Estimation of the phase content of SAR images of the ocean using 2D bispectrum estimation

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Abstract - Phase information is relevant either for detecting and quantifying nonlinearities in a stochastic process or for reconstructing nonminimal phase linear systems. Higher Order Spectra (contrary to the spectrum) preserve the phase information and consequently can be used for both problems quoted previously. Knowing Higher Order Statistics have already provided good results in sea surface classification [1], the natural following step is to take into account spatial context by using the bispectrum in order to study nonlinearities in the S.A.R. mapping process of the ocean surface.

INTRODUCTION

We have divided this paper into three parts, the first one deals with moment and bispectrum definitions and properties and also about nonlinearities detection and quantification problem, the second part is devoted to study SAR images nonlinearities in the case of waves traveling in the range direction and comparison with theoretical ocean surface SAR images bispectrum is roughed out in the third and last part.

BISPECTRUM DEFINITIONS AND PROPERTIES

During these last ten years, Higher Order Spectra analysis has allowed to progress in nonlinear systems detection and identification. As a matter of fact, the capability of Higher Order Spectra to estimate the signal phase, permits to detect and quantify phase coupling induced by nonlinearities as described below. For a 2D signal, third order moment can be defined as:

$$M_3^X(n_1,n_2,n_3,n_4) = E\{X(i,j).X(i+n_1,j+n_2).X(i+n_3,j+n_4)\}$$

The bispectrum can be defined by two different ways [2], [3], either as the Fourier transform of the third order moment.

$$B(\omega_1,\omega_2,\omega_3,\omega_4) = \sum_{n_1=-\infty}^{+\infty}\sum_{n_2=-\infty}^{+\infty}\sum_{n_3=-\infty}^{+\infty}\sum_{n_4=-\infty}^{+\infty} M_3^X(n_1,n_2,n_3,n_4)e^{-\imath\left(\sum_{i=1}^{4}\omega_i n_i\right)}$$

or as the mathematical expectation of the Fourier coefficients triple product.

$$B(\omega_1,\omega_2,\omega_3,\omega_4) = E\{\tilde{X}(\omega_1,\omega_2).\tilde{X}(\omega_3,\omega_4).\tilde{X}^*(\omega_1+\omega_3,\omega_2+\omega_4)\}$$

With $\tilde{X}(\omega_1,\omega_2) = DFT(X(i,j))$. To explain, as clearly as possible, how bispectrum can provide information about the process nonlinearity, one can consider the following signal:

$$X(i,j) = \cos(k_x^1.i + k_y^1.j + \varphi_1) + \cos(k_x^2.i + k_y^2.j + \varphi_2)$$
$$+\cos((k_x^1+k_x^2).i+(k_y^1+k_y^2).j+\varphi_3)+\cos(k_x^4.i+k_y^4.j+\varphi_4)$$
$$+\cos(k_x^5.i+k_y^5.j+\varphi_5)+\cos((k_x^4+k_x^5).i+(k_y^4+k_y^5).j+\varphi_4+\varphi_5)$$

Where φ_i are random phases uniformly distributed over $[0, 2.\pi]$.
Sinusoids of wave vector k^4, k^5, k^6 are said to be "quadratically phase coupled". For such a signal, the third order moment is equal to:

$$M_3^X(n_1,n_2,n_3,n_4) = \frac{1}{4}\Big[$$
$$\cos(k_x^5.n_1 - (k_x^4+k_x^5).n_3 + k_y^5.n_2 - (k_y^1+k_y^5).n_4)$$
$$+cos(k_x^5.n_3 - (k_x^4+k_x^5).n_1 + k_y^5.n_4 - (k_y^4+k_y^5).n_2)$$
$$+cos(k_x^4.n_1 - (k_x^4+k_x^5).n_3 + k_y^4.n_2 - (k_y^4+k_y^5).n_4)$$
$$+cos(k_x^4.n_3 - (k_x^4+k_x^5).n_1 + k_y^4.n_4 - (k_y^4+k_y^5).n_2)$$
$$+cos(k_x^4.n_1+k_x^5.n_3+k_y^4.n_2+k_y^5.n_4)+cos(k_x^4.n_3+k_x^5.n_1+k_y^4.n_4+k_y^5.n_2)\Big]$$

Only the phase coupled signal part appears in the third order moment and thus, from several signal trials (i.e. with different phases), the bispectrum can discriminate the signal part with phase coupling from the one without phase coupling. It must be noted that the bispectrum presents a peak at the bifrequency: $\omega_1 = k_x^4, \omega_2 = k_y^4, \omega_3 = k_x^5, \omega_2 = k_y^5$ (i.e. the joined frequency of the first two sinusoids). Sinusoids passing through a quadratic filter generate sinusoids which phases and frequencies are the sum of the phases and frequencies of the original sinusoids. So output sinusoids and input sinusoids are quadratically phase coupled, and for this reason, we assume a nonlinear system model which is divided into a linear system in parallel with a quadratic system as depicted in figure 1. It can be interesting to quantify which part of energy is provided by each system with the bicoherency function defined as [2], [3]:

$$P(\omega_1,\omega_2,\omega_3,\omega_4) = \frac{B(\omega_1,\omega_2,\omega_3,\omega_4)}{\sqrt{S(\omega_1,\omega_2).S(\omega_3,\omega_4).S(\omega_1+\omega_3,\omega_2+\omega_4)}}$$

For instance, in the following signal:

$$X(i,j) = A_1.\cos(k_x^1.i + k_y^1.j + \varphi_1) + A_2.\cos(k_x^2.i + k_y^2.j + \varphi_2)$$
$$+ A_3.\cos((k_x^1+k_x^2).i + (k_y^1+k_y^2).j + \varphi_1 + \varphi_2)$$
$$+B.\cos((k_x^1+k_x^2).i+(k_y^1+k_y^2).j+\theta)$$

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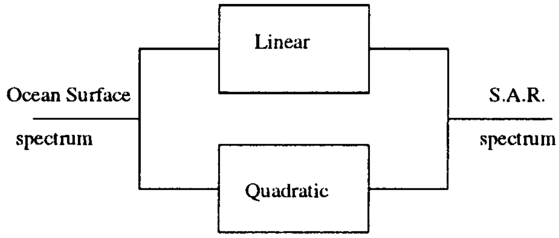

Figure 1: Nonlinear model.

where φ_i and θ are random phases uniformly distributed over $[0, 2.\pi]$, the first of the two sinusoids which frequency is expressed as a sum can be seen as the quadratic filter output generated by sinusoids of wave vector k^1, k^2 and the second one as the linear filtering of the sinusoid of wave vector $k^1 + k^2$. A second order analysis does not separate each part, whereas a bispectral analysis takes into account only the quadratic phase coupled part and thus using previous results, the bicoherency is equal to:

$$P(\omega_1 = k_x^1, \omega_2 = k_y^1, \omega_3 = k_x^2, \omega_4 = k_y^2) = \frac{(A_3)^2}{(A_3 + B)^2}$$

More generally a signal is said "linear", if its bicoherency is flat [2], [3]. However to extract information from a four dimensional structure remains difficult, but a solution already given in [4] uses bicoherency standard deviation in two dimensional slices to visualize peaks in the bicoherency.

Study of bicoherency in a special case

When waves propagate perpendicularly to the satellite track (i.e. in the range direction) a well known phenomenon is the spectrum splitting due to the mapping process nonlinearities [5]. In the spectrum, two peaks can be observed with a rather weakest one. Our purpose is to compare phases of both peaks, using the bicoherency, to estimate if the waves have been generated by two independent waves on the sea surface, or by a unique one. If SAR image waves arise from the mapping of two independent ocean waves, no constant phase relation between the waves exists in several signal trials, whereas this relation exists in the case where SAR image waves are produced by an unique ocean wave. To estimate the bispectrum and consequently the bicoherency, we used two conventional methods presented in [2], [3] and extended to the two dimensional signal case in [6]. The direct method consists in the averaging of the Fourier coefficients (second definition), meanwhile bispectrum is obtained by Fourier transform of third order moment in the indirect method (first definition). Bicoherency is estimated using spectrum periodogram estimator with direct method and correlogram estimator with indirect method. In order to work on several trials we have divided our original 256X256 image into 128X128 nine subimages with overlap, and assumed thus to analyze independent trials. Bispectrum and consequently bicoherency are estimated on a $[64]^4$ grid, the frequency samples number being dictated by memory limitations and time computation, and so the spectrum is also estimated on a $[64]^2$ grid. In table 1 and table 2, we have summarized four information obtained from six ERS-1 256X256 subimagettes, the spectrum peaks localization, the maximum bicoherency peak localization, the maximum bicoherency value and the bicoherency value at the bifrequency formed by the joined two spectrum peak frequencies. For legibility reasons, we give the peak position in the grid (i.e. in integer), for the spectrum two pairs are related (one for each peaks!) and in each pair the first integer is the frequency along the range (real frequency is obtained by dividing the integer by 64 multiplied by the corresponding ERS-1 resolution, 20 meters for the range and 16 meters for the azimuth), for bicoherency integers in odd position correspond to the range frequencies.

First conclusion is that the two methods don't give

	Spectrum peaks	Maximum peak	Maximum value	Bifrequency value
1	5 -3 8 0	5 -4 7 0	0.36	0.23
2	3 -4 8 -2	8 -2 8 -2	0.36	0.18
3	6 -3 7 -2	8 6 -5 3	0.32	0.24
4	5 -3 7 1	5 -4 8 2	0.39	0.24
5	5 -2 6 1	8 2 4 2	0.31	0.21
6	5 -3 7 1	0 6 9 2	0.37	0.27

Table 1: bicoherency peaks localization (direct method)

	Spectrum peaks	Maximum peak	Maximum value	Bifrequency value
1	5 -3 8 0	1 0 9 0	0.85	0.62
2	4 -4 7 -2	8 2 -11 2	0.90	0.52
3	6 -3 8 -2	3 -3 1 8	0.97	0.68
4	5 -3 8 2	7 -3 6 0	1.57	1.09
5	3 -2 6 1	14 1 -9 1	1.00	0.85
6	5 -4 7 1	5 -5 9 1	1.05	0.84

Table 2: bicoherency peaks localization (indirect method)

the same bicoherency rate, but it remains more or less equal for a given estimation method. But the important point is that bicoherency is rather high in both methods (especially at the bifrequency formed by the joined spectrum peak frequencies) and two independent waves which phases have been estimated over nine independent trials can't provide a so high bicoherency rate (on simulated images of [6], we have found a bicoherency rate of 10^{-2} for both methods). As it was already well-known, owing to the phase content, we have found that both waves have been generated by an nonlinear process from an unique original wave. However some results remain unexplained, for instance, in some images a rather high bicoherency at high frequencies (waves less than 100 meters). This is one example of the possible use of phase information for SAR image of ocean surface study, and in the following section, we introduce the theoretical bispectrum of SAR image of sea surface.

THEORETICAL BISPECTRUM OF SAR IMAGES

Study of the spectrum allows to recover some parameters linked to sea surface parameters, but this study remains limited the mere fact that all the information is contained only in the spectrum magnitude, meanwhile it is transcribed in the magnitude and the phase of the bispectrum. For this reason it is interesting to calculate theoretical bispectrum of SAR images of ocean surface, using the classical RAR-SAR decomposition of [5] and the second definition of the bispectrum, as for the spectrum is calculated. Thus theoretical bispectral expression is equal to:

$$B(k_1,k_2) = I_o^3 \iint e^{-i(k_1.x)} . e^{-i(k_2.x')} . e^{-\left[(k_1^2+k_2^2+k_1.k_2)\right].\rho_{dd}(0)}$$
$$e^{\left[\rho_{dd}(x-x').(k_1+k_2).k_1+\rho_{dd}(x).(k_1+k_2).k_2-\rho_{dd}(x').k_1.k_2\right]} .$$
$$\Big(H_1(x,x') + H_3(x,x',k_1,k_2)$$
$$+ i\big(H_2(x,x',k_1,k_2) + H_4(x,x',k_1,k_2)\big)\Big) dx.dx'$$

Where $H_i(x,x',k_1,k_2)$ is given in appendix A, I_o is the RAR image intensity average, $\rho_{dd}(x)$ is the correlation function of the stochastic field of azimuth shift induced by the orbital surface velocity. A special interesting case is when waves propagates in the azimuthal direction, (i.e. $k_y^1 \simeq 0$ and $k_y^2 \simeq 0$), thus the bispectrum is given by the Fourier transform of an even function, and consequently the bispectrum is real. A verification of our assumptions and the ones of [5] is given in table 3, in which phase average and phase standard deviation are related for both conventional methods. Results on four images agree with the expected theoretical behavior of the phase (and with our assumptions) with an average about equal to zero and a weak standard deviation, even if for some frequencies, the phase is quite different from zero. Beyond this example, combination of bispectrum phase and magnitude contents (and spectrum magnitude) can provide new interesting results about SAR mapping process of the ocean surface.

	Direct Method		Indirect Method	
	Average	Standard deviation	Average	Standard deviation
1	−3°21	10°61	0°22	10°86
2	−3°15	10°63	0°17	10°85
3	−3°81	10°44	−0°16	10°87
4	−3°69	10°48	−0°11	10°87

Table 3: Phase average and phase standard deviation

CONCLUSIONS

As already said in introduction, Higher Order Spectra are a theory which has been developed during these last ten years and 2D signal bispectrum applications are very seldom. However, phase content is a very interesting field of research and an attractive alternative tool to the spectrum analysis. But, and it is an important limitation, Higher Order Spectra estimators suffer of higher variance than spectrum ones, and phase content is extracted with more difficulty than spectrum magnitude. Moreover, for 2D signals, information is distributed over a four dimensional structure and its exploitation is not immediate. Nonlinearities quantification and nonlinear system identification remain for the moment the more promising field of Higher Order Spectra analysis for sea surface SAR images.

Appendix A

$$H_1(x,x') = 1 + \rho_{II}(x'-x) - \rho_{II}(x') - \rho_{II}(x)$$

$$H_3(x,x',k_1,k_2) =$$
$$-\left[k_1^y(\rho_{Id}(0) - \rho_{Id}(x'-x)) + k_2^y(\rho_{Id}(x') - \rho_{Id}(x'-x))\right].$$
$$[k_1^y(\rho_{Id}(x-x') - \rho_{Id}(0)) + k_2^y(\rho_{Id}(x) - \rho_{Id}(0))]$$
$$-\left[k_1^y(\rho_{Id}(0) - \rho_{Id}(x'-x)) + k_2^y(\rho_{Id}(x') - \rho_{Id}(x'-x))\right].$$
$$[k_1^y(\rho_{Id}(-x') - \rho_{Id}(-x)) + k_2^y(\rho_{Id}(0) - \rho_{Id}(-x))]$$
$$-[k_1^y(\rho_{Id}(x-x') - \rho_{Id}(0)) + k_2^y(\rho_{Id}(x) - \rho_{Id}(0))].$$
$$\left[k_1^y(\rho_{Id}(-x') - \rho_{Id}(-x)) + k_2^y(\rho_{Id}(0) - \rho_{Id}(-x))\right]$$

$$H_2(x,x',k_1,k_2) =$$
$$\left[1+\rho_{II}(x)\right].\left[k_1^y.(\rho_{Id}(0) - \rho_{Id}(x'-x)) + k_2^y.(\rho_{Id}(x') - \rho_{Id}(x'-x))\right]$$
$$+\left[1+\rho_{II}(x')\right].\left[k_1^y.(\rho_{Id}(x-x') - \rho_{Id}(0)) + k_2^y.(\rho_{Id}(x) - \rho_{Id}(0))\right]$$
$$+\left[1+\rho_{II}(x-x')\right].\left[k_1^y.(\rho_{Id}(-x') - \rho_{Id}(x)) + k_2^y.(\rho_{Id}(0) - \rho_{Id}(-x))\right]$$

$$H_4(x,x',k_1,k_2) =$$
$$\left[k_1^y.(\rho_{Id}(x-x') - \rho_{Id}(0)) + k_2^y.(\rho_{Id}(x) - \rho_{Id}(0))\right].$$
$$\left[k_1^y.(\rho_{Id}(-x') - \rho_{Id}(-x)) + k_2^y.(\rho_{Id}(0) - \rho_{Id}(-x))\right].$$
$$\left[k_1^y.(\rho_{Id}(0) - \rho_{Id}(x'-x)) + k_2^y.(\rho_{Id}(x') - \rho_{Id}(x'-x))\right]$$

$\rho_{II}(x)$ is the correlation function of the RAR image intensity field (assumed to be gaussian), and $\rho_{Id}(x)$, the cross correlation function between the RAR image intensity field and the azimuth shift field.

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