



Stokes waves modulation by internal waves

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[1] The effect of subsurface currents induced by internal waves on nonlinear surface waves is theoretically analyzed. An analytical and numerical solution of the modulation equations are found under the conditions close to the group velocity resonance. It is shown that smoothing of the down current surface waves is accompanied by a relatively high-frequency modulation while the profile of the opposing current is reproduced by the surface wave's envelope. Long surface waves can form the wave modulation forerunner ahead of the internal wave, while the relatively short surface waves create the trace of the internal wave. **Citation:** Lee, K.-J., H.-H. Hwung, R.-Y. Yang, and I. V. Shugan (2007), Stokes waves modulation by internal waves, *Geophys. Res. Lett.*, *34*, L23605, doi:10.1029/2007GL031882.

1. Introduction

[2] Internal waves (IW) represent one of the most interesting phenomena in ocean dynamics. Field investigations carried out by conventional contact oceanography methods are complicated by significant technical problems.

[3] Currently, another line of inquiry is very common: the remote study of IWs by their exhibition at the sea surface along with contact measurements. The ability of IWs to substantially change the wave structure on that surface had long been explored [Hughes and Grant, 1978; Osborne and Burch, 1980; Alpers, 1985], confirmed by many observations, and is reliably established now. Remote sensing instruments (radars, traditional optical devices, and laser locators–lidars) can detect reliably the IW surface exhibitions of the ocean surface. At the same time, the quantification of IW parameters by remote sensing of the sea surface calls for solving a number of theoretical problems. First, construct a model of the sea surface imaging and establish mechanisms of surface wave (SW) perturbation by IWs.

[4] The following are the basic properties of sea surface anomalies established experimentally during combined oceanographic missions as a result of the interaction between IWs and the sea surface [Basovich *et al.*, 1987; Hughes and Grant, 1978; Gasparovic *et al.*, 1988]: (1) Wide bands of slicks and rough sea move at a phase velocity of the IW train. These bands are most distinct at moderate and light wind speeds (lower than $5 \text{ m} \cdot \text{s}^{-1}$). (2) The maximum modulation is observed at co-propagation of sea wind waves

and IWs, especially for the sea spectral components whose group velocity c_g is close to the IW phase velocity c ($c_g \sim c$). (3) An individual slick is not uniquely arranged with respect to the IW phase. It can be both above the IW trough and above the IW crest. (4) The distance between the bands usually corresponds to the IW length. (5) A lot of experiments [Basovich *et al.*, 1987; Bakhanov *et al.*, 1994] demonstrated the wind waves anomalies on the long (about 1 km) distance from the space filled by the internal waves.

[5] In general, the interaction between surface and internal waves was studied both theoretically and experimentally in the work of the last three decades. Nevertheless, we have no exhaustive theory of this phenomenon up until now, though several mechanisms are proposed for such an interaction in various SW spectral ranges.

[6] A basic modulation mechanism in meter and decimeter SW ranges is the hydrodynamic impact of a subsurface current induced by IWs on the SWs [Lewis *et al.*, 1974; Phillips, 1977]. The quasi-steady model of such an impact on a linear surface wavepacket was first proposed by Gargett and Hughes [1972] and Phillips [1977]. The analysis of modulational equations had shown the packets with a group velocity approaching the IW phase velocity to be most sensitive to the subsurface current. The SW amplitude in the group resonance range grows infinitely, showing inapplicability of the developed linear modulation model to quantitative description of the wave interaction.

[7] Recently the unsteady propagation of short surface waves in the presence of IW current was studied [Donato *et al.*, 1999; Stocker and Peregrine, 1999] using both simple ray theory for linear waves and a fully nonlinear numerical potential solver. For ray theory, the occurrence of focusing is examined in some detail and it is shown that in these regions the waves steepened and may break. Comparisons are made between ray theory and the more accurate numerical simulations.

[8] Transformation of gravity capillary surface waves on the current created by a large-amplitude internal wave is observed by Kropfli *et al.* [1999] and Bakhanov and Ostrovsky [2002]. In particular, the location of the maxima and minima of the surface wave spectral density with respect to the IW profile is studied. It is shown that for sufficiently large-amplitude internal solitary waves (solitons) propagating in the same direction as the surface wave the minimum of density for all SW lengths is situated over the crest of the soliton. These observations conflict with the expectation that the highest surface roughness would be near the region of the greatest surface gradient [Gasparovic *et al.*, 1988; Hogan *et al.*, 1996]. It can be mentioned, however, that the last conclusions were made for sufficiently small IW currents, $\sim 1\text{--}2 \text{ cm/sec}$, whereas currents may be many times bigger for large amplitude IW. This does clearly emphasize the importance of the wave's nonlinearity in the modeling of an IW impact on the sea surface.

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[9] The goal of the present work is to construct a uniformly valid steady model of the IW interaction with nonlinear Stokes surface waves under the condition close to the group velocity resonance and to describe a number of observed experimental effects.

[10] The paper consists of five sections. General equations of the one-dimensional interaction between an SW train and a nonuniform current induced by IWs are derived in section 2. Section 3 is dedicated to an analysis of the traveling solution for the resonant interaction. The data on analytically and numerically calculated interaction for various-type of IWs and initial SW parameters are presented in Section 4. Section 5 contains concluding remarks and inferences.

2. Modulational Equations of a One-Dimensional Interaction

[11] Internal waves in the Ocean only produce very small vertical displacements of the free surface. If $N(0)$ is the Brunt-Vaisala frequency at the sea surface $z = 0$ (z is the vertical coordinate in the upward direction), then near the surface, the vertical velocity component is $w = W(z)\exp[i(Kx - nt)]$, $W(z) \propto \exp\{(N^2(0)/n^2 - 1)^{1/2} Kz\}$, where n and K – frequency and horizontal wave number of internal wave [Phillips, 1977]. The condition of constant pressure at the free surface leads to

$$n^2 W_z - gk^2 W = 0 \quad \text{at } z = 0. \quad (1)$$

and therefore the free surface condition can be fixed as $W = 0$ at $z = 0$ (solid cover condition), provided

$$\frac{n^2}{gK} |N^2(0)/n^2 - 1| \ll 1$$

Since the frequency n of internal waves is always much less than the frequency of a free surface wave with the same wave number, namely $(gK)^{1/2}$, this condition is usually strongly satisfied.

[12] The typical case here is the existence of a strong seasonal thermocline which separates two relatively homogeneous water masses. $N(z)$ consequently has a single sharp maximum at the thermocline and is very small elsewhere. By using the continuity equation $U_x + W_z = 0$ (U and x are the value of horizontal velocity and coordinate, respectively) and the dynamic boundary condition (1) we'll have the following estimation for the vertical velocity near the free surface: $W/U \sim n^2/gK \sim \delta\rho/\rho_0$ ($\sim 10^{-3}$ characteristically), where $\delta\rho$ and ρ_0 are the maximal variation of the density inside the thermocline and the mean density, correspondingly.

[13] As about 200–500 SW lengths and more can be settled within one IW length, the interacting waves have very different scales. Therefore, the problem can be considered as the SW propagation in a slowly varying moving medium. The first set of complete equations to describe short waves propagating over much larger scale nonuniform currents were given by Longuet-Higgins and Stewart [1964]. Wave energy is not conserved and the concept of 'radiation stress' was introduced to describe the averaged

momentum flux terms which govern the interchange of momentum with the current. In this present model, it is also sufficient to consider the energy exchange in terms of wave action conservation law of the second order and neglect the effect of this momentum transfer on the form of the surface current because it will be an effect of the highest order [Stocker and Peregrine, 1999].

[14] We construct the model of the IW effect on propagation of the narrow-band weakly nonlinear Stokes packet of gravity SWs based on the following assumptions: (1) Surface and internal waves propagate along a common x -direction. (2) The current horizontal velocity in the subsurface layer is set as the traveling wave $U = U(K\xi) = U(K(x - ct))$, where $\xi = x - ct$ is the accompanying coordinate. (3) Horizontal subsurface current, $U(K\xi)$, is induced by the internal wave we'll define by the velocity potential: $\Phi_0(K\xi)$: $d\Phi_0(K\xi)/d\xi = U(K\xi)$. Fluid motion is accepted to be the potential since the horizontal current varies slowly enough ($U_\xi/kU = O(\varepsilon^2)$, a_0 is a characteristic free-surface displacement, $2\pi/k_0$ is a typical surface wavelength, and $\varepsilon = a_0 k_0 \ll 1$ is the conventional small average wave steepness parameter). The ratio, U/c , is assumed to be small, $U/c \sim \varepsilon$, which is usually confirmed by experimental data ($U/c \sim 0.1-0.3$) [Hughes and Grant, 1978; Gasparovic et al., 1988]. This, along with the assumption $c \sim c_p$ (c_p is the SW phase velocity), satisfies to the continuity equation in the second order. Finally, the velocity potential function will be represented as a sum of wave's and current's potential $\phi(x, z, t) + \Phi_0(K\xi)$.

[15] The set of equations for potential motion of an ideal incompressible infinite-depth fluid with the free surface is given by the Laplace equation:

$$\phi_{xx} + \phi_{zz} = 0, \quad -\infty < z < \eta(x, t) \quad (2)$$

the boundary conditions at the free surface:

$$g\eta + \phi_t + \frac{1}{2}(\phi_x^2 + \phi_z^2) = 0, \quad z = \eta(x, t), \quad (3)$$

$$\eta_t + \phi_x \eta_x = \phi_z, \quad z = \eta(x, t), \quad (4)$$

$$\text{and at the bottom: } \phi = 0, \quad z = -\infty. \quad (5)$$

[16] Here, $\eta(x, t)$ is the free-surface displacement, g is the gravity acceleration, and t is time.

[17] The variables are normalized as follows:

$$\begin{aligned} \phi &= a_0 \sqrt{\frac{g}{k_0}} \phi' = \varepsilon \sqrt{\frac{g}{k_0^3}} \phi', \quad \eta = a_0 \eta' = \frac{\varepsilon}{k_0} \eta', \quad t = \frac{1}{\sqrt{gk_0}} t', \\ z &= \frac{z'}{k_0}, \quad x = \frac{x'}{k_0}, \quad U(K(x - ct)) = U'(K/k_0(x' - c/c_p t')) \\ c_p &= U'(\varepsilon_1(x' - c' t')) c_p, \end{aligned} \quad (6)$$

where $\varepsilon_1 = K/k_0$ – another small parameter characterizing the ratio of the surface and internal wavelengths and the dimensionless quantities are primed. It is noteworthy that

normalization (6) explicitly specifies the principal scales of sought functions $\phi = O(\varepsilon)$ and $\eta = O(\varepsilon)$. Then, the set (2)–(5) is reduced to the form

$$\phi_{xx} + \phi_{zz} = 0, \quad -\infty < z < \varepsilon\eta(x, t) \quad (7)$$

$$-\eta = \phi_t + U\phi_x + \varepsilon \frac{1}{2}(\phi_x^2 + \phi_z^2), \quad z = \varepsilon\eta(x, t), \quad (8)$$

$$\eta_t + U\eta_x + \varepsilon\phi_x\eta_x = \phi_z, \quad z = \varepsilon\eta(x, t), \quad (9)$$

$$\phi = 0, \quad z = -\infty, \quad (10)$$

where the primes are omitted. The weakly nonlinear surface wave train is described by a solution to equations (7)–(10), expanded into a Stokes series in terms of small parameter ε .

[18] Assuming the wave motion phase $\theta = \theta(x, t)$ in the presence of a slowly varying current U , we define the local wavenumber k and frequency σ as:

$$k = \theta_x, \quad \sigma + k \cdot U = -\theta_t \quad (11)$$

These main wave parameters together with the first-order velocity potential amplitude, ϕ_0 , will be considered further as slowly varying with the typical scale, $O(\varepsilon^{-1})$, longer than the primary wavelength and period [Whitham, 1974]:

$$\phi_0 = \phi_0(\varepsilon x, \varepsilon t), \quad k = k(\varepsilon x, \varepsilon t), \quad \sigma = \sigma(\varepsilon x, \varepsilon t) \quad (12)$$

On this basis, we attempt to recover the effects of long-scaled current and nonlinear wave dispersion additional (having the same order) to the Stokes' term with the wave steepness squared.

[19] The solution to the problem, uniformly valid to $O(\varepsilon^3)$, is found by a two-scale expansion with the differentiation:

$$\frac{\partial}{\partial t} = -(\sigma + k \cdot U) \frac{\partial}{\partial \theta} + \varepsilon \frac{\partial}{\partial T}, \quad \frac{\partial}{\partial x} = k \frac{\partial}{\partial \theta} + \varepsilon \frac{\partial}{\partial X}, \quad T = \varepsilon t, \\ X = \varepsilon x. \quad (13)$$

Substitution of the velocity potential in its linear form, $\phi = \phi_0 e^{kz} \sin \theta$, satisfies the Laplace equation (7) in the first order in ε due to (11) and gives the following additional terms in the second order $O(\varepsilon^2)$:

$$\varepsilon(2k\phi_{0X} + k_X\phi_0 + 2kk_X\phi_0 z)e^{kz} \cos \theta + \dots = 0 \quad (14)$$

To satisfy the Laplace equation in the second order, Yuen and Lake [1982] and Shugan and Voliak [1998] suggested additional phase shifted term with a linear and quadratic z correction in the representation of the potential function ϕ :

$$\phi = \phi_0 e^{kz} \sin \theta - \varepsilon \left(\phi_{0X} z + \frac{k_X \phi_0}{2} z^2 \right) e^{kz} \cos \theta + \dots \quad (15)$$

Exponential decaying of wave's amplitude with z is accompanied by the second order subsurface jet due to

slow horizontal variations of the wave number and amplitude of the wave packet.

[20] The free-surface displacement $\eta = \eta(x, t)$ is also sought as an asymptotic series,

$$\eta = \eta_0 + \varepsilon\eta_1 + \varepsilon^2\eta_2 + \dots, \quad (16)$$

where η_0 , η_1 , and η_2 are $O(1)$ functions to be determined. Using expressions (15)–(16) subject to the dynamic boundary condition (8), we consequently find the components of the free-surface displacement

$$\eta_0 = \sigma\phi_0 \cos \theta, \quad (17)$$

$$\eta_1 = -\phi_{0T} \sin \theta - U\phi_{0X} \sin \theta + \frac{1}{2}\sigma^2 k\phi_0^2 \cos(2\theta), \quad (18)$$

$$\eta_2 = -\frac{3}{8}\sigma k^3 \phi_0^3 \cos \theta. \quad (19)$$

Only the self-action term with fundamental wave phase θ is included in the third-order displacement (19).

[21] Substitution of velocity potential (15) and displacement (16)–(19) into the kinematics boundary condition (9) gives the following relationships between the wave modulation characteristics:

$$\sigma^2 = k + \varepsilon^2 k^4 \phi_0^2 + \varepsilon^2 (\phi_{0TT} + 2U\phi_{0XT} + U^2\phi_{0XX})/\phi_0, \quad (20)$$

$$[\phi_0^2 \sigma]_T + \left[\left(U + \frac{1}{2\sigma} \right) \phi_0^2 \sigma \right]_X = 0. \quad (21)$$

The first of these formulas represents the wave dispersion relation with the total second-order amplitude-phase dispersion included in the presence of the current, U . Equation (21) yields the known wave action conservation law. Modulation equations (20)–(21) are closed by the equation of wave phase conservation that follows from (11) as a compatibility condition [Phillips, 1977]:

$$kT + (\sigma + kU)_X = 0 \quad (22)$$

The derived set of equations (20)–(22) in the absence of a current coincide with those of Shugan and Voliak [1998].

3. Traveling Wave Solution

[22] Let us analyze the traveling wave solutions to the problem (20)–(22) supposing all the unknown functions to depend on the single coordinate $\xi = X - cT$, where the waves are stationary. Then, after integrating (21) and (22) the problem has the form:

$$\sigma^2 = k + \varepsilon^2 k^4 \phi_0^2 + \varepsilon^2 (U(\xi) - c)^2 \phi_{0\xi\xi} / \phi_0, \quad (23)$$

$$(U(\xi) - c + c_g) \phi_0^2 \sigma = A, \quad (24)$$

$$\sigma + k(U(\xi) - c) = \Omega, \quad (25)$$

where $c_g = 1/(2\sigma)$ is the linear group velocity of surface waves, A and Ω are the integration free constants with the physical meaning of wave action flux and frequency, respectively, measured in the moving coordinate frame. The values of constants are determined from the boundary conditions for the uniform Stokes wave at infinity. In the dimensional form these values are

$$A = \left(\frac{1}{2} \sqrt{\frac{g}{k_0}} - c \right) \frac{a_0^2}{\sqrt{gk_0}}; \Omega = \sqrt{gk_0} \left(1 + \frac{1}{2} (a_0 k_0)^2 \right) - k_0 c,$$

where a_0 , k_0 are the constant amplitude and wave number of the Stokes wave in the absence of current at the infinity.

[23] The analysis of the problem in the linear statement [Gargett and Hughes, 1972; Lewis et al., 1974] where the dispersion relation (23) is independent of the SW amplitude describes the wave modulation under conditions far from the group resonance $c \approx c_g$ well. Amplitude distribution can be found from the energy equation (24) after solving the “kinematic” closed set of equations (23), (25) for the wavenumber and frequency. In the resonance vicinity.

$$U(\xi) - c + c_g = 0, \quad (26)$$

the solution found has singularities related to an infinite amplitude growth at the points of SW “blockage” by the current (see equation (24)), which defines a “barrier” for SW propagation over it. In this case, the linear steady solution becomes incorrect.

[24] After eliminating the wavenumber k and frequency σ the set (23)–(25) transforms into a single equation for the first-order potential amplitude ϕ_0 :

$$\frac{1}{(U-c)^2} \left(\frac{A}{\phi_0^2} \right)^2 = \frac{(1+4\Omega(U-c))}{4(U-c)^2} + \varepsilon^2 \left(\left(\frac{1}{2} - \frac{A}{\phi_0^2} \right) \frac{1}{(U-c)^2} + \left(\frac{\Omega}{(U-c)} \right)^4 \cdot \phi_0^2 + \varepsilon^2 (U-c)^2 \phi_{0\xi\xi} / \phi_0 \right) \quad (27)$$

[25] The main feature of the model is clearly visible from equation (27): the solution in the zero order on ε exists for positive values of expression

$$1 + 4\Omega(U(\xi) - c), \quad (28)$$

depending only on the flow velocity and the basic parameters of the problem. For sufficiently large negative values of $U(\xi)$, expression (27) can become equal to zero or even negative so the highest orders of dispersion relation have to be taken into account. It can be shown that the zero value of expression (28) corresponds to the condition of zero local velocity of the wave action flux and correspondingly to unlimited growth of the wave amplitude (see equation (24)).

4. Stokes Surface Wave, Resonantly Modulated by the Current

[26] First we’ll analyze the derived set (23)–(25) under the resonant boundary conditions (26). At $A = 0$ the energy equation immediately yields the following result: the SW energy flux through any section $\xi = \xi_0$ is zero in the frame

of reference related to the IWs. The above mentioned blockage condition (26) defines the direct modulation of the surface wavenumber by the current

$$k = k_0 \left(1 + \frac{3U(\xi)}{c} \right) \quad (29)$$

Hence, the SW length in the resonant interaction mode is modulated rather weakly by the current induced by IWs. In this case, the SWs shorten and lengthen respectively at down- and countercurrents.

[27] Modulation of an initially homogeneous Stokes wave will be considered under the following boundary conditions:

$$\xi \rightarrow -\infty, \quad U(\xi) \rightarrow 0, \quad \phi_0 \rightarrow 1, \quad k \rightarrow 1. \quad (30)$$

Free parameters in the set (23)–(25) can be determined by using the boundary condition (30):

$$c = \frac{1}{2} - \frac{\varepsilon^2}{4}; \quad \Omega = \frac{1}{2} + \frac{3\varepsilon^2}{4}. \quad (31)$$

It can be mentioned here, that no dimensional value of velocity $c = 1/2$ corresponds in the leading order to linear group velocity of surface waves and due to the condition of phase synchronism – to phase velocity of the internal wave.

[28] The equation (27) up to $O(\varepsilon^2 U/c, \varepsilon_1^2 U/c)$ takes on the form

$$G_2 \phi_{0\xi\xi'} + (G_0 u(\xi') - 1) \phi_0 + \phi_0^3 = 0, \quad (32)$$

where ξ' - is the renormalized variable to the space scale of IW: $\xi' = (\varepsilon_1/\varepsilon)\xi$ and primes will be omitted further; $u(\xi) = U(\xi)/|U_0|$ - current velocity function, normalized by its maximum amplitude, $G_2 = \left(\frac{K}{2k_0} \right)^2 \frac{1}{(a_0 k_0)^2}$ is the relationship between surface and internal typical wavelengths and SW initial steepness, $G_0 = \frac{2|U_0|}{(a_0 k_0)^2} \sim O(\varepsilon^{-1})$ is a big parameter characterizes the value of subsurface current relative to the unperturbed SW steepness.

[29] Equation (32), subject to the boundary conditions (30), was analyzed analytically and numerically for various values of controlling dimensionless parameters G_0 and G_2 . Current velocity function, $u(\xi)$, is presented in equation (32) in the normalized form (both space and amplitude scales are presented in the dimensionless parameters G_0 and G_2). Therefore in analytical simulations, we’ll assume function $u(\xi)$ to be a solitary smooth function with a nonzero value inside the interval $(-1, 1)$ and maximal amplitude equal to the unit.

[30] Let’s first consider the modulation of SW on a counter flow when $u(\xi) < 0$. The equation (32) contains the big parameter $G_0 \gg 1$ and the solution is sought as an asymptotic series:

$$\phi_0(\xi) = G_0^{1/2} \phi_{00}(\xi) + \phi_{01}(\xi) + \dots \quad (33)$$

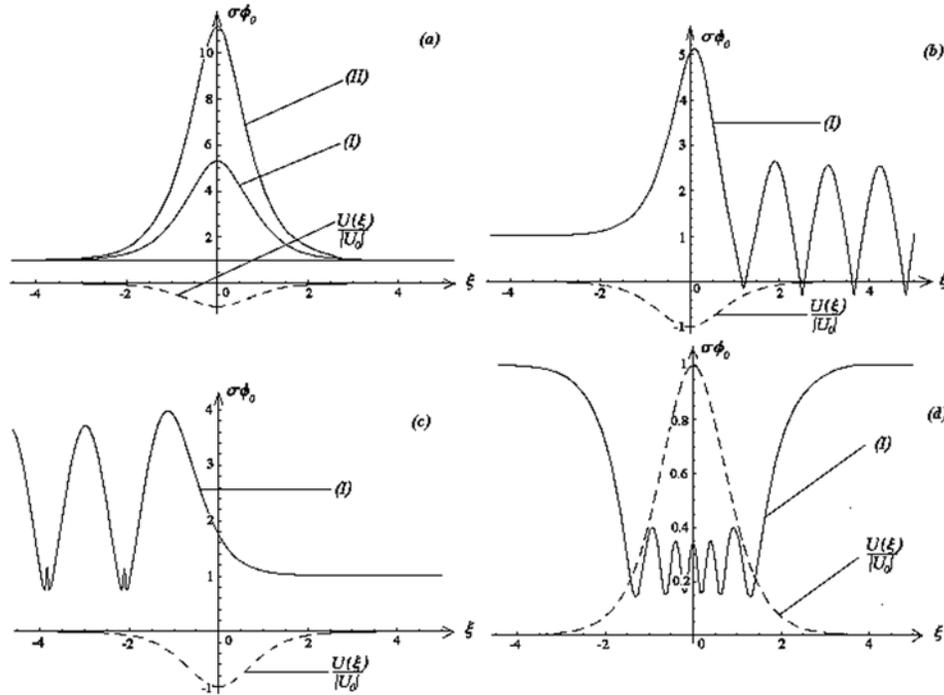


Figure 1. Surface wave envelope $\sigma\phi_0 = \sigma\phi_0(\xi)$ at (a) $G_2 = 1, c = 0.5, \varepsilon = 0.1$; $G_0 = 10$ – curve (I) and $G_0 = 20$ – curve (II), respectively; (b) $G_2 = 5, c = 0.45, \varepsilon = 0.1, G_0 = 10$ – curve (I); (c) $G_2 = 5, c = 0.6, \varepsilon = 0.1, G_0 = 10$ – curve (I); (d) $G_2 = 1, c = 0.5, \varepsilon = 0.1; G_0 = 10$ – curve (I). The dashed curve is the current profile $u(\xi)$.

After substitution of expression (33) into (32), the solution in the main order looks like:

$$\phi_{00}(\xi) = (1 - G_0 u(\xi))^{1/2} / G_0^{1/2} \tag{34}$$

The next order of approximation results to the equation

$$\phi_{01\xi\xi} + q^2(\xi)\phi_{01} + \phi_{00\xi\xi}G_0^{1/2} = 0, \tag{35}$$

where $q^2(\xi) = 2G_0\phi_{00}^2(\xi)/G_2$. This is the linear non-uniform Schrödinger equation, containing the big parameter $q^2(\xi) = O(G_0)$ at the potential. The potential function does not change the sign. Consequently, here we apply a standard method of stationary phase [Nayfeh, 1981] to find its asymptotic solution:

$$\begin{aligned} \phi_{01} = & -\phi_{00\xi\xi}G_0^{1/2}/q^2 + C_1 \frac{\text{Sin}\left(\int_{-1}^{\xi} q(\tau)d\tau\right)}{q^{1/2}} \\ & + C_2 \frac{\text{Cos}\left(\int_{-1}^{\xi} q(\tau)d\tau\right)}{q^{1/2}}, \end{aligned} \tag{36}$$

where C_1 and C_2 are the constants that are to be determined from the boundary conditions for the smooth merging of the solution (36) with the constant velocity potential function: $\phi_0 = 1, \phi_{0\xi} = 0$ at $\xi \leq -1$. Within $o(G_0^{1/4})$, values of constants are the following: $C_2 = 0, C_1 = -\phi_{00\xi}(-1)G_0^{1/2}/q^{1/2}(-1)$.

[31] We consider the data calculated for the modulation of SW by the typical solitary type IW negative current $u(\xi) = -\text{Sech}^2[\xi]$ for the near resonant conditions of wave interaction. The results of numerical calculations of the general equation (27) are shown in Figure 1a for two different values of the current intensity parameter G_0 . The above asymptotic solution sufficiently describes the basic properties of SW modulation. The main property of the solution is that the envelope of the SW on a counter flow may increase considerably, surpassing its initial value several times. For sufficiently large scale IW ($G_2 \leq 1$), the SW envelope reproduces the shape of the subsurface current. The amplitude of modulation grows with the intensity of the IW-induced current and is symmetric relative to the current profile axis.

[32] The data found for shorter IWs ($G_2 \geq 5$) and a slight mismatch from the resonance regime ($c < 1/2$) are shown in Figure 1b. The following interaction features are noteworthy: the SW steepness modulation maximum shifts to the IW head slope as the IW horizontal size decreases. Another strong particularity of the solution is the arising of the IW forerunner: ahead of the IW, in the region with no current, an intensive periodic SW train propagates with low-frequency modulation exceeding the entering wave amplitude. This means that the relatively long SW ($c < 1/2$), after crossing the region of internal wave current, keeps the low frequency amplitude modulations. That property is confined by field measurements [Basovich et al., 1987; Bakhanov et al., 1994].

[33] Another possibility of arising the trace of the IW while crossing the field of short SW ($c > 1/2$), also in the form of SW low frequency modulations, is shown at

Figure 1c. The intensity of modulations in the trace of the IW is comparable with the main peak modulations.

[34] Resonant solution for the SW modulation on a down flow $u(\xi) > 0$ can be found in the following form:

$$\phi_0(\xi) = 1 + \phi_{01}(\xi) + \dots \quad (37)$$

where $\phi_{01}(\xi) = o(1)$. Substitution of the expression (37) into (32) gives for $\phi_{01}(\xi)$:

$$\phi_{01\xi\xi} + q^2(\xi)\phi_{01} + G_0/G_2u(\xi) = 0, \quad (38)$$

where $q^2(\xi) = (2 + G_0u(\xi))/G_2$. This is the linear non-uniform Schrödinger equation, containing the big parameter $q^2(\xi) = O(G_0) \sim O(\varepsilon^{-1})$ at the potential. The potential function does not change in sign and consequently, we can apply a standard method of stationary phase [Nayfeh, 1981] to find an asymptotic solution on the interval $(-1, 1)$:

$$\phi_{01} = C_1 \frac{\text{Sin} \left(\int_{-1}^{\xi} q(\tau) d\tau \right)}{q^{1/2}} + C_2 \frac{\text{Cos} \left(\int_{-1}^{\xi} q(\tau) d\tau \right)}{q^{1/2}} - \frac{G_0u}{2 + G_0u}. \quad (39)$$

where C_1 and C_2 are the constants, determined from the boundary conditions for the smooth merging of the solution (39) with the constant velocity potential function: $\phi_0 = 1$, $\phi_{0\xi} = 0$ at $\xi \leq -1$: $C_2 = 0$, $C_1 = G_0u_\xi(-1)q^{-1/2}(-1)/(2 + G_0u(-1))$.

[35] Numerical calculations of the SW variation at the positive current $u(\xi) = \text{Sech}^2[\xi]$ are shown in Figure 1d. The analytical solution to the problem conforms well to numerical calculations. Here, the basic effect is a strong smoothing of the sea surface over the developed current. The wave amplitude decreases and accompanies by formation of the SW envelope local maxima, the surface smoothing strengthens, and some maxima grow with the intensity for the IW-induced current. As the interaction length grows (G_2 decreases), the smoothing strengthens increasingly. Moreover, high-frequency modulation of the SW envelope grows as well. Numerical simulations show that a significant forerunner or trace of the IW will not appear for a positive subsurface currents.

[36] Numerical simulations of the SW modulation by IW-induced currents were also performed for various ranges of SW frequencies on the basis of the general modulation equation (27). Results clearly show that the maximal modulations take place for the SW in the vicinity of resonance conditions $c_g \sim c$ and sharply decreased far from the group resonance frequencies.

5. Conclusions

[37] The account of SW nonlinearity allows constructing of the uniformly valid stationary solutions to the problem of the SW–IW interaction in the vicinity of their group velocity resonance and to describe a number of observed experimental effects. The basic properties of the solution correspond to the results of observations of SW modulations by large amplitude internal waves [Kropfli et al., 1999;

Bakhanov and Ostrovsky, 2002] and by the relatively short internal waves [Basovich et al., 1987; Bakhanov et al., 1994].

[38] We note the basic features of the near resonance SW modulation:

[39] (1) The surface wave number vector in the current region varies insignificantly and the SWs shorten and lengthen on following and counter currents, respectively.

[40] (2) The interaction with large-scale solitary IWs lead to sufficiently strong modulations of the SW envelope. The surface countercurrent causes the SW steepness growth and the envelope shape follows the current profile. The steepness of modulation grows with the intensity of IW-induced current and is symmetric relative to the current profile axis. A moderate high-frequency oscillation is imposed on an algebraic part of the solution.

[41] (3) Modulation by a large-scale positive solitary IW-induced current leads to the surface smoothing accompanied by relatively high-frequency modulation.

[42] (4) An IW forerunner can arise ahead of the IW train for a long SW, manifesting itself as a modulated SW train with the period comparable to the IW spatial scale. Leaving the internal wave field, the surface waves save the obtained modulation.

[43] (5) A trace of IW in the case of negative subsurface current can also arise for the relatively short SW in the form of a modulated SW train with a comparable intensity.

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