A numeric evaluation of attenuation from ambient noise correlation functions Jesse F. Lawrence^{1*}, Marine Denolle¹, Kevin J. Seats¹, Germán A. Prieto²

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The ambient noise correlation function (NCF) calculated between seismic stations contains, under appropriate conditions, accurate travel-time information. However, NCF amplitudes are highly debated due to noise source intensity and distribution, seismic intrinsic attenuation, scattering, and elastic path effects such as focusing and defocusing. We prove with various numerical simulations that the NCFs calculated for a uniformly dispersive medium using the coherency method preserve accurate geometrical spreading and attenuation decay. We show that for a wide range of noise source distributions, the coherency of the noise correlation functions matches a Bessel function decaying exponentially with a specific attenuation coefficient. Conditions needed to obtain these results include averaging over a long enough time intervals, a uniformly distributed seismic network and a good distribution of far-field noise sources. We also show that the estimated attenuation coefficient corresponds to the interstation and not the noise-source to receiver structure.

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Introduction:

In order to better understand the Earth structure and to predict ground shaking, seismologists have to face the foremost challenge of explaining seismic-wave amplitudes from scattering and anelasticity. Affecting in particular surface waves, attenuation is traditionally measured from earthquake records. Earthquakes occur at specific times and location around the Earth, which limits the path coverage of surface-waves and thus the areas of potential studies. To overcome this shortage of data, recent developments in seismology have led to the use of the ambient seismic field to extract elastic and anelastic propagation effects.

The cross-correlation of the ambient seismic field recorded at two stations, which we refer as the noise correlation function (NCF), shows remarkably similar properties to the Green's function between the two receivers [e.g., *Weaver and Lobkis*, 2001, 2004; *Derode et al.*, 2003a,b; *Shapiro and Campillo*, 2004; *Wapenaar* 2004, 2006; *Sánchez-Sesma and Campillo*, 2006]. This powerful novel technique provides new opportunities to extensively explore the Earth [e.g., *Shapiro et al.*, 2005; *Sabra et al.*, 2005a,b; *Yao et al.*, 2006; *Zheng et al.*, 2008, *Prieto et al*, 2010]. Among a wide range of applications, researchers have been able to produce higher resolution images of the crustal and upper mantle structure than with traditional approaches [e.g., *Shapiro et al.*, 2005; *Sabra et al.*, 2005b; *Yao et al.*, 2006; *Lin et al.*, 2008; *Gudmundsonn et al.*, 2008; *Yao and van der Hilst*, 2009; *Stehly et al.*, 2009].

Ambient seismic noise is a ubiquitous signal that is recorded worldwide at seismic stations. At periods between 5 - 30 s, it is excited by the interaction between the ocean and the crust [e.g., *Longuet-Higgins*, 1950, *Hasselmann*, 1963; *Tanimoto et al.*, 1998; *Rhie and Romanowicz*, 2004,2006; *Shapiro et al.*, 2006; *Kedar et al.*, 2008; *Tanimoto*, 2007; *Zheng et al.*, 2008; *Nashida et al.*, 2009; *Roux et al.*, 2010; *Ardhuin et al.*, 2011]. At short periods, *Longuet-Higgins*, [1950] suggests that the interaction of the oceanic waves with the shallow sea floor forms the first microseism peak (10-20s) and the interference of those waves with their coastal reflection forms the second microseism peak (5-10s). More recently, [Kedar et al., 2008; *Landés et al.*, 2010] found strong spatial and temporal correlation between energetic microseism and powerful oceanic storms.

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The quality of the NCFs seems to depend on the distance separating the receivers [e.g., *Young et al.*, 2011, *Bensen et al.*, 2008], with faster convergence toward the Green's function for short distances [*Sabra et al.*, 2005; *Larose* 2005; *Weaver and Lobkis* 2005]. This is true when using long-duration one-bit normalization or running-normalization techniques [e.g., *Bensen et al.*, 2008], as compared to the largely increased convergence rate when using short time windows [*Seats et al.*, 2011]. Scattering due to complex 3D structure and intrinsic attenuation, however, strongly affect the bandwidth of the coherent content. The observed NCF amplitude variations are extremely similar to the relative variation in Green's function amplitudes [*Weaver and Lobkis*, 2001; *Larose et al.*, 2005; *Colin de Verdière*, 2006; *Prieto and Beroza*, 2008; *Cupillard and Capdeville*, 2010; *Denolle et al.*, 2012].

The purely elastic surface-wave amplitude decay with distance, or geometrical spreading, is expressed in cylindrical coordinates with a Bessel function of the first kind. The SPatial AutoCorrelation (SPAC) method introduced by [*Aki*, 1957] predicts the variation of the amplitudes with frequency and distance found with the cross-correlation techniques [e.g., *Sánchez-Sesma and Campillo*, 2006; *Ekström et al.*, 2009; *Prieto et al.*, 2011]. The relation between the spectrum of the vertical-to-vertical NCF, or spectrum of the vertical-to-vertical elastic Green's function, and the Bessel function is:

$$\operatorname{Re}[\gamma(\omega)] = J_0\left(\frac{\omega r}{c(\omega)}\right), \qquad 1$$

where *r* is the receiver-receiver separation (later referred to as r_{xy}) and $c(\omega)$ is the phase velocity for a laterally homogeneous medium [e.g., *Sanchez-Sesma and Campillo*, 2006].

Whether or not the attenuation can be extracted from the ambient-noise SPAC technique is still controversial. *Prieto et al.*, [2009] observed stronger amplitude decay with increased station separation than predicted by pure elasticity in homogenous media. From a onedimensional approximated medium [*Prieto et al.*, 2009] or a laterally varying medium [*Lawrence and Prieto*, 2011] the additional decay is modeled as a decreasing exponential (equivalent to attenuation (α) measurements with traditional earthquake studies):

$$\operatorname{Re}[\gamma(\omega)] = J_0\left(\frac{\omega r}{c(\omega)}\right) \exp(-\alpha(\omega)r).$$
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This method provides reasonable geological and geophysical results from the scales of reservoirs [*Weemstra et al.*, 2011*a*,*b*] to continents [*Lawrence and Prieto*, 2011]. *Lin et al.*

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[2011] confirmed that the NCFs amplitude decay with receiver separation, in agreement with numerical evaluations of [*Cupillard and Capdeville*, 2010]. Further support has been presented by Zhang and Yang [2013] using the C^3 (C-cubed) methodology [Stehly et al., 2008]. *Nakahara* [2012] showed that the expression above (eq. 2) is approximately correct for small attenuation, providing a theoretical basis to estimate attenuation from the coherency of ambient noise records.

With concerns about the effects of ambient noise sources distribution on NCF amplitudes, the accuracy of the NCF anelastic measurements are actively discussed. *Weaver* [2011], *Tsai* [2011], and *Buckingham* [2012] suggest that NCF amplitudes depend on noise source distribution and that azimuthal averaging of coherency estimates may not be enough for retrieving reliable attenuation coefficients. Numerical simulations [*Cupillard and Cadeville*, 2010] and theoretical derivations [*Tsai*, 2011; *Nakahara*, 2012] corroborated that for uniformly distributed noise sources, NCF amplitude decays depend on the attenuation of the medium.

Much of the debate regarding the recovery of attenuation from NCFs may largely depend on the different assumptions made regarding the distribution of the ambient source field [*Weaver*, 2011]. We stress that the Earth's ambient source field is not uniformly distributed; and rather that its origin varies spatially and temporally, depending on seasonal effects and proximity to the coasts [e.g., *Schulte-Pelkum et al.*, 2004; *Stehly et al.*, 2006; *Chevrot et al.*, 2007; *Aster et al.*, 2008; *Kedar et al.*, 2008; *Ardhuin et al.*, 2011]. In practice, normalizing the ambient seismic field records and averaging of normalized correlations over time is thought to alleviate the strong directionality of the ambient seismic field. The NCFs can be considered as a summation of displacements measured after propagation (with geometrical spreading and attenuation) from many sources.

In this work, we present a numerical approach to express our understanding of estimated attenuation coefficients from NCFs and the effect of appropriate signal processing to reduce biases introduced by non-uniform source distributions. We first build the synthetic ambient seismic field for a dispersive medium. We then compute synthetic coherencies (or spectra of the NCFs) at all station pairs for comparison with theoretical model of the Green's function (or the Bessel function). We evaluate the impact of the noise-source and receiver distributions

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on attenuation measurements by numerically demonstrating that synthetic NCF amplitudes are consistent with the expected results from *Prieto et al.* [2009]. We show that for a wide range of noise source distributions we are able to recover accurate attenuation coefficients. We further show that the attenuation measured is sensitive to the local structure beneath the array, instead of the structure between the sources and the receiver array.

Analytical Framework

Realistic ambient seismic field recorded at a given receiver can be considered as the sum of all wave fields propagating from all sources to the receiver. We assume a laterally homogenous medium with vertical only variations, and hence dispersive properties. In the far field approximation, the response of the medium to a single point-force source located at s and of spectrum F becomes the displacement spectrum u

$$u_{x}(s,\omega) = F(s,\omega) e^{i(\omega t_{0} - kr_{sx})} e^{-\alpha(\omega)r_{sx}} / \sqrt{r_{xx}}, \qquad 3$$

where ω is the frequency, k is the wavenumber, t_0 is the source time [after Sánchez-Sesma and Campillo, 2006], $\alpha(\omega)$ is the frequency-dependent attenuation coefficient, and r_{sx} is the source-receiver distance. In cylindrical coordinates, this is a far-field approximation to the elastic surface-wave displacement eigenfunction with attenuation incorporated as an exponential decay. As in *Prieto et al.* [2009], we consider only the vertical component of the Rayleigh waves.

We follow the conventions in *Sanchez-Sesma and Campillo* [2006] with two receivers, located at x and at the origin y, that record the response of the medium to a point source s of origin time t = 0. The cross-spectrum (or cross-correlation in frequency domain) of those two records becomes:

$$C_{\mathbf{x}\mathbf{y}} = u_{\mathbf{x}}(\mathbf{s},\omega)u_{\mathbf{y}}^{*}(\mathbf{s},\omega) = \left|F(\mathbf{s},\omega)\right|^{2} e^{-ik\tau_{\mathbf{x}\mathbf{y}}\cos[\varphi-\theta]} e^{-\alpha(\omega)\tau_{\mathbf{s}\mathbf{x}}} e^{-\alpha(\omega)\tau_{\mathbf{s}\mathbf{y}}} / \sqrt{\tau_{\mathbf{s}\mathbf{x}}} \sqrt{\tau_{\mathbf{s}\mathbf{y}}}, \qquad 4$$

where θ and φ are the respective azimuths between receivers in y and in x, to the source. Posing $r_{xy} = |\mathbf{x}-\mathbf{y}| = |\mathbf{x}|$ gives $r_{sx} = r_{sy} + r_{xy}\cos(\psi-\theta)$ and yields $e^{-ikr_{xx}}e^{ikr_{sy}} = e^{-ik(r_{sy}+r_{xy}\cos[\varphi-\theta])}e^{ikr_{sy}} = e^{-ikr_{xy}\cos[\varphi-\theta]}$. In practice, we compute the crosscorrelation of time windows with finite duration, which limits the number of source per time window. We overcome this by summing correlation functions over large duration of time. φ 2013 The Author. Journal of Geographysical Becauch: Solid Earth published by Wiley on babalf of the American

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We indicate this using the notation $\langle \rangle$, and write the cross-spectrum that results from the contribution of sources uniformly distributed on a circle centered in **y**,

$$\langle \mathcal{C}_{xy} \rangle = \langle u_x(s,\omega) u_y^*(s,\omega) \rangle = \frac{1}{2\pi} \iint \frac{|\mathcal{P}(s,\omega)|^2}{\sqrt{r_{sx}}\sqrt{r_{sy}}} e^{-ikr_{sy}\cos(\varphi-\theta)} e^{-ik(\omega)r_{sx}} d\varphi d\theta \quad (5)$$

Without attenuation ($\alpha = 0$), assuming a uniform source field with a white source spectrum (i.e., $F(s, \omega) = \overline{F}(\omega)=1$ and $r_{sx} = \overline{r}$, constant for all sources), and within the far field approximation ($r_{xy} \ll r_{sx} \sim r_{sy}$), equation 5 becomes:

$$\langle \mathcal{C}_{xy} \rangle = \langle u_x(s,\omega) u_y^*(s,\omega) \rangle = \frac{|F(s,\omega)|^2}{2\pi} \iint e^{-ikx_{xy}\cos\{\omega-\theta\}} d\varphi d\theta \quad (6)$$

The assumption of $r_{sx} \approx r_{sy} \approx \overline{r}$ likely introduces some degree of inaccuracy (see Figure 1) – but is needed to simplify equation 6 into the analytical solution of *Sánchez-Sesma and Campillo* [2006] that converges to the Green's function:

$$\left\langle C_{xy}(\omega) \right\rangle = \frac{\left| \overline{F}(\omega) \right|^2}{2\pi \overline{r}} J_0(k(\omega) r_{xy}),$$
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where J_0 is the Bessel function of the first kind, order zero, and the wavenumber $k(\omega) = \omega c(\omega)$ with $c(\omega)$ the phase velocity of the medium. Sánchez-Sesma and Campillo [2006] express the linear relation between the real part of the cross-spectrum $\langle C_{xy}(\omega) \rangle$ and the imaginary part of the Green's function between the two receivers (the Bessel function). This remarkable result led to direct imaging of the North American crustal structure with 2D images of phase velocity [*Ekström et al.*, 2009].

In most analytical expressions of the noise correlation function noise sources are assumed to be uncorrelated in time and space and the contribution of each source to the correlation function can be determined independently. In this study, because of attenuation we do not make this assumption and calculate the correlation including the contribution of all sources in each time window. For realistic noise sources and seismic arrays, the geometrical spreading and attenuation terms affect the amplitudes of the recorded ambient seismic field,

$$u_{x}(\omega) = \int \frac{F(s,\omega)}{\sqrt{r_{sx}}} e^{-i(\omega t_{s} - kr_{sx})} e^{-\alpha(\omega)r_{sx}} ds, \quad (8)$$

with the surface integral representing the summation of a continuum of noise sources that occur at origin times t_s and locations on ds.

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In order to measure appropriate amplitudes for attenuation analysis, *Prieto et al.* [2009] computed the time-averaged coherence,

$$Y_{xy}(\omega) = \left\langle \frac{u_x(\omega)u_y^*(\omega)}{\left\{ |u_x(\omega)| \right\} \left\{ |u_y(\omega)| \right\}} \right\rangle, \qquad 9$$

where |.| indicates the power spectral density in the L₂ sense, { . } refers to a whitening process using running average (to ensure deconvolution stability). $\langle . \rangle$ indicates the time averaged ensemble, which is accomplished by dividing a pair of long (>90-day) time series into many short (1800-7200s) time series and averaging the coherence for all such time series (see *Seats et al.*, [2011] for more discussion of equation 9). Even though the ambient noise source distribution is not uniform on Earth, *Prieto et al.* [2009] empirically showed that the real part of equation 9 matches a Bessel function (equation 7) modified by an attenuation term (equation 2 repeated here),

$$\operatorname{Re}[\gamma_{xy}(\omega)] \approx J_0(\omega r_{xy}/C(\omega))e^{-\alpha(\omega)r_{xy}}.$$
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Prieto et al. [2009] use raw signal and do not pre-process the data with the typical approaches of one-bit or running normalizations [e.g., *Bensen et al.*, 2007]. The raw coherency provides reliable Green's function amplitude variations for short duration of stacked correlations.

Unfortunately, deriving equation 10 from equations 8 and 9 is not trivial (e.g., equations 12, 16, and 18 from *Tsai* [2011] or equation 9 from *Colin de Verdière* [2006]). Most attempts that simplify this proof rely on significant assumptions (e.g., independent sources) and collapse to variations of equation 3 (e.g., equation 23 in *Tsai* [2011]). Under these assumptions, the NCF reveals a strong dependence on source and receiver distributions [*Tsai*, 2011].

Using the ambient seismic field displacement expressed in equation 8, the crossspectrum for a receiver pair located at (x,y) becomes

$$u_{x}(\omega)u_{y}^{*}(\omega) = \int \frac{F(s,\omega)}{\sqrt{r_{sx}}} e^{-i(\omega t_{x}-kr_{xx})} e^{-i(\omega)r_{sx}} ds \int \frac{F^{*}(s,\omega)}{\sqrt{r_{sy}}} e^{-i(\omega t_{y}-kr_{sy})} e^{-i(\omega t_{y}-kr_{sy})} e^{-i(\omega t_{y}-kr_{sy})} ds, \quad (11)$$

Equation 11 differs from equation 5 or 6 in that we assume that multiple sources are contributing to the records at both stations (x and y). The coherency (equation 9) then becomes

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$$\gamma_{xy}(\omega) = \langle \frac{\int F(s,\omega)/\sqrt{r_{sx}} e^{-i(\omega t_s - kr_{sx})} e^{-\alpha(\omega)r_{sx}} ds \int F^*(s,\omega)/\sqrt{r_{sy}} e^{-i(\omega t_s - kr_{sy})} e^{-\alpha(\omega)r_{sy}} ds}{\left(\left| \int F(s,\omega)/\sqrt{r_{sx}} e^{-i(\omega t_s - kr_{sx})} e^{-\alpha(\omega)r_{sx}} ds \right| \right\} \left(\left| \int F(s,\omega)/\sqrt{r_{sy}} e^{-i(\omega t_s - kr_{sy})} e^{-\alpha(\omega)r_{sy}} ds \right| \right\}} \rangle, (12)$$

Solving equation 12 analytically is laborious and we turn to numerical approaches to compute the coherencies.

Numerical solution:

For numerical calculations, we discretize equation 8 with a finite number of sources per time window N_s ,

$$u_{x}(\omega) = \sum_{i=1}^{N_{s}} F(\mathbf{s}_{i}, \omega) e^{i(\omega_{s_{i}} - kr_{s_{i}x})} e^{-\alpha(\omega)r_{s_{i}x}} / \sqrt{r_{s_{i}x}}.$$
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The medium is laterally homogeneous, and we assume Rayleigh-wave phase velocity $C(\omega)$ and attenuation $\alpha(\omega)$ dispersions from *Prieto et al.* [2009] for southern California. We explore the impact of different source scenarios (varying distributions and spectra) on the stacks of coherency NCFs of numerically generated data calculated with equation 13.

Another widely discussed factor entering analyses on Green's function retrieval is the role of incoherent (or non-correlated) noise sources such as site effects, electrical noise, wind, thermal effects, cultural noise, micro-seismicity, and anything that can create vibrations well recorded at one sensor compared to others. We add incoherent signal to each numerically generated record, which could represent site effects, electrical noise, wind, thermal effects, cultural noise, micro-seismicity, and anything that can create vibrations well recorded at one sensor compared to others. The incoherent noise is added as a white uniformly random signal with constant magnitude at all times. We vary the level of source amplitudes to span wide range of coherent-signal-to-incoherent-noise ratios.

To validate the technique of *Prieto et al.* [2009], we need to recover from $\text{Re}[\gamma_{xy}(\omega)]$ the phase velocities and attenuation coefficients used to generate the numerical records. We adopt similar approach to *Prieto et al.* [2009] and average all $\text{Re}[\gamma_{xy}(\omega)]$ for each station separation, sampled at a 2 km interval, regardless of the array aperture. We then conduct a 3-stage grid search for the appropriate value of phase velocity and attenuation coefficient by

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minimizing the L1 norm residuals derived from equation 10 at a given frequency. Our analysis focuses on the period band of the microseism ambient field (7 - 24 seconds) used in prior studies [*Prieto et al.*, 2009; *Lawrence and Prieto* 2009].

We explore through various noise source scenarios when equation 10 is valid. We identified the cases where this condition failed and stress the need to incorporate sufficient sources (found to be above 16 sources per time window) to overcome local incoherent noise, the sources need to be in the far-field, and the number of receivers has to be appropriate to average spatially the coherency and form the Bessel function.

Synthetic Sources:

We seek to reflect the reality of the ambient seismic field to account for different source types such as oceanic waves, wind on mountains, or directly earthquakes, micro-earthquakes, and cultural noise, with their typical signatures. We consider linear source arrays for oceanic waves crashing on the shore and earthquakes along a local fault line. We impose the oceanic sources to be more frequent than the tectonic sources. At a global scale (greater distances), we assume that ocean waves crashing on a coast (or teleseismic sources at plate boundaries) can be roughly approximated as circular source arrays. We model the coherent cultural noise as clusters within which the sources are randomly distributed, but periodically triggered. We simulate storm-driven ocean waves coupled to the sea bottom as broad areas of randomly distributed sources varying in source amplitude through time. We allow amplitudes to vary by of magnitudes for all source types. The source times are stochastically generated with a uniform distribution such that on average N_s sources appear in each time 1800s time window of the 90-day synthetic record.

We impose a scaled delta function for source time functions. We model the sources as a stochastic logarithmic Gutenberg-Richter magnitude frequency relationship [*Gutenberg and Richter*, 1954]. While a Gutenberg-Richter relationship may not be exactly accurate for describing all vibrations in the microseism band, we merely suggest that stronger sources are less common than weaker sources [*Ardhuin et al.*, 2011]. We experiment with the number of sources per time window (N_s) that could yield a reasonable ambient field. We compute the synthetic coherency using a series of spatial source distributions (Figure 1):

A) a uniformly random distribution along a circle.

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- B) a uniformly random set of sources along a line.
- C) a uniformly random source area.
- D) a uniformly random source shell or ring area.

Receivers:

We fix the uniformly distributed receiver array to have $N_x = 100$ stations and a typical array radius of $L_x = 100$ km, which is realistic for regional network such as in Southern California or Europe. Using an uneven receiver spacing ensures a better sampling of the distance between 1 and 200km when sampled at a 2km interval (albeit unevenly covered with respect to receiver spacing).

Generalized Parameterization:

We explore the parameter domain within which equation 10 is valid to characterize the appropriate ambient seismic field. The three dimension scales, the source-length scale L_s , the receiver-length scale L_x and the source-receiver separation Δ_{sx} reflect the radiation aperture with the ratios L_s/Δ_{sx} and L_s/L_x (i.e, Figure 1). Another factor to consider is the number of sources, N_s , relative to the magnitude distribution, M_s . The larger the range of source amplitudes used to generate the synthetic spectra the fewer the sources dominate any given seismogram. Finally, Due to geometrical spreading, the range of source-to-receiver distances also plays an important role in the number of dominant sources: $[max(r_{sx})-min(r_{sx})]$.

For a given scenario, we generate synthetic ambient noise seismograms which we divide into 1800s time windows, as found to be optimal for fastest convergence toward a stable NCF [*Seats et al.*, 2011]. We then compute and stack the coherencies (in the frequency domain) for each station pair and stack Re[$\gamma_{xy}(\omega)$] over all station pairs to estimate the coherency at various distances. For each frequency, we estimate phase velocity and attenuation coefficients and we finally evaluate the match between the synthetic, or "observed", coherency and the predicted values from equation 2. Table 1 summarizes the parameters simulated by the various scenarios.

The synthetic noise seismograms are a good visual control of similarity between the generated noise and the true ambient seismic field. Figure 3 shows examples of the generated ambient seismic field with different noise source distributions and the resulting variability in

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the signals. Figure 3 presents example synthetic noise seismograms for the four source distributions illustrated in Figure 2, namely circular, linear, uniform, and uniform shell. We imposed 128 sources per time window (N_s =128) because it provides a reasonable balance between synthetics seismograms with identifiable phases above the noise and those without. With 128 sources per time window, Δ_{sx} =500km, L_s =1000km, and L_x =100km, only 14± $_{5}^{90}$ % of the time windows contain events that have a short-term average (STA) over 4s to long-term average (LTA) over 60s ratio (STA/LTA) greater than 2. We find that the number of detectable phases increases (23 ± $_{9}^{62}$ %) as N_s decreases (1< N_s <64), but only marginally decreases (10 ± $_{4}^{7}$ %) as N_s increases ($N_s > 256$).

Figure 4 shows encouraging results for the time-domain NCFs for selected receiver pair for the uniformly random source distribution. The move out is consistent with the input phase velocity and the amplitudes decrease with increasing station separation as expected due to geometrical spreading and attenuation. In Figure 5, we compare the real portion of the synthetic coherency (equation 12) with the theoretical elastic (equation 7) and the attenuated Bessel function (equation 10). If the coherencies qualitatively fit the expected Bessel function, their amplitudes better match the attenuated Bessel function. We proceed with a 3-stage grid search for phase velocity and attenuation following *Prieto et al.* [2009] and *Lawrence and Prieto* [2011].

Figure 6 qualitatively illustrates the accuracy of the recovered dispersion and attenuation curves for all of the source types. Figure 6 confirms that retrieving correct and accurate phase velocities is possible for a wide range of source distributions. The estimated attenuation coefficients exhibit less accuracy that the phase velocities, but nonetheless reliable measurements. Most importantly, our attenuation estimates are consistent for all source distribution tested.

Discussion:

Our results suggest that it is possible to retrieve reliable attenuation coefficient estimates from the ambient seismic field using the coherency technique. We tested a wide range of source distribution length scales, varying the source-receiver distance Δ_{sx} and source lengths L_s , with respect to a fixed receiver distribution. We estimate the best fitting estimates of phase velocity and attenuation measurements by minimizing the misfit in the L2 norm sense of the objective function $\Phi(c,\alpha) = || \operatorname{Re}(\gamma(\omega)) - J_0(\omega r/c(\omega)) e^{-\alpha(\omega)r} ||_2$.

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The L2 norm misfit between the initial and estimated phase velocities and attenuation coefficients provides a quantitative measure of the method's accuracy/precision, which we illustrate in Figure 7. Despite those encouraging results, we note particular scenarios for which the coherency method will fail to provide accurate attenuation measurement.

The first and obvious scenario that fails to provide the correct answer occurs when the sources are confined in the near field ($\Delta_{sx} < L_x$ and $L_s < L_x$) and the sources are seen from a narrow angle. This result further supports the analytical studies. The second case for which the coherency is poorly reconstructed highlights the destructive role of incoherent phases. With a fixed level of incoherent, or local noise, the signal-to-noise ratio of coherent signal decreases as a) the distance between the receivers and b) the noise sources increases (very far field). We confirmed this results by simply increasing the magnitude of the very-far-field sources (as tested with factors from 10 to 1000) or by further stacking over time (> 90 days), thus retrieving correct attenuation coefficients when amplitudes are high.

The source-receiver geometries under which we obtained accurate velocities from the NCF techniques span a large range of length scales. Receiver array scales vary from dozens of meters [e.g., *deRidder and Dellinger*, 2011; *Young et al.*, 2011] to thousands of kilometers [e.g., *Lin et al.*, 2008; *Bensen et al.*, 2008, 2009]. This suggests that our configurations where L_s/L_x and Δ_{sx}/L_r ratios are large may be more representative of the Earth's ambient-noise source field. Although none of the idealized distributions are likely to be best representing the Earth's ambient field, we obtain the most accurate estimations when the sources are randomly distributed over a broad range of distances and azimuths.

One very critical part of testing this technique is the effect of the number of noise sources compared with the level of incoherent noise surrounding the receiver array. We vary the number of noise sources per time window from 1 to 2048, and the maximum amplitude of the incoherent noise from 10^{-6} to 10^1 (Figure 8). We obtain more accurate estimates for simulations with more than ~16 noise sources per window (e.g., $N_s > 16$) and inaccurate estimates for simulations with fewer noise sources per window ($N_s < 16$). When the incoherent noise level is below ~1, the accuracy is sufficient to recover accurate attenuation. Within these two confines, there is little trade-off between these parameters for a wide range of dimensional values.

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Ultimately, a combination of the proposed source configuration is more plausible than any individual one. We therefore created a case that evenly draws from the four source scenarios shown in Figure 2. This general source distribution provides results similar to those of the uniformly random (shell) distribution (Figure 9), where the phase velocity and attenuation results are reliable for a broad range of parameters. This strongly suggests that sources with sufficient distribution, both in aperture and range, will be sufficient to construct Green's functions with reliable amplitude information.

One major result from these simulations reveals that our synthetic coherencies calculated with two or fewer sources per time window better match the unattenuated Bessel function. This is particularly true for nearly symmetric far-field distributions (i.e. circular and uniform sphere). Since we tested those cases for long durations of stacking (>4years), we cannot explain this result by simply stating the lack of sources. It is worth noting that two sources per time window delimit a sharp boundary between the best fit to the Bessel function with or without attenuation. We recall the simple solution of equation 7 for a single source ($N_s = 1$). Theoretical analysis where $N_s=1$ suggests the stacked coherency is independent of attenuation. Our results are then in good agreement with both the analytic work of *Weaver* [2011] and *Tsai* [2011] and the empirical work of *Prieto et al.* [2009] and *Lawrence and Prieto* [2011].

Including attenuation inherently alters the recorded ambient field. Due to attenuation, small local sources contribute to high frequency more than larger far-field sources. The synthetic seismogram is a sum of sources triggered at various locations, and each source is recorded with different amplitude due to attenuation. Normalization by a smoothed amplitude spectrum "whitens" the records of all sources, not each source individually. Consequently, the coherency represents a normalization that preserves the relative damping of each source's propagated spectrum contained with the data record. While the above examples show the importance of including multiple sources in equation 12, there is no straightforward analytical solution proving that stacked coherency should equate to a damped Bessel function.

It remains to be shown that the estimated attenuation coefficient is sensitive to the structure near the receivers, not near the sources or the path between sources and receivers. We compare the attenuation coefficients measured from the synthetic NCFs calculated with

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different attenuation coefficients locally around the sources and the receivers. In the first case we set attenuation coefficients to zero for a 200km radius region surrounding the receivers, while maintaining the same attenuation coefficients for the source region as above, and opposite in the second case. We perform this test only for the uniform shell scenario with small source offset ($\Delta_{sx} = 100$ km) and intermediate-to-large source length ($L_s = 1000$ km) in order to separate the sources and receivers. This simple test confirms that the method measures attenuation near the receivers, and not the attenuation near the sources, as illustrated in Figure 10.

Future analyses with the method presented in *Prieto et al.* [2009] should benefit from considering additional amplitude constraints. For example, 3D focusing effects bias amplitudes significantly within a heterogeneous medium [e.g., *Lin et al.*, 2011]. *Prieto et al.* [2009] and *Lawrence and Prieto* [2011] attempted to reduce the focusing and defocusing effects by azimuthally averaging over large arrays (and all directions). However this averaging does not reduce all amplification effects for most 3D heterogeneous structures. Accounting for amplification with adjoint or finite frequency kernels for amplitude may help reduce such biases. Similarly, repeating the analyses conducted here with full 3D elastic [e.g., *Stehly et al.*, 2011] wave propagation with intrinsic attenuation decay may illuminate potential biases in attenuation estimates due to structures between the sources and receivers.

While the results of this study support the link between the empirical interpretations of NCF energy decay as seismic attenuation, there remains a lack of any analytical proof linking the increased amplitude decay to the functional form $e^{-a(\omega)r_{xy}}$. A recent paper by *Nakahara* [2012] gives theoretical proof that this assumption is nearly correct. Our simulations indicate that the functional form, which is the intuitive solution, holds for a large variety of noise source distributions.

Conclusion:

This study numerically demonstrates that the *Prieto et al.*, [2009] method to measure seismic attenuation from ambient noise correlation functions is viable. In contrast, these results disagree with analytical studies, which may suggest that 1) assuming a "white homogeneous ambient seismic field" is not equivalent to whitening the recorded ambient field, 2) correlating multiple sources is different from independently correlating each source,

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and 3) incoherent noise between the sensors may contribute to the NCF distance dependant decay. Our numerical results illustrate that attenuation coefficients are recoverable for a wide range of appropriate source conditions: 1) a reasonable azimuthal distribution of far-field sources, 2) multiple sources recorded per time window, 3) sufficient stacking and 4) a receiver distribution that provides ample station-to-station azimuths and station separations. For a variety of source conditions, including those with non-uniform source distributions, we accurately recover the phase velocity and attenuation from synthetic NCFs. However, when we include few sources per time window ($N_s \le 2$) in the simulations, our measured attenuation coefficients are negligible, which corresponds to the analytical assessment of *Weaver* [2011] that single-source NCFs are independent of attenuation. With more sources per time window (e.g., $N_s = 128$), the NCFs are only sensitive to the attenuation in the region of the receiver array. In elasticity, the correspondence between the well-constructed NCF and the elastic Green's function is well accepted [*Weaver and Lobkis*, 2004]. In anelasticity, we can now account for attenuation and include the exponential decay $e^{-a(\omega)/w}$ to the Green's function.

Along with other studies [e.g., Yang and Ritzwoller, 2008; Cupillard and Capdeville, 2010], our synthetic ambient field seismograms yield NCFs that contain appropriate amplitude information (e.g., Figures 3 and 4). We investigated the accuracy to which those amplitudes are retrieved with respect to frequency and the scale-lengths for the different source distribution configurations (Figure 7). Given the scenario listed above, our synthetic coherencies match really well the attenuated Bessel functions and we estimate correctly the attenuation of the medium, which disagrees with analytical results of *Tsai* [2011] and *Weaver* [2011]. The origin of the ambient seismic field sources still needs further investigation due to the complexity of the spatial and temporal distribution of noise sources and their respective contribution to the ambient seismic field [Longuet-Higgins, 1950; Webb and Constable, 1986; Cessaro, 1994; Chevrot et al., 2007; Kedar et al., 2007; Ardhuin et al., 2011]. Furthermore, Seats et al., [2011] demonstrated that coherency stacks from more overlapping short time windows (30min-1hour rather than day-long) yields better source distributions than those from longer time windows. This can result from a better averaging process where large-magnitude sources do not dominate over the other sources recorded in successive time windows.

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	Source Array	N _s /1800s	Days	Overlap	L _x	Ls	$\Delta_{\mathbf{sx}}$	Ms
	Geometry*				(km)	(km)	(km)	
	A (151 runs)	25	30	50%	100	10^{1} - 10^{4}	10^{1} - 10^{4}	2
	B (151 runs)	25	30	50%	100	10^{1} - 10^{4}	$10^{1}-10^{4}$	2
	C (151 runs)	25	30	50%	100	10^{1} - 10^{4}	10^{1} - 10^{4}	2
	D (151 runs)	25	30	50%	100	10^{1} - 10^{4}	$10^{1}-10^{4}$	2
	B (24 runs)	2^{0-8}	30	50%	100	10^{3}	5×10 ²	1,2,4,8
	B (24 runs)	2^{0-4}	365	50%	100	10^{3}	5×10 ²	1,2,4,8

Table 1: Description of Synthetic Scenarios

* The labels correspond to the sub-panels (A-D) in Figure 2. A is a circle source. B is a linear source. C is a uniformly random filled circle. D is a uniformly random filled circular shell.

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Figure 1: This figure illustrates the variables used in the set of equations, including source location **x** and **y**, source locations **s**, source-receiver distances r_{sx} and r_{sy} , and receiver separation r_{xy} , the angle between receiver-source and receiver-receiver, θ . In the far-field assumption the source-receiver distance r_{sy} is calculated as if **s** were effectively a line source (grey lines).



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Figure 2: This figure illustrates the geometries tested here, and the geometrical parameters. The source distribution (green) has a scale length (radius or half length) of L_s . The receiver distribution (red) has a scale length (radius) of L_x . The distance between the source center and the receiver center is given by Δ_{sx} (blue). The source geometries are referred to in the text as A) circular, B) linear, C) randomly uniform, and D) randomly uniform shell. When $\Delta_{sx} < L_x < L_s$, the source distribution is approximately azimuthally uniform with respect to receivers (except for the linear source). When $L_x < L_s < \Delta_{sx}$, the sources are azimuthally limited with respect to the receivers. When $L_s < L_x < \Delta_{sx}$, the sources are azimuthally limited area source. When $\Delta_{sx} < L_s < L_x$, all sources are located within the receiver array.

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Figure 3: This figure illustrates three ambient seismograms generated using a stochastic set of sources with random source amplitudes and source geometries: A) circular source, B) linear source, C) uniformly random source, and D) Uniformly random source shell as shown in Figure 2. The same sensor location was used in each synthetic seismogram. The source length was $L_s=2000$ in each case and the source-receiver offset was $\Delta_{rs}=0$ in each case. The number of sources used to generate this window was $N_s = 128$. The reduced high-frequency amplitudes in seismograms A) and D) are caused by greater attenuation accrued over a greater minimum distance between the sources and receivers.

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Figure 4: The noise correlation functions for a subset of synthetic seismograms generated using equation 14, a uniformly random source location distribution with source length L_s =2000 (panel C of Figure 2), source-receiver separation Δ_{rs} =0, and stacked for coherencies of 1800 s time windows over 90 days with 128 sources per time window. These NCFs were bandpass filtered between 0.02 and 0.2 Hz.

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Figure 5: A) The real portion of the stacked coherencies, $\text{Re}[\gamma_{xy}(\omega)]$, calculated in equation 14 for a uniformly random source location distribution with source length $L_s=2000$ (panel C of Figure 2), source-receiver separation $\Delta_{rs}=0$, and stacked for coherencies of 1800s time windows over 90 days with 128 sources per time window. Each coherency is stacked according to station separation at 2km intervals. The real coherency values of are similar to B) the theoretical Bessel function, but better match the amplitudes of the C) attenuated Bessel function. The final panels illustrate the difference between the observed real coherency measured from the synthetics as compared to the D) Bessel function and E) attenuated Bessel function.

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Figure 6: The A) phase velocity and B) attenuation coefficients input into the synthetic calculation (black) are recovered with differing degrees for the (red) uniformly random, (blue) circular, and (grey) linear source distributions. There is greater percent error for the attenuation measurements than phase velocity measurements, but for the rough values are recovered. These curves are calculated using source location distributions with source length $L_s=2000$ (panel C of Figure 2), source-receiver separation $\Delta_{rs}=0$, and stacked for coherencies of 1800s time windows over 90 days with 128 sources per time window. Each coherency is stacked according to station separation at 2km intervals (as in Figure 5) and fit with a 3-stage grid search.

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Figure 7: The RMS misfits for (a-d) phase velocity and (e-h) attenuation coefficients vary as a function of the length scales, source length L_s , receiver length Lx, and source-receiver separation Δ_{sx} . As expected, when source length and source-receiver separation are small (i.e. the sources are within the array – dashed box), the phase velocity and attenuation are not well recovered. When the sources are all over an order of magnitude more distant from the receivers than the receivers are from each other, the phase velocity and attenuation coefficients are not recovered well. The parameter-space of well-recovered phase velocity and attenuation estimates lie within the bold polygons. The NCF stacks were calculated for coherencies of 1800s time windows over 90 days with 128 sources per time window. Note: panels (a-d) and (e-f) correspond to the geometries illustrated in panels (a-d) in Figure 2.

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Figure 8: This figure illustrates the misfit tradeoff between the number of sources per window (N_s) and the maximum level of uniform incoherent white noise, n_{max} , at each receiver. The colors illustrate the misfit between the estimated and known input values for A) phase velocity, $C(\omega)$, and B) attenuation coefficients, α . The misfit is consistently low for $N_s > 16$ and $n_{max} < 1$. The misfits are calculated for simulations having uniformly random sources and receivers with geometric parameters $L_s=1000$, $\Delta_{sx}=100$ and $L_x=100$, where the NCFs were stacked for coherencies of 1800s time windows over 90 days.

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Figure 9: Similar to Figure 7 except, illustrating misfit between known and measured a) phase velocity, and b) attenuation coefficient for a single set of source distributions generated as a random amalgamation of the four source types illustrated in Figure 2.



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Figure 10: This figure illustrates the attenuation coefficients measured from synthetic NCF stacks calculated with (red) attenuation only in the 200km radius region containing the receivers, and (blue dashed) attenuation only outside the 200km radius region containing the receivers (i.e. source-side attenuation). The synthetics were calculated with a uniformly random shell source distribution, where the source offset and length scales are given by L_s =1000 and Δ_{sx} =100, and the receiver length scale is L_x =100. The NCF stacks were calculated for coherencies of 1800s time windows over 90 days with 128 sources per time window and an input attenuation (black) from *Prieto et al.* [2009].



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