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OCEAN WAVES: A SURVEY OF SOME RECENT RESULTS*

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Abstract. A broad survey of the theory of ocean waves is presented, with special emphasis on recent developments. After a brief review of the classical wave modes in the ocean, a discussion of several recently discovered modes of oscillation is given. This is followed by a description of (1) finite-amplitude effects on each of these modes and (2) nonlinear interactions between different modes. Various types of wave-media interactions are next reviewed: refraction, scattering and diffraction, critical layer absorption and shear flow instability. A number of ocean wave problems which involve statistical and probabilistic aspects are also discussed. The different mechanisms for wave generation are described, but only the wind-wave generation problem is discussed in detail. The paper concludes with a list of challenging problems that are of current interest to the oceanographic community.

1. Introduction. The subject of ocean waves has been studied by mathematicians, physicists and engineers for centuries. In spite of these extensive efforts, many new phenomena relating to the familiar types of ocean waves have been discovered only in the last two decades. Also, during this same period, whole new classes of ocean wave modes have been found. The main purpose of this review is to introduce the reader to a number of these recent developments in the theory of ocean waves.

Many of the new theoretical developments can be directly attributed to experiments conducted in the laboratory and the ocean. Further, many recent theoretical discoveries have in turn provided the impetus for carrying out new, more refined experiments. This active interplay between theory and experiment is best illustrated by three examples.

1. The Stokes' expansion for a propagating two-dimensional sinusoidal surface gravity wave is a regular perturbation solution of an inherently nonlinear problem involving a moving, a priori unknown free surface (Lamb [135, p. 417]). It was proved by Levi-Civita [145] and Struik [292] that this expansion converges provided the wave slope ak (a = wave amplitude, k = wavenumber) is sufficiently small. However Benjamin and Feir [18] and Feir [59] discovered by experiment and theory that the convergence of this solution did not guarantee its stability. Indeed, they found that such a wave becomes unstable to side-band perturbations whenever $kH > 1.363$ (H = mean water depth), the instability being due to nonlinear interactions. This instability is now referred to as "side-band instability" and can be understood within the framework of wave-wave interactions (see § 3). Yet even this discovery proved to be only part of the story. Just recently it was found by numerical and laboratory experiments that after a period of exponential growth, the above unstable solution would demodulate and return to a near-uniform state (Yuen and Lake [326], Lake et al. [134]). The energy in the wave train, which is initially confined to a few low modes centered about the carrier wave with wavenumber k, would spread to many higher modes, but would eventually regroup into the original low modes. Such a process repeats periodically in time and is known as "recurrence", named after the now familiar Fermi–Pasta–Ulam recurrence phenomenon (see [200] for a detailed discussion and references). In a numerical study of the long time evolution of a nonlinear mass-spring system, Fermi, Pasta and Ulam

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found that the energy deposited initially in the lower modes did not spread irreversibly (thermalize) to all higher modes by the nonlinear interactions, but periodically returned to the original low modes.

2. In the course of analyzing tide gauge data along the east Australian coast, Hamon [92], [93] observed a northward-propagating, low-frequency signal in the daily mean sea level. Subsequently Robinson [260] showed that a narrow continental shelf/slope region could indeed act as a wave guide for the unidirectional propagation of such low-frequency (several days period) energy in the ocean. He called such low-frequency wave motions "continental shelf waves". (It is of historical interest to note here that a similar type of trapped wave (a "quasi-geostrophic wave") on a sloping shelf of semi-infinite width was discovered theoretically several years previously by Reid [253].) After this discovery of continental shelf waves off the east Australian coast, they were observed at several other locations in the world: the west and east coasts of the United States (Mooers and Smith [203], Mysak and Hamon [219]), the west coast of Scotland (Cartwright [42]), the north Mediterranean coast (Saint-Guily and Rouault [267]), Lake Ontario (Csanady [51]), and the Florida Strait (Schott and Düing [271], Brooks and Mooers [36]). Although first regarded primarily as interesting curiosities by the oceanographic community, continental shelf waves are now believed to play an important role in such phenomena as the meandering of western boundary currents and coastal upwelling (see the review by LeBlond and Mysak [141] for details).

3. Rossby or planetary waves consist of westward-propagating lateral meanders in the current field of a thin layer of fluid lying on a rotating sphere (Rossby [264], Longuet-Higgins [154], [155]). In the atmospheric context they are sometimes called "weather waves" or "cyclone waves", and through their interactions with the zonal winds at mid-latitudes they are known to play an important role in the general circulation of the atmosphere (Starr [284], Pedlosky [231], Dickinson [56]). It had been conjectured for some time that Rossby waves also occur in the open ocean. However, it was not until two large-scale, multi-buoy experiments were performed in the North Atlantic (the so-called Polygon (Brekhovskikh et al. [28], Koshlyakov and Grachev [131]) and MODE (Gould et al. [84], Freeland et al. [63]) experiments) could such a hypothesis be thoroughly tested. The data collected in these experiments indicate that the long-period fluctuations of the open ocean are, in fact, most aptly described in terms of large two-dimensional eddies that slowly drift westward. It has been shown that such eddies can be described kinematically as a superposition of several plane Rossby waves (McWilliams and Robinson [191], McWilliams and Flierl [190]). Also, it has been argued that they could originate from baroclinically unstable Rossby waves extracting energy from the mean flow (Gill et al. [80]) or be due to a resonant triad of Rossby waves (Pedlosky [232]). Finally, it has been suggested that these eddies may not be Rossby waves at all but may represent the decay of two-dimensional turbulence (Rhines [258]) or the transient response of the open ocean to meteorological forcing (Philander [235]). To test these various theories, a new multi-national large-scale experiment (called POLYMODE) has been conducted in the central North Atlantic during 1978.

Before discussing the properties of ocean waves and their interactions, we briefly review the basic restoring forces that exist in the ocean, giving rise to different wave types. The ocean basins are filled with a slightly compressible and electrically conducting liquid lying on the surface of a weakly magnetized rotating sphere. Compressibility allows for the existence of sound waves. Electrical conductivity in the presence of a magnetic field leads to the possibility of Alfvén waves and also splits the basic sound wave into a fast and slow mode (Cabannes [41]). However, because the Earth's magnetic field is so weak, the associated electromagnetic restoring forces are insignificant compared to elastic, gravitational and the other restoring forces in the ocean. Gravity waves arise through the restoring action of gravity on water particles displaced vertically from equilibrium levels, such as a free surface or a geopotential surface in the ocean's interior. At the interface between two fluids (such as air and water), surface tension also acts as a restoring force and gives rise to capillary waves. Rotation introduces the Coriolis force which allows for the existence of inertial or gyroscopic waves. Finally, variations in the equilibrium potential vorticity (see § 2) give rise to Rossby or planetary waves.

The five basic types of oceanic waves identified above—sound, capillary, gravity, gyroscopic and planetary—generalty occur simultaneously, with the five basic restoring forces acting together to produce more complicated mixed types of waves. The relative importance of each restoring force in any given case depends on the properties of the fluid, the geometry of the container and the period and wavelength of the oscillation. In particular, the effects of bottom topography and the continental margins are most important. As well as modifying several of the basic wave types introduced above, topographical and coastal features can also give rise to whole new classes of trapped waves such as Kelvin, edge, shelf and bottom-intensified waves.

The choice of topics that can be discussed in a review paper on the theory of ocean waves is clearly far from unique. Our choice was based on both our own experience and research interests and on the fact that it is on these topics (outlined below) that much of the current research on ocean wave theory is being done by applied mathematicians and physical oceanographers. However, we note here that there are two important topics that are not discussed in this review: (1) oceanic sound waves, a subject of considerable technological importance (e.g., see Caruthers [43]) and (2) the existence and uniqueness theory of evolution equations that describe different wave classes (e.g., see Marchuk [176], Kordzadze [129], Bona and Smith [23], Shinbrot [275], Ton [303], [304], [305]).

In § 2, we give a brief overview of the basic classical linear wave modes in the ocean, together with a short introduction to several new classes of waves discovered during the last two decades. In a few cases, these "new" waves are in fact only embellishments of earlier results long forgotten in the literature. In § 3, the effects of the nonlinear terms on a number of these linear modes are discussed. In particular we give a brief survey of some of the recent work on solitary waves and wave-wave interactions in the oceanic context. In § 4, we examine various interaction processes that commonly occur in the ocean: refraction of waves by topography and mean currents; scattering of waves by topographic and coastal features and by inhomogeneities in the medium; absorption of waves by mean currents; and the growth of waves due to unstable mean shear flows. In § 5, we introduce the concept of a wave spectrum and other statistical and probabilistic ideas used in ocean wave theory. In § 6, we discuss the general energetics of the ocean and give a brief survey of our present day understanding of surface wave generation. Finally, in § 7 we discuss a number of research topics now being actively pursued by oceanographers and wave theoreticians.

2. Oceanic normal modes.

Plane waves. It is well known (Yih [324]) that a stably density-stratified fluid in a gravity field can support propagating vibrations, commonly called *internal gravity* waves, which arise because of the restoring effect of buoyancy. Some of the properties of internal gravity waves have been illustrated most vividly in the experiments of Mowbray and Rarity [205]. In a rotating fluid (see Greenspan [85, p. 51]) the Coriolis force keeps displaced particles in orbit about their equilibrium position and permits the propagation of *gyroscopic* waves; these may be viewed as vibrations of the vortex lines which permeate a rotating fluid. In both physical systems, the wave propagation is anisotropic, there existing preferred directions along the gradient of density or the rotation vector, respectively. The strong analogies between stratified and rotating systems have been reviewed by Veronis [312].

The ocean is rotating and stratified: free waves travelling in its midst (and of scales smaller than the ocean depth, so as not to be affected by boundaries) will be a mixture of internal gravity and gyroscopic waves. The angular rate of rotation of the earth about the polar axis is specified by the constant vector Ω; the density stratification is commonly expressed in terms of the *Brunt–Väisälä* frequency, defined for an incompressible fluid by $N = [\mathbf{g} \cdot \nabla\rho/\rho]^{1/2}$ where $\mathbf{g}$ is the acceleration of gravity and ρ the basic-state density of water. Plane mixed gyroscopic-internal gravity waves (of angular frequency ω and wavenumber vector $\mathbf{k}$) satisfy the dispersion relation (see Tolstoy [302], LeBlond and Mysak [142, p. 46]):

$$\omega^2 = \{|\mathbf{N} \times \mathbf{k}|^2 + (2\Omega \cdot \mathbf{k})^2\}/k^2 \tag{2.1}$$

where the vector $\mathbf{N}$ has magnitude N and the same direction as $\nabla\rho$ and N is assumed to be constant or slowly varying.

The dispersion relation (2.1) describes the propagation of plane wave Fourier components emerging from an arbitrary disturbance through a uniformly rotating spherical shell of stratified fluid. The progress of the waves away from their source may be followed by using ray-theory. Although $\mathbf{\Omega}$ is a uniform vector, $\mathbf{N}$ is a function of position and refraction takes place, so that the rays are curved. The rays soon intersect the boundaries of the ocean and must be extended by reflection rules appropriate to a rigid boundary (at the ocean bottom) or to a free surface (at the air-sea interface). These reflection rules are discussed by Phillips [239] and Sandstrom [269]. Equation (2.1) has been used by Hughes [114] to trace the propagation of gyroscopic-internal waves from low latitudes up to *critical latitudes* ($\pm\theta_c$) at which the polewards component of the group velocity vanishes. Free waves of frequencies such that $|\theta_c| < \pi/2$ cannot propagate poleward of these critical latitudes.

The ocean is relatively thin (compared to its lateral dimensions) and, over much of its depth, strongly stratified ($N \gg 2\Omega$). It may be shown that this combination of circumstances inhibits the effect of that component of $\mathbf{\Omega}$ which is parallel to the Earth's surface (see Veronis [310], Needler and LeBlond [224]). On a local plane approximation to the Earth's surface (the f-plane), the rotation vector $\mathbf{\Omega}$ has components $(0, \Omega \cos\theta, \Omega \sin\theta)$ in the eastward, northward and upward directions, respectively (θ = latitude). Dropping the northward component of Ω from (2.1) and using subscripts V and H to denote the vertical and horizontal components of the wavenumber vector, (2.1) takes the f-plane form

$$\omega^2 = \{N^2 k_H^2 + 4\Omega^2 k_V^2 \sin^2\theta\}/k^2. \tag{2.2}$$

This specialized form of the dispersion relation for gyroscopic internal waves has been used by Magaard [172], Sandstrom [270] and Baines [6] to investigate wave propagation in an ocean of nonuniform depth; it has also been used by Rattray et al. [250], Prinsenberg and Rattray [248] and Baines [7] to examine the generation of internal tides on continental slopes. Note that for $k_H = 0$, $\omega = \pm 2\Omega \sin\theta$; this is the "inertial frequency" associated with purely horizontal movements of the water and which plays an important role in oceanography.

Normal modes. Most of the interest in oceanic wave motion lies in the horizontal propagation of waves from one area of the ocean to another. The ocean may then be considered as a wave guide for gyroscopic-internal gravity waves. Solutions satisfying top and bottom boundary conditions may then be found by superposing pairs of waves satisfying (2.2). Since N is in general a function of depth, the rays are curved and this superposition is not usually straightforward. For a flat-bottomed ocean, however, the linearized momentum and mass conservation equations may be separated, following a method introduced by Taylor [294] (see also Kamenkovich [122]), into their vertical and horizontal dependence. For motions $\propto e^{-i\omega t}$, the vertical dependence $W(z)$ of the vertical velocity in a fluid of depth H is then found to satisfy (see LeBlond and Mysak [142, p. 70])

$$(\rho W_z)_z + \rho\left(\frac{N^2-\omega^2}{gh_n}\right)W = 0 \tag{2.3}$$

subject to the boundary conditions

$$W = 0 \quad \text{on a flat bottom} \quad z = -H \tag{2.4a}$$

$$W_z - \frac{W}{gh_n}\left[g + \frac{\sigma(\omega^2 - 4\Omega^2\sin^2\theta)}{g\rho h_n}\right] = 0 \quad \text{at the free surface,} \quad z = 0. \tag{2.4b}$$

The z subscripts in (2.3) and (2.4) denote differentiation. The surface boundary condition (2.4b) includes the effects of gravity (through g) and of surface tension (through the coefficient σ).

The quantity h_n is a separation constant which depends on the frequency as well as on the form of the stratification $N(z)$; h_n appears also in the horizontal dependence equation

$$P_{xx} + P_{yy} + \left(\frac{\omega^2 - 4\Omega^2\sin^2\theta}{gh_n}\right)P = 0 \tag{2.5}$$

where $P(x, y)$ is the horizontal dependence of the vertical velocity.

Given a vertical stratification $N(z)$ and a frequency ω, the eigenfunctions $W(z)$ and eigenvalues h_n of (2.3)–(2.4) form a complete and nondegenerate set. The eigenfunctions are called vertical modes and include a barotropic (or surface) mode (horizontally propagating for frequencies such that $\omega \geqq 2\Omega \sin\theta$) and a series of baroclinic (or internal) modes (horizontally propagating in the range $2\Omega\sin\theta \leqq \omega \leqq N_{\max}$ where $N_{\max}$ is the maximum value of $N(z)$ in $-H \leqq z \leqq 0$). The barotropic mode is strongly trapped near the free surface at high frequencies (wind waves), but nearly depth-independent at low frequencies: there is no reversal of direction of horizontal currents with depth for the surface mode. The baroclinic modes exhibit a vertical structure which increases in complexity with mode number: the nth mode has n nodes of horizontal velocity over the depth. Of particular interest is the strong concentration of baroclinic mode motions which can occur in the vicinity of maxima of $N(z)$, i.e. in layers of density gradients (pycnoclines). In the limit of discontinuous changes in density, the baroclinic modes become interfacial waves, such as are commonly observed in sharply stratified coastal areas (see Apel et al. [4] or Samuels and LeBlond [268]). An example is shown in Fig. 1.

All the vertical modes obey the same type of horizontal dependence given by (2.5), the only difference being the value of h_n entering that equation. Wave-type solutions of (2.5), representing horizontal propagation, are all normally dispersive, with speeds decreasing with increasing vertical mode number. Note that for long surface waves,

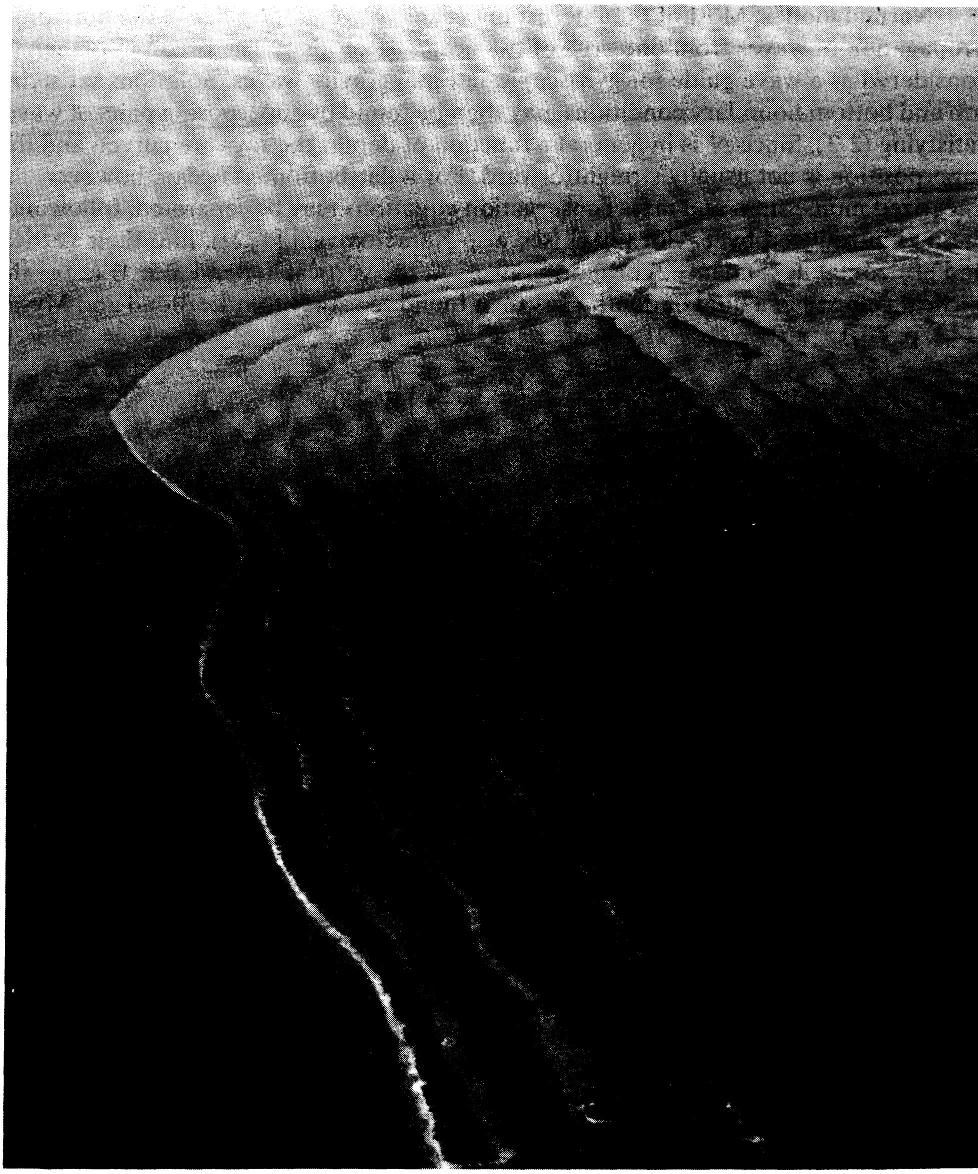

FIG. 1. *Aerial photograph of internal waves, as seen by their surface effects, in the southern Strait of Georgia, British Columbia. Photo taken on May* 29, 1968, *from an altitude of approximately* 750 m. *The camera points eastward and the distance between the first two bands in the wave group is about* 100 m. *From Samuels and LeBlond* [268].

$h_n \to H$ (the depth of the fluid), and it is thus natural to call h_n the "equivalent depth" for this as well as for the other modes.

The solutions of the system (2.3)–(2.5) include all the small-amplitude free wave solutions of super-inertial frequency ($\omega \geqq 2\Omega \sin \theta$); forced wave solutions may also be represented in terms of such free waves. These waves are basically gravity waves (except for capillary effects) modified by the Earth's rotation; they can also exist in a nonrotating fluid ($\Omega \equiv 0$). This class of waves is often referred to as "first-class" waves; "second-class" waves are defined below.

First-class surface waves include, at their high frequency (periods less than 0.075 sec) and short wavelength ($\lambda < 17$ mm) limit, the capillary waves due to the surface tension term (σ) appearing in (2.4b) (see Kinsman [127, p. 167]). Such waves form the familiar wave pattern observed around a fishline in a quiet flow. Wind waves are found at longer periods (0.5–10 sec) and wavelengths (10 cm–100 m); a detailed account of their properties is found in Kinsman [127] and in Phillips [240]. Recent field measurements of wind waves are discussed by Hasselmann et al. [102] and Regier and Davis [252]. Tsunamis (waves produced by submarine earthquakes) have even longer periods (15–90 min) and wavelengths. The waves observed around the periphery of the Pacific Ocean following the 1964 Alaska earthquake may be seen in Spaeth and Berkman [283]. Finally, the longest-period first-class waves are the tides, produced by small local imbalances in the lunar and solar attraction. The tide may be represented in terms of free and forced long-period gravity waves (also called Poincaré waves, after Poincaré [247]; see also Platzmann [242]). Tidal generation will be discussed in more detail in § 6.

All the surface waves have, at least in theory, possible internal counterparts of higher vertical mode numbers. That internal waves are less well documented is not surprising since they are more difficult to observe. Much progress has been made in recent years following the spectral model of Garrett and Munk [68], [69] and considerable observational effort (see Briscoe [35], Thorpe [300], Wunsch [320], Müller et al. [207] for recent reviews and discussions).

Free first-class waves have super-inertial frequencies ($\omega \geqq 2\Omega \sin \theta$). There is, however, one exception: the Kelvin wave, which propagates along a lateral boundary with amplitude decaying exponentially away from its support (LeBlond and Mysak [142, p. 211]). In this special solution, the Coriolis force is at all times balanced exactly by a pressure gradient (a surface slope for the barotropic mode) along its crests: the wave propagates as if there were no rotation and is not limited to super-inertial frequencies. An important fraction of the surface tide along coastlines is made up of a Kelvin wave (see Munk et al. [211]).

Second-class waves. Oceanic waves of sub-inertial frequency are also possible; although originally called "second-class waves" by Hough [112], they are also commonly referred to as *planetary* waves (because of their long wavelengths) or as *Rossby* waves (after C. G. Rossby [264], who first clearly elucidated their nature). The second class waves are obtained formally by improving the local geometrical approximation of the Earth's surface beyond the f-plane introduced above. The local vertical component of the Earth's rotation vector (of magnitude $\Omega \sin \theta$) is a function of latitude. Rossby [264] was the first to describe long-period oscillations on the *β-plane*, a local planar approximation to the globe in which the dominant effect of the variation of $\Omega \sin \theta$ with latitude is retained. The approximations involved in using the β-plane are discussed by Veronis [309], [310], and Grimshaw [87]. From a physical point of view, second-class waves arise as a consequence of the conservation of angular momentum. In the absence of frictional torques, the integrated angular momentum (or *potential vorticity*) of a fluid layer of uniform density and variable thickness H is conserved along a path line:

$$\frac{D}{Dt}\left[\frac{\zeta + 2\Omega \sin \theta}{H}\right] = 0 \tag{2.6}$$

where ζ is the local vertical component of the fluid vorticity. This conservation law is a direct consequence of Ertel's vorticity theorem (see Krauss [133, p. 61] for a derivation). The vortex stretching arising from gradients of the ratio $2\Omega \sin \theta/H$ gives rise to restoring torques which bring displaced fluid columns back towards their equilibrium position.

Second-class waves have sub-inertial frequencies ($\omega < 2\Omega \sin \theta$). In a stratified ocean with a flat bottom, a vertical normal mode decomposition proceeds as on the f-plane. The vertical dependence of the vertical velocity is found as the solution of a problem of the form (2.3)–(2.4). The simplest horizontal dependence equation is that obtained for the north-south velocity component V (LeBlond and Mysak [142, p. 121]):

$$V_{xx} + V_{yy} + \frac{i\beta}{\omega} V_x + \left(\frac{\omega^2 - 4\Omega^2 \sin^2 \theta}{gh_n}\right) V = 0 \tag{2.7}$$

where $\beta = (1/R)\ (\partial/\partial\theta)\ (2\Omega \sin \theta)$ (with R the radius of the Earth), is the local derivative with respect to latitude of the vertical component of the Earth's rotation. In the classical β-plane introduced by Rossby, both $\sin \theta$ and β are held constant in (2.7); other β-plane approximations are appropriate near the equator (see Matsuno [179]) and the poles (see LeBlond [140]). For plane waves, (2.7) is a cubic in ω with coefficients dependent on the properties of the medium (β, $\sin \theta$, h_n) and the horizontal wavenumber components. This cubic has three real roots; two high-frequency roots ($\omega > \Omega \sin \theta$) correspond to the first-class modes discussed above; the low frequency root ($\omega < 2\Omega \sin \theta$) is a second-class wave. The second-class wave may be examined directly by neglecting ω^2 with respect to $4\Omega^2 \sin^2 \theta$ in (2.7).

The sub-inertial (second-class) plane wave solutions of (2.7) are strongly anisotropic: all phase propagation is towards the west. The group velocity is not so restricted, however, and the influence of low frequency events may be transmitted toward all parts of the ocean by planetary waves. The great importance of planetary waves in atmospheric dynamics has long been recognized (see Dickinson [56]), but their role in the ocean is still a subject of intense investigation (see Rhines [257], [258]).

Topographic waves. The battery of first- and second-class vertical modes of an ocean of uniform depth make up the classical arsenal of theoretical research in time-dependent oceanic phenomena. These modes may be superimposed to describe free and forced waves in basins of uniform depth and various physical shapes. Numerous examples may be found in the literature (see Lamb [135, pp. 320 ff.], Veronis and Stommel [313], Platzmann [242], Rao [249], Longuet-Higgins [156], Pnueli and Pekeris [246]).

Many areas of the ocean floor can, however, by no stretch of the imagination be approximated as flat, and it has long been recognized that bathymetric variations must have an important effect on the properties of oceanic wave motions. The most straightforward bathymetric influence is that exerted by bottom slopes on second class waves, where, as mentioned above, H variations are equivalent to the inverse of $\sin \theta$ variations of $2\Omega \sin \theta$ with latitude (see Veronis [311]). The restriction to small bottom slopes ensures that no significant coupling between the vertical (w) and horizontal ($\mathbf{u}$) components of velocity takes place through the bottom boundary condition $w = \mathbf{u} \cdot \nabla H$ and that the separability of horizontal and vertical dependences of the motion remains possible.

The first truly topographic wave was discovered by Stokes [288]; the Stokes edge wave is a modified surface gravity wave (i.e. a first class wave) trapped along a coast by a uniformly sloping bottom. This edge wave propagates along a coast with amplitude decaying away from it and can exist in many horizontal modes ($n = 0, 1, 2, \cdots$) corresponding to as many nodal lines of surface displacement running parallel to the coast. Long considered a mere theoretical curiosity, the importance of edge waves in nearshore dynamics has gradually been recognized: Munk et al. [210] have noted that edge waves may be excited by tsunamis; Bowen and Inman [25], [26], Guza and Davis [90], and Guza and Inman [91] have shown how edge waves may be excited by surf and how they may determine the occurrence of beach cusps and rip-currents. The influence of the Earth's rotation (see Reid [253], Kajiura [121] and Mysak [215]), density stratification (see Greenspan [86]) and of the form of the bottom profile (see Ball [9], Odulo [229] and Huthnance [117]) on the properties of edge waves have also been examined. We note that the vertical and horizontal dependences of the wave motion cannot in general be separated in edge waves or other topographic waves, and that the following description applies only to an unstratified fluid, in which only the barotropic mode is possible.

The simplest account of the edge wave is in terms of the refraction of pure surface gravity waves by the sloping bottom. As the speed of surface waves increases with water depth, rays travelling towards deep water are refracted back onshore where they are reflected back offshore to be refracted anew. This interpretation suggests that processes which may be represented in terms of refraction or reflection of short waves in regions of extrema of wave properties may account for other geometrical modifications of the basic flat-bottom first class waves. Trapping of waves by ridges has been studied by Buchwald [38], and the trapping of waves around islands and seamounts has been discussed by Longuet-Higgins [157], [160], Summerfield [293] and, more specifically in terms of ray theory, by Shen et al. [274].

Geometrical trapping of second class waves by regions of strong bathymetric gradients is also possible (see Smith [278] and Rhines [254], [255]). The continental shelf waves discussed by Robinson [260] and Mysak [213] and observed along many coasts (see references earlier) are of such a nature. Similarly, second class waves travelling along a steep escarpment from which their amplitude decays on both sides are akin to continental shelf waves, the only difference being that the depth of the ocean remains nonzero on both sides of the escarpment instead of vanishing along a coast which parallels it. Such waves have rather inappropriately been called "double-Kelvin" waves (see Rhines [254], Longuet-Higgins [158], [159], Saint-Guily [266]), in analogy to the first class Kelvin waves, which decay on one side of a boundary. All these trapped first and second class waves may exist in a series of horizontal modes which describe their structure in the "potential well" formed by the bathymetric slope.

A rather special form of topographic trapping occurs in the "bottom-trapped' wave discovered by Rhines [256], which depends on the Coriolis force, a bottom slope and a density stratification. This wave may be considered as a short wavelength (and low frequency) limit of the barotropic second class wave on a uniformly sloping bottom (see also Needler and LeBlond [224]), for which the vertical structure gradually increases from a horizontal velocity profile uniform with depth at long wavelengths to an exponential decay from the bottom up for short wavelengths. Thompson and Luyten [296] have seen some evidence of this type of wave in long time series of current measurements on the New England continental slope.

Equatorial trapping. Second class waves are subject to refraction through variations of the ratio $\sin\theta/H$. Even in an ocean of uniform depth, one would thus expect wave trapping about the equator (where the parameter β entering (2.7) is a maximum). This possibility was first explored by Stern [285], Bretherton [30], Matsuno [179] and Blandford [22]. It is now well established that the equatorial region behaves like a wave guide along which Kelvin and planetary waves can propagate quickly across the ocean.

The structure of these waves across the equator is given in terms of parabolic cylinder functions, decaying at high latitudes. The consequences of equatorial propagation have been discussed by Lighthill [148] and Anderson and Rowlands [2], [3] in connection with the seasonally varying Somali Current, and by McCreary [182] and Hurlburt et al. [116] with respect to the episodic occurrence of El Niño on the Peruvian coast.

The combined case of topographic and equatorial trapping has recently been treated by Mysak [217], [330] and Geisler and Mysak [72]. In these papers the properties of zonally propagating, long-period barotropic waves trapped over a number of topographic profiles were studied. In general it was found that the speeds of propagation were very dependent on whether the topographic variations straddled the equator or were entirely contained in one hemisphere.

The classification of ocean waves into linear vertical modes and topographic modes provides a useful general framework for thinking about the time-dependent variations of the ocean. However it does not explain the amounts in which the various modes contribute to the power spectrum (see § 5) of oceanic motions. A gastronomic analogy would identify the various modes as the ingredients of the oceanic dish; the recipe for mixing them up in the right proportions to reproduce the radiation field of the sea would be provided by appropriate energy sources for each mode and by interaction rules for the coupling between modes. The specification of this recipe, or at least of what is known of it at this time, is the subject of most of the following sections of this review.

3. Nonlinear effects. Anyone who has stood on the beach observing the crashing surf must be aware that linear theory cannot describe all aspects of ocean waves. Nonlinear effects arise in fluid motions from the advective accelerations present in the momentum and mass conservation equations; in addition, nonlinear terms occur in the boundary conditions at a free-moving surface. Nonlinearity modifies the properties of individual linear modes; it also leads to coupling between these modes. Because the best known large-amplitude phenomena occur in surface gravity waves, where they manifest themselves as solitary waves, surf and hydraulic jumps, our discussion will begin with this type of wave.

Surface waves. Surface gravity waves (the barotropic mode) may be characterized in terms of the three length scales a, H, λ, which are respectively the amplitude of the surface displacement, the depth of the fluid and the wavelength ($\lambda = 2\pi/k$). For gravity waves, as for other types of waves, a simple measure of the importance of nonlinearity is the ratio of particle velocity to phase speed (essentially the ratio of nonlinear acceleration terms to the local time derivative). For deep-water waves ($H/\lambda \gg 1$) this ratio is of order a/λ; for shallow-water waves, of order a/H. Thus, wind waves in deep water break (the ultimate consequence of nonlinearity) when their slope is large ($a/\lambda \simeq \frac{1}{7}$); surf breaks on a shallow beach when its height becomes comparable to the water depth ($a/H \simeq 0.4$).

A general classification of the importance of nonlinearity, valid for all water depths, may be presented in terms of the parameters $\varepsilon = a/H$ and $\mu = (H/\lambda)^2$, which are respectively measures of the importance of amplitude and phase dispersion (see LeBlond and Mysak [142, p. 94]). Amplitude dispersion accelerates the larger waves and leads to shock formation; phase dispersion spreads out disturbances with the larger waves leading the shorter ones. The ratio of these two effects (ε/μ) has been called the Ursell number.

For small Ursell number ($\varepsilon \ll \mu = O(1)$), ε is the only small parameter of the problem. Expansions of surface wave solutions in powers of ε [or more conveniently, $\varepsilon\sqrt{\mu}$] were initially calculated by Stokes [289]; they have been carried out to fifth order in $\varepsilon\sqrt{\mu}$ by Skjelbreia and Hendrickson [277]. The convergence of the series solutions was proven by Nekrasov [225], [226], Levi-Civita [145] and Struik [292]. Nonlinearity sharpens the wave crests and flattens the troughs into a wave profile approaching a trochoidal shape (see Lamb [135, p. 418]). The travelling wave of maximum steepness was found by Stokes [290] to have a sharp crest of angle 120°. For such a wave, the particle speed at the crest (u) is exactly equal to the phase speed (c): a steeper wave, with $u > c$ would topple forward and break. The steepest standing wave crest has an angle of 90° (see Taylor [295] and Longuet-Higgins [153]; the acceleration at the crest is equal to that of gravity: steeper waves just fly apart. Longuet-Higgins [161] has shown that, to a good degree of approximation, the form of the steepest travelling and standing waves is given, in scaled variables, by

$$Z = \ln \sec x, \tag{3.1}$$

where Z is the elevation above the trough (at which $x = 0$). Travelling waves occupy $|x| \leqq \pi/6$; standing waves, $|x| \leqq \pi/4$. Longuet-Higgins [162] has also discovered that the wave of maximum energy has an amplitude somewhat smaller than the possible maximum, and pointed out the implications of this startling result on the initiation of wave breaking.

The discovery, by Benjamin and Feir [18], that surface gravity waves (in the small Ursell number regime) are unstable, as illustrated in Fig. 2, came as a surprise to cohorts of wave dynamicists lulled into a confident mood by the convergence proofs of Levi-Civita and others (as referred to above). Convergence of a series solution does not of course imply the stability of this solution! Surface wave instability manifests itself for wavelengths less than 4.61 times the depth of the fluid, i.e. $kH > 1.363$ (see Feir [59], Lake et al. [134], and Lake and Yuen [333] for descriptions of the observational work, and Benjamin [17], Benney and Newell [331], Whitham [332] and Lighthill [147] for early theoretical treatments). The instability may be understood in terms of the

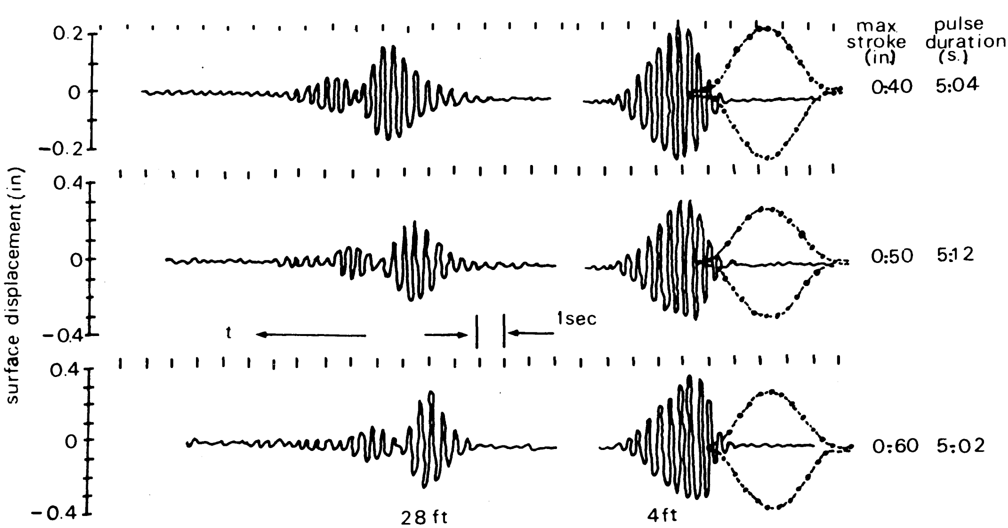

FIG. 2. *The disintegration of surface gravity wave pulses in deep water. The envelope of the wave pulse, as generated by the wavemaker, is shown in dotted lines on the right. The surface displacement, observed after the pulse has propagated* 4 *ft. and* 28 *ft. from the wavemaker, are shown on the left, as a function of time. Three cases are shown, with nearly equal pulse durations but increasing wave amplitudes* (*note the different amplitude scales*). *Adapted from Feir* [59].

nonlinear interaction of small sidebands with the primary wave and is a special case of the nonlinear wave-wave interactions discussed below.

Since its discovery in 1967, the Benjamin-Feir instability phenomenon has attracted the attention of many applied mathematicians. For example, it was first pointed out by Benney and Roskes [334] that the above instability criterion ($kH > 1.363$) does not carry over to three-dimensional perturbations. Also, there were several attempts at obtaining a mathematical description of the long-time evolution of the modulation of a two-dimensional wave train using the method of multiple scales. Chu and Mei [335], [336] found a description in terms of a pair of coupled differential equations, whereas Hasimoto and Ono [337] showed that the slowly-varying envelope satisfied a nonlinear Schrödinger equation. An important step forward was taken by Davey and Stewartson [338], who derived two nonlinear partial differential equations that described the evolution of the modulation of a *three*-dimensional wave train. From these equations, new stability criteria were obtained. It turns out, however, that these equations can be derived as a limiting case of equation 1.19, p. 379, in Benney and Roskes [334]. Also, a simplified form of the Davey–Stewartson equations was used recently by Longuet-Higgins of gravity waves (see also Fox [62]). For discussions of shear flow effects and the valid form of the modulation equation when kH is near 1.363, see Johnson [340] and [341] respectively. Finally, for very recent theoretical work related to gravity wave instabilities and the modulation equations, see Longuet-Higgins [342], [343], Ankar and Freeman [344] and Stuart and DiPrima [345].

When the Ursell number is of order unity ($\varepsilon \simeq \mu$), amplitude and phase dispersion have comparable influences on surface gravity wave behavior. The various formulations for long waves ($\mu \ll 1$) of small, but finite, amplitude ($\varepsilon \ll 1$) have been reviewed by Peregrine [233]. Of particular interest is the equation derived by Korteweg and de Vries [130] (KdV equation) to describe unidirectional wave propagation. In terms of the displacement of the nondimensionalized free surface η, this equation has the form

$$\eta_t + \eta_x + \frac{3\varepsilon}{2}\eta\eta_x + \frac{\mu}{6}\eta_{xxx} = 0. \tag{3.2}$$

The third term represents the influence of nonlinearity; the last, that of phase dispersion. Equation (3.2) has solutions of permanent form, travelling at a constant speed, and expressed in terms of elliptic cosine functions. The long wave limit of these "cnoidal" waves is a wave consisting of a single positive hump; it is called a solitary wave and has the form

$$\eta = \eta_0 \operatorname{sech}^2\left[\left(\frac{3\varepsilon\eta_0}{4\mu}\right)^{1/2}(x-ct)\right] \tag{3.3}$$

in which the (nondimensional) speed is given by

$$c = 1 + \tfrac{1}{2}\varepsilon\eta_0. \tag{3.4}$$

This wave was first observed by Russell in 1844 [265] who called it the "great wave of translation". The largest solitary wave has a sharp crest of angle 120° and a height reaching 0.827 that of the undisturbed water depth (see Yamada [322], Lenau [144], and Longuet-Higgins and Fenton [163]). It has been found by Longuet-Higgins and Fenton [163] that, as in Stokes' waves, the maximum energy corresponds to wave heights slightly smaller than the maximum value. It took more than fifty years after Russell's observations before Korteweg and de Vries, in 1895, arrived at a theoretical explanation of the solitary wave, and yet another half century was to pass before the KdV equation was to find application in other fields of physics and become the object of renewed interest. Applications to wave propagation in plasmas (see Gardner and Morikawa [64], Berezin and Karpman [20], Washimi and Taniuti [315]) and to the Fermi–Pasta–Ulam problem (see Zabusky [327], [328]) have been discovered, and the KdV equation has been extensively studied as a general prototype of nonlinear wave propagation. The development of research on the KdV equation and on the remarkable properties of its solutions have been reviewed in this journal by Miura [200] and in a short book by Karpman [123]. The insights provided by observations obtained from the relatively simple hydrodynamic context have proved extremely fruitful in other physical situations, and it is expected that continuing experiments on nonlinear surface gravity waves, such as those of Yuen and Lake [325] and Maxworthy [180], may continue to play a seminal role in advancing our understanding of nonlinear dispersive wave propagation.

Along with the many attractive properties of the KdV equation, however, there are also a number of shortcomings, which have recently been reviewed by Benjamin et al. [329]. Among these are the difficulties in obtaining numerical solutions of (3.2) and the problems of establishing the existence and uniqueness of solutions corresponding to general classes of initial values. To circumvent these problems, Benjamin et al. [329] showed that a rational alternative to (3.2) is obtained by replacing the last term by $-\eta_{xxt}$. This modified KdV equation was termed "the regularized long wave equation". For the modified equation they were readily able to establish that a classical ("regular") solution exists and that the solutions are unique and depend continuously on the initial values.

In the limit of large Ursell number ($\varepsilon \gg \mu$), amplitude dispersion dominates: the formulation for surface gravity waves in this "hydraulic limit" is formally identical to that of sound waves (see Stoker [287, chap. 10]) and all the methods developed in acoustics apply (e.g., see Whitham [317]). Hydraulic shock waves, also called *bores*, are primarily of interest in channel or river flow. Tidal bores occur in many coastal streams in regions of high tidal amplitudes (see Tricker [306] for illustrations and Abbott [1] and Whitham [317] for theoretical discussions). The other common instance of a bore is found in surf (see Biesel [21] and Ho et al. [109]).

Internal waves. The internal modes of a stratified ocean are also modified by nonlinearity. As for surface waves, the relevance of nonlinearity may be examined in three ranges according to the relative importance of amplitude to phase dispersion. The phenomena encountered and the nonlinear distortions of the wave profiles are similar in most respects to those discussed above for surface waves. The distortion of wave profiles (as given by isopycnal displacements) for small Ursell numbers have been discussed by Thorpe [298], Long [149], Griscom [89] and Magaard [173]. Internal cnoidal and solitary waves in continuously and discontinuously stratified fluids have been studied by Long [150], [151], Davis and Acrivos [53], Benjamin [15], [16], Gargett [65] and Lee and Beardsley [143]. Of particular interest is the modified KdV equation discussed first by Benjamin [16] in connection with wave propagation at the interface between fluid layers of different densities, one layer being much thicker than the other. This equation for the interfacial displacement η has the form

$$\eta_t + \eta_x - 2\varepsilon r \eta \eta_x - \mu s \mathcal{F}(\eta_x) = 0, \tag{3.5}$$

where r and s are parameters describing the density structure, ε is defined as the ratio of wave displacement to layer thickness D and $\mu = D/\lambda$. The operator $\mathcal{F}$ is defined by

(3.6)
$$\mathcal{F}(\phi)=\frac{1}{2\pi}\int_{-\infty}^{\infty}|k|\left\{\int_{-\infty}^{\infty}\phi\, e^{ikx'}\,dx'\right\}e^{-ikx}\,dx.$$

Equation (3.5) also has solitary wave solutions (not of course of the form (3.3)) arising from a balance between amplitude and phase dispersion effects. Finally, in the hydraulic limit, internal bores also exist; their properties are reviewed by Yih [324].

Capillary waves. Capillary waves exhibit anomalous phase dispersion (short waves travel faster than long waves); their amplitude dispersion is also anomalous: large amplitude waves of permanent form thus have flat crests and sharp troughs: just the opposite of what is found for surface gravity waves. An exact solution for nonlinear capillary wave motion has been presented by Crapper [48].

Planetary waves. Planetary waves were first recognized in the ocean as meanders of strong western boundary currents (such as the Gulf Stream in the Atlantic and the Kuroshio off the coast of Japan) and their nonlinear properties were first studied in that context (see Moore, [204]). As was subsequently found by Larsen [139] and Clarke [47], finite amplitude planetary waves of permanent form (such as cnoidal and solitary waves) can also exist. More recently, Maxworthy and Redekopp [181] and Redekopp [251] have re-examined the properties of large amplitude planetary waves in shear currents and have shown that, under certain conditions, stationary eddies may form; they have suggested that such conditions might account for the presence of Jupiter's Great Red Spot. Other recent studies of large amplitude planetary waves superimposed on mean currents are those of Larichev and Reznik [137], [138].

Weak wave-wave interactions. The studies of nonlinear wave properties referred to above describe the features of *single* nonlinear waves; they do not explain the mutual interaction of the broad spectra of waves of various kinds which coexist on the sea surface and in its depths. Weak wave-wave interactions, occurring in time and space scales greatly exceeding those characterizing the interacting waves, are treated by perturbation techniques; strong interactions require techniques similar to those used in the study of turbulent flow.

The theory of weak resonant wave-wave interactions, as presented in the works of Hasselmann [97], [98], [100] (one of its principal expositors) is, at least in principle, quite simple. Consider the elementary case of two interacting "primary" waves of the form $\varepsilon u_1=\varepsilon a_1 e^{i(\mathbf{k}_1\cdot\mathbf{x}-\omega_1 t)}$ and similarly for εu_2, where ε is an appropriate small parameter. These waves satisfy the linearized equation $L_0(\varepsilon u_i)=0(i=1,2)$; for plane waves, this equation reduces to the dispersion relation $D(\omega_i,\mathbf{k}_i)=0$ $(i=1,2)$, where $\mathbf{k}_i$ is a two-dimensional wavenumber vector. An interaction product between the two primary waves may be found to next order from

(3.7)
$$L_0(\varepsilon^2 u_3)=-L_1(\varepsilon u_1,\varepsilon u_2),$$

where the operator L_1 involves products of εu_1 and εu_2 or their derivatives. The interaction product remains of $O(\varepsilon^2)$ unless resonance occurs, i.e. unless $D(\omega_3,\mathbf{k}_3)=0$ where $\omega_3=\omega_1+\omega_2$, $\mathbf{k}_3=\mathbf{k}_1+\mathbf{k}_2$ say (see also (3.8)). Resonant solutions will, after a sufficient time has elapsed, dominate the interaction product by growing to become comparable to the basic interacting waves εu_1 and εu_2. One then speaks of a resonant triad of waves, exchanging energy between each other.

The frequency ω_3 and the wavenumber $\mathbf{k}_3$ of the interaction product are determined by the forcing term on the right hand side of (3.7); allowing for appropriate choices of signs for ω_j, $\mathbf{k}_j$, one may relate the frequency and wavenumber of the interaction product and those of the basic waves through

$$(3.8) \qquad \sum_{j=1}^{3} \omega_i = 0; \qquad \sum_{j=1}^{3} \mathbf{k}_j = \mathbf{0}.$$

For resonant interactions, $D(\omega_j, \mathbf{k}_j) = 0$ for $j = 1, 2, 3$. Resonant interactions between four or more waves (three or more basic waves and their interaction product) are also possible and will be found for higher orders in ε. The strength of the interaction is given by the magnitude of the forcing term in (3.7), called the coupling coefficient. Whether resonant interactions between a given number of waves are possible depends on the form of the dispersion relation. For nondispersive waves, resonant interactions are possible between any number of waves since ω is a linear function of $\mathbf{k}$. For dispersive waves, one must prove the existence of and construct interactions loci in the $\mathbf{k}$-plane.

The resonant interaction processes may be illustrated by a technique borrowed from the field of particle physics. Feynman diagrams (see Schweber [272]) represent the interaction as the convergence of n-arrows, representing the interacting waves, onto a vertex from which emerges the interaction product (examples are shown in Hasselmann [97], [98]). Resonant waves are analogous to real particles, forced waves (not obeying the dispersion relation) to virtual particles. One should finally note that interactions of waves with spatial modulations of the medium of propagation (usually characterized by a zero frequency but finite wavenumber) may also be handled through the wave-wave interaction framework.

Resonant interactions between planetary waves occur at the second order (i.e. involve a triad of waves). Longuet-Higgins and Gill [164] have examined this interaction in detail; Newell [228] has shown that interactions between planetary waves are capable of generating steady zonal flows (see also Plumb [245]). Interactions between topographic planetary waves also occur at second order (Mysak [216]). Internal gravity wave triads also interact resonantly (see Thorpe [297] and Kenyon [126]); these interactions have been studied experimentally by Martin et al. [177], McEwan [183] and McEwan et al. [184]. Surface gravity waves on the other hand, interact only at the third order, as shown by Phillips [238]. Benney [19] and Bretherton [31] have examined in detail the energy transfer between resonant quartets of surface gravity waves. McGoldrick et al. [187] have illustrated these resonances in the laboratory. Hasselmann [99] has shown that the instability discovered by Benjamin and Feir [18] and already referred to above, may be interpreted as an interaction between a basic wave of finite amplitude and two side bands (the basic wave occurs twice in the interactions, so that the interaction, although involving only three waves, is nevertheless at the third order). The relevance of wave-wave interactions on wind-wave generation and shape of the wind-wave spectrum will be discussed in § 6.

Resonant interactions also occur between different wave modes. McGoldrick [185], [186] and Simmons [276] have discussed the resonant coupling between capillary and gravity waves. The coupling between surface and internal gravity wave modes has been examined by Thorpe [297] and Brekhovskikh et al. [29]. The special case of a two-layer fluid has been discussed by Ball [8] and, in the laboratory, by Lewis et al. [146]. Interactions occurring in the nearshore area between incoming swell and edge waves have been related by Guza and Davis [90] and Guza and Inman [91] to the formation and spacing of rip currents. Finally, as an example of an interaction between ocean waves and other types of waves one might mention the generation mechanism proposed by Longuet-Higgins [152] for microseisms, where two surface gravity waves interact resonantly with an elastic wave at the surface of the Earth's crust (the generation theories for microseisms are reviewed by Hasselmann [96]).

Ocean waves of various modes are found in continuous spectra rather than in single Fourier components. In a continuous spectrum, no resonant multiplet may be considered isolated, since one or more of its components may be a member of one or more other resonant groups. In such a spectrum, resonant interactions tend to redistribute energy between all interacting multiplets. The evolution of wave spectra, subject to weak resonant wave-wave interactions, may be described through a "radiation balance" equation (see LeBlond and Mysak [142, p. 324]) analogous to the Boltzmann transport equation of statistical thermodynamics (see Kittel, [128, p. 406]). The collision term analogous to that found in Boltzmann's equation includes contributions from all resonant couplings, as well as any external energy source and dissipation terms. Hasselmann [100] and Willebrand [318] have applied the radiation balance method with considerable success to the study of surface gravity waves (for which the best data are available). Müller and Olbers [206] have examined the radiation balance of the internal gravity wave spectrum. This topic will be discussed in more detail in § 5.

Strong interactions. Strong interactions between waves are those which take place in space or time scales comparable to the wavelength or period of the interacting waves. The large amplitude waves discussed at the beginning of this section may be thought of as a result of strong self-interaction in a single wave. In the language of weak wave-wave interaction theory, the coupling of finite amplitude waves may be regarded as a case where the parameter ε introduced before (3.7) is of order unity. In that case, the forced wave solutions of (3.7) are just as large or larger than the resonant solutions and energy is exchanged not only between selected multiplets but between all waves of a continuous spectrum. This type of strong coupling is that found in turbulent interactions.

Because of their symmetry, plane internal gravity waves and barotropic planetary waves are *exact* solutions of the nonlinear equations dictating their properties. Such plane wave solutions are thus not limited in their amplitude and may be large enough for strong coupling to play an important role in their interactions. For internal waves, a turbulent energy cascade, strongly affected by buoyancy effects, can arise through strong interactions. The "buoyancy subrange" of turbulence characterizing this type of interaction has been explored by Lumley [170].

The planetary wave synthesis of the Polygon and MODE data by McWilliams and Robinson [191] and McWilliams and Flierl [190] have shown that the particle velocities observed exceed the propagation speeds of the best-fit planetary waves. Thus, the nonlinear advection terms must exceed the local accelerations in these waves, a sure indication of strong nonlinearity. The interactions of a field of large amplitude barotropic planetary waves shares the peculiar properties of two-dimensional turbulence, with an energy cascade towards large scale motions, as discussed at length by Rhines [257], [258] (see also § 5).

4. Interactions of waves with the medium. In this section we review the ways in which ocean waves are modified due to their interaction with shear flows, variable bottom topography and other inhomogeneities of the medium. If the interactions occur over length and time scales that are large compared with the characteristic wavelength and wave period respectively and in the absence of dissipation, then they can be described as being "action-conserving" and the techniques of ray theory (geometric optics) can be used in their analysis (e.g. see Bretherton [33], Shen [273]). Examples of such weak interactions are the refraction of surface waves by a horizontally sheared current and the refraction of internal waves by a Brunt–Väisälä frequency that varies slowly with depth. On the other hand, when the interactions occur over length and time scales that are comparable to those of the waves, they can be described as being strong; in such cases ray theory is not applicable. Typical examples of strong interactions are the scattering of long gravity waves by islands and seamounts, the absorption of internal waves by a vertically sheared current, and the amplification of planetary waves in the presence of large-scale flows with vertical and horizontal shear.

Conservation of wave action density. A variational approach to the study of slowly varying wave trains in an inviscid fluid was introduced by Whitham [316] and further discussed by Bretherton and Garrett [34], Garrett [67], Bretherton [33] and Whitham [317]. In the papers by Bretherton and Garrett, attention is focused on small-amplitude, nearly plane waves of the form

$$a(\mathbf{x}, t) \exp [i\theta(\mathbf{x}, t)].$$

For this wave form, the local wavenumber vector $\mathbf{k}(\mathbf{x}, t)$ and local frequency $\omega(\mathbf{x}, t)$ are defined (relative to an inertial frame) by

$$\mathbf{k} = \frac{\partial \theta}{\partial \mathbf{x}}, \qquad \omega = -\frac{\partial \theta}{\partial t}, \tag{4.1a}$$

from which the conservation of wave crests follows:

$$\frac{\partial \mathbf{k}}{\partial t} + \frac{\partial \omega}{\partial \mathbf{x}} = \mathbf{0}. \tag{4.1b}$$

The underlying dynamics imply a local dispersion relation of the form

$$\omega = \Omega[\mathbf{k}(\mathbf{x}, t); \mathbf{x}, t]. \tag{4.2}$$

The wave rays are the curves obtained by integrating the equations

$$\frac{d\mathbf{x}}{dt} = \mathbf{c}_g, \tag{4.3}$$

where $\mathbf{c}_g = \partial\omega/\partial\mathbf{k}$ is the group velocity, obtained from (4.2). For a fluid moving with slowly varying velocity $\mathbf{U}(\mathbf{x}, t)$, it was shown by Bretherton and Garrett [34] that the fundamental quantity conserved along the rays is the wave-action density A, defined as

$$A = E_0/\omega_0, \tag{4.4}$$

where E_0 is the wave energy density (averaged over a period) measured in the moving system and $\omega_0 = \omega - \mathbf{k} \cdot \mathbf{U}$ is the intrinsic (also sometimes called Doppler-shifted) frequency, the wave frequency observed in the moving system. The conservation of A along rays takes the simple form

$$\frac{\partial A}{\partial t} + \nabla \cdot (\mathbf{c}_g A) = 0. \tag{4.5}$$

As can be seen from (4.5), wave action is a more fundamental quantity than energy for wave propagation since the former is conserved along rays on both moving and stationary systems. Indeed, by straightforward differentiation it follows from (4.4) that the following energy balance equation holds (LeBlond and Mysak [142, p. 33]):

$$\frac{\partial E_0}{\partial t} + \nabla \cdot (\mathbf{c}_g E_0) + T_{ij}\frac{\partial U_i}{\partial x_j} = 0, \tag{4.6}$$

where $T_{ij}\, \partial U_i/\partial x_j$ represents the rate of energy exchange per unit volume along a ray in the moving system. T_{ij} is a symmetric, second-order interaction stress tensor. The third term in (4.6) can be interpreted as the rate of working of the interaction stress against the spatial rate of strain of the mean flow. In the earlier studies of the interaction between surface waves and currents, (4.6) was used in the analysis rather than (4.5) and T_{ij} was identified as the radiation stress tensor (the momentum flux of the waves) by Longuet-Higgins and Stewart [165], [166], [167].

The basic ray theory equations (4.2), (4.3) and (4.5) have been used extensively in recent years to study the refraction of surface waves by slowly varying currents lying above a gently sloping sea bed. Work on this combined problem has been reviewed recently by Jonsson [119]; a number of elementary examples involving either current refraction or topographic refraction separately may be found in LeBlond and Mysak [142, chap. 6]. Ray theory can also be used to describe the interaction between two waves provided the period and wavelength of one of the waves greatly exceeds those of the other, so that the shorter, higher frequency wave may be regarded as propagating in a current which is slowly varying in space and time (Gargett and Hughes [66]).

The ray theory briefly described above for small-amplitude, nearly plane waves in an inviscid fluid has many obvious limitations. For example, as surface waves are refracted in shallow coastal zones, wave breaking (due to increasing amplitudes) and dissipation (due to bottom friction) occur, both of which invalidate the assumptions of ray theory. To handle finite-amplitude and dissipative effects, other techniques have to be used, such as perturbation expansions (Jonsson [119]) and numerical integrations (Peregrine and Thomas [234]). Also, ray theory breaks down at a caustic, defined as an envelope of rays along which total internal reflection takes place. As a wave approaches a caustic, the wavelength increases and becomes comparable to the length scale of the variations in the medium. At the caustic itself there is an abrupt transition from oscillatory to decaying behavior and special matching techniques have to be employed (Ludwig [169]). For details of the solution near a caustic for different ocean wave refraction problems, we refer the reader to McKee [189], Hughes [115] and Smith [280].

Another limitation of the classical theory is that it only treats the case of single Fourier components, i.e., line spectra. As mentioned in connection with wave-wave interactions, in the ocean one is generally interested in the evolution of continuous wave spectra as they are modified by refraction and other interaction processes. For simplicity, consider a time-independent medium, for which ω is invariant along a ray; then it is possible to introduce a slowly varying wavenumber energy spectrum $S(\mathbf{k};\mathbf{x},t)$ (analogous to the energy density E_0 for a single Fourier component—a more precise definition is given in § 5). The action density spectrum (analogous to A) is then defined as

$$n(\mathbf{k};\mathbf{x},t)=S(\mathbf{k};\mathbf{x},t)/\omega_0, \tag{4.7}$$

where ω_0 is the intrinsic frequency of a wave as observed in a fluid moving with velocity $\mathbf{U}(\mathbf{x})$. The amount of wave action contained in a small element δk of wavenumber space is $n\delta k$ and therefore in place of (4.5) we have

$$\frac{\partial}{\partial t}(n\,\delta k)+\nabla\cdot(\mathbf{c}_g n\,\delta k)=0. \tag{4.8}$$

Upon simplifying (4.8), it follows that (LeBlond and Mysak [142, p. 323])

$$\frac{\partial n}{\partial t}+\mathbf{c}_g\cdot\nabla n=0, \tag{4.9}$$

which shows that the action density spectrum remains constant along rays. Equation (4.9) is the most simple form of the "radiation balance" equation referred to at the end of § 3; its more general form is discussed in § 5.

Finally, we note that ray theory does not describe the reflection, diffraction and scattering of waves by moderate or abrupt changes in the medium. However, it should be noted that an extension of ray theory to a geometrical theory of diffraction has been developed (Keller [125], Christiansen [45], [46]).

Wave diffraction and scattering. Waves are diffracted and/or scattered when they encounter obstacles whose boundaries have radii of curvature comparable to or less than the incident wavelength. Although the two words are often used interchangeably, strictly speaking, diffraction occurs when waves encounter a sharp edge or corner, and scattering, when they encounter blunt shaped obstacles or irregular boundaries. Here we shall give a brief survey of a few selected ocean wave diffraction and scattering problems that for the most part have been solved by standard techniques as found, for example, in Jones [118] and Roseau [263].

The classical Sommerfeld diffraction problem of long surface waves incident upon a vertical semi-infinite barrier was first treated by Crease [50]. Because of the presence of rotation, it was found that in addition to the usual reflected and cylindrically scattered waves, the diffracted field included a Kelvin wave propagating parallel to the barrier in the shadow zone. The diffraction of planetary waves by a semi-infinite barrier and by a gap in an infinite barrier have been treated by Mysak and LeBlond [222] and by McKee [188]. Because of the anisotropy of planetary waves, it was found that extra care had to be given to the radiation condition, namely that the diffracted field must consist of waves whose energy propagates away from the scattering region.

A problem of considerable interest in the theory of tsunamis is the scattering of long gravity waves by islands and seamounts. Such problems have recently been thoroughly discussed by Jonsson et al. [120]. The scattering of planetary waves by islands and seamounts has been treated by Rhines [255].

In recent years considerable attention has been devoted to studying the effects of coastal variations on the propagating of the barotropic tide, as modelled by Kelvin waves and surface gravity waves of tidal periods. The types of coastal variations considered generally fall into two categories: (1) sharp bends; and (2) small irregularities on an otherwise rectilinear boundary. Most of the studies dealing with coastlines of the first category have focussed on the diffraction of a Kelvin wave by a corner (e.g. see Buchwald [39] and LeBlond and Mysak [142, chap. 4] for other references). Pinsent [241] on the other hand, was the first to study the scattering of long waves and the attenuation of Kelvin waves by a coastline of the second category. However, his solutions, being based on the Born approximation, break down when the coastal irregularities extend over an extensive coastline. In such a case, it is convenient to treat the coastal irregularities as a stationary random function of position along the coast and then use a modified form of the theory of wave propagation in an infinite random medium (Howe and Mysak [113], Mysak and Tang [223], Mysak and Howe [221]).

Finally, we mention that there have been a number of recent studies dealing with the propagation of various types of ocean waves in media with random variations, such as temperature fine structure, bottom topographic irregularities and current fluctuations. In these studies it has been generally found that energy is continuously scattered from the coherent wave field into the incoherent (or random) wave field, with the energy transfer being in the direction of the group velocity. For a recent survey of wave propagation in random media, with applications to surface, internal, inertial, edge and planetary waves, we refer the reader to the recent survey by Mysak [218]. A brief discussion of the mathematical problems involved in this topic is given in § 5.

Critical layer absorption. Internal gravity waves in a nonrotating stratified fluid can be severely attenuated in the presence of a vertically sheared flow $U(z)$ as they propagate across that layer (or level) $z = z_c$ at which the intrinsic or Doppler-shifted frequency vanishes. At such a layer, the vertical component of the group velocity also vanishes and there is a substantial transfer of momentum to the mean flow. The behavior of an internal wave near a critical layer itself was determined by Booker and Bretherton [24] by the method of Frobenius.

These and other earlier studies were motivated by a desire to understand the behavior of internal waves propagating into the upper atmosphere in the presence of the mean wind shear. There still is considerable interest in various aspects of critical layers for internal waves (e.g., dissipative effects (Hazel [103], Breeding [27]), nonlinear effects (Maslowe [178]) and rotational effects (Grimshaw [88]), and in their relevance in atmospheric dynamics (Geller et al. [73])). However, the importance of critical layer absorption for internal waves in the ocean has only recently been examined (e.g., see Bell [14] and Thorpe [300]).

Critical layers can also exist for planetary waves in the presence of a large-scale, laterally sheared flow. They were first discussed in the atmospheric context by Dickinson [54], [55] and only recently discussed in the oceanic context (Geisler and Dickinson [71], Mofjeld and Rattray [201], Yamagata [323]). In Geisler and Dickinson [71], the absorption of a planetary wave by a north-south geostrophic current $V(x)$ was considered and the results applied to the problem of planetary wave reflection at western boundary currents. A nonlinear theory of critical layer absorption of planetary waves has recently been presented by Redekopp [251].

Stability of parallel flows. The topic of stability of parallel flows has had a long history in fluid mechanics. The fundamental problem is to determine whether a given shear flow is stable to travelling wave perturbations. In the oceanic context there has been considerable interest in the stability of two different classes of flows: (1) relatively small-scale, vertically sheared flows in a nonrotating stratified fluid; and (2) large-scale flows with horizontal and/or vertical shear on the β-plane. In case (1), the wave perturbations are modified internal waves which grow exponentially with time if the flow is unstable, leading to overturning and hence vertical mixing over scales of meters to tens of meters. In case (2), the wave perturbations are modified planetary waves; for unstable flows, the waves develop into large eddy-like motions with horizontal scales of tens to hundreds of kilometers.

The study of the stability of stratified shear flows has been largely motivated by a desire to understand the initiating mechanisms for the production of small-scale turbulence in the ocean. If a mean flow with vertical shear is unstable with respect to internal wave perturbations, then the latter may grow, by extracting kinetic energy from the mean flow, into finite-amplitude billows which in turn break and produce vertical mixing (turbulence) on a scale that completely dominates molecular diffusion (Turner [307]). Clearly, a full understanding of the stability of a prescribed shear flow ultimately depends on the numerical integration of a nonlinear system. Nevertheless, it is very useful to be able to predict the onset of shear flow instabilities based on linear analysis. The starting point in such linear studies is the Taylor–Goldstein equation (LeBlond and Mysak [142, p. 398]) for the vertical dependence of a vertical velocity having a horizontal wave-like dependence of the form $e^{ik(x-ct)}$ with $k>0$. This equation is a second order differential equation whose coefficients involve the mean stratification and current, and the eigenvalue c. One of the most celebrated theorems that follows from this equation (under appropriate boundary conditions) is Miles' [196] sufficiency condition for stability: "If the Richardson number $\mathrm{Ri} \geqq \frac{1}{4}$ (where $\mathrm{Ri} \equiv N^2/U_z^2$), then the flow $U(z)$ is stable (i.e., c is real)." Unlike many shear flow stability criteria, this result has a very simple physical interpretation. Since the Richardson number represents the ratio of buoyancy to inertia, Miles' theorem effectively states that if the stabilizing influence of stratification dominates the destabilizing influence of the nonlinear terms, then the flow is stable. The simplicity of Miles' sufficiency condition for stability also makes it easy to check in the laboratory (Thorpe [299]).

The application of stratified flow instability (also sometimes dubbed Kelvin–Helmholtz instability) has had a long history in geophysical fluid dynamics. In addition to its possible role in the production of small-scale turbulence, it has been proposed as a mechanism for the generation of surface waves, especially at an air-oil interface (Miles [194]). Also, clear air turbulence in the atmosphere is now commonly believed to be generated by Kelvin–Helmholtz instabilities, especially at its nonlinear stages (Atlas et al. [5]). Recently observed large billows in the thermocline and in Loch Ness have also been interpreted as Kelvin–Helmholtz instabilities (Thorpe et al. [301]).

Studies of the stability of large-scale flows that are maintained geostrophically (i.e., the Coriolis force is balanced by the pressure gradient in the horizontal momentum equations) generally fall into one of two categories: (1) barotropic instability and (2) baroclinic instability. In case (1), the mean current has lateral shear only and the unstable wave perturbations grow at the expense of the kinetic energy of the mean flow. In case (2), the mean flow has vertical shear only and the unstable waves grow at the expense of the available potential energy due to the sloping isopycnals that arise in conjunction with a geostrophic, vertically sheared flow (the so-called thermal wind relation—see LeBlond and Mysak [142, p. 417]). The unstable waves in both cases generally have wavelengths of order 100 km; consequently, it is conceivable that these waves may be an initiating mechanism for meso-scale eddies that occur in different parts of the ocean (see Rhines [258] for a summary of such eddy observations). These eddies have been observed, for example, in the vicinity of intense boundary currents such as the Gulf Stream and also in the Polygon and MODE regions of the North Atlantic (see LeBlond and Mysak [142, p. 448] for a summary of these observations). In the same way that small-scale, vertically oriented eddies arising from shear flow instabilities can transport heat, salt and energy in the vertical direction, these meso-scale eddies can transport such quantities over large horizontal distances in the ocean. It thus seems possible that they may play a central role in the mean circulation of the ocean, as similar eddies do in the atmosphere.

A lucid account of the theory of barotropic and baroclinic instability has been given by Pedlosky [231]. For large-scale flows with both lateral and vertical shear, the governing linearized eigenvalue equation corresponding to the Taylor–Goldstein equation for small-scale stratified shear flows is a nonseparable partial differential equation. Further, the eigenvalue c (the horizontal wave phase speed) occurs both in the equation and in the bottom boundary condition. The problem of solving this equation either exactly or approximately by analytic techniques remains a challenge to this day. Likewise, because this is a partial differential equation, the stability criteria for flows with both lateral and vertical shear are generally rather complicated (e.g., see Pedlosky [230], LeBlond and Mysak [142, chap. 7]). However, if one considers flows with either lateral shear or vertical shear only (cases (1) and (2) above), then the governing partial differential equation reduces to an ordinary differential equation and the problem of determining the eigenvalues and corresponding eigenfunctions for a given flow profile becomes relatively straightforward. Nevertheless, the combined problem of both lateral and vertical shear is important in many applications. At present there are essentially two simple ways of handling this situation: (1) To discretize the vertical structure into two (Pedlosky [230]) or three (Davey [52], Wright [319]) layers, in each of which the mean velocity is independent of depth; this leads to a system of ordinary differential equations. (2) To introduce only weak lateral shear together with a strong vertical shear and hence use multiple scale techniques to obtain an approximate analytic solution (Stone [291], Gent [74], [75]).

5. Statistical and probabilistic aspects. In the last two sections, we referred to the concept of a power spectrum and briefly introduced the topic of wave propagation in random media. During the last three decades, these as well as other statistical and probabilistic topics have played an increasingly important role in the study of ocean waves. These topics will now be discussed along with a number of applications to different aspects of ocean wave theory.

The spectrum (also called a "power spectrum" or "energy spectrum") has its origin in the theory of random noise (Rice [259]). Let $f(t)$ be a real stationary random function of t with zero ensemble mean. Then the frequency spectrum $S(\omega)$ of $f(t)$ is defined as

$$S(\omega)=\lim_{T\to\infty}\frac{1}{2\pi T}|\hat{f}_T(\omega)|^2, \tag{5.1}$$

where $\hat{f}_T(\omega)$ is the truncated Fourier transform of $f(t)$:

$$\hat{f}_T(\omega)=\int_{-T/2}^{T/2} f(t)\,e^{-i\omega t}\,dt. \tag{5.2}$$

Since $\hat{f}_T$ represents a Fourier coefficient of f at frequency ω, $S(\omega)$, being proportional to the square of this coefficient, represents the "power" or "energy" per unit frequency interval. In analogous ways, wavenumber and wavenumber-frequency spectra can be defined for real zero-mean stationary random functions $f(\mathbf{x})$ and $f(\mathbf{x}, t)$ respectively.

The auto-covariance function for a zero-mean real stationary random function $f(t)$ is defined as

$$\Gamma(\tau)=\langle f(t+\tau)f(t)\rangle, \tag{5.3}$$

where $\langle\cdot\rangle$ denotes the ensemble average. The quantity $\Gamma(0)$ is called the variance of f. A powerful result, on which the Blackman–Tukéy method for computing spectra is based, is the Wiener–Khinchin theorem: "For any ergodic stationary zero-mean real random function $f(t)$, the spectrum $S(\omega)$ is the Fourier transform of $\Gamma(t)$". That is,

$$S(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Gamma(\tau)\,e^{i\omega\tau}\,d\tau. \tag{5.4}$$

These results can be readily generalized to handle the case of the zero-mean stationary random functions $f(\mathbf{x})$ and $f(\mathbf{x}, t)$.

In practice the spectra and auto-covariance functions of observed current, temperature and amplitude fluctuations associated with different ocean wave types usually change slowly with space and time, since these fluctuations are only approximately stationary (e.g., see Fig. 4 in § 6). The natural way to study the evolution of such spectra is through the "radiation balance" equation, which was briefly referred to in §§ 3 and 4. For wave propagation in a time-independent medium, in which case the wave frequency ω is invariant along a ray, the radiation balance equation is most conveniently written in terms of the (wavenumber) action density spectrum $n(\mathbf{k}; \mathbf{x}, t) = S(\mathbf{k}; \mathbf{x}, t)/\omega_0$ (see (4.7)), where the wavenumber spectrum $S(\mathbf{k}; \mathbf{x}, t)$ is obtained from the wavenumber-frequency spectrum $S(\mathbf{k}, \omega; \mathbf{x}, t)$ by integrating the latter over ω. The equation for n is

$$\frac{dn}{dt} = \Sigma, \tag{5.5}$$

where $d/dt = \partial_t + \mathbf{c}_g \cdot \boldsymbol{\nabla}$ denotes differentiation along a ray and Σ represents the "source" of wave action; the latter includes terms representing forcing effects, dissipation and interaction processes (such as the wave-wave and wave-current interactions discussed above). When there are no source terms, (5.5) reduces to (4.9) and the radiation balance equation describes only refractive processes. On the other hand, if neither refraction nor nonlinear interactions occur, then (5.5) represents a balance between a prescribed input or forcing spectrum $S_i(\mathbf{k}, \omega)$, the response spectrum $S_r(\mathbf{k}, \omega)$ and dissipation. This balance results in an equation of the form

$$S_r(\mathbf{k}, \omega) = |H(\mathbf{k}, \omega)|^2 S_i(\mathbf{k}, \omega), \tag{5.6}$$

where $H(\mathbf{k}, \omega)$, the transfer function, characterizes the underlying wave dynamics. Equations of the form (5.6) (with the inclusion of one spatial variable to describe the relevant horizontal or vertical modes in the problem) have been used by Mysak [214], Käse and Tang [124], Wunsch and Gill [321], and Goodman and Levine [83] to study respectively the generation of continental shelf waves, internal waves, equatorially trapped waves and internal Kelvin waves by broad-banded atmospheric disturbances.

To date the most extensive application of the radiation balance equation (also called a transport or kinetic equation) has been in the study of surface gravity waves, for which the most data are available. For this reason, we defer the discussion of this topic until § 6, where a treatment of the generation and interaction of surface gravity waves is given. The energy or radiation balance of the internal wave field in the deep ocean, characterized by the Garrett and Munk [68] spectrum, has been examined recently by Müller and Olbers [206]. Although their paper starts out with a lengthy list of all the possible source terms that can be included in Σ, in the final analysis they provide estimates of only the following energy transfers: (1) an input into the internal wave field at low wavenumbers from a mean shear flow, (2) the dissipation at high wavenumbers due to wave breaking, and (3) the transfer of energy at intermediate wavenumbers to high and low wavenumbers due to resonant interactions. In their study, the question remains however as to how energy is initially fed into the wave field in the intermediate wavenumber range. Käse and Tang [124] have suggested that energy may be fed into internal waves at these scales by a randomly varying isotropic wind stress applied at the sea surface. Other internal wave studies, which have dealt with simplified forms of (5.5), include the generation of internal waves by flows over irregular bottom topography (Bell [14]) and the scattering of internal wave energy by randomly varying microstructure (Mysak and Howe [220]).

The radiation balance equation (5.5) is generally limited to describing "slow" energy transfers within the wave spectrum and hence cannot be used to describe strong wave-wave interactions which take place over time scales comparable to the wave period. As mentioned in § 3, strong interactions are believed to take place between planetary waves in the ocean. However a description of these interactions in a baroclinic ocean with topography is not yet available, although significant advances have been made by Rhines [257], Holloway and Hendershott [111] and Holloway [110] for the case of barotropic planetary waves. Using a mixture of analytical, numerical and qualitative results, Rhines [257] showed that on a constant depth β-plane, the energy cascade of two-dimensional eddies towards large scales ceases at a critical wavenumber $k_\beta = (\beta/2U)^{1/2}$, where U = r.m.s. particle speed. At larger wavelengths, the energy cascade is by means of weak resonant planetary wave interactions. However, as a result of the anisotropic dispersion relation for planetary waves, the final tendency is for a zonal (east-west) motion. In the work of Holloway and Hendershott [111], a unified analytical treatment of the above turbulent and wave-wave interactions is given. In this paper, the "test field model" of statistical turbulence theory (Kraichnan [132]) is extended to the case of two-dimensional flow on the β-plane. From an analysis of an evolution equation for the amplitude of a Fourier component of the vorticity, they were able to recover Rhines' results in the limit of short and long wavelengths. In the work of Holloway [110], a general formalism of wave interactions is proposed which contains weak wave-wave interactions when the waves are of infinitesimal amplitude, but which also accounts for strong wave interactions by means of a resonance broadening or "propagator renormalization" approach. In the limit of arbitrarily strong planetary waves, the formalism recovers a theory of two-dimensional turbulence.

In the study of wave propagation in random media, which was briefly discussed in § 4, one is mainly interested in finding the mean solution of a stochastic partial differential equation, i.e., an equation with randomly varying coefficients whose statistics are known. These random coefficients are a model of the inhomogeneities in the wave bearing medium. It is assumed that these inhomogeneities are not affected by the propagation of the waves themselves. The mathematical problem of solving stochastic partial differential equations is far from trivial. However, if the inhomogeneities are weak, which is often the case in oceanic wave applications, then various perturbation methods can be used (e.g., see Chow [44], Mysak [218]). Alternatively, if the fluctuations are relatively large, a numerical simulation of the random medium can be used (Bell [13]).

To conclude this section, we note that statistical and probabilistic concepts play an important role in our description and understanding of wave propagation in the real ocean. This has been especially true for the case of surface gravity waves, which have been studied intensively by both theoreticians and experimentalists. A large part of the next section will be devoted to this topic.

6. The generation of ocean waves. The sources of momentum and energy needed to generate the observed oceanic radiation field are to be sought inside the ocean as well as on its boundaries. Within the ocean waters, waves may extract energy from the mean flow or from the potential energy of the stratification through some of the instability mechanisms discussed in § 5. Wave-wave interactions or topographic coupling may also feed energy from one kind of wave to another, or transfer energy from one mode to another (as in the generation of internal tides through interaction of the barotropic tide with topography). These internal processes are difficult to observe and their presence is inferred from theoretical considerations rather than from direct observation.

Wave generation at oceanic boundaries is much better documented. There are many ways in which energy may be transferred through oceanic boundaries in a time-dependent or spatially modulated form which leads to wave generation. Direct bodily injection of fluid carrying with it kinetic and/or potential energy, for example, results from pulsated sources of momentum (such as a variable river flow) or buoyancy (heat and/or salt). Surface wave generation by a varying current issuing from a narrow channel, as treated by Voit [314] and Buchwald [40], is an example of a variable momentum source. The problem of internal wave generation by buoyancy fluctuations, treated by Magaard [174], provides an instance of a varying potential energy source.

The tides. Energy may also be added to the ocean waters by the action of two kinds of forces: body forces and surface stresses. Body forces, such as gravity or the Coriolis force act throughout the body of the fluid. The Coriolis force always acts at right angles to the flow and can do no work. The only body force generating oceanic waves is that of gravity, and more specifically, the small imbalance of the combined solar and lunar gravity fields which exist at the surface of the Earth is commonly called the tidal force. It is the component of this force parallel to the Earth's surface which produces the tides. Reviews of the theory and results on tides have been presented by Hendershott [106], [107] and Hendershott and Munk [108].

The tidal force is known quite accurately in terms of the orbital parameters of the Sun and the Moon; it may be decomposed into a number of "tidal constituents" which make up a line spectrum of forcing terms. As was already recognized by Laplace [136], tidal constituents occur in three groups of frequencies: semi-diurnal tides, diurnal tides and longer period tides. A lengthy compilation of about 300 constituents was presented by Doodson [57]; shorter lists of the principal tides are found in many oceanography texts (for example, see Neumann and Pierson [227]).

The problem of tidal generation then consists of reconciling the observed distribution of tidal amplitudes and phases with the known forcing spectrum. The ocean responds to the tidal impulses in a forced motion which may be represented (at least in a linear description) as a superposition of the free modes of oscillation of the ocean basins. Tidal oscillations on a global scale are best described in spherical coordinates and obey the shallow water equations known as Laplace's Tidal Equations (LTE) (see Miles' [198] for a recent discussion). Free solutions of the LTE for the Atlantic and the combined Atlantic and Indian Oceans have been presented by Platzmann [243], [244]. Forced solutions of the LTE for the whole ocean or for partial oceanic basins have been calculated by a number of authors for some of the dominant tidal constituents (see Hendershott [107] for a review of the subject). A comparison of the free and forced solutions shows that some oceanic basins (the Atlantic for example) have free periods of oscillation which are near some of the forcing frequencies and are near resonance. The agreement between the numerical solutions of the LTE and the observed tides is on the whole satisfactory (considering what little data are available on deep ocean tides) but limited by the spatial resolution of the grid of integration. Higher resolution models have been constructed for various coastal areas; these local models are usually driven by the adjacent oceanic tides rather than by direct astronomical forcing (for example, see Crean [49], Godin [81] and Heaps [104]).

The question of local tidal prediction, at a given harbor or similar point of interest, is a much simpler matter. The predictions of the local sea level variations and currents at tidal frequencies does not require any understanding of the spatial structure or propagation characteristics of the tidal response. Assuming a line spectrum of forcing frequencies, the local response may be resolved into contributions from each of the forcing constituents by "harmonic" analysis (see Godin [82]) or by the response method of Munk and Cartwright [208]. Predictions of the tide then follows by extrapolation of the fitted time series into the future.

Long waves. Surface stresses include normal (basically, the pressure) and tangential stresses (due to friction). Although pressure forces due to bottom displacements may cause tsunamis, these spectacular waves are too rare to contribute significantly to the average oceanic wave field (see Murty [212] for a review of tsunamis). The dominant

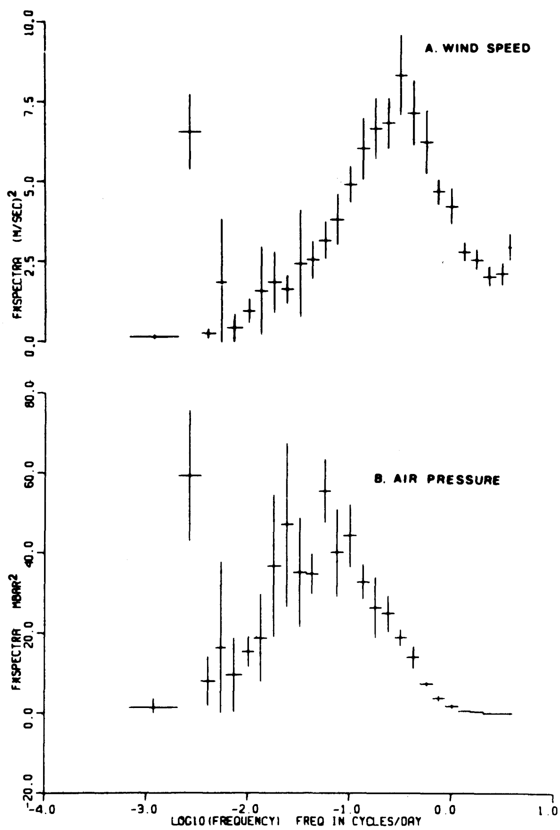

FIG. 3. *Autospectra of A) wind speed and B) air pressure, at Ocean Weather Station* P *in the northeast Pacific. The vertical error bars represent approximately* 95%*-confidence intervals about the mean of each spectral estimate. From Fissel et al.* [61].

source of pressure fluctuations and tangential stresses on the oceanic boundaries is of course the atmosphere.

Typical mid-latitude spectra of atmospheric pressure and wind speed (from Fissel et al. [61]) are shown in Fig. 3. Sharp peaks occur at the annual frequency and broad maxima at time scales of about 3.1 and 15 days for wind and pressure fluctuations respectively, reflecting the passage of synoptic disturbances of horizontal scales of the order of hundreds of kilometers. Since the atmospheric forcing is predominantly at subinertial frequencies, the oceanic response would be expected to consist of second class waves, described in terms of the vertical normal modes discussed above. Veronis and Stommel [313] studied the response of mid- and high-latitude ocean regions to atmospheric input and found that, because the internal modes propagate so slowly, the baroclinic adjustment to changes in wind conditions would take place over time scales of the order of decades. In the equatorial regions, on the other hand, planetary waves propagate much more rapidly and the response time is measured in weeks (see Lighthill [148], Fieux and Stommel [60]). A different approach has been used by Philander [235], in a recent review of the response of bounded basins, wherein the oceanic motion is represented in terms of vertically propagating modes satisfying lateral conditions.

Wind waves. It is well known that an apparently steady wind can generate short period "wind-waves". These waves are of great practical interest because of their relevance to navigation and to coastal processes and have been studied more intensively than any other kind of oceanic waves. The data on wind-waves are indeed good enough to allow quantitative comparisons with the extensive body of theory which has been elaborated to explain their observed properties. Detailed accounts of wind-waves are found in Kinsman [127] and Phillips [240]; a more recent review of progress in the field is given by Barnett and Kenyon [11].

Wind-wave spectra in a well developed sea are typically strongly peaked (at a frequency ω_m, say). The evolution of spectra with fetch X (the distance over which the wind blows) is shown in Fig. 4 (taken from Hasselmann et al. [102]). Figure 4 illustrates the gradual decrease of ω_m with fetch, corresponding to the peak of the spectrum occurring at gradually longer wavelengths; it also shows the initial overshoot of energy levels at short fetches to values which exceed those existing at the same frequencies at longer fetches. Observations, such as those of Hasselmann et al. [102], indicate that $\omega_m \propto X^{-1/3}$, while the energy of the spectrum increases as X. The rapid decay of spectral energies at frequencies $\omega > \omega_m$ (see curve 11 in Fig. 4) often follows a ω^{-5} power law, as expected in a sea where the wave amplitude is dominated by wavebreaking (see Phillips [237]). A more complete review of wind-wave properties is to be found in LeBlond and Mysak [142, pp. 482 ff].

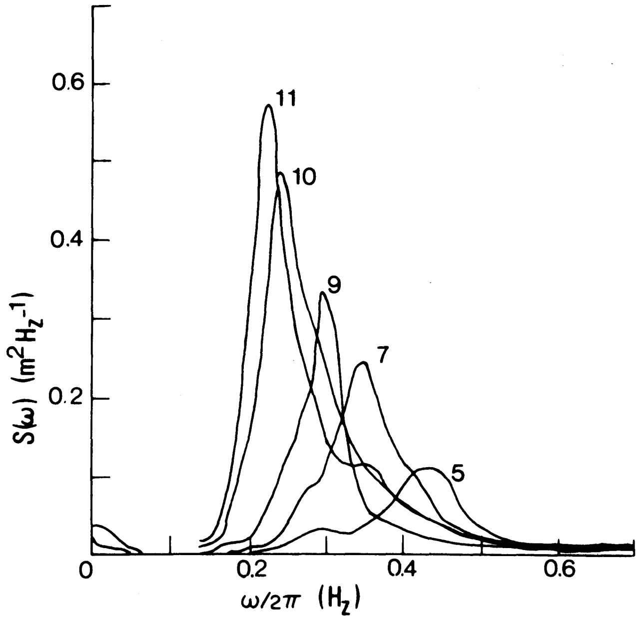

FIG. 4. *The evolution of wind-wave spectra with fetch, from the JONSWAP observations. The fetch increases with the number labeling the spectral peak. From Hasselmann et al.* [102].

The process of wind-wave generation is intimately associated with the characteristics of the air flow over the moving, undulating ocean surface. The air flow over the ocean is turbulent and the presence of the turbulence determines the shape of the vertical profile of the wind over the sea (see Lumley and Panofsky [171] on the atmospheric boundary layer). It is thus only an *apparently* steady wind which raises surface waves: the short time- and space-scale unsteadiness, in the form of turbulence, plays both a direct and an indirect role (through its effect on the mean wind profile) in wave generation.

Two basic mechanisms have been advanced to explain wind-wave generation. Phillips [236] suggested that turbulent pressure variations, advected by the mean flow, could generate waves on an initially calm ocean by keeping in phase with surface displacements, i.e., through travelling resonant forcing. The bimodal angular distribution of wave energy, with a peak on each side of the direction of the wind speed, expected from this resonance mechanism, has been observed by Gilchrist [78] under short fetch conditions. The linear rate of energy increase predicted by the Phillips' theory (see also Stewart and Manton [286] for refinements) is adequate only to explain the initial appearance and growth of wind-waves; real waves soon grow at a much faster rate!

The generation mechanism suggested by Miles [192] assumes that small waves already present at the sea surface would induce perturbations in the mean atmospheric shear flow which could modify the surface air pressure in such a way as to transfer energy to the waves and amplify them. Miles' shear flow instability mechanism yields an exponential rate of growth of wave energy; it was combined (Miles [195]) with Phillips' resonance mechanism to account for the initial appearance and subsequent growth of wind waves through the action of pressure forces at the sea surface.

In spite of a number of refinements (Miles [193], [194], [197]), the rate of energy input predicted by the above mechanisms remained insufficient by about an order of magnitude to explain the rate of growth of the spectral peak, measured by Snyder and Cox [282] and Barnett and Wilkinson [12]. Direct numerical computations of the wind field over a wavy sea, by Gent and Taylor [76], [77], and hence of the work done in the sea surface by pressure as well as tangential stresses, also cannot reconcile the computed energy input with the observed growth.

A number of nonlinear wave-wave interaction mechanisms (see for example, Hasselmann [101], Garrett and Smith [70]) have been invoked to feed energy from the

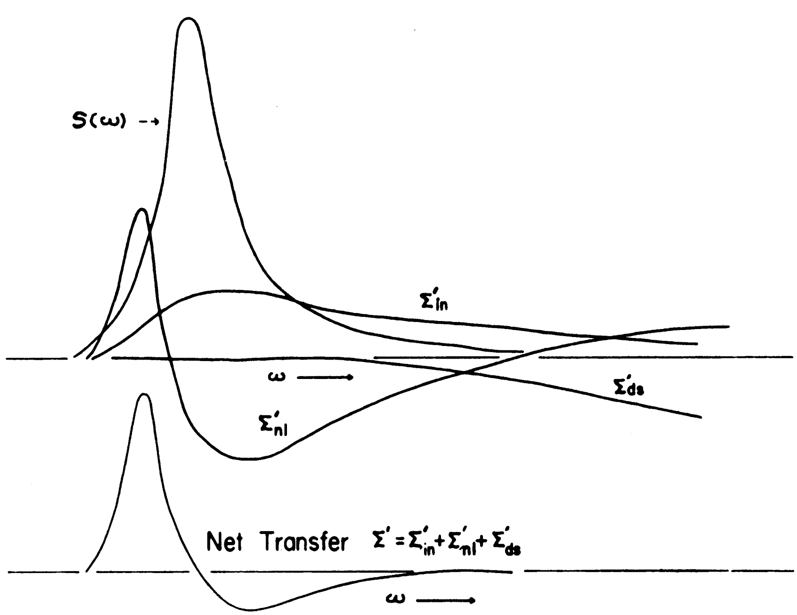

FIG. 5. *Schematic energy balance for a wind-wave spectrum* $S(\omega)$. $\sum'_{in}$ *represents direct energy input from the wind,* $\sum'_{ds}$ *energy removed by dissipation, and* $\sum'_{nl}$ *the result of the nonlinear wave-wave interaction mechanisms. From Hasselmann et al.* [102].

high frequency (ω^{-5}) part of the spectrum to the peak in order to explain its rapid rate of growth. Calculations of the effect of nonlinear interactions on the evolution of the JONSWAP spectra (measured by Hasselmann et al. [102]), as evaluated from the source terms in the radiation balance equation (5.5), shows that energy may actually be fed into the spectral peak by wave-wave interactions (see Fig. 5). Theoretical calculations (see Fox [62]) also lead to this somewhat surprising conclusion. This is not the end of the story, however, and other effects, such as that of the three-dimensional structure of the wind field (as suggested by Dorman and Mollo-Christensen [58]), or the enhanced momentum transfer rates associated with flow separation over breaking waves (as noted by Banner and Melville [10]), may also play a significant role in wind-wave generation.

The dissipation of wind-waves is dominated by wave breaking which, as mentioned above, plays a significant role in the high frequency region of the wind-wave spectrum. Once waves travel beyond their domain of generation, they become "swell" and attenuate only very slowly, as shown by Snodgrass et al. [281]. The ultimate demise of swell comes in a catastrophic climax in the onshore surf zone, a process which is difficult to study and is still poorly understood.

7. Unresolved problems. Ongoing problems concerning ocean waves fall into two broad, overlapping categories: (1) those problems relating to the properties of a particular type of wave: its generation, propagation, nonlinear distortion and eventual dissipation; (2) broader problems concerning the general oceanic radiation field, involving interactions between many types of waves and their environment. The first type of problems are mainly deterministic in nature; problems of the second type are involved with a statistical specification of the wave climate of the ocean and are dominantly stochastic.

Examples of the first type of problem abound in the theory of surface gravity waves. A number of questions concerning the nature of nonlinear surface waves, as embodied in the properties of the KdV equation, have been mentioned in Miura's recent review [200]. The phenomenon of wave breaking is also very poorly understood; although an interesting model has been presented by Longuet-Higgins and Turner [168] and numerical simulations have been attempted (see Harlow et al. [95] and Harlow and Shannon [94]), the role of breaking on wave generation and the dynamics of real surf still present challenging problems. Another interesting problem, involving wave-current interactions is that of the occurrence of the "giant waves" mentioned by Mallory [175] and studied by Smith [279]. These waves have caused damage to very large ships off the southeast coast of South Africa and are obviously of great concern to navigation.

The topic of tidal dissipation may also be considered as a problem of the first type. The total rate of energy dissipation in the Earth-Moon system is known from astronomical data (see Munk and MacDonald [209], Rochester [262]). The bulk of this energy loss is thought to occur in the ocean (Hendershott [105]), but present estimates, such as that of Miller [199], seem insufficient to account for the observed dissipation rate. A different approach to the problem, which calculates the retarding tidal torques directly rather than estimating the dissipation, has been pioneered by Brosche and Sündermann [37]; this approach looks promising and will hopefully be pursued further.

Much still has to be learned about the properties of various classes of long-period oceanic waves. The possible origin of meso-scale oceanic eddies as planetary waves drawing energy from the mean oceanic circulation through some instability mechanism has received considerable attention (see, for example, Robinson and McWilliams

[261]), but the situation is far from well resolved. A large multinational experimental programme, POLYMODE [308], is now underway to provide more extensive data on oceanic eddy dynamics. Equatorially trapped waves have been found to exert considerable influence on subtropical processes and to provide a rapid access path for information to cross the oceans (see McCreary [182] and Hurlburt et al. [116]). The study of these waves and their modifications due to topography (e.g. see Mysak [217], [330] and Geisler and Mysak [72]) are likely to continue to be pursued actively. Similarly, coastally trapped waves are related to upwelling phenomena and to variations in sea level (Gill and Clarke [79]) and are under intensive study.

The second type of problem is part of the specification of the ocean climate. A sensor immersed at some point in the ocean measures fluctuations in currents and other water properties which stretch over a broad spectrum of periods (from seconds to months!). Most of the observed variability may be attributed to waves of some kind or other (on this matter, see Monin et al. [202]). The wave field observed at a fixed point consists of a superposition of waves which have reached it from wide areas of the ocean. Some of the time variations, such as the tides, are of a deterministic nature and are readily predicted. On the other hand, a large part of the variability is of a stochastic nature. Thus, much of the wave field consists of waves generated by atmospheric sources, propagating at different speeds according to their type and modal structure, and subject to mutual interactions and to refraction and energy exchanges with currents. The full specification of the time variability at a point is then practically impossible and one must be happy with a statistical representation of the wave field. The simple spectral power models obtained for surface waves (in a fully developed sea) by Phillips [237], for internal waves by Garrett and Munk [68], [69], and for planetary waves (viewed as two-dimensional turbulence) by Rhines [257] are examples of this kind of statistical description. A number of empirical relations between wind speed and fetch and the properties of wind-waves provide some statistical information on the energy level of surface agitation, but no such relations are available for other kinds of waves. This kind of information is sorely needed in many practical problems and one would hope to see it emerge from a proper synthesis of the observations in the near future.

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