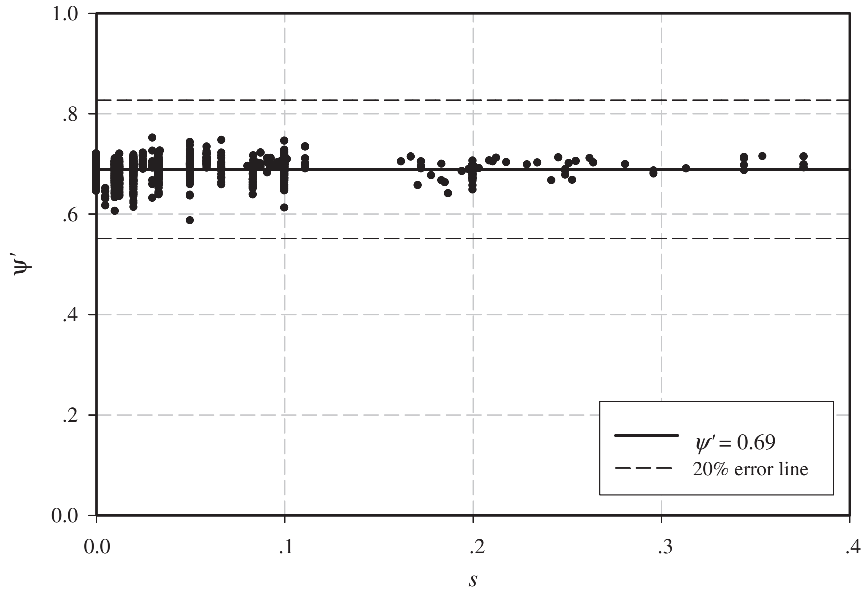


Fig. 10. Modified breaking index.

breaking wave celerity, λ_b is the breaking wave steepness, γ_b is the relative breaking wave height, and g is the gravity acceleration, is introduced. The incipient condition of wave breaking can then be given by $\psi'_b = 0.69$. This condition is a significant improvement to the existing formulas in terms of the accuracy. In addition, it is a local relation. The incident wave conditions and the beach slope are not included. Further verifications on the proposed formula are also carried out by referring to the field data of Nakamura et al. (1968) and the laboratory data for waves on gravel bottom by Lee and Mizutani (2010). The agreements are all excellent.

Notation

The following symbols are used in this paper:

Variables

C	wave celerity
h	water depth
H	wave height
L	wavelength
s	bottom slope
λ	wave steepness
γ	relative wave height
ψ	new breaking index
ψ'	modified new breaking index

Subscripts

b	for variables at the breaking point
0	for variables under deep water conditions

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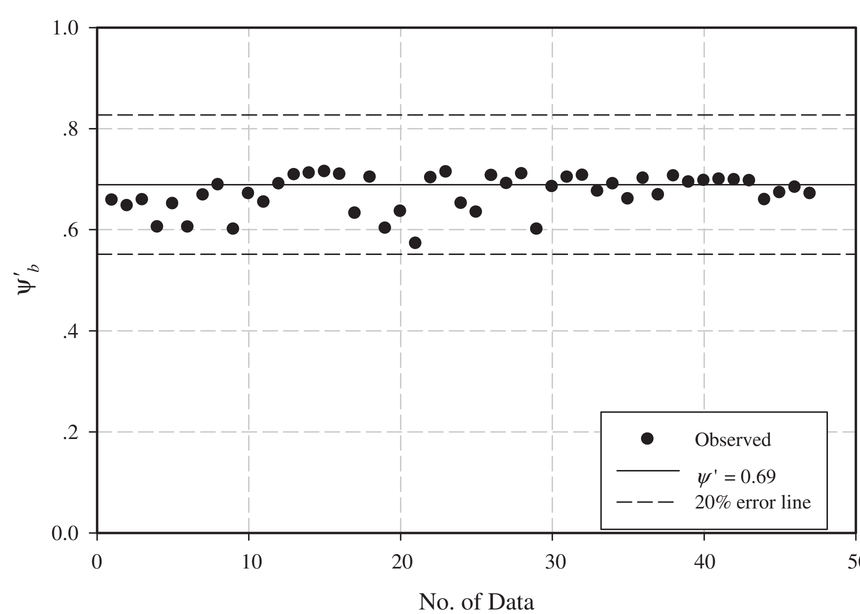

Fig. 11. Verification of new formula by field data of Nakamura et al. (1968).

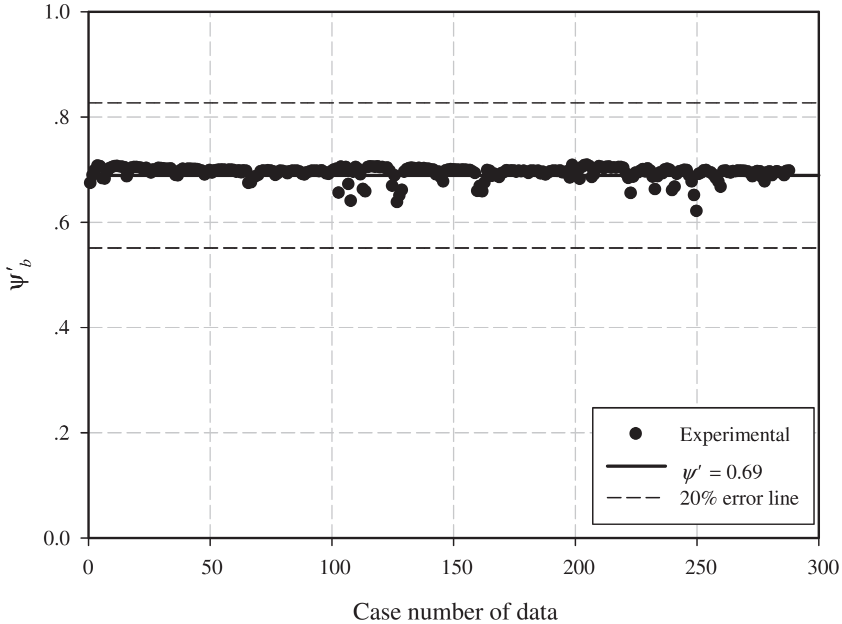

Fig. 12. Verification of new formula by laboratory data of Lee and Mizutani (2010).

providing us the valuable experimental data they collected. We also thank Dr. Takao Shimizu from CRIEPI, Japan for providing the valuable experimental data measured in a large scale wave flume. The research is partially supported by State Key Laboratory of Hydrosicience and Engineering of China under grant no. 2011-ky-1 and by the Natural Science Foundation of China (NSFC) under grant no. 10772099.

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