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Technical note

Is the wind wave frequency spectrum outdated

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Abstract

This paper presents a detailed examination of the practice of using the frequency spectrum to characterize wind waves. In particular, the issue of stationarity and Gaussian random process in connection with wind wave studies is addressed. We describe a test for nonstationarity based on the wavelet spectrum. When this test is applied to wind wave time series, the results significantly diverge from those expected for a Gaussian random process, thus casting critical doubts on the conventional concept of the wind wave frequency spectrum. © 1999 Published by Elsevier Science Ltd. All rights reserved.

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1. Introduction

Posing the title of a paper as a question usually implies a definitive answer is at hand. This is not necessarily the case here. The question is only an attempt to call attention to some of the recent developments in wind wave spectrum analysis. The concept of frequency spectrum analysis was first introduced to ocean wind wave studies around 1950. Over the following five decades, Fourier spectrum analysis was the standard procedure used by atmospheric and oceanic scientists as well as coastal and marine engineers to analyze and predict wind-generated ocean waves.

The long lasting usefulness of wind wave spectrum analysis is clearly not acciden-

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tal but well warranted. Here are some of the notable successes of wave spectrum analysis:

- The wave spectrum provides an appropriate representation of energy distribution at the ocean or lake surface.
- The wave spectrum encompasses all the Fourier components of a spatially homogeneous and temporally stationary wave field.
- Most of the common measures of wind wave characteristics are conveniently related to the moments of a wave frequency spectrum.
- The concept of wave spectrum has also contributed significantly to progress in numerical modeling for wind wave predictions.

The emerging application of the wavelet transform and time–frequency analysis and the advent of simultaneous wind and wave measurement at the same high resolution, however, have cast a wary shadow over the conventional notion of a wind wave frequency spectrum. Fourier-transformed spectral information in the frequency domain can conceal the information about the temporal variability of wave activity. The wave frequency spectrum simply cannot provide spectral information on actions occurring at any specific time of interest within the time series. The lack of information about the time localized spectra has generally been taken for granted for practical reasons. However, ignoring the temporal variability of the spectrum can suppress the details of crucial local processes occurring at short time scales. To offset this difficulty, almost all the analyses and published studies have basically contended that the wave field was practically a stationary Gaussian random process. On the other hand, the professed stationarity clearly contradicts the well-known and well-established observable fact that wind waves always appear in groups, i.e., higher waves occur in successively separated sequences. It is certainly not unreasonable to assert that stationarity and wave grouping cannot coexist in the same wave field; therein lies the problem with the traditional wind wave frequency spectrum.

The wavelet analysis is an approach of more recent inception (Daubechies, 1992). Initially aimed at providing an easily interpretable visual representation of signals, it has evolved as an effective alternative to the standard Fourier analysis (Combes et al., 1989). Based on wavelet analysis of continuously recorded simultaneous wind and wave measurements made in Lake Michigan and in the Pacific Ocean, this paper attempts to examine the fundamental assumptions associated with the concept of the wave frequency spectrum. In light of the practical applications of wavelet time–frequency spectrum analysis to the wind waves, it appears likely that the answer to the question posted by the title of this paper is yes, the wind wave frequency spectrum is probably now outdated.

2. The genesis of the wind wave frequency spectrum

The period from the late 1940s to the early 1950s perhaps marked the beginning of the modern study of wind waves and the era of wave frequency spectrum. Notably, Barber and Ursell (1948) published the first measurements and analysis of ocean wave spectra, and Pierson and Marks (1952) adopted the works of Tukey (1949) and introduced power spectrum analysis to the ocean wave data analysis. Now five decades later, and with the development of the Fast Fourier Transform (FFT) technique, spectrum analysis has become a routine starting point for any time series data analysis. But in the early day the adaptation of spectrum analysis to wind wave studies was by no means simple and straightforward.

Kinsman (1965) described the conceptual struggles that were encountered at the time before the wave frequency spectrum approach was applied:

1. The conviction that the problem of bringing law to the confusion of the sea was, in its essence, a statistical problem. This required a firm departure from the classic approach to waves and commitment to the then-new-and-unfamiliar discipline of stochastic processes.
2. The realization that even under the new formulation the motion obeys the classical equations. This is not trivial. There is no a priori reason to suppose that a statistic must propagate in the classical manner. In fact, there are some which do not.
3. The identification of the energy spectrum as the ordering and governing principle in the apparent confusion.
4. The conception that the space–time function describing a given sea state must have a certain multivariate probability structure which, if stationary, can depend only on time and space coordinate differences.

It seems that in the beginning, adapting of stochastic process and energy spectrum as the practical approach to follow required conviction and commitment, but everything was basically uncertain. It was amply clear, however, that using the energy spectrum should be the approach to follow, and the conceptual basis required the process to be stationary.

Decades later, in a recent definitive publication that summarized the current state-of-the-art knowledge of wind waves and modeling, Komen et al. (1994) echoed and elucidated those earlier viewpoints as

> In practice one never considers the deterministic initial value problem for a realistic sea basin. The reason is that it is virtually impossible to determine the Fourier modes with the correct phases. To overcome this problem one resorts to a statistical description

and emphasized that in so doing “it is essential that the wave field is statistically stationary and homogeneous” while “the probability distribution of the sea surface is nearly Gaussian”. Thus, Komen et al. further secured the notion that the *wave spectrum* is still the basic element of wind wave processes, just as it was identified in the early years.

So in essence, the application of wave spectrum analysis and the subsequent numerical wave modeling developments all initially stem from an approach that was basically a recourse for convenience and expediency rather than for intrinsic and deterministic dynamical reasons. This certainly does not preclude us from pursuing other possible approaches, now or in the future. The fundamental assumption that the wave process is stationary and Gaussian should not be taken for granted. In practical applications, however, it is usually expected or assumed that the requirement is generally fulfilled. But efforts to check the process to ascertain whether the required conditions actually exist have been rarely carried out.

3. Wind wave frequency spectrum and wind wave wavelet spectrum

Spectral analysis transforms a time series of instantaneous water surface measurements into an energy spectrum in the frequency domain. If the water surface measurements represent wind wave data, then the distribution of wave energy with respect to frequency, the frequency corresponding to the peak energy, the total energy, and various spectral moments can all be readily obtained from the frequency spectrum. What the frequency spectrum does not provide, however, is an indication of the variability of the various energy measures within the time series. In a truly stationary process, time localization should not be important. But in the real world where stationarity is only a remote idealization, then time localized information, if needed, cannot be extracted from a frequency spectrum. This is where the usefulness of the recently advanced approach of *wavelet transform* emerges.

As shown by Liu (1994), Torrence and Compo (1998), and others, wavelet transforms can be considered a broadened extension of the commonly used Fourier transform. Both methods transform the function that is representing the process in one domain to some different domain. For the Fourier transform, the new domain consists of basis functions that are sines, cosines, or complex exponential functions. For the wavelet transform, the new domain contains basis functions called mother wavelets, or analyzing wavelets, that can be constructed from other specified functions for particular applications. In the Fourier transform, the basis functions are localized in the frequency domain. In the wavelet transform, the basis functions are localized in both the frequency and the time domain. It is this expanded capability in time localization that makes the wavelet transform a useful tool for studying nonstationary processes.

Fig. 1 presents an example comparing Fourier spectrum and wavelet spectrum analyses. A typical 10-min time series data of surface wave fluctuations, sampled at 1.7 Hz, is shown in part (a). Its energy frequency spectrum is given by the solid line curve in part (b). Part (c) is the corresponding time–frequency wavelet spectrum, calculated using the Morlet wavelet (Liu, 1994). The wavelet spectrum is illustrated as energy density contours in the two-dimensional time–frequency plane. An average of the wavelet energy densities in part (c) leads to the equivalent wavelet frequency spectrum plotted as the knotted curve in part (b) for comparison. Prior to the development of wavelet spectrum analysis, if a frequency spectrum like the solid curve in part (b) was calculated for a particular time series, one probably would envision some approximately constant energy distribution during the 10-min recording period similar to the equivalent time–frequency spectrum shown in part (d), instead of the intermittent quality of the wavelet spectrum as shown in part (c).

The time–frequency wavelet spectrum in part (c) carries more detailed information

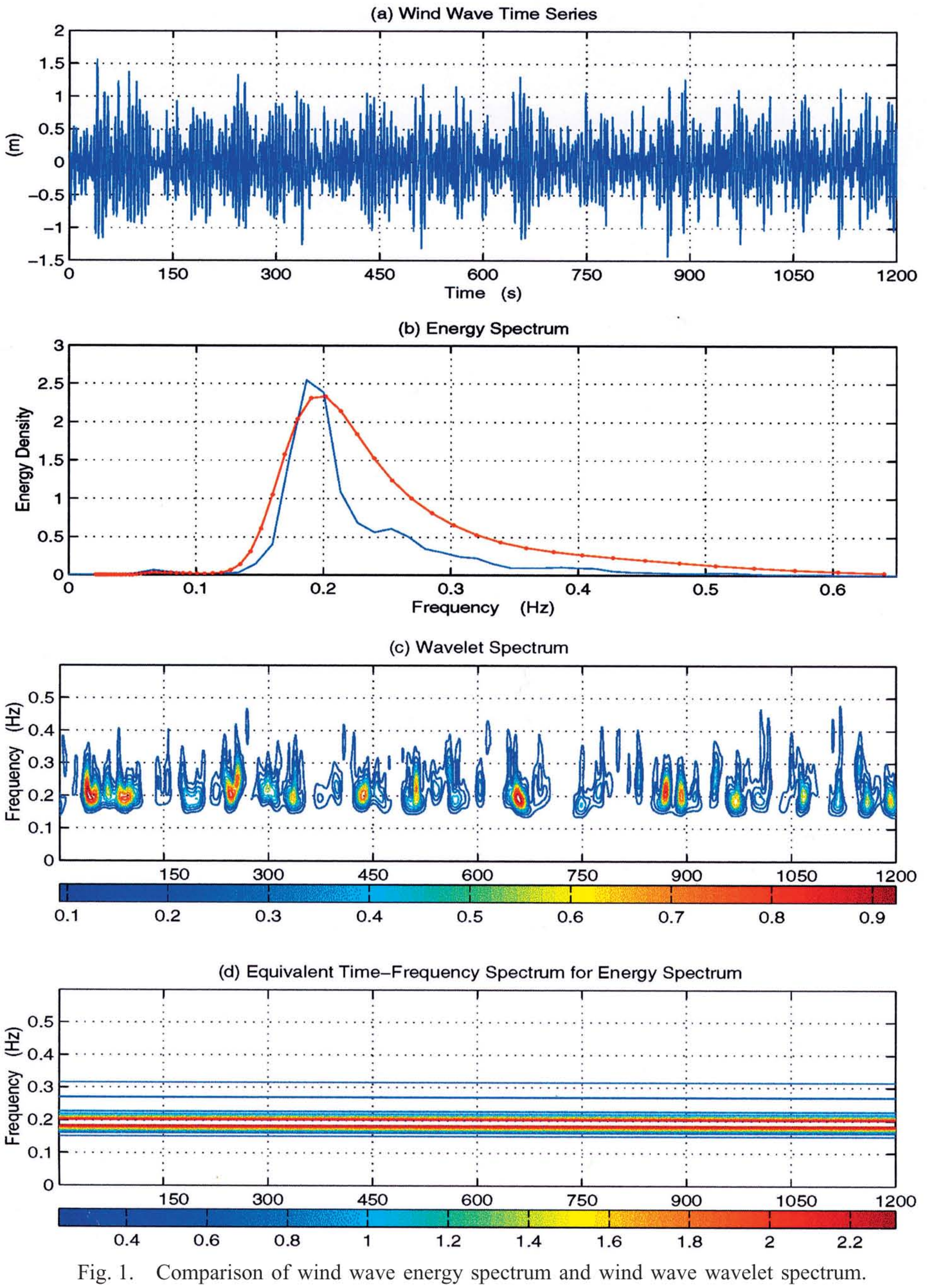

Fig. 1. Comparison of wind wave energy spectrum and wind wave wavelet spectrum.

about time variability than the frequency spectrum of part (b), as well as the intuitive equivalent time–frequency spectrum in part (d). What new information is provided by the wavelet spectrum? Clearly the energy distribution with frequency is not at all approximately constant throughout this 10-min recording period. Rather it increases and decreases intermittently corresponding to the surface fluctuations. The intermittent nature of wave groupings, an unmistakable characterization of typical wind wave recordings, is totally hidden in the conventional frequency spectrum. Yet one basic characteristic of wind waves is really the intermittent quality of wave groups. Even though we do not currently have a good understanding of wave groupings, efforts to explore these grouping characteristics will be the key to improve understanding of wind wave processes.

4. How do wind waves grow?

Perhaps one of the striking results in the application of the conventional frequency spectrum analysis to wind waves is the presumed confirmation of the theoretical exponential growth in wind waves. Based on the measurements of wind waves in the laboratory and oceans, Plate et al. (1969), Barnett and Wilkenson (1967) and others showed that by examining the magnitude of the peak-energy frequency component in a set of growing spectra as a function of time or space, the component appeared to grow exponentially. The rapid exponential growth of the energy level tends to slow down and eventually dip slightly, which was characterized by Barnett and Sutherland (1968) as an "overshoot" effect. Phillips (1977) adopted these results and postulated the existence of four distinct phases in the development of a wave component: (1) an initial phase followed by (2) a rapid exponential growth phase that led to (3) the overshoot phase before (4) the saturation phase was finally attained. Phillips further commented that:

> It would be pleasant if each phase could be associated with a single dynamical process, but the threads seem often to be more tightly interwoven than this.

The exponential growth of the wind wave spectrum has evolved as the basic, central element in formulating the source function for numerical wind wave modeling.

To further examine the notion of the wind wave growth process, we will examine an 8-h continuous time series of wind waves sampled at 1.7 Hz using an instrumented NDBC buoy in western Lake Michigan. The time series corresponds to a moderate storm in November 1995 during which wind speeds increased from 8 to 15 m/s and wind direction was fairly constant. The resulting surface wave fluctuations are presented in Fig. 2(a). Fig. 2(b) presents the sequence of frequency spectra obtained from wavelet spectrum analysis every 20 min of data consecutively. The results show a rather familiar picture of spectral growth of wind waves where the peak-energy frequency shifts continuously toward the lower frequencies as the energy densities grow. Now concentrating on the frequency component for which the highest peak energy is achieved in this episode, its development is plotted on a semi-logarithmic

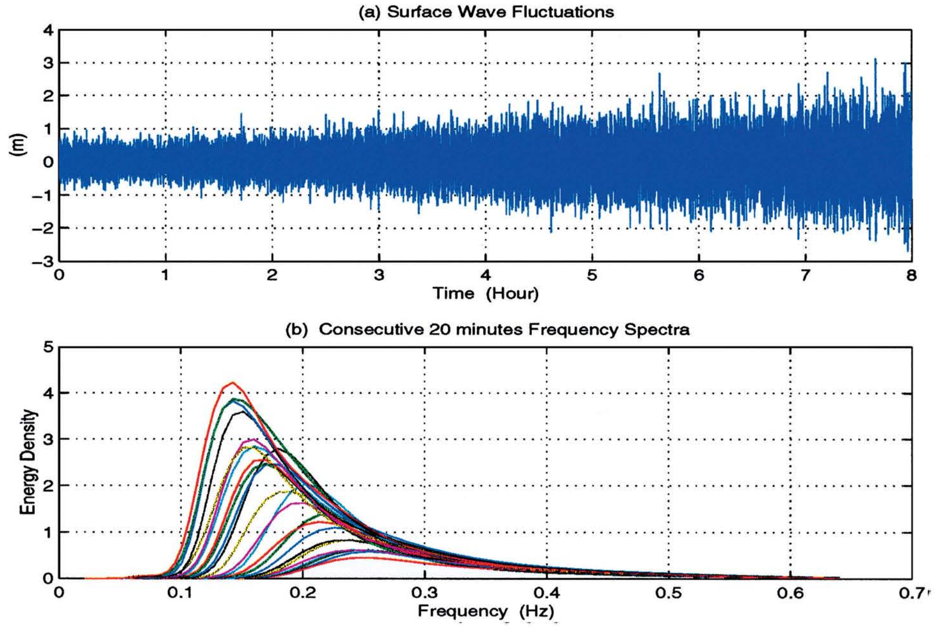

Fig. 2. (a) An episode of surface wind waves; and (b) consecutive 20-min frequency spectra during the episode.

scale with respect to time as shown by the solid line connecting the asterisks in Fig. 3. The episode appears to have grown from an existing decayed wave system, so that the initial phase led directly to distinctive exponential growth. There is no indication of overshoot effect in this episode, but the exponential growth does slow down and seemingly approach saturation. The line connecting the asterisks shown in Fig. 3 appears to confirm once again the conventional notion of exponential wave growth.

With the advance of wavelet spectrum analysis, much more detailed information on the variability of the wave energy spectrum can now be obtained. The detailed variations of the same frequency component, according to the wavelet spectrum, are also plotted and superimposed in Fig. 3. The portrayal of the wave growth process is totally different. The simple exponential growth curve is now replaced by a rapidly fluctuating, highly intermittent time history. The trend of growth is still there and unmistakable, but it no longer lends itself to a simple, straightforward interpretation. Threads are not only "tightly interwoven" as Phillips envisioned, but the actual process of wave growth may be much more complex than even Phillips' vision.

Now an interesting question might be raised: for practical applications can the details of the complex pattern of wave growth be ignored and replaced with the previous familiar growth patterns? The answer would necessarily be yes, because the familiar pattern has served reasonably well and in many numerical models of wave growth that provide useful results. On the other hand, what does the familiar, conventional frequency spectrum really represent in light of the details of time variability available from wavelet spectrum analysis in the time–frequency domain? The

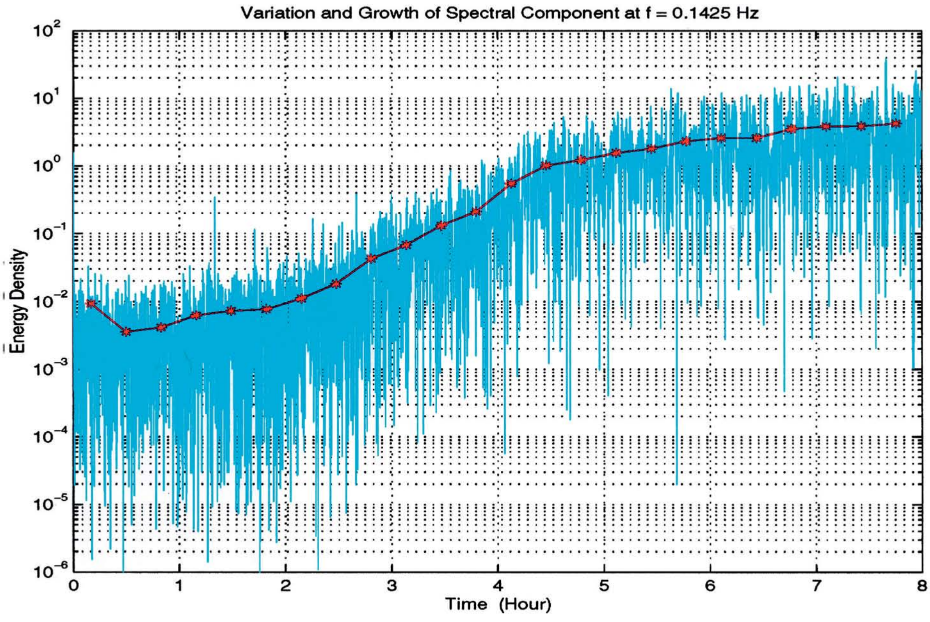

Fig. 3. The time history of the spectral component at frequency 0.1425 Hz.

difference between the complex picture of wave growth indicated by the wavelet analysis and the simple exponential growth is equivalent to the difference between Fig. 1(c) and Fig. 1(d) as being more representative of the actual wind wave growth process.

5. A test for stationarity

The conceptual difference between Fig. 1(c) and Fig. 1(d) is not significant if Fig. 1(d) can be used in practice to represent Fig. 1(c). This assertion can only be true if the wind wave growth process is completely stationary. So the key to all the questions concerning the adequacy of the conventional frequency spectrum is basically a question of the stationarity of the process. On the other hand, almost all studies of wind waves have ignored the issue of stationarity. Most have simply taken for granted that wind waves are stationary, as exemplified by this far-reaching statement in Komen et al. (1994): "Fortunately, the probability distribution of the sea surface is nearly Gaussian", without any substantiation or validation.

Strictly speaking, stationarity means that the statistical properties of the process remain the same at all times. This definition of complete stationarity was in general too stringent to be practical, so efforts to relax the requirements were introduced. For instance: weak or wide-sense stationarity requires only the second order moment, i.e. the variance, be the same at all times. Wide-sense stationarity is basically the requirement for frequency spectral analysis. Mathematical and statistical theories on stationary processes are well established. Surprisingly, a search of text books as well as published literature reveals that no simple techniques readily available for assessing the stationarity of a given time series.

Examining the differences between Fig. 1(c) and Fig. 1(d) has suggested a new simple test of stationarity. Since the wavelet spectrum in Fig. 1(c) provides the local frequency spectrum at every time point while Fig. 1(d) gives a constant local frequency spectrum across the time points, the total sum of their differences should be small if the process is in fact stationary. An index measure can be defined using the wavelet spectrum for the wind waves as $W(\omega_i, t_j)$ in terms of frequency, ω_i, and time, t_j, and the equivalent frequency spectrum obtained from integrating the wavelet spectrum with respect to time given by

$$\Phi_m(\omega_i) = \frac{1}{N} \sum_{j=1}^{N} W(\omega_i, t_j)$$

where N is the total number of data points. The *Nonstationarity Index* (N.I.) can then be defined as

$$\text{N.I.} = \sum_i \sum_j \left[\frac{W(\omega_i, t_j) - \Phi_m(\omega_i)}{\Phi_m(\omega_i)} \right]^2 .$$

Under this definition the N.I. will be a positive number and every time series will have an N.I. attached to it. A time series with a larger N.I. will be more nonstationary than the one whose N.I. is smaller. Or alternatively, a time series with a smaller N.I. will probably be more stationary than the one with a larger N.I. number. It must be noted here that this is a totally empirical approach, which may or may not be related to mathematical concepts of stationarity. In general, stationarity is usually a property of a theoretical process, and a time series is simply a realization of a process that can either be stationary or nonstationary. On the other hand, a time series with a relatively large N.I. would not likely be a realization of a stationary process.

Five different time series are selected to evaluate the feasibility of the simple testing procedure proposed here. These time series consist of a widely used Doppler signal, a composite of grouped sine waves, actual surface wind wave fluctuations, actual wind speeds, and a Gaussian random signal. All are set at 28 800 s long, with a 1.7 Hz sampling rate. A sample segment for each of the five time series is shown in Fig. 4.

In calculating the nonstationarity indices, each time series is first subdivided into different segment lengths to calculate their N.I., and then an average N.I. is obtained for each predetermined segment length. The results, shown in Fig. 5, are presented as N.I. versus segment length on a log–log scale to encompass all the outcomes.

Without a preconceived notion of what might transpire in the process, the Nonstationarity Index performed credibly well. The clearly nonstationary Doppler signal exhibited the highest N.I, which increased exponentially with increasing segment length. The Gaussian random signal, known to be stationary, exhibited the lowest N.I., about 1, which was generally unaffected by the segment length, as one would expect for a stationary time series. The grouped sine wave series also had a high

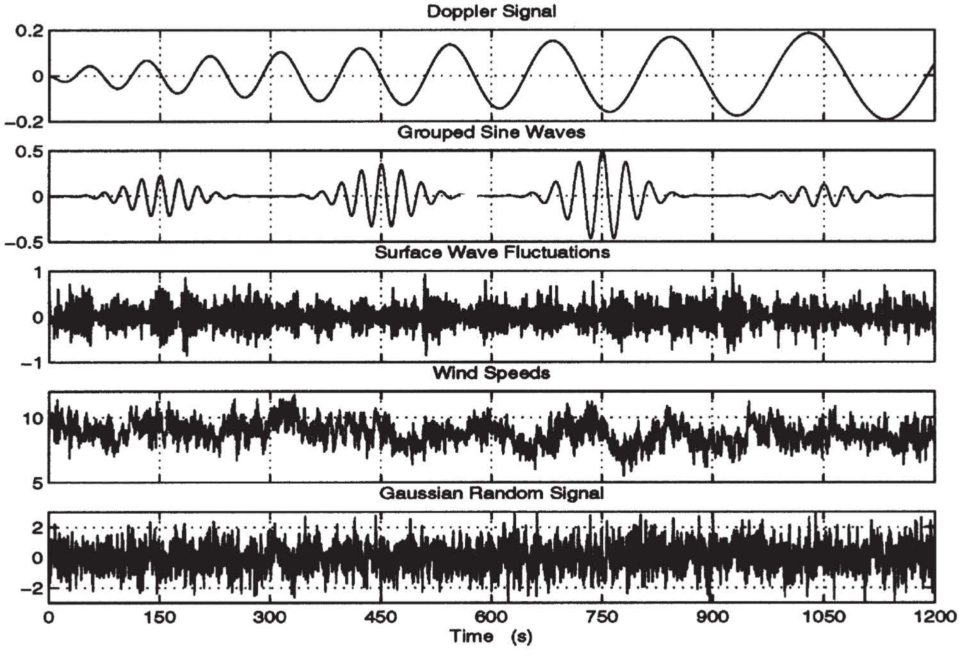

Fig. 4. Segment samples of selected time series.

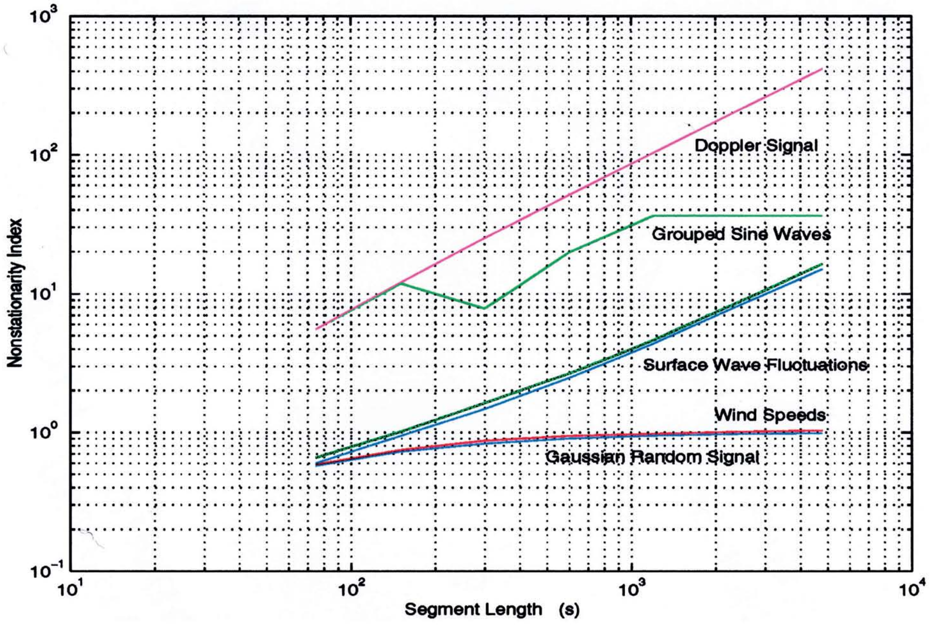

Fig. 5. Nonstationarity Index for different time series.

N.I. The wave and wind speed time series, however, are the most interesting to this study. It is a surprise to see that the high-resolution time series of wind speeds turned out to exhibit practically the same stationarity as a Gaussian random signal. It may also be a surprise to some that the time series of wind waves was not at all close to the Gaussian random signal, at least for segment lengths of 5 min or more. Note that in addition to the measured surface wave fluctuations from eastern Lake Michigan shown in Fig. 4, a separate surface wave fluctuations time series measured by a NDBC buoy from the Pacific Ocean has also been used in the stationarity test. It is the second line near the “surface wave fluctuations” label in Fig. 5, close to the Lake Michigan result.

It is highly significant that the results for surface wave fluctuations in Fig. 5 are clearly diverging from the results for Gaussian random signal. It shows that a knowledge of the stationarity of a time series is important and it should not be taken for granted or ignored. It also shows that a wind wave time series is unlikely to be a realization of the Gaussian random process and the current prevailing concept of wind waves that is based on describing wind waves as a Gaussian random process might be very much in question.

6. Concluding remarks

After nearly half a century of dominating wind wave study as the empirical governing principle, the energy frequency spectrum of wind waves has now undergone some previously unexplored scrutiny. The emerging availability of new analysis techniques like the wavelet transform has weakened the once uncontended significance enjoyed by the wave frequency spectrum. Current state-of-the-art models for wind waves are still based on the frequency wave spectrum concept and still face unsatisfactory error margins. Based on the results presented here, it may now be necessary to seek alternative conceptual paradigms for wind waves to make further progress in understanding wind wave processes. In conclusion, to answer the question posed in the title of the paper, it is my opinion that the current wind wave frequency spectrum is outdated.

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