Available online at www.sciencedirect.com
SCIENCE DIRECT®
Coastal Engineering 52 (2005) 197–200
Coastal Engineering
An International Journal for Coastal, Harbour and Offshore Engineers
www.elsevier.com/locate/coastaleng

Discussion

Discussion of "Wave transformation by two-dimensional bathymetric anomalies with sloped transitions" [Coast. Eng. 50 (2003) 61–84]

Huan-Wen Liu[a], Peng-Zhi Lin[b,*]

[a]*Department of Mathematics and Computer Sciences, Guangxi University for Nationalities, Nanning, Guangxi 530006, PR China*
[b]*Department of Civil Engineering, National University of Singapore, Singapore 119260, Singapore*

Received 26 August 2004; accepted 16 November 2004
Available online 28 December 2004

Bender and Dean (2003) investigated the interaction of linear water waves with two-dimensional trenches and shoals of finite width with sloped transitions between the depth changes (Fig. 1) using three methods: the step method, the slope method and the numerical method. Among them, the step method is valid in arbitrary water depth while the slope method and numerical method are valid only in the shallow water region. The slope method is an extension of the earlier analytical solution by Dean (1964) for long wave modification by linear transitions. The slope method is a very useful and important approach that makes the exact solution of wave reflection and transmission possible. However, possibly due to the complex nature of the resultant system of equations, instead of obtaining the closed-form analytical expression, Bender and Dean (2003) computed the reflection and transmission coefficients numerically with the use of standard matrix technique for the linear system of equations with eight unknown amplitudes and eight unknown phases.

Recently, Lin and Liu (2005) has found the closed-form analytical solution for wave interaction with an obstacle of general trapezoidal shape based on the shallow water theory. Their approach can be easily extended to the trench cases discussed in Bender and Dean (2003). With the use of Lin and Liu's (2005) method, the explicit expression can be obtained and the reflection coefficient is given as follows

$$K_R = |z_1 - iz_2|/|z_1 + iz_2|, \tag{1}$$

where

$$z_1 = [J_1(\alpha_1)Y_1(\alpha_2) - J_1(\alpha_2)Y_1(\alpha_1)](PX + QX') + i[J_1(\alpha_1)Y_0(\alpha_2) - J_0(\alpha_2)Y_1(\alpha_1)](PX' + QX), \tag{2}$$

$$z_2 = [J_1(\alpha_2)Y_0(\alpha_1) - J_0(\alpha_1)Y_1(\alpha_2)](PX + QX') + i[J_0(\alpha_2)Y_0(\alpha_1) - J_0(\alpha_1)Y_0(\alpha_2)](PX' + QX), \tag{3}$$

DOI of original articles 10.1016/j.coastaleng.2003.08.002, 10.1016/j.coastaleng.2004.11.003.

* Corresponding author. Fax: +65 6779 1635.
E-mail address: cvelinpz@nus.edu.sg (P.-Z. Lin).

doi:10.1016/j.coastaleng.2004.11.002

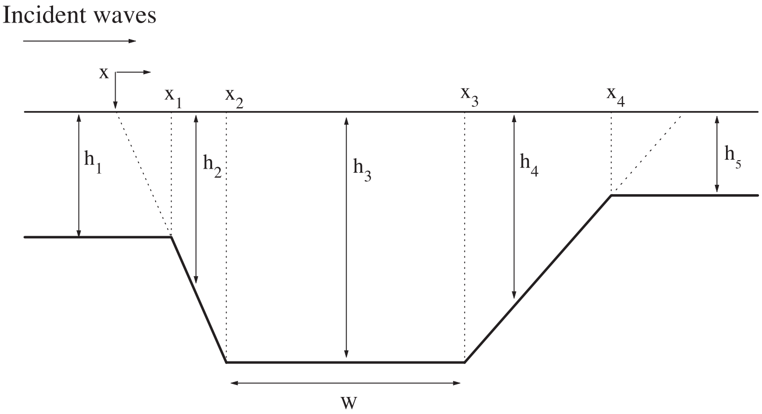

Fig. 1. Definition sketch for trench with sloped transitions.

$$X = e^{-ik_3W} + e^{ik_3W}, \quad X' = e^{-ik_3W} - e^{ik_3W}, \tag{4}$$

$$P = J_0(\beta_1)Y_1(\beta_2) - J_1(\beta_2)Y_0(\beta_1) + i[J_0(\beta_2)Y_0(\beta_1) - J_0(\beta_1)Y_0(\beta_2)], \tag{5}$$

$$Q = J_0(\beta_2)Y_1(\beta_1) - J_1(\beta_1)Y_0(\beta_2) + i[J_1(\beta_2)Y_1(\beta_1) - J_1(\beta_1)Y_1(\beta_2)], \tag{6}$$

with $r_{13} = \sqrt{\frac{h_1}{h_3}}$, $r_{53} = \sqrt{\frac{h_5}{h_3}}$, $\alpha_1 = 2k_1x_1, \alpha_2 = \alpha_1/r_{13}, \beta_1 = 2k_3h_3(x_4 - x_3)/(h_3 - h_5), \beta_2 = r_{53}\beta_1$, and k_1 and k_3 being wave numbers related to water depths h_1 and h_3, respectively.

As a validation, four cases shown in Fig. 16, Fig. 17, Fig. 19 and Fig. 20 in Bender and Dean (2003) are calculated again here by using the analytical solution (1), and the comparisons are presented in Figs. 2(a)–(b) and 3(a)–(b), respectively. It can be seen that the agreement among the present analytical solution and solutions of the numerical method and the slope method are quite good.

It is worth indicating that the analytical solution (1) can be reduced to several well-known special cases, namely, waves past a rectangular obstacle with a finite length by Mei (1989), a step with an infinite length by Lamb (1932) and an infinitely long step behind a

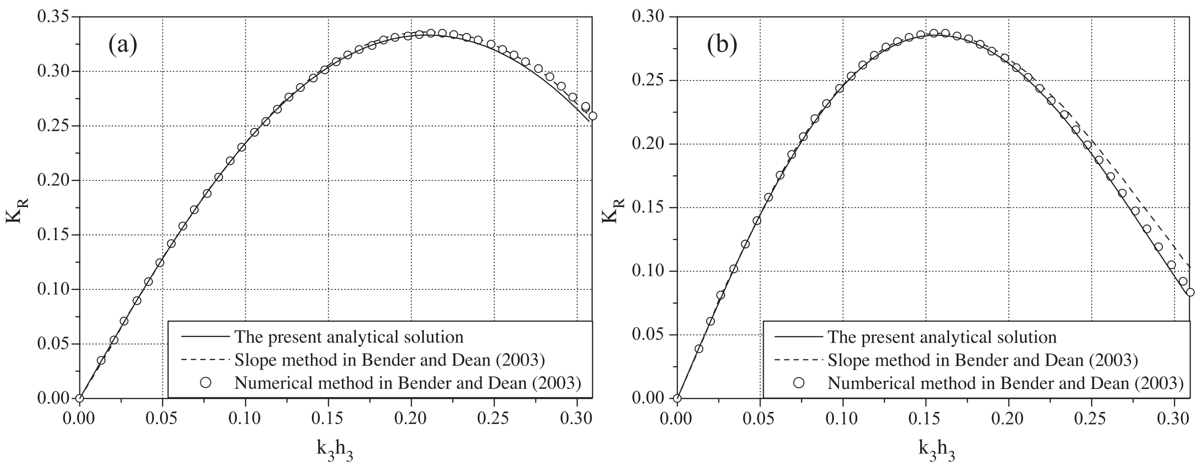

Fig. 2. Comparison among the present analytical solution, the numerical solution and slope method solution for two cases studied in Bender and Dean (2003): (a) Fig. 16 and (b) Fig. 17.

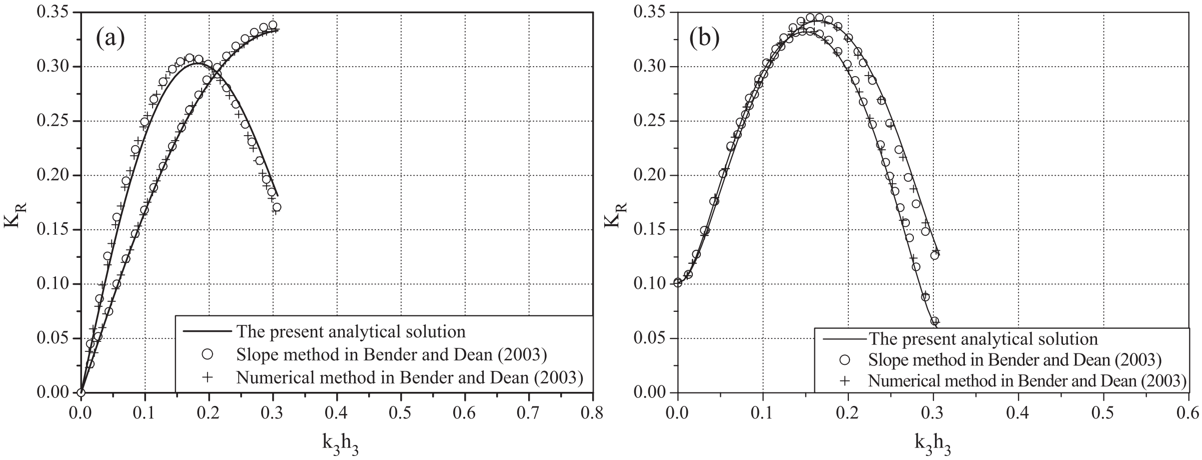

Fig. 3. Comparison among the present analytical solution, the numerical solution and slope method solution for two cases studied in Bender and Dean (2003): (a) Fig. 19 and (b) Fig. 20.

linear slope by Dean (1964). They are shown as follows.

When $x_2 \rightarrow x_1$ and $x_4 \rightarrow x_3$, then $\alpha_1 \rightarrow 0$, $\alpha_2 \rightarrow 0$, $\beta_1 \rightarrow 0$ and $\beta_2 \rightarrow 0$, thus

$$P \approx -\left(\frac{2}{\pi\beta_2} + \frac{\beta_2}{\pi\Gamma(2)}\ln\frac{2}{\beta_1}\right) + i\frac{2\ln r_{53}}{\pi\Gamma(1)}, \tag{7}$$

$$Q \approx -\left(\frac{2}{\pi\beta_1} + \frac{\beta_1}{\pi\Gamma(2)}\ln\frac{2}{\beta_2}\right) + i\frac{\Gamma(1)}{\pi\Gamma(2)}\left(\frac{1}{r_{53}} - r_{53}\right), \tag{8}$$

$$z_1 \approx \frac{\Gamma(1)}{\pi\Gamma(2)}\left(\frac{1}{r_{13}} - r_{13}\right)(PX + QX') + i\frac{1}{\pi}\left(\frac{\alpha_1}{\Gamma(2)}\ln\frac{2}{\alpha_2} + \frac{2}{\alpha_1}\right)(PX' + QX), \tag{9}$$

$$z_2 \approx \left(\frac{2}{\pi\alpha_2} + \frac{\alpha_2}{\pi\Gamma(2)}\ln\frac{2}{\alpha_1}\right)(PX + QX') - i\frac{2\ln r_{13}}{\pi\Gamma(1)} \times (PX' + QX), \tag{10}$$

therefore

$$K_R \approx \frac{|(1-r_{13})(1+r_{53})e^{-ik_3W} - (1+r_{13})(1-r_{53})e^{ik_3W}|}{|(1+r_{13})(1+r_{53})e^{-ik_3W} - (1-r_{13})(1-r_{53})e^{ik_3W}|}, \tag{11}$$

which coincides with the reflection coefficient given by Mei (1989) (see pp. 130–131) for this special case, where carefulness must be taken as there is a pen slip in Eq. (4.17) in Mei's (1989) solution.

Furthermore, if $r_{13}=1$ or $r_{53}=1$, then (11) degenerates into

$$K_R = (1 - r_{53})/(1 + r_{53}), \tag{12}$$

$$K_R = (1 - r_{13})/(1 + r_{13}), \tag{13}$$

which are the reflection coefficients for an infinite step given by Lamb (1932).

On the other hand, when $r_{53} \rightarrow 1$, then $\beta_1 \rightarrow \infty$ and $\beta_2 \rightarrow \infty$ and

$$P = Q = -\frac{2}{\pi}\frac{1}{\sqrt{\beta_1\beta_2}}e^{i(\beta_2-\beta_1)} + O\left(\frac{1}{\beta_1^2}\right), \tag{14}$$

we therefore have

$$K_R \approx \frac{|z_3 - iz_4|}{|z_3 + iz_4|} = \frac{\sqrt{|z_3|^2 + |z_4|^2 + 2[Re(z_3)Im(z_4) - Im(z_3)Re(z_4)]}}{\sqrt{|z_3|^2 + |z_4|^2 - 2[Re(z_3)Im(z_4) - Im(z_3)Re(z_4)]}}, \tag{15}$$

where Re(z) and Im(z) denote the real and imaginary part of z, respectively, and

$$z_3 = J_1(\alpha_1)Y_1(\alpha_2) - J_1(\alpha_2)Y_1(\alpha_1) + i[J_1(\alpha_1)Y_0(\alpha_2) - J_0(\alpha_2)Y_1(\alpha_1)],$$

$$z_4 = J_1(\alpha_2)Y_0(\alpha_1) - J_0(\alpha_1)Y_1(\alpha_2) + i[J_0(\alpha_2)Y_0(\alpha_1) - J_0(\alpha_1)Y_0(\alpha_2)].$$

By denoting that

$$\begin{aligned}&\Delta_1 = J_0^2(\alpha_1) + J_1^2(\alpha_1), \quad \Delta_2 = J_0^2(\alpha_2) + J_1^2(\alpha_2),\\ &\mu_1 = Y_0^2(\alpha_1) + Y_1^2(\alpha_1), \quad \mu_2 = Y_0^2(\alpha_2) + Y_1^2(\alpha_2),\\ &v_1 = J_0(\alpha_1)Y_0(\alpha_1) + J_1(\alpha_1)Y_1(\alpha_1),\\ &v_2 = J_0(\alpha_2)Y_0(\alpha_2) + J_1(\alpha_2)Y_1(\alpha_2),\end{aligned}$$

the Eq. (15) can be further simplified as

$$K_R \approx \frac{\sqrt{\Delta_1\mu_2 + \Delta_2\mu_1 - 2v_1v_2 - \frac{8}{\pi^2}\frac{r_{13}}{\alpha_1^2}}}{\sqrt{\Delta_1\mu_2 + \Delta_2\mu_1 - 2v_1v_2 + \frac{8}{\pi^2}\frac{r_{13}}{\alpha_1^2}}}, \tag{16}$$

which coincides with the analytical solution by an infinite step behind a linear slope, Dean (1964) (see pp. 19–20) for long wave modification by an infinite step behind a linear slope, although Dean considered waves incident from the opposite direction.

Acknowledgement

This project is supported by the grant from the National University of Singapore (R-264-000-121-112) and the grant from the Meteorological Services of Singapore (R-264-000-121-291). The first author likes to acknowledge the support by the Natural Science Foundation of P.R. China (NSFC10162001, NSFC10462001), the Excellent Young Teachers Program of Ministry of Education P.R. China (EYTP1784), Guangxi Shi-Bai-Qian Scholars Program (2001224) and the Education Department of Guangxi.

References

Bender, C.J., Dean, R.G., 2003. Wave transformation by two-dimensional bathymetric anomalies with sloped transitions. Coast. Eng. 50, 61–84.

Dean, R.G., 1964. Long wave modification by linear transitions. J. Waterw. Harb. Div., ASCE 90 (1), 1–29.

Lamb, H., 1932. Hydrodynamics. Dover (Section 176).

Lin, P., Liu, H.-W., 2005. An analytical study for linear long wave reflection by a two-dimensional obstacle of general trapezoidal shape. J. Eng. Mech. ASCE (to appear).

Mei, C.C., 1989. The Applied Dynamics of Ocean Surface Gravity Waves. World Scientific, Singapore.