On the Direction of Approach of Microseismic Waves
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mental value. The scattering is almost isotropic (in the relative co-ordinate system) for all neutron energies up to about 40 million volts. Only for still higher energies, which are at present unavailable, an experimental determination of the sign of the anisotropy would decide whether the force between neutron and proton is of the exchange type or an ordinary force.

On the Direction of Approach of Microseismic Waves

By A. W. Lee, M.Sc., D.I.C., Kew Observatory

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1—Introduction

It has been thought for some time that an examination of the relation between the phases of the horizontal and vertical displacements in microseisms would be of interest in showing how closely the oscillations compare with Rayleigh waves, but a practicable scheme for making the observations has only recently been developed. In the earlier attempts the turning points of consecutive oscillations were timed during several minutes, but the accuracy attained by interpolation between the minute breaks was not high enough for reliable comparisons between the components. A solution of this difficulty has now been found in a modification of the method adopted by Leet,* who has examined the relation between the horizontal and vertical phases of the microseisms recorded at Harvard Observatory, using comparisons of the movements *exactly at the minute breaks*. The application of this new method to the seisograms of Kew Observatory is described in the present paper.

2—Tabulation of the Phases of the Microseisms

Fig. 1 shows portions of the records obtained from the Galitzin seismographs at Kew on January 11, 1930, when the microseisms were very large. Upward movements on the seismograms correspond with ground

* ‘ Gerl. Beitr. Geophys.,’ vol. 42, p. 232 (1934).

movements to the north, to the east, and upwards. The direction of recording is from right to left.

For the purposes of this investigation the phases at the beginning of the minute breaks have been allotted numbers according to a scale, fig. 2, from 0 to 15. The entries 0 and 8 indicate that the beginning of

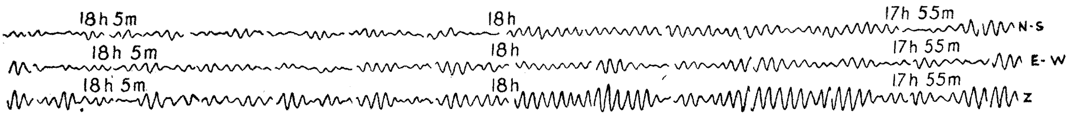

FIG. 1—Large microseisms recorded at Kew Observatory, January 11, 1930

the break coincides with the crest and trough respectively. Four examples illustrating readings of 12, 0, 7 and 3 are shown in the figure. The procedure adopted in the tabulations is to estimate the phases at fifty consecutive time breaks, the series beginning 25 minutes before the specified hour.

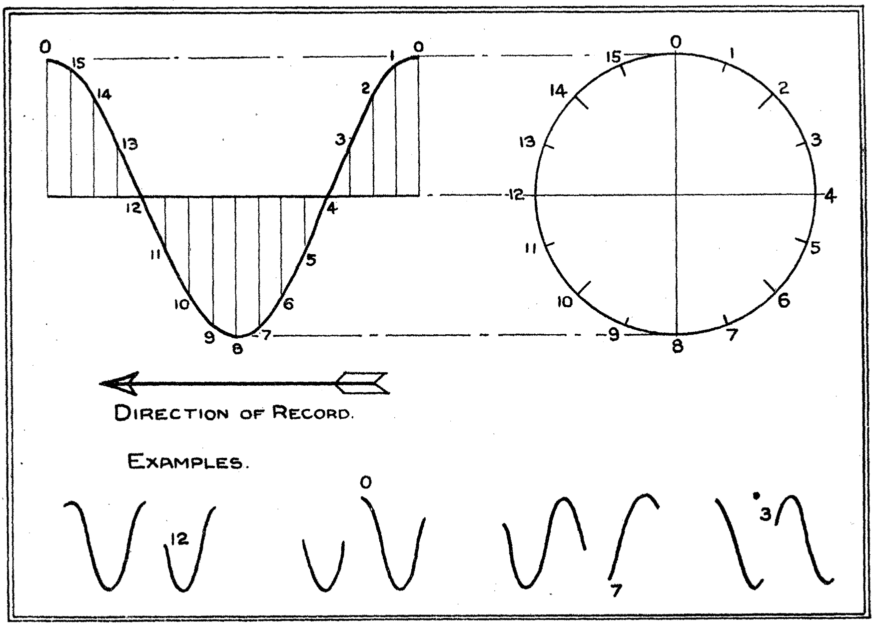

FIG. 2—Scale of tabulation for phase angles

The free periods of the horizontal and vertical seismographs being 25 sec and 13 sec, the " lag " of the ground movements behind the corresponding oscillations in the records is not the same for the three components, and the phases of the horizontal and vertical ground movements are not directly comparable from the seismograms.

In Galitzin's notation the formula for the time lag with simple harmonic earth movements is :—

$$\text{Time lag } (\tau + \tau_1) = \frac{T_p}{2\pi} \cdot \tan^{-1}\left\{\sqrt{(1 - \mu^2)} \cdot \frac{2u}{u^2 - 1}\right\} + \frac{T_p}{2\pi}\left\{\tan^{-1}\left(\frac{2u_1}{u_1^2 - 1}\right) + \frac{\pi}{2}\right\}.$$

Here T_p is the period of the earth wave, μ^2 the damping coefficient, u and u_1, the ratios of period of the earth-wave to the periods of the pendulum and of the galvanometer, and the inverse tangents are between 0 and π.*

The phase differences between earth movements of periods 0 to 10 seconds and the recorded oscillations, for the Galitzin seismographs at Kew are as given in Table I.

TABLE I

Period of earth movement, seconds	0	2	4	6	8	10
Horizontal phase correction, degrees	450	431	413	395	378	362
Vertical phase correction, degrees	450	415	382	351	324	300

These values are appropriate for critical damping and with the free period of each pendulum equal to that of the corresponding galvanometer; the effects of slight departures from these ideal conditions are not appreciable. The differences between the phase corrections of the horizontal and vertical seismographs, for oscillations of periods 4, 6 and 8 seconds, are 31°, 44° and 54°; consequently in dealing with the microseisms this difference may be taken as 45°. The sign of the difference is such that when the horizontal and vertical records are in phase the vertical earth movement is 45° in advance of the horizontal movement. To allow for this difference between the instruments 45° is subtracted from the phases of the vertical component, before the values are compared with the phases of the horizontal components.

A specimen of the tabulations, showing values for 1930, January 11d 18h, is given in Table II.

3—DATA

The microseisms associated with depressions over different parts of the eastern Atlantic and western Europe have been investigated. The

* The phase difference is called a " lag " implying that the movements of the recorder follow the corresponding movements of the ground. Actually when the movements are not simple harmonic the changes in amplitude of the recorder precede the changes in amplitude of the ground movement. The lag, $\tau + \tau_1$, given by the Galitzin formula should therefore be decreased by 2π. In other words the lag of the earth movement behind the movement of the light spot of the seismograph is $2\pi - (\tau + \tau_1)$. *Cf.* Scrase, ' London Proc. Phys. Soc.,' vol. 43, p. 259 (1931).

TABLE II—TABULATION OF PHASES OF THE MICROSEISMS AND PHASE DIFFERENCES BETWEEN THE COMPONENTS. 1930. JANUARY 11D 18H, PHASE UNIT = $\pi/8$

Time, G.M.T.		Phases as read			Z phase corrected	Phase differences		
		N—S	E—W	Z		Z and N—S	Z and E—W	N—S and E—W
h	m							
17	35	15	9	5	3	12	6	10
	36	9	14	14	12	13	2	5
	37	15	11	9	7	8	4	12
	38	0	6	5	3	13	3	6
	39	8	2	15	13	11	5	10
	40	3	4	6	4	15	0	1
	41	13	6	5	3	10	3	9
	42	1	5	7	5	12	0	4
	43	7	11	9	7	0	4	4
	44	0	6	3	1	15	5	6

occasions were selected from the synoptic weather maps (British Daily Weather Reports). The occasions chosen,* together with the positions of the depressions and the bearings from Kew to the disturbed regions, are set out in Table III.

TABLE III—OCCASIONS SELECTED FOR TABULATION OF THE PHASES OF THE MICROSEISMS

Date	Figure	Postion of depressions	Bearing of centres of depressions from Kew
1930, January 11d 18h ..	5	North-west of Scotland	North-west.
		North-west of Azores	West.
1932, December 10d 7h	6	Atlantic Ocean off northern Portugal	South-west.
		Barent's Sea	North-north-east.
1933, December 13d 7h	7	Bay of Biscay to Ligurian Sea	South.
		Northern Norway	North-north-east.
1934, January 18d 7h....	8	Between the Shetlands and Norway	North-north-east.
1934, January 23d 7h....	9	Greenland Sea	North.
		Atlantic Ocean west of Ireland	West.
1934, February 8d 7h ..	10	Southern Norway and Kanai Peninsula	North-north-east.

* A comparison of the microseismic amplitudes and the meteorological conditions on the first of these dates has already been published—London, Meteorological Office, Geophysical Mem., No. 62 (1934).

Fifty measurements of the phases were made in each case, and after applying the " lag correction " of 45° to the values for the vertical component, the phase differences between the components were tabulated. The significance of a few of the measurements is uncertain owing to juxtaposition of two periods at the minute breaks, but such values do not occur often enough to have an appreciable effect upon the complete series.

Table IV shows the distribution of the phase differences between the components, and the numbers of the values from 1 to 7 and from 9 to 15; the totals for the six occasions are given at the bottom of the table. There is a striking contrast between the data for the three pairs of components, the phase differences of most frequent occurrence being 270° between Z and N—S, 90° between Z and E—W and 180° between N—S and E—W. For Z and N—S, 82 of the differences are from 1 to 7 and 188 from 9 to 15, whilst for Z and E—W 217 are from 1 to 7 and 58 from 9 to 15. The N—S and E—W phase differences are more evenly distributed, the numbers in these groups being 126 and 131 respectively. The data for the separate days are examined in Section 5.

With each series containing only 50 measurements, irregularities in the distributions among the sixteen possible phase differences are inevitable and the values must be smoothed. The method adopted is to take overlapping groups of five values, giving double weight to the second and fourth, and treble weight to the third; thus, if F_n denotes the number of observations of any phase difference (n), the smoothed frequency expressed as the percentage of the total number of values is

$$\{F_{n-2} + 2F_{n-1} + 3F_n + 2F_{n+1} + F_{n+2}\}\frac{100}{450}.$$

These smoothed frequencies for the six occasions are represented by ordinary graphs in fig. 3, and by vectorial diagrams in the insets to the weather maps, figs. 5–10. The figures on the axes of the vectorial diagrams represent frequencies of 5 and 10% of the total number of values per $22\frac{1}{2}$°.

4—Relation Between the Phases of the Oscillations

In the curves of fig. 3 clearly defined maxima are shown in the region of 270° for the frequencies of the Z and N—S phase differences on 1930, January 11d 18h, 1934, January 18d 7h, January 23d 7h, and February 8d 7h, and of 90° for the Z and E—W differences of 1930, January 11d 18h, 1932, December 10d 7h, 1934, January 18d 7h, January 23d 7h, and February 8d 7h. There are maxima around 180° for the N—S and E—W

TABLE IV—NUMBER OF OCCURRENCES OF SPECIFIED PHASE DIFFERENCES BETWEEN THE COMPONENTS OF THE MICROSEISMS

Date	Components	Phase difference*																Number of values	
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1–7	9–15
1930, January 11d 18h ..	Z and N—S	1	0	1	1	2	2	1	0	2	6	7	8	8	3	6	2	7	40
	Z and E—W	3	4	7	12	7	5	4	3	1	1	1	1	0	1	0	0	42	4
	N—S and E—W	2	3	2	1	3	5	3	4	6	7	4	2	3	4	0	1	21	21
1932, December 10d 7h ..	Z and N—S	3	3	4	2	6	3	1	2	3	5	5	1	4	3	3	2	21	23
	Z and E—W	3	3	7	10	6	6	5	3	1	2	1	0	0	1	2	0	40	6
	N—S and E—W	2	2	3	2	5	4	4	2	5	2	3	3	0	6	4	3	22	21
1933, December 13d 7h ..	Z and N—S	4	4	5	2	6	5	0	4	1	2	3	1	3	3	5	2	26	19
	Z and E—W	2	4	7	4	2	2	2	4	4	1	3	1	3	4	4	3	25	19
	N—S and E—W	7	5	0	2	3	4	3	6	1	4	3	1	2	3	1	5	23	19
1934, January 18d 7h	Z and N—S	4	0	1	2	2	2	2	2	3	4	4	7	4	5	6	2	11	32
	Z and E—W	1	3	1	3	11	5	6	6	3	4	2	1	0	1	2	1	35	11
	N—S and E—W	4	3	2	1	3	1	2	4	5	2	3	5	2	6	5	2	16	25
1934, January 23d 7h	Z and N—S	2	0	2	1	3	0	1	2	2	5	6	6	6	3	6	5	9	37
	Z and E—W	4	5	2	9	8	4	3	6	1	3	1	0	0	0	2	2	37	8
	N—S and E—W	2	5	2	0	6	5	2	3	4	6	3	2	1	5	3	1	23	21
1934, February 8d 7h	Z and N—S	1	1	1	0	0	0	3	3	4	2	2	10	11	5	4	3	8	37
	Z and E—W	1	2	7	4	9	5	9	2	1	0	2	2	3	0	1	2	38	10
	N—S and E—W	1	3	3	2	2	3	3	5	4	2	5	6	2	3	3	3	21	24
Total for the six occasions	Z and N—S	15	8	14	8	19	12	8	12	15	24	27	33	**36**	22	30	16	82	188
	Z and E—W	14	21	31	42	**43**	27	29	24	11	11	10	5	6	7	11	8	217	58
	N—S and E—W	18	21	12	8	22	22	17	24	**25**	23	21	19	10	27	16	15	126	131

* The unit in the scale of phase differences is $\pi/8$.

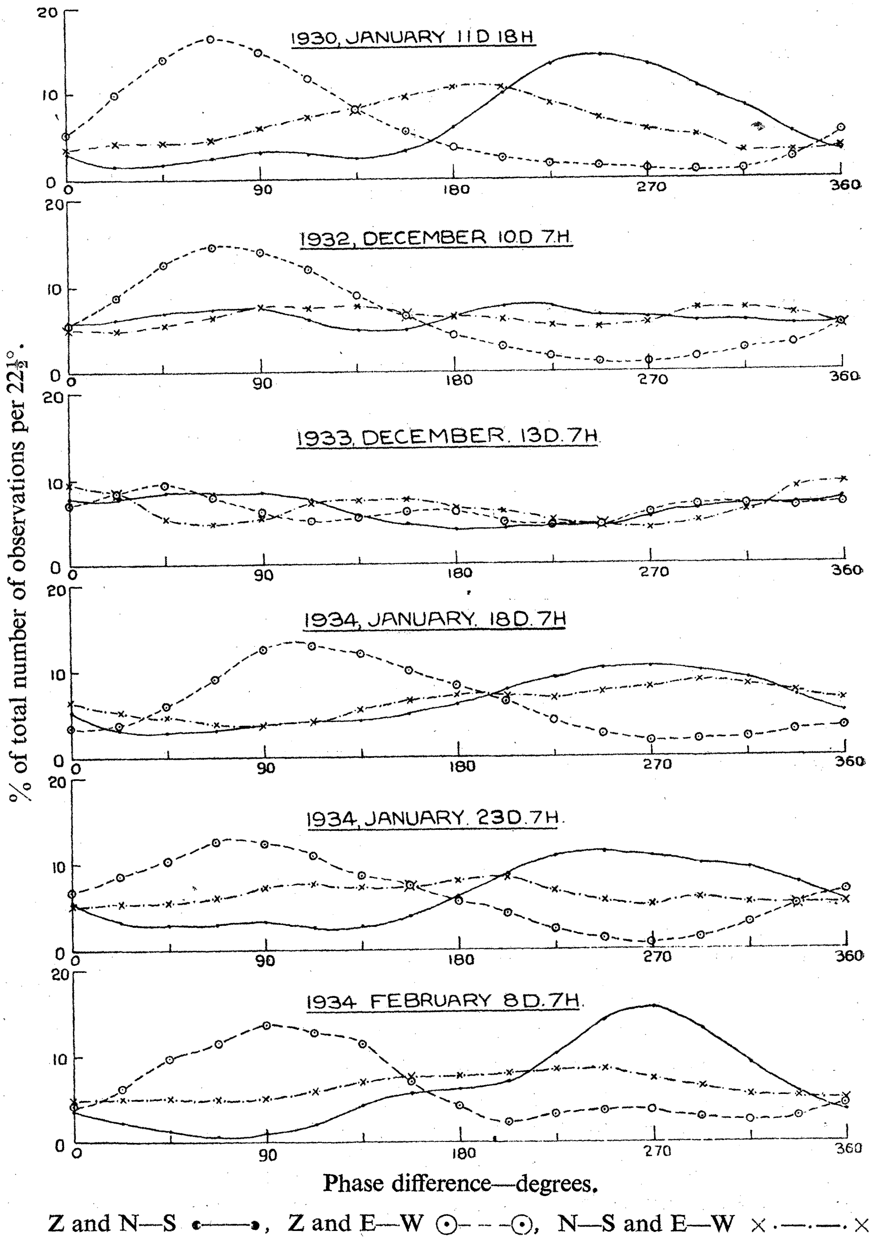

FIG. 3—Smoothed frequencies of specified phase differences between the components of the microseisms

differences on 1930, January 11d 18h, 1934, January 23d 7h, and February 8d 7h; on 1934, January 18d 7h differences between 180° and 360° predominated between these components. The variations shown in the three curves for 1933, December 13d 7h, and in two of the curves for 1932, December 10d 7h, (Z and N—S, N—S and E—W) are comparatively small.

The motion in Rayleigh waves is in the direction of propagation and in the vertical, and these components differ in phase by 90°, each earth particle moving in an elliptic orbit with the motion at the lowest point of the path in the direction of propagation of the waves. The movements for

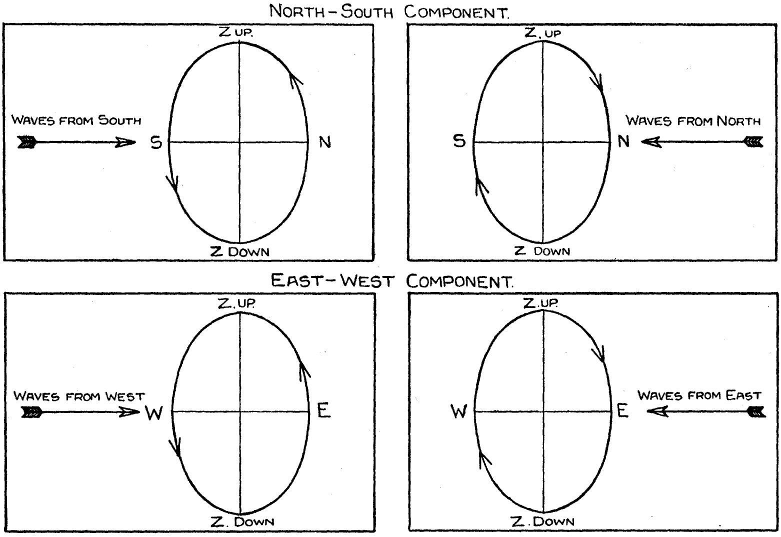

FIG. 4—Ground movements with Rayleigh waves from different directions

Rayleigh waves arriving from north, south, east and west would, therefore, be of the types shown in fig. 4.* For the waves from south and from west the vertical movement lags by 90° behind the horizontal, and for waves from east and from north the vertical precedes the horizontal by 90°. It is shown in an appendix that when waves from adjacent directions interfere the most probable phase differences between the N—S, E—W, and Z components are those for waves travelling in an intermediate direction, but the differences may vary over a considerable range. Hence

* The diagram illustrates the movement in simple Rayleigh waves on the surface of a homogeneous solid. The ratio of the amplitudes of horizontal and vertical microseismic movements varies from place to place according to the stratification of the underlying rocks. At Kew Observatory these amplitudes are nearly equal. (London, Meteorological Office, 'Geophysical Mem.,' No. 66 (*in the press*).)

the preponderance of phase differences of 270° between Z and N—S, of 90° between Z and E—W and of 180° between N—S and E—W, agrees with the theory that the microseisms are Rayleigh waves which are most frequently travelling from north-west to south-east.*

5—Comparisons of the Microseismic Phase Differences with the Weather Maps

(*a*) 1930, *January* 11*d* 18*h*, *fig.* 5—The microseisms at this time were much larger and the periods were longer than on any of the other dates,

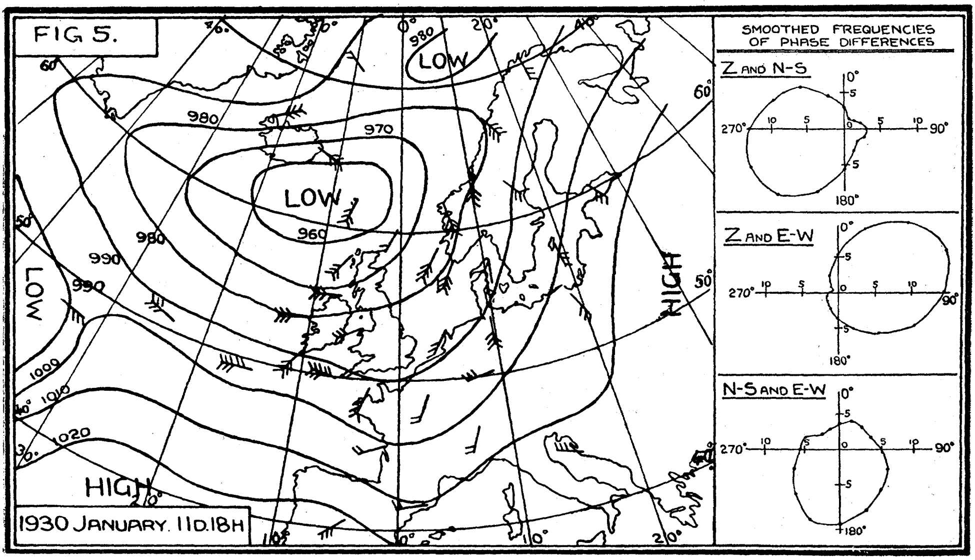

FIG. 5

the amplitude and period for the N—S component being 10·0 μ and 7·7 seconds.†

The distributions of the phase differences between the components are consistent with movements arriving from the north-west, indicating that

* The conclusion reached by Don Leet (*loc. cit.*) was that the microseisms registered at the Harvard Observatory could not be Rayleigh waves, since the vertical component apparently agreed in phase with the dominant horizontal component. When allowance is made, however, for the difference between the types of seismograph at that observatory, the supposed anomaly disappears, and the phase differences are characteristic of Rayleigh waves reaching Harvard from the north-east and from the south-east.

† The amplitudes and periods are taken from the Observatories Year Books; the values for 6h G.M.T. are given in the comparisons with weather maps for 7h.

they were caused by the more northerly depression. If the depression near the Azores had been the more effective in generating the microseisms, the disturbances would have arrived at Kew from the west; the Z and E—W phases would tend to differ by 90°, but the N—S component would be affected by waves from the south as well as from the north of west, and its phases would be variable with little tendency to differ from those of Z by 270°, or from those of the E—W component by 180°.

The axes of the frequency distributions are not exactly on the 90° and 270° lines. No theoretical explanation of this discrepancy has been found, but it may possibly be due to the differences between the actual wave forms and the simple harmonic motion which has been assumed in computing the lag corrections of the seismographs.

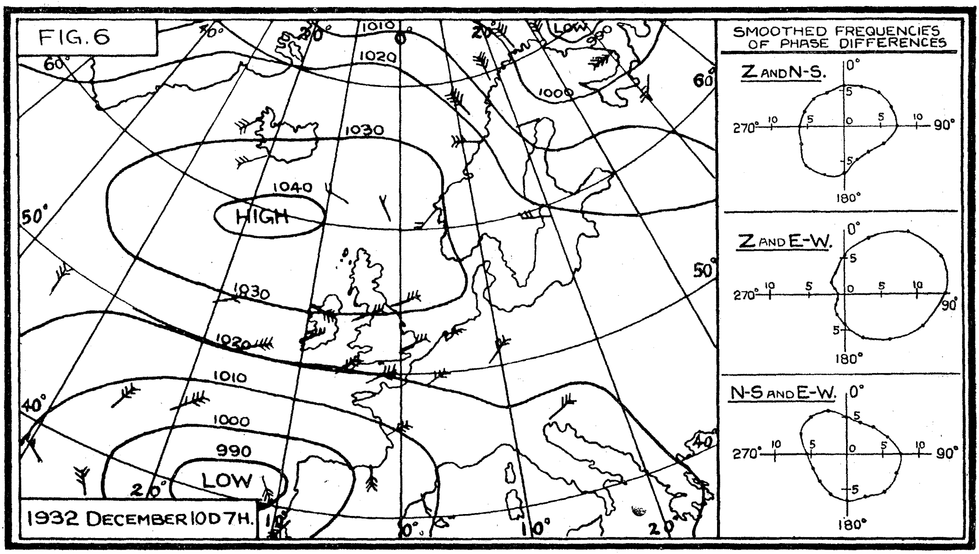

Fig. 6

(*b*) 1932, *December* 10*d* 7*h*, *fig*. 6—For a winter month the microseisms were small, the N—S tabulations giving an amplitude of 1·6 μ and period 6·7 seconds.

Two depressions appear in the weather map. The more southerly, off Portugal, gave strong winds over the Atlantic Ocean south-west of Britain, and around the English Channel, the Bay of Biscay and western Hibernia. Stormy conditions around the Norewegian coast were associated with the depression in the Barent's Sea; the wind reached gale force at Röst and very rough seas† were reported from Ingöy and Röst. Such conditions are very favourable for the generation of miscroseisms

† " New " International Code.

according to the German seismologists, who accept Wiechert's hypothesis that the oscillations are caused by surf breaking against rocky coasts. The small amplitude at this time does not support this hypothesis.

The distribution of the phase differences between Z and E—W corresponds with that on 1930, January 11, so the waves generally arrived at Kew from a westerly direction. There is no striking asymmetry in the other two diagrams, the inference being that the N—S component was affected by waves from south and north of west. It appears therefore that the Kew microseisms were generated from the northern sector of the Atlantic depression.

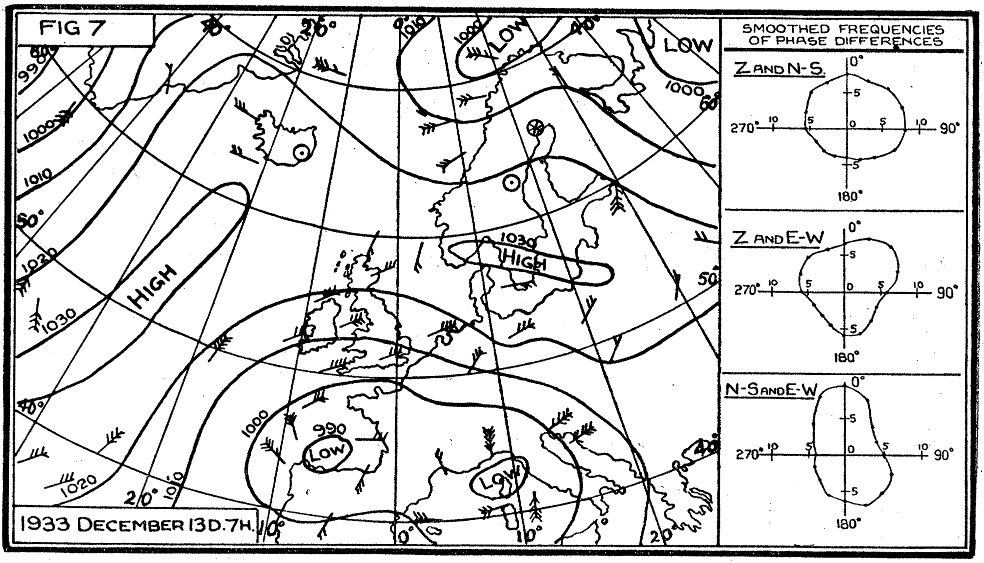

FIG. 7

(*c*) 1933, *December* 13*d* 7*h*, *fig*. 7—The amplitude (0·6 μ) was smaller and the period was shorter (4·0 seconds) than on either of the other five occasions.

A complex depression covered south-western Europe, with centres over the Bay of Biscay and the Ligurian Sea, and another depression was situated off Northern Norway. Winds were strong round the north of Norway, over the North Sea, southern Britain, the English Channel and the Bay of Biscay.

The phase differences between Z and N—S suggest that movements from southerly azimuths were more frequent than those from northerly azimuths, and the differences between Z and E—W suggest interference between waves from easterly and westerly sources.

(*d*) 1934, *January* 18*d* 7*h*, *fig.* 8—The microseisms, of amplitude 3·6 μ and period 6·0 seconds, are larger than the average for January.

The depression, centred between the Shetlands and Norway, raised gales over the North Sea and the south of Norway; sea disturbance was 5 (very rough) at Utsire and Bornholm, and 6 (high) at The Scaw and Blaavands Huk. A secondary depression west of Ireland was associated with stormy conditions on the Atlantic and from southern England to the Bay of Biscay.

The phase diagrams imply that the microseisms arrived at Kew from north-west. The axis of the Z and E—W diagram is skewed beyond the

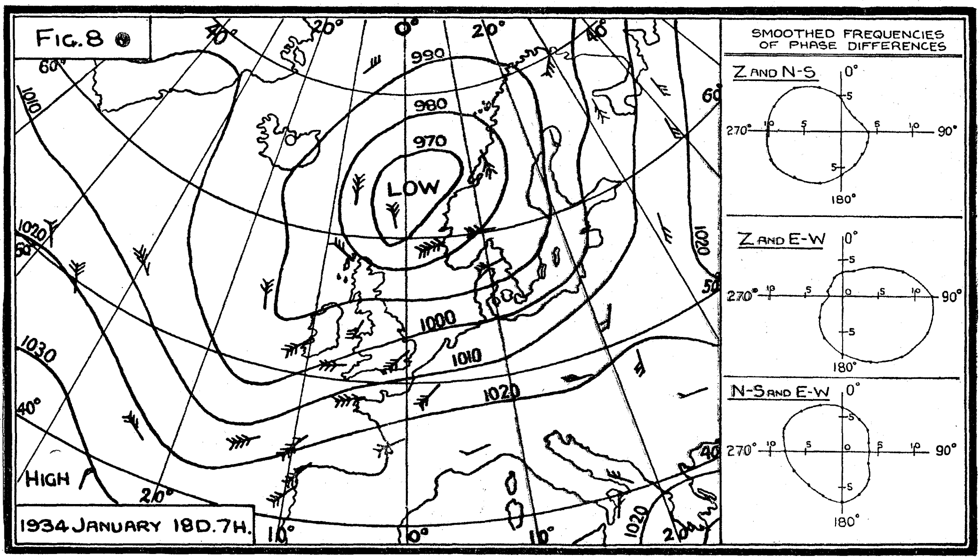

FIG. 8

90°, but the distribution strongly favours movements from the west rather than from the east of north.

(*e*) 1934, *January* 23*d* 7*h*, *fig.* 9—The microseisms (amplitude 2·3 μ, period 7·0 seconds) were about normal for January.

The map shows depressions over the Greenland Sea, and over the Atlantic west of Ireland. The depressions caused gales over the Atlantic and on the Norwegian coast; very rough seas were reported from Ingöy and Röst.

Movements from the west and from the north predominate in the phase diagrams, indicating that the Kew disturbances came chiefly from the north of west. With sea disturbances from south-west to north-west and from north to north-east of Britain, the absence of microseisms from the east of north and from the south of west is notable; the inference is that the region most favourable for the generation of the microseisms was in the Atlantic north-west of Kew.

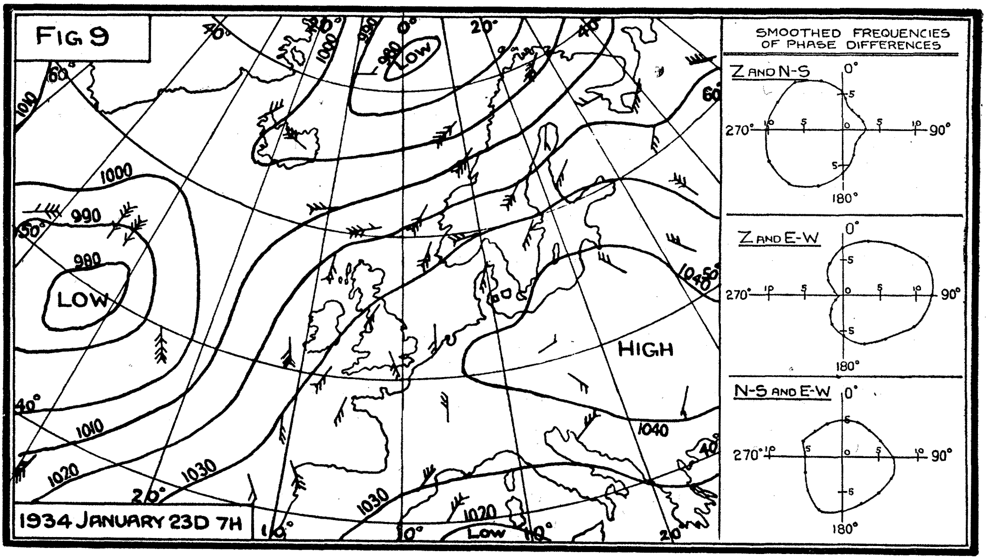

FIG. 9

(*f*) 1934, *February 8d 7h*, *fig.* 10—The microseisms were much larger than the average for February, the amplitude being 4·8 μ and the period 6·7 seconds.

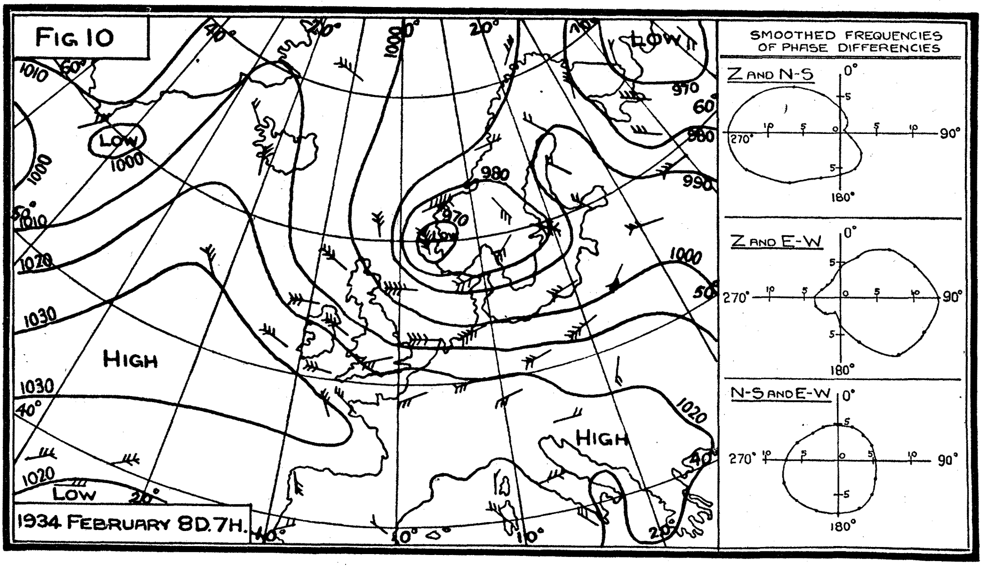

FIG. 10

A complex depression with centres over southern Norway and the Kanai Peninsula caused gales round the southern coasts of Norway and over the North Sea. The sea disturbance was high (6) at Ingöy, Utsire and The Scaw, very high (7) at Bornholm, and precipitous (8) at Blaavands Huk. Even under these conditions there is very little evidence of microseisms being produced north-east of Kew. Phase differences of 270° between Z and N—S, of 90° between Z and E—W, and to a lesser extent of 180° between N—S and E—W, predominate. It appears, therefore, that the microseisms must have arrived from north-west, having originated in regions well to the west where the wind and sea disturbance were less than near the centre of the depression.

6—Conclusions

The results of the comparisons of the microseismic phase differences and the weather maps are summarized in Table V.

Table V—Comparisons of Microseismic Phase Differences and the Weather Maps

Date	Figure	Centre of depression	Microseisms "Preponderance" of component
1930, January 11d 18h ..	5	N.W., W.	N., 0·7 ; W., 0·8
1932, December 10d 7h	6	S.W., N.N.E.	N.S., 0·0 ; W., 0·7
1933, December 13d 7h	7	S., N.N.E.	S., 0·1 ; W., 0·1
1934, January 18d 7h....	8	N.N.E.	N., 0·4 ; W., 0·5
1934, January 23d 7h....	9	N., W.	N., 0·6 ; W., 0·6
1934, Febrary 8d 7h	10	N.N.E.	N., 0·6 ; W., 0·6

The figures for estimating the " preponderance " of the components of the microseisms have been computed from the numbers of phase differences from 1 to 7 and from 9 to 15, the difference between these numbers being expressed as a fraction of the total number. For example, of the 50 tabulated differences between Z and E—W for 1930, January 11d 18h, 42 are from 1 to 7 and 4 from 9 to 15, and the value is (42–4)/50 or 0·8. The largest possible value (1·0) would occur with all 50 observations in one of the two groups, and the least (0·0) when the numbers in these groups were equal.

For two occasions, figs. 6 and 7, when depressions were centred to the south-west and south, the " preponderance " coefficients of the N—S component are very small, and the microseisms were generated over an area which included regions north and south of Kew. On the other four occasions movements from the north-west predominate, and the regions in which the microseisms were produced are not affected by the position of the depressions. Figs. 8, 9 and 10 show stormy conditions around Norway, but the phases of the microseisms emphasize that the movements must have originated north-west of Kew. The results agree with the distributions of " standard " amplitudes in north-west Europe which were obtained from the " Survey of Microseismic Disturbances Recorded during January, 1930," these amplitudes being greatest north-west of Britain. Apparently the microseisms are generated in deep water. The directions of arrival are inconsistent with the theories that the oscillations are caused by the action of wind or waves on steep coasts, or by the motion of waves over shallow water.

The results have shown that although the phase differences between the components are variable, they are generally grouped around certain values, in accordance with the theory of Rayleigh waves approaching from neighbouring directions. The absence of Love waves (inferred from earlier investigations of the microseismic amplitudes*) is confirmed, for if Love waves of appreciable magnitude had been superposed upon the Rayleigh waves, the phases of the horizontal movements would have been unrelated to those of the vertical movements.

It appears, therefore, that the microseisms are generated in regions where the water is deep, and the mechanism must be such that no shearing waves are developed.

In conclusion I wish to express my thanks to Dr. F. J. W. Whipple, Superintendent of Kew Observatory, who suggested a number of improvements which have been incorporated in this paper.

7—Summary

A new method has been developed for tabulation of the phases of the microseisms. It is found that the phase differences between the components are variable, but certain values predominate in accordance with the theory of Rayleigh waves approaching from adjacent directions; the distribution of the phase differences indicates the direction of arrival of the waves.

The phase differences between the N—S, E—W and Z components at Kew Observatory are compared with the weather maps for six occasions when depressions were located over different parts of the eastern Atlantic and western Europe. In two cases, with depressions to the south-west

* ' Mon. Not. R. Astr. Soc.,' Geophys. Suppl. 3, p. 84 (1932).

and south, the microseisms were generated over an area which included regions north and south of Kew. On the other four occasions movements from the north-west predominate, and the regions in which the microseisms were produced are not affected by the position of the depressions. Apparently the microseisms are generated in deep water. There is no support for the theories that the oscillations are caused by the action of wind or waves on steep coasts, or by the motion of waves over shallow water.

The absence of Love waves is confirmed, showing that the mechanism by which the microseisms are generated must be such that no shearing waves are developed.

Appendix

The Theory of Rayleigh Waves arriving from Adjacent Directions

Since all the theories concerning the origin of microseisms agree that they are due to causes which operate over a large area, the motion must be due to trains of waves arriving from adjacent directions. The theoretical relations between the components of the motion are then more complicated than for a single train of Rayleigh waves. As an example we may combine two trains of simple Rayleigh waves (*i.e.*, waves in a homogeneous medium) differing slightly in period. The argument can be extended to cover any number of trains of waves of this type.

In these waves the movements are in vertical planes parallel to the direction of propagation. Let the constant ratio of the amplitudes of the vertical and horizontal movements be denoted by K, and let the bearings of the directions of propagation for the two trains of waves be α and β, measured from north through east. If the time $t = 0$ is chosen so that the waves are then in phase, the horizontal and vertical components of the displacements may be written:—

$$\mathrm{A}\cos[(p+\delta p)\,t] \quad \text{and} \quad \mathrm{KA}\sin[(p+\delta p)\,t]$$

$$\mathrm{B}\cos[(p-\delta p)\,t] \quad \text{and} \quad \mathrm{KB}\sin[(p-\delta p)\,t]$$

The displacements of the ground to the north, to the east and upwards are:—

$$\mathrm{N} = \mathrm{A}\cos\alpha\cos[(p+\delta p)\,t] + \mathrm{B}\cos\beta\cos[(p-\delta p)\,t],$$
$$\mathrm{E} = \mathrm{A}\sin\alpha\cos[(p+\delta p)\,t] + \mathrm{B}\sin\beta\cos[(p-\delta p)\,t],$$
$$\mathrm{Z} = \mathrm{K}\{\mathrm{A}\sin[(p+\delta p)\,t] + \mathrm{B}\sin[(p-\delta p)\,t]\}.$$

These equations may be rewritten in the form:—

$$N = A_N \cos(pt + \varepsilon_N),$$
$$E = A_E \cos(pt + \varepsilon_E),$$
$$Z = A_Z \sin(pt + \varepsilon_Z),$$

where

$$\left.\begin{aligned} A_N{}^2 &= A^2 \cos^2 \alpha + B^2 \cos^2 \beta + 2AB \cos \alpha \cos \beta \cos \\ A_E{}^2 &= A^2 \sin^2 \alpha + B^2 \sin^2 \beta + 2AB \sin \alpha \sin \beta \cos \\ A_Z{}^2 &= K^2 \{A^2 + B^2 + 2AB \cos \end{aligned}\right\} (2t\,\delta p),$$

$$\left.\begin{aligned} \tan \varepsilon_N &= \frac{A \cos \alpha - B \cos \beta}{A \cos \alpha + B \cos \beta} \tan \\ \tan \varepsilon_E &= \frac{A \sin \alpha - B \sin \beta}{A \sin \alpha + B \sin \beta} \tan \\ \tan \varepsilon_Z &= \frac{A - B}{A + B} \tan \end{aligned}\right\} (t\,\delta p).$$

The resultant motion shows a quasi-frequency p, which is the mean of the component frequencies. The motion differs from that with simple Rayleigh waves. The amplitudes of the ground movements in either component show beats, alternating between the sum and the difference of the component amplitudes. The values of ε_N, ε_E and ε_Z pass through zero and through $\pi/2$ together, but the differences between ε_N, ε_E and ε_Z can oscillate through considerable ranges. The phase difference between the components depends upon t and the direction of propagation, but the dominant differences are those for a single train of waves from an intermediate direction. The two horizontal components are in phase, and the phase difference between these components and the vertical is 90°, if waves with the same frequency are arriving from adjacent directions, or if waves differing in frequency are travelling in the same direction.

With a greater number of sources, or when the waves are propagated through a stratified medium, other terms must be included in the equations for the motion and the interference phenomena are more complicated. The effects of interference are minimized in tabulations made according to the usual procedure, the amplitudes being measured from the largest oscillations during intervals of at least 10 minutes.