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A simple method to determine breaker height and depth for different deepwater wave height/length ratios and sea floor slopes

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Abstract

Existing, easily applicable methods to calculate the depth and height of breaking waves are hampered by two obstacles. First, the breaker depth is usually required to compute its height, and vice versa. Second, the equations take into account either the deepwater height to wavelength ratio or the sea floor slope, but not both. A simple iterative procedure is therefore proposed which incorporates both elements. For fully developed waves breaking over a nearly horizontal bottom, the breaker height and depth are also direct functions of the deepwater wavelength.

Keywords: Waves; Breakers; Coastal engineering; Sediment transport

1. Introduction

The height and depth of breaking waves are of considerable interest to coastal engineers, oceanographers and sedimentologists. Wave height is one of the most important factors influencing the design of coastal constructions. An overly conservative estimation can greatly increase costs and make projects uneconomical, whereas underestimation could result in structural failure or significant maintenance costs (Vincent et al., 2002). Horizontal water particle velocities also reach their maximum values at the breakpoint, so that the sea floor beneath the breaker zone is where the coarsest sediments are entrained or brought into suspension. Because this zone migrates with tides and variations in the wave climate, a relatively wide coastal swath is ultimately affected. It is therefore important that the depth of the breaker zone as a function of the sea floor slope and wave climate be determined as accurately as possible.

Although a number of equations have been proposed to this effect, these generally suffer from two serious drawbacks. Most equations express the breaker height/breaker depth ratio (H_b/d_b) as a function of other variables, which means that either the breaker height is required to obtain the depth, or vice versa. A second shortcoming of existing methods is that they do not employ all the variables affecting the breaker height and depth, with the result that they apply only to limited conditions. Here, a simple procedure is proposed that incorporates the most relevant variables and simultaneously solves for H_b and d_b.

2. Previous methods

Some of the techniques discussed here were not explicitly developed to predict breaker height and depth, but rather to express the maximum H_b/d_b ratio as a function of other variables. However, these ratios can be used for such predictions if one of the two variables can be determined by other means.

Keulegan and Patterson (1940) noted that the H_b/d_b ratio is related to wave breaking, which they considered to take place at values between 0.71 and 0.78.

Miche (1944) gave the maximum steepness for waves propagating in water depths less than half the deepwater wavelength ($L_o/2$) as

$$H_b/L_b = 0.142 \tanh(2\pi d_b/L_b) \qquad (1)$$

where L_b is the wavelength at breaking (i.e., immediately seaward of the breaker). The value of 0.142 was based on the theoretical deepwater limit for wave steepness, as proposed by Michell (1893). Eq. (1) is in agreement with an envelope curve to laboratory measurements (Dean and Dalrymple, 1991), but is strictly valid only for a horizontal bottom.

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Collins (1970) was among the first to consider the effect of the bottom slope (α, in degrees) on wave breaking, but did not take other variables into account. His equation was expressed as

$$H_b/d_b = 0.72 + 5.6 \tan\alpha \tag{2}$$

which yields a ratio of 0.72 over a horizontal bed, that increases to 1.21 for a 5° slope.

Komar and Gaughan (1973) derived the following semi-empirical relationship from linear wave theory:

$$H_b/H_o = 0.56(H_o/L_o)^{-1/5} \tag{3}$$

The subscript o denotes deepwater conditions. Although this equation considers the (H_o/L_o) ratio, it does not take the bottom slope into account.

One of the most useful equations has been that of Weggel (1972), who considered the effect of the sea floor slope in addition to the gravity constant g and wave period T_w.

$$H_b/d_b = E_1 - E_2 H_b/gT_w^2 \tag{4}$$

where

$$E_1 = 1.56/\left(1 + e^{-19.5 \tan\alpha}\right) \tag{5}$$

$$E_2 = 43.75\left(1 - e^{-19 \tan\alpha}\right) \tag{6}$$

Eq. (4) is valid for $\tan\alpha \leq 0.1$ (a slope of $\leq 5.7°$) and $H_o/L_o \leq 0.06$. For low steepness waves over a nearly horizontal bottom, it resolves to an H_b/d_b ratio of 0.78, but for any positive angle the water depth is required to solve for H_b. Eq. (4) can also be used to calculate d_b if H_b is known.

Komar (1998) proposed two separate equations for H_b and d_b, respectively.

$$H_b = 0.39 g^{0.2}\left(T_w H_o^2\right)^{0.4} \tag{7}$$

$$d_b = H_b\left\{1.2\left[S/(H_b/L_o)^{0.5}\right]^{0.27}\right\} \tag{8}$$

where S is the sea floor gradient ($\tan\alpha$).

Sakai and Battjes (1980), based on the theory of Cokelet (1977), plotted a curve of the wave breaking limit as a function of H_b/H_o against H_o/L_o. This curve can be described by the following equations:

$$H_b = H_o\left[0.3839(H_o/L_o)^{-0.3118}\right] \text{when } H_o/L_o < 0.0208 \tag{9}$$

$$H_b = H_o\left[0.6683(H_o/L_o)^{-0.1686}\right] \text{when } 0.0208 \leq H_o/L_o < 0.1 \tag{10}$$

$$H_b = H_o \text{ when } 0.1 \leq H_o/L_o \tag{11}$$

The same curve in Sakai and Battjes (1980) also represents the ratio of H_b/H_o against d_b/L_o, which can be recast into the equations

$$H_b = H_o\left[27429(d_b/L_o)^2 - 773.71(d_b/L_o) + 7.4343\right] \text{ when } d_b/L_o < 0.011 \tag{12}$$

$$H_b = H_o\left[0.3976(d_b/L_o)^{-0.3834}\right] \text{ when } 0.011 \leq d_b/L_o < 0.049 \tag{13}$$

$$H_b = H_o\left[21.867(d_b/L_o)^2 - 7.06(d_b/L_o) + 1.5573\right] \text{ when } 0.049 \leq d_b/L_o < 0.6 \tag{14}$$

$$H_b = H_o \text{when } 0.6 \leq d_b/L_o \tag{15}$$

Although Eqs. (9)–(15) consider the deepwater wave height and length, the sea floor slope is not taken into account.

Fenton and McKee (1990) determined the greatest (unbroken) wave that could exist as a function of both wavelength and depth over a nearly horizontal bottom as

$$H_b = d_b\left[0.141063(L_b/d_b) + 0.0095721(L_b/d_b)^2 + 0.0077829(L_b/d_b)^3\right] / \left[1 + 0.078834(L_b/d_b) + 0.0317567(L_b/d_b)^2 + 0.0093407(L_b/d_b)^3\right] \tag{16}$$

Again, the sea floor slope is not considered.

3. An iterative approach

In the equations above, the variables used to determine d_b or H_b include L_o, H_o, L_b, α (or S), T_w and g. However, in no case are all these parameters considered together. The acceleration due to gravity g is constant for any latitude (an average value of 9.81 m s^{-2} is used here), whereas the wave period T_w does not change from deep into shallow water. Therefore, only the first four variables need to be considered.

The deepwater wavelength L_o is a direct function of the wave period and acceleration of gravity, as given by the standard equation:

$$L_o = gT_w^2/2\pi \tag{17}$$

The deepwater wave height H_o depends on the sea state, which is a function of the wind velocity, fetch and duration. For a fully developed sea (FDS), the wave period, length or height do not increase, regardless of the fetch or duration of the wind blowing at any specific velocity. However, for fetch or duration-limited conditions both the wave height and period may have lower values. Resio et al. (2003), based partly on data gathered during the Joint North Sea Wave Project (JONSWAP), provide nomograms plotting wave periods and heights against fetch and

Table 1
Data used in calculating breaker heights and depths for the different equations

U_{a10}	T_w	H_{oo}	H_{oc}Eq. (18)	L_{oc}Eq. (17)	L_b
2.5	1.6	0.14	0.14	4.00	2.67
5.0	3.3	0.6	0.6	17.00	11.33
7.5	5.1	1.4	1.4	40.61	27.07
10.0	6.6	2.5	2.4	68.01	45.34
12.5	8.4	4.1	3.9	110.17	73.45
15.0	11.1	6.4	6.8	192.37	128.25
17.5	11.8	8.2	7.7	217.40	144.93

Observed wave periods (T_w) and deepwater fully developed wave heights (H_{ob}) are derived from nomograms (Figs. II-2-25 and II-2-26) in Resio et al. (2003). Deepwater wavelengths (L_{oc}) and wave heights (H_{oc}) are calculated from Eqs. (17) and (18), respectively, and breaker wavelengths are obtained from $2L_o/3$ (Le Roux, submitted for publication).

duration for different wind speeds. Table 1 shows their wave periods and heights for FDS conditions (i.e. where the curves level off). An analysis of Table 1 indicates that the fully developed wave height H_{oFDS} under such conditions is given by

$$H_{oFDS} = L_o/9\pi \tag{18}$$

The observed and calculated values correspond exactly for wind velocities up to 7.5 m s^{-1} (Fig. 1), but differ by up to 6% for higher wind speeds, which can be attributed to the fact that the nomograms become less accurate and more difficult to read for higher wind velocities due to the logarithmic scale employed. However, separate curves fitted to the two sets of data give exactly the same equation ($H_{oFDS}=0.0542T_w^{2.0156}$).

Combining Eqs. (17) and (18), this means that the H_o/L_o ratio under FDS conditions is given by $1/9\pi$ or 0.0354.

The breaker wavelength L_b should be a direct function of the sea floor gradient. For fully developed waves over nearly horizontal slopes, Le Roux (submitted for publication) showed that

$$L_b = 2L_o/3 \tag{19}$$

This corresponds to within 5% to the breaker wavelength according to cnoidal theory. Under these circumstances L_b can therefore be calculated and used in Eq. (1), for example. On steep slopes, however, the breaker would be in shallow water while the next wave crest seaward thereof would still be in deep water. This crest would therefore be affected less by bottom friction, advancing faster than would be the situation on a nearly horizontal slope. L_b should therefore be shorter on steep slopes than on gentle slopes. Although the exact amount of shortening is uncertain, L_b should nevertheless be a direct function of L_o and α, so that it should be sufficient to consider only the latter two parameters together with H_o as the most important variables.

Sakai and Battjes (1980), based on Cokelet (1977), published a 3-dimensional graph showing plots of different H_o/L_o ratios against H_w/H_o and d/L_o. Recast into equations considering all three ratios simultaneously, H_w (which is the wave height in any water depth) can be calculated by

$$H_w = H_o\{A \exp[(H_o/L_o)B]\} \tag{20}$$

where A and B are coefficients given by

$$A = 0.5875(d/L_o)^{-0.18} \text{ when } d/L_o \leq 0.0844 \tag{21}$$

$$A = 0.9672(d/L_o)^2 - 0.5013(d/L_o) + 0.9521 \quad \text{when } 0.0844 < d/L_o \leq 0.6 \tag{22}$$

$$A = 1 \text{when } d/L_o > 0.6 \tag{23}$$

$$B = 0.0042(d/L_o)^{-2.3211} \tag{24}$$

However, the actual breaking limit for any specific H_o/L_o ratio is determined by the bottom slope, which is not considered in these equations.

Laboratory tests of periodic waves with periods from 1–6 s on slopes varying from 0° to 11.3° (Shore Protection Manual, 1984) showed that the H_b/d_b ratio varied from 0.83 to 1.32. Using their tabulated values, this can be expressed as

$$H_b = d_b\left(-0.0036\alpha^2 + 0.0843\alpha + 0.835\right) \tag{25}$$

which levels off at 1.3282 and a slope of 12° (Fig. 2). Grilli et al. (1997) found that no wave breaks on slopes steeper than 12°, which means that H_b/d_b cannot exceed 1.3282. Eq. (2) of Collins (1970) can therefore only be valid up to a slope of 6.2°.

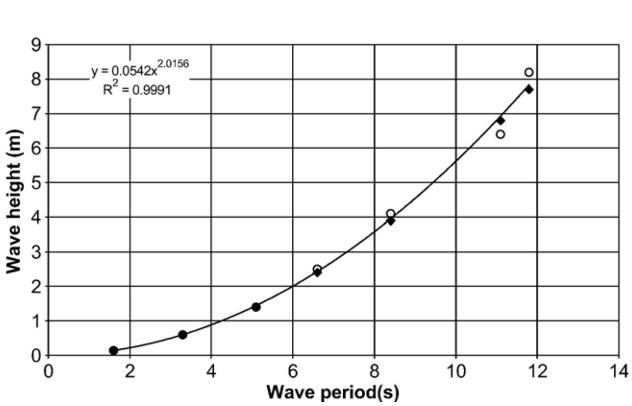

Fig. 1. Plot of wave heights as predicted by Eq. (18) (black diamonds) and wave heights indicated by Resio et al. (2003) (open circles) against wave period.

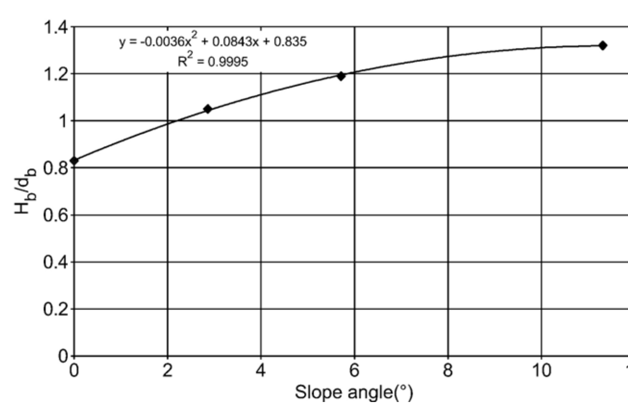

Fig. 2. Plot of H_b/d_b ratio as a function of bottom slope (data from Shore Protection Manual, 1984).

Table 2
Comparison of breaker heights and depths for fully developed waves (H_o/L_o=0.0354) with different periods over different slopes

T_w	Col	Mic	S&B1	S&B2	K&G	Kom	F&M	LR	Weg	Kom	LR
	H_b	H_b	H_b	H_b	H_b	H_b	H_b	H_b	d_b	d_b	d_b
1 × 10^−6° slope											
1.6	0.14	0.17	0.16	0.18	0.15	0.15	0.15	0.17	0.22	0.00	0.20
3.3	0.61	0.71	0.70	0.76	0.66	0.66	0.63	0.71	0.91	0.01	0.85
5.1	1.46	1.69	1.69	1.81	1.57	1.58	1.50	1.69	2.17	0.02	2.03
6.6	2.44	2.82	2.83	3.04	2.63	2.65	2.50	2.83	3.63	0.04	3.39
8.4	3.96	4.57	4.58	4.91	4.26	4.29	4.06	4.59	5.88	0.06	5.50
11.1	6.91	7.98	7.98	8.56	7.43	7.47	7.09	8.02	10.28	0.11	9.60
11.8	7.81	9.02	9.03	9.68	8.40	8.45	8.01	9.06	11.62	0.13	10.85
5° slope											
1.6	0.20		0.16	0.19	0.15	0.15		0.19	0.18	0.15	0.16
3.3	0.84		0.70	0.81	0.66	0.66		0.81	0.77	0.64	0.69
5.1	2.00		1.69	1.95	1.57	1.58		1.93	1.83	1.52	1.65
6.6	3.35		2.83	3.27	2.63	2.65		3.23	3.07	2.55	2.77
8.4	5.43		4.58	5.29	4.26	4.29		5.23	4.97	4.13	4.49
11.1	9.48		7.98	9.22	7.43	7.47		9.14	8.69	7.20	7.83
11.8	10.71		9.03	10.43	8.40	8.45		10.33	9.82	8.14	8.85
10° slope											
1.6			0.16	0.20	0.15	0.15		0.20		0.18	0.15
3.3			0.70	0.84	0.66	0.66		0.85		0.77	0.65
5.1			1.69	2.00	1.57	1.58		2.04		1.84	1.55
6.6			2.83	3.36	2.63	2.65		3.41		3.08	2.59
8.4			4.58	5.43	4.26	4.29		5.53		4.99	4.19
11.1			7.98	9.47	7.43	7.47		9.65		8.70	7.32
11.8			9.03	10.71	8.40	8.45		10.91		9.84	8.27

Col = Collins (1970); Mic = Miche (1944); S&B1 = Eqs. (9)–(11), Sakai and Battjes (1980); S&B2 = Eqs. (12)–(15), Sakai and Battjes (1980); K&G = Komar and Gaughan (1973); Kom = Komar (1998); F&M = Fenton and McKee (1990); LR = this paper; Weg = Weggel (1972).

In Eqs. (20)–(25), the values of L_o and H_o are fixed by the deepwater wave climate, so that only d and H_w can change as the wave moves into shallow water. For any H_o/L_o ratio, the H_w/H_o ratio would thus vary with depth as expressed in d/L_o, until the breaking limit is reached and H_w becomes H_b. The latter would lie along the trajectory of any specific H_o/L_o ratio as modeled in Eq. (20), but would ultimately be determined by the bottom slope. Iterating d in Eqs. (20) and (25) until the calculated breaker heights coincide, thus provides a simultaneous solution for both d_b and H_b. This can easily be done on a spreadsheet.

4. H_b and d_b for fully developed waves over slopes of 0–10°

As shown above, the H_o/L_o ratio of fully developed waves is 0.0354. Using this ratio and iterating Eqs. (20) and (25) with different water depths until the breaker heights coincide, yield the heights and depths shown in Table 2. First, a nearly horizontal slope of $1\times10^{-6\circ}$ was used, this value being chosen only to avoid dividing by zero and to actually cause wave breaking, which would not occur on an absolutely horizontal bottom.

An analysis of the obtained values indicates that there is a direct relationship of H_b with the deepwater wavelength L_o. The breaker height is given by

$$H_b = L_o/24 \tag{26}$$

Further analysis of Table 2 indicates that the breaker depth can be obtained by

$$d_b = L_o/20.0392 \tag{27}$$

Combining Eq. (26) with Eq. (19) also shows that

$$H_b/L_b = 1/16 = 0.0625 \tag{28}$$

Similarly, Eq. (27) can be combined with Eq. (19) to yield

$$d_b/L_b = 0.0749 \tag{29}$$

Finally, it can be shown from Eqs. (26) and (19) that

$$H_b/d_b = 1/1.2 = 0.8333 \tag{30}$$

Eq. (30) agrees with the experimental value of 0.83 reported for a 0° slope in the Shore Protection Manual (1984).

It should be emphasized that Eqs. (19), (26)–(30) are only valid for fully developed waves breaking over a nearly horizontal bottom.

Table 2 also compares the obtained values with those given by the other equations mentioned above. The only of these equations strictly applicable to a horizontal bottom are those of Miche (1944) and Eqs. (9)–(11) of Sakai and Battjes (1980), in which case the breaker heights show an excellent correlation with

Eqs. (20) and (25) or (26) throughout the range of wave climates, with the difference never exceeding 4 cm.

For this situation, the equation of Collins (1970) consistently yields the lowest breaker height, followed by Fenton and McKee (1990). Eqs. (12)–(15) derived from Sakai and Battjes (1980) yield the highest breakers, showing a maximum difference of 66 cm with Miche (1944) for an 11.8 s wave period. The equations of Komar and Gaughan (1973) and Komar (1998) give very similar H_b values that fall between those of Fenton and McKee (1990) and Miche (1944).

As concerns breaker depth, the method proposed here gives consistently lower values than that of Weggel (1972), but the difference does not exceed 77 cm. The H_b/d_b ratio of Weggel is about 0.78, as compared to 0.83 for this paper, the latter being in accordance with experimental measurements over a horizontal bottom (Shore Protection Manual, 1984). The equation of Komar (1998) does not yield realistic values for a horizontal bed.

For a slope of 5°, Eqs. (20) and (25) indicate a significant increase in breaker height as compared to a horizontal bottom, as also shown by the equation of Collins (1970) and Eqs. (12)–(15) derived from Sakai and Battjes (1980). Eqs. (9)–(11) of Sakai and Battjes (1980), Komar and Gaughan (1973), and Komar (1998) record no difference.

Eqs. (20) and (25) indicate that the breaker depth decreases for the same wave periods at higher slopes, as confirmed by the equation of Weggel (1972). The H_b/d_b ratio for Weggel varies around 1.05, as compared to 1.16 for Eqs. (20) and (25), the latter being in accordance with laboratory observations (Shore Protection Manual, 1984). Komar (1998) in this case yields more realistic values.

For a 10° slope, the different equations record only very moderate changes in the breaker height as compared to a 5° slope, with the exception of a dramatic increase shown by the equation of Collins (1970). However, as mentioned above, Eq. (2) should only be valid for slopes up to 6.2°. In this case Eqs. (12)–(15) of Sakai and Battjes (1980) continue to correlate well with this paper, with a maximum difference of 20 cm. Komar and Gaughan (1973), Komar (1998), and Eqs. (9)–(11) of Sakai and Battjes (1980) record no changes.

The method proposed here underestimates the breaker depths given by Weggel (1972) by up to 102 cm, but such a slope is outside the original range considered valid by the latter author. However, the H_b/d_b ratio of 1.32 is in accordance with laboratory observations for a slope of about 10° (Shore Protection Manual, 1984), whereas this ratio as obtained from Weggel (1972) would be about 1.16. Komar (1998) now seems to somewhat overestimate the breaker depths.

5. H_b and d_b for developing waves over slopes of 0–10°

Fetch or duration-limited (developing) waves are normally shorter and steeper than fully developed waves. For example, a wave forming at a wind velocity of 10 m s^{-1} blowing over a fetch of 10 km, will have a height of 0.5 m as compared to 2.4 m under FDS conditions (Resio et al., 2003, Fig. II-2-23). The

Table 3
Comparison of breaker heights and depths for developing waves (H_o/L_o=0.05) with different periods over different slopes

T_w	Col	Mic	S&B1	S&B2	K&G	Kom	F&M	LR	Weg	Kom	LR
	H_b	H_b	H_b	H_b	H_b	H_b	H_b	H_b	d_b	d_b	d_b
1 × 10^{-6}° slope											
1.6	0.13	0.17	0.16	0.18	0.15	0.15	0.13	0.15	0.22	0.00	0.18
3.3	0.56	0.71	0.70	0.78	0.66	0.66	0.58	0.65	0.91	0.01	0.80
5.1	1.32	1.69	1.69	1.88	1.57	1.58	1.37	1.53	2.17	0.07	1.83
6.6	2.24	2.82	2.83	3.12	2.63	2.65	2.32	2.60	3.63	0.04	3.12
8.4	3.65	4.57	4.58	5.05	4.26	4.29	3.77	4.23	5.88	0.06	5.06
11.1	6.35	7.98	7.98	8.81	7.43	7.47	6.58	7.37	10.28	0.11	8.83
11.8	7.19	9.02	9.03	9.96	8.40	8.45	7.44	8.34	11.62	0.13	9.99
5° slope											
1.6	0.18		0.16	0.20	0.15	0.15		0.17	0.16	0.15	0.15
3.3	0.76		0.70	0.85	0.66	0.66		0.73	0.68	0.64	0.63
5.1	1.78		1.69	2.04	1.57	1.58		1.72	1.59	1.52	1.47
6.6	3.04		2.83	3.39	2.63	2.65		2.93	2.72	2.55	2.51
8.4	4.93		4.58	5.49	4.26	4.29		4.75	4.41	4.13	4.08
11.1	8.60		7.98	9.58	7.43	7.47		8.29	7.70	7.20	7.11
11.8	9.73		9.03	10.82	8.40	8.45		9.38	8.71	8.14	8.04
10° slope											
1.6			0.16	0.20	0.15	0.15		0.18		0.18	0.14
3.3			0.70	0.87	0.66	0.66		0.77		0.77	0.58
5.1			1.69	2.10	1.57	1.58		1.81		1.84	1.37
6.6			2.83	3.49	2.63	2.65		3.08		3.08	2.34
8.4			4.58	5.64	4.26	4.29		5.00		4.99	3.79
11.1			7.98	9.85	7.43	7.47		8.71		8.70	6.61
11.8			9.03	11.13	8.40	8.45		9.86		9.84	7.48

Abbreviations as in Table 2.

wave period in this case will be 2.2 s (Resio et al., 2003, Fig. II-2-24), so that L_o will be 7.56 m and H_o/L_o 0.0661 as compared to the FDS ratio of 0.0354. Such steeper waves will behave differently from fully developed waves, e.g. manifesting as plunging instead of spilling breakers on the same slope.

The total energy of a wave is the sum of the potential energy, which is directly proportional to its height, and the kinetic energy, which is a direct function of its celerity. Developing waves have shorter wavelengths than fully developed waves, whereas their wave heights should not exceed those of the latter for the same wind speed. The total energy of such high-steepness waves will therefore be less than that of fully developed waves, so that they should have lower breakers on the same slope. To test this hypothesis, the same deepwater wave heights were used as in Table 2, but the H_o/L_o ratios were increased to 0.05 by shortening the wavelength. Table 3 shows the results of this exercise.

On a nearly horizontal bottom, there is a decrease of about 8% in the breaker height compared to fully developed waves, with a similar decrease in the breaker depth. A decrease in breaker height is also shown by Collins (1970) and Fenton and McKee (1990).

For a 5° slope, the wave height is about 12% higher than for a nearly horizontal bottom, but there is a decrease in breaker height and depth of about 9% with respect to fully developed waves. Such a decrease in breaker height is also shown by Collins (1970). The breaker depths given by the method proposed here correspond closely to those given by Komar (1998), but somewhat underestimate Weggel (1972).

Over a slope of 10°, breaker heights increase only slightly (about 5%) with respect to a 5° slope, whereas the breaker depth decreases further. Eqs. (12)–(15) of Sakai and Battjes (1980) also show a slight increase in breaker height.

6. Discussion and conclusions

For fully developed waves breaking over a nearly horizontal bottom, the breaker wavelength, height and depth show definite and simple relationships with the deepwater wavelength. This indicates that the d_b/L_b, H_b/L_b and H_b/d_b ratios also have fixed values under these conditions, as supported by recent experimental studies using video image processing (comment by anonymous reviewer). From Table 3 it can also be seen that the H_b/d_b ratio apparently does not change much (if at all) for high-steepness waves over a nearly horizontal bottom, averaging 0.8315 as compared to 0.8333 for fully developed waves. However, for sloping bottoms the H_b/d_b ratio increases with the slope angle and L_b changes by an as yet unknown factor.

The equation of Collins (1970) correlates well with the method proposed here on a slope of 5° for both developing and fully developed waves, but appears to underestimate breaker heights somewhat over a nearly horizontal bottom. It also predicts lower breaker heights for developing waves. However, this method is only valid up to a bottom slope of 6.2°. Miche (1944) likewise correlates very well with the proposed method for fully developed waves over a nearly horizontal bottom, but does not take the H_o/L_o ratio into account and therefore shows no difference between developing and fully developed waves. Furthermore, as this method requires the breaker wavelength L_b, which presently can only be determined accurately for a nearly horizontal bottom (Eq. (19)), it cannot be applied to slopes. Eqs. (9)–(11) derived from Sakai and Battjes (1980) also show an excellent correlation with the proposed method for fully developed waves over a nearly horizontal bottom, but because these equations take neither the slope nor the H_o/L_o ratio into account, no differences are indicated for higher slopes or developing waves. Eqs. (12)–(15) deduced from Sakai and Battjes (1980), on the other hand, show an excellent correlation with the proposed method for fully developed waves on any positive slope, although they seem to overestimate the breaker heights somewhat for a nearly horizontal bottom. For developing waves, however, this method indicates higher breaker heights than for fully developed waves, which is contrary to the expected trend. Komar and Gaughan (1973), and Komar (1998) also do not consider the bottom slope, and although these methods give reasonable values for a horizontal bottom, they increasingly underestimate the breaker height for higher slopes. They also show no difference between developing and fully developed waves. The equation of Fenton and McKee (1990) appears to consistently underestimate breaker heights in comparison with the other methods and also requires the breaker wavelength, so that it can be applied only to nearly horizontal bottoms.

As concerns breaker depth, Weggel (1972) shows a decrease in depth with an increase in slope, in accordance with the method proposed here. However, it is applicable only to a maximum slope of 5.7°. Komar (1998) gives completely unrealistic values for a nearly horizontal bottom, but shows a fair correlation with the present method over a 5° slope. For higher slopes, however, it seems to overestimate the breaker depth. It also indicates no difference between developing and fully developed waves.

An obvious concern in using the method proposed here is whether laboratory observations can be applied directly to field conditions. For a nearly horizontal bottom, Eqs. (20) and (25) yield a H_b/d_b ratio of 0.83, which is considerably higher than the ratios of 0.71–0.78 normally considered to be the breaking limit (Keulegan and Patterson, 1940; Wiegel, 1960; Collins, 1970; Weggel, 1972; Fenton and McKee, 1990). A possible explanation for this discrepancy may be that the still water level (SWL) under field conditions may be considered to lie halfway between the breaker crest and trough. However, in the case of cnoidal waves (which generally represent the wave form just before breaking) the water surface is distributed asymmetrically about the SWL and the wave trough actually lies closer to the SWL than the wave crest (Korteweg and De Vries, 1895; Demirbilek and Vincent, 2002). For example, for 6.6 s waves formed under FDS conditions, the breaker height and depth over a sea floor with a 1° slope would be 2.93 m and 3.20 m, respectively. Using cnoidal theory (see Figs. II-1-13 and II-1-14 in Demirbilek and Vincent, 2002, for example) the height of the wave crest and trough are calculated to be 5.52 and 2.59 m from the sea floor, respectively. Taking the SWL at $(5.52-2.59)/2+2.59$ would place the SWL at 4.06 m. The calculated H_b/d_b ratio would therefore be 0.72 instead of 0.92, possibly accounting for the low ratios reported by some authors.

Other factors that probably have some influence on the breaker height and depth, such as the bottom roughness, the type of wave, and the effect of waves reflected from the beach onto the incident waves, were not examined for the purposes of this paper. The four major types of breakers generally recognized, namely spilling, plunging, collapsing, and surging breakers (Patrick and Wiegel, 1955; Wiegel, 1964; Galvin, 1968), also depend on the beach slope, wavelength and period (Brown et al., 1999) and may therefore be accounted for in the method proposed here. However, this aspect needs to be investigated further. Advanced numerical models of Boussinesq, Navier–Stokes or RANS type were also not considered for the purpose of this paper, which was to provide an easily applicable method that takes only the most important controls on breaker height and depth into account.

For engineering applications, it should be stressed that the maximum breaker heights occur during fully developed sea conditions and on higher slopes, whereas the breaker depth decreases for developing waves and on increasing slope angles.

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