# DISCUSSION OF TRIPARTITE MICROSEISMIC MEASUREMENTS*

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ABSTRACT

There has been a tendency to overemphasize the accuracy of a tripartite station's determination of the bearing of a microseismic wave. Such a determination is based on the assumption that the time of passage of the crest of a specific advancing wave is uniquely observed at each instrument of the network. This is not the case, in general, where waves are approaching from more than one direction. There is compelling evidence that they are doing this in many, if not all, storms.

Specific cases are discussed of waves of the same period and waves of different periods crossing a tripartite network. It is concluded that routine averaging of intervals from time marks to the nearest crest or trough can lead to serious errors if the pattern and character of groups at the three stations are not taken into account.

Velocity determinations for microseisms are open to question when they are based on records from horizontal components only, which do not permit distinguishing between Rayleigh and Q waves.

In 1884, John Milne set up seismographs at the corners of a triangle on the campus of the Tokyo Engineering College. He supplied them with a circuit for simultaneous time marks at all three stations, and endeavored to determine the direction of travel of earthquake waves crossing the network. The first experiments were not satisfactory because the stations were too close together, but Japanese seismologists later got better results by enlarging the triangle until distances between stations ranged from 7,500 to 35,750 ft.[1] By selecting a wave that could be recognized at all stations of the network, and measuring the time of passage of the same phase of that wave at each station, these investigators determined the direction of approach for waves from a number of earthquakes. This method was applied to the study of microseisms by F. Kishinouye and N. Nasu, of the University of Tokyo,[2] H. D. Krug, at Göttingen,[3] E. Ramirez, at St. Louis,[4] and M. Gilmore, of the U. S. Navy.[5]

Such three-station arrays have become known as tripartite networks, or simply tripartite stations. Interpretation of their records has assumed, in the words of Gilmore, "that any microseismic wave or any particular peak of a wave passing over the tripartite station could be accurately recorded, and

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[1] A. Imamura, "Seismic Triangulation in Tokyo," *Publications of the Earthquake Investigation Committee in Foreign Languages*, No. 7, pp. 5–24 (1902).

[2] F. Kishinouye, "On Microseisms of Four Second Period," *Bull. Earthquake Res. Inst., Tokyo Imp. Univ.*, 13: 146–154, 608–615 (1935).

[3] H. D. Krug, "Ausbreitung der natürlichen Bodenunruhe (Mikroseismik) nach Aufzeichnungen mit transportablen Horizontal-Seismographen," *Zeitschr. für Geophysik*, 13: 328–348 (1937).

[4] J. Emilio Ramirez, S.J., "An Experimental Investigation of the Nature and Origin of Microseisms at St. Louis, Missouri," *Bull. Seism. Soc. Am.*, 30: 35–84, 139–178 (1940).

[5] Marion H. Gilmore, "Microseisms and Ocean Storms," *Bull. Seism. Soc. Am.*, 36: 89–119 (1946).

the arrival times determined. If the very small differences in times of arrival at the three stations are accurately measured, the absolute direction of propagation of the wave can be calculated, *regardless of whether it is a true Rayleigh wave or the product of some very complicated combination of waves.*"[6]

Ramirez, in discussing variations in bearings during a microseismic storm, said: "The variation may also be due to other microseismic waves, of smaller amplitudes, coming from various directions and overridden by the larger waves. This interference of waves is evident in certain double or superposed storms of different directions and more or less equal amplitudes. Finally, the variation may be due to a rather extensive area as source of origin. The focus or origin of microseisms does not seem to be a point, as in the first motion in earthquakes, but rather a region several hundred kilometers in diameter."[7]

The reference to bearings being distorted by "other microseismic waves, of smaller amplitudes," does not state the problem fully. There will be no interference pattern, or evidence in the record of more than one set of waves, if they differ only in amplitude. If two groups of waves of equal period overlap, they combine to form simple sine waves of the same period but with maxima displaced in phase from those of the component waves. As a matter of fact, in general, if two sine waves combine, the recorded maxima and minima do not coincide in phase with either of the components. This is a situation which the tripartite station with single-component seismographs at each point is powerless to resolve and one which leads to indeterminate errors in bearings.

For example, consider two waves: $y_1 = A_1 \sin (p_1t + k_1x)$, and $y_2 = A_2 \sin (p_2t + k_2x)$, with $p$ the angular frequency in radians per second, so $p = 2\pi/T$, where $T$ is the period in seconds; $t$ is time, $x$ is distance, and $k$ is in radians per unit distance, so $k = 2\pi/\lambda$, where $\lambda$ is the wave length; $y$ is the displacement of a particle in the path of the wave, and may be regarded as representing the recorded motion on a seismogram.

If these waves have $p_1 = p_2$, that is, have the same period, and reach a network from different angles and out of phase, say with

$$\underset{\text{radians}}{(p_2t + k_2x)} - \underset{\text{radians}}{(p_1t + k_1x)} = \pi/2 \text{ radians} = 90^\circ$$

regardless of the ratio of $A_1{:}A_2$, they combine to form a simple sine wave $y = y_1 + y_2$, with its maximum 45° after that of $y_1$ and 45° before that of $y_2$. As these waves sweep on across the tripartite network, their phase relationships will change and the recorded maximum at each of the other corners of the network will lead or lag the true maxima of the components by some angle other than the 45° computed above. Accordingly, the observed time of maxi-

[6] Gilmore, *op. cit.*, p. 92.
[7] Ramirez, *op. cit.*, p. 60.

mum at each station of the network will not correspond to any individual wave, and the computed bearing will be in error.

Interference patterns appear only when $p_1$ and $p_2$ are different. Two which are suggestive of many seen in the records of microseisms are shown in figure 1. For the top trace, $T_1 = 2.08$ sec., $T_2 = 1.56$ sec., and $T_2/T_1 = p_1/p_2 = 0.75$. For the bottom trace, $T_1 = 3.13$ sec., $T_2 = 2.61$ sec., and $T_2/T_1 = p_1/p_2 = 0.83$. Here again the recorded maxima are at times which differ from those of

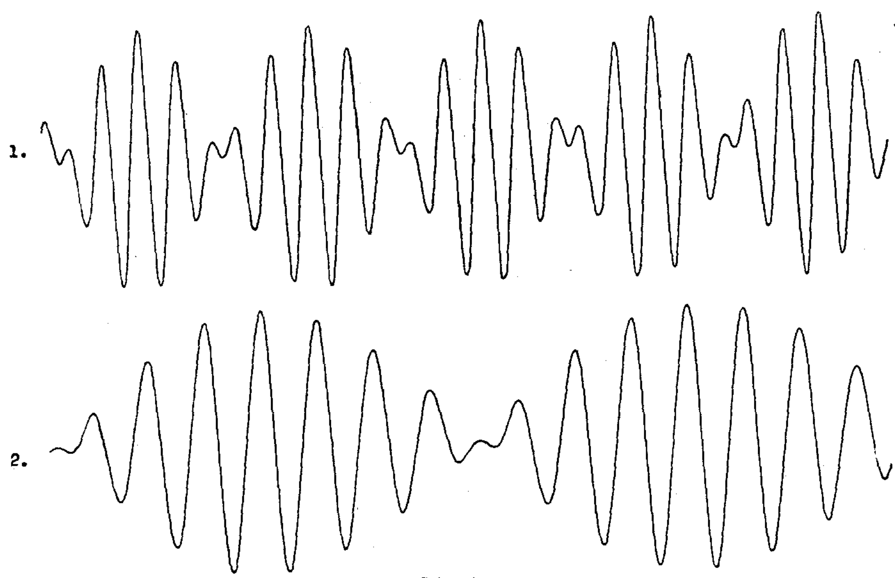


Fig. 1

the true maxima of the component waves, and the phase relationships change as the waves sweep across a tripartite network at different angles.

A detailed calculation of a special case has been given by Galitzin[8] for $A_1 = 1$; $A_2 = 1/3$; $T_2 = 1/3\ T_1$, and the two waves starting 90° out of phase. The combined curve is shown in figure 2. Phase angles at which successive maxima and minima occur are tabulated as follows:

Successive maxima and minima occur at (degrees)

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y_1$.................. | | | | | 90 | | | | | | | 270 | | |
| $y_2$.................. | 0 | | | 60 | | | 120 | 180 | | | 240 | | | 300 |
| $y\ (= y_1 + y_2)$........ | | 22.5 | 45 | | | 112.5 | | | 202.5 | 225 | | | 292.5 | |

These show again that the record, $y$, shows crests and troughs which are not at the times of maximum or minimum for either of the component waves, $y_1$

[8] F. B. Galitzin, "Zur Frage der Analyse Zusammengezetzter Harmonischer Schwingungen," *Bull. de l'Académie Impériale des Sciences de St. Pétersbourg*, pp. 449–474 (1913).

and $y_2$. If $y_1$ and $y_2$ sweep across a tripartite network at different angles, the relationship of $y$ at each corner of the net to the true crests of $y_1$ and $y_2$ will vary. There is compelling evidence that microseismic waves frequently approach a station from different directions at the same time.[9]

Another aspect of the problem is illustrated by the first microseismic storm studied in detail by Ramirez at St. Louis.[10] It was the largest investigated. It ran from October 23 to 28, 1938. On October 24, Ramirez reported waves

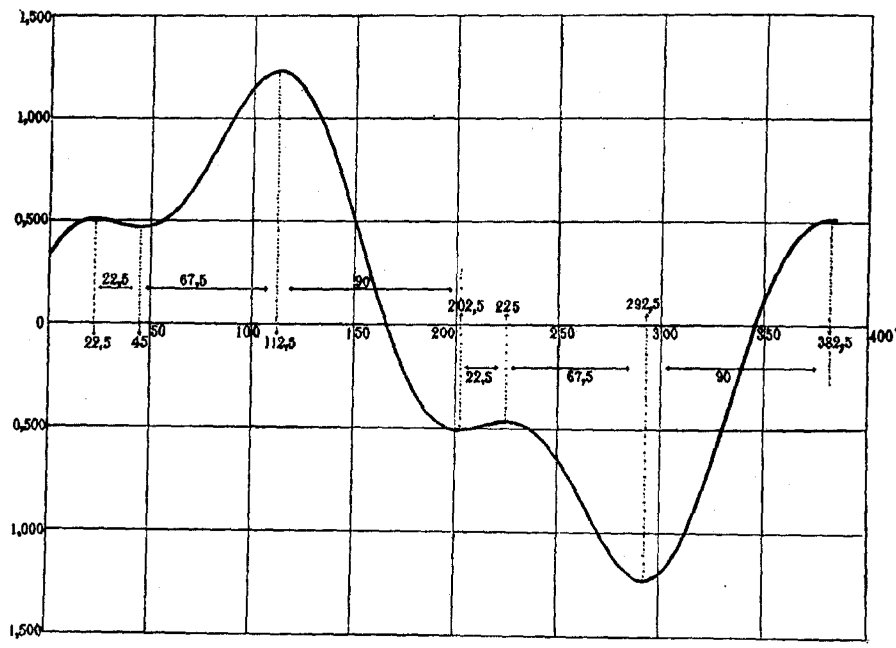


Fig. 2

from N 8° W, and on October 25 a few from N 45° W as well. There was a slight decrease in recorded amplitudes late on the 25th, then a strong resurgence leading to maximum trace amplitudes of 14 mm. on the 26th, computed as waves arriving from N 33° E. He published a section from the E–W record at St. Louis University between 2:56 and 2:59 P.M., October 26, 1938, and one covering the same time interval from the E–W record at Washington University, 4 miles nearly due west. He stated that these records "give a very fair idea of the microseismic storm of October 26, 1938," and that they "bring out the fact that on this date all the regular waves were arriving at the St. Louis University station earlier than at Washington University. The microseisms were

[9] L. Don Leet, "Microseisms in New England—Case History of a Storm," *Geophysics*, 12: 639–650 (1947).

[10] Ramirez, *op. cit.*, p. 59.

coming from the northeast." These record sections are shown in figure 3. The St. Louis record was printed reading from right to left in the Ramirez report and has here been recopied to read from left to right for direct comparison with the Washington record. Time marks were placed on the original records every 6 sec., but they do not all show on the printed copies. They were numbered from 1 through 36 by Ramirez, but only 1, 10, 20, and 30 are marked in figure 3. An extra minute mark shows at 20.

Ramirez measured the time between the nearest crest or trough and each of twenty-three time marks for each station. The difference between this interval at St. Louis and at Washington ranged from 0.3 sec. to 2.0 sec., but was always positive. This was interpreted as meaning that *in every case the*

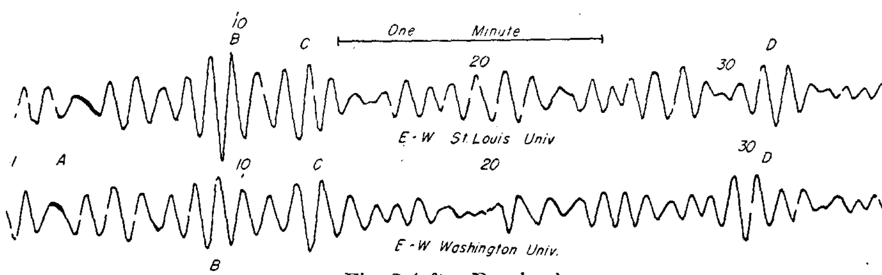


Fig. 3 (after Ramirez)

*wave reached St. Louis first.* These time differences represented phase differences in degrees from about 20° to 130° between the two stations.

An average of ninety-four readings of this kind between 2:45 P.M. and 3:15 P.M. local time, which included the time covered by figure 3, gave a time difference of 1.15 sec., corresponding to waves approaching the stations from N 33° E.

Examination of figure 3, however, opens some questions about the validity of this averaging procedure. The break in sinusoidal character at A occurred at the Washington University station before it did at St. Louis, and the pattern of group B looks like a group that reached Washington University first. C reached St. Louis first, by this criterion, but D, again, passed Washington University first. Meanwhile, between C and D there is a complete breakdown of the point-for-point comparability of the two records. This is most striking in the vicinity of time mark 20. On the basis of record character, even this brief three-minute sample makes it clear that these microseisms are not simple trains of waves sweeping past St. Louis and Washington universities from one or even close to one point of the compass and a common origin of no greater extent than "a region several hundred kilometers in diameter." Under those conditions, averages of differences in passage time at the two stations, conditioned only by the requirement that they be taken from regular sinusoidal waves, would seem to have limited significance.

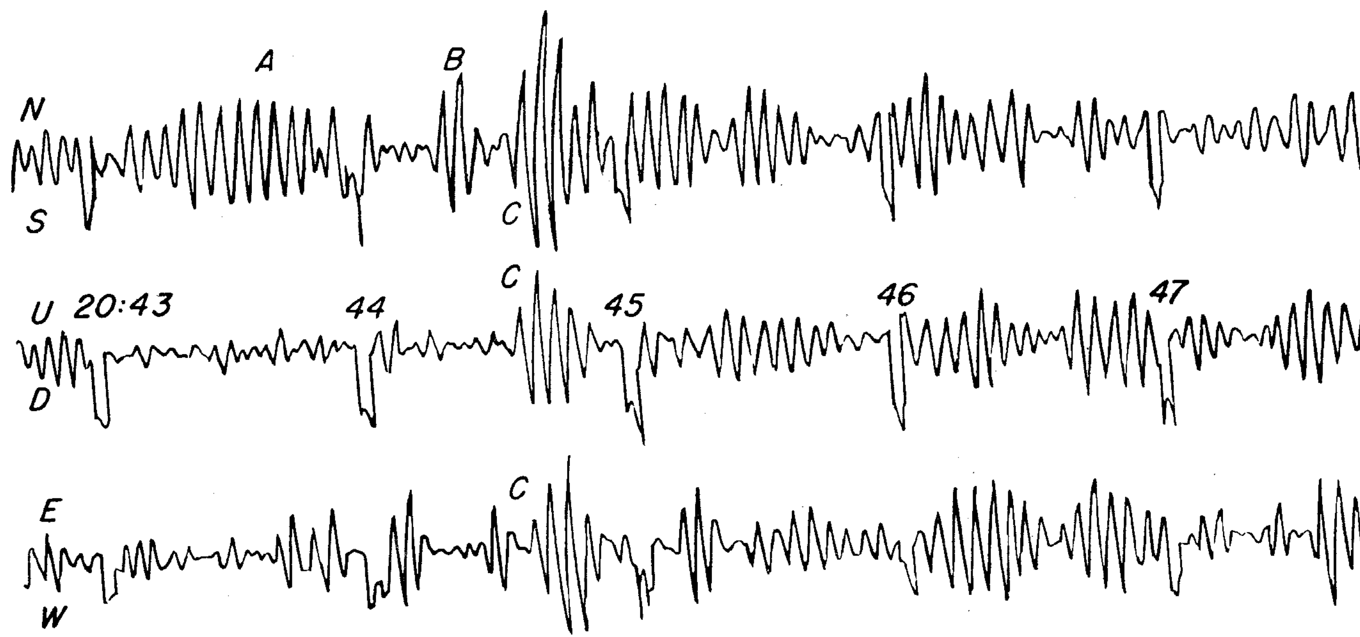


Fig. 4

In determinations of the velocity of microseisms a further problem is presented by the employment of horizontal-component seismographs for registration at the corners of a tripartite network, as practiced by Ramirez and Gilmore. Such instruments are unable to distinguish between Rayleigh and Q waves and there can be no assurance that velocity determinations are being made on the same wave type in all cases, even if the same recorded crest at each corner represents a true wave. If only one instrument were available for each station, a vertical would have the merit of not responding to Q waves.

Values for the velocity of microseisms have been reported over so wide a range as to make it seem highly probable that different wave types as well as phase changes instead of progressive wave advance have been measured:

| *km/sec.* | *Location* |
|---|---|
| 1.1 | Göttingen |
| 2.66 | St. Louis |
| 3.16 | Guam |
| 3.25 | Richmond, Florida |
| 4.00 | Puerto Rico |

It is possible that an ultimate solution of this problem will be found with a tripartite station equipped with three-component registration at each corner of the triangle. Rayleigh waves could then be used to check directions of approach,[11] and the character of the records and wave types could be analyzed in such a way as to spot the occasional isolated microseisms that pass without interference from other directions. When this has been done, the direction of its travel can be fixed precisely by the relative times of its passage at the three corners of the network.

Figure 4 illustrates some of the complications of wave pattern recorded at the Harvard station during five minutes of a microseismic storm, primarily to point up the necessity for having three-component records. Group A represents a train of two or more nearly pure Q waves. At B is a simple, isolated Q wave from nearly due east or west of the station. This could be used for a tripartite determination of direction of approach. Group C is apparently a practically pure Rayleigh wave approaching from roughly SSE, which could also be used for a tripartite measurement.

[11] Leet, *op. cit.*

Harvard, Massachusetts,
April 8, 1949.