Asian and Pacific Coasts 2005
September 4-8, 2005, Jeju, Korea

DIFFRACTION PARAMETERIZATION IN SPECTRAL WIND WAVE MODEL

Igor V. Lavrenov[1] and Sang Wook Park[2]

ABSTRACT**:** The present paper is devoted to development and numerical simulation of the parameterization of wave diffraction around islands. The approach for the treatment of the problem raised due to frequency-angular discretization is applied here. The energy balance equation includes additional correction term presented as second derivatives over directions. This can be regarded as diffusive operator producing some energy exchange between nearest angular direction.

KEYWORDS**:** Wind Wave, Garden-Sprinkler Effect, Diffraction

1.Introduction

Successful problem solution of hindcasting and forecasting of a sea wind wave depends on the quality of physical description presented in model, numerical realization of the wave energy balance equation and on the accuracy of wind data input (Komen et. al., 1994). One of the most typical errors of wind wave model realization is connected with absence of wave diffraction consideration. There are many small islands and submerged obstacles in the ocean. They produce the wind wave diffraction. The absence of diffraction realization in modern wind wave models produces large wave shadows behind islands in wave numerical simulation. It is caused by the fact that wave evolution is described by the spectral energy balance equation based on the Boltzmann Equation initially derived for the gas kinetic theory in 1872. In wind wave application the continuous frequency-angular wave spectrum is prescribed as a specific number of frequency-angular components propagated along the trajectories. The equations of wave propagation in ocean are defined rays described by geometric optics approximation. This approach does not take into account diffraction. Actually wave field evolution is considered as elementary particle propagation (Lavrenov, 2003).

The finite width of spectral frequency and angular bands introduces some complications for the wave propagation numerical simulation. Ideally, the initial wave energy contained originally within some area must propagate quite smoothly over the ocean surface in time. However, in most wind wave models the spectral resolution is so rough that it induces the so-called “Garden-Sprinkler Effect” (Booij and Holtuisen, 1987). It results in the energy spreading from the source along the directions prescribed in advance by the discrete spectrum representation in the initial area. At some distances from it an anomaly increased concentration of wave energy is manifested itself in these directions, but explicitly insufficient in the others. Thus limited angular resolution

[1]Head, Oceanography Department, Artic and Antarctic Research Institute, 38 av. Bering St., Petersburg, 199397, Russia. lavren@aari.nw.ru
[2]Research Scientist, Numerical Weather Prediction Division, Korea Meteorological Administration, 460-18 Shindaebang-Dong Dongjak-Gu, Seoul 156-720, Republic of Korea. swpark@kma.go.kr

of the model introduces artificial anisotropy in the spatial wave energy distribution. As a consequence of this phenomenon, the model prediction of swell propagation from a distant storm is unsatisfactory.

The existence of this problem was mentioned by many scientists (Ocean Wave Modeling SWAMP group, 1985; Booij and Holtuijsen, 1987; The WAM model, 1988; Ryabinin, 1991a,b; Tolman, 1992), but no sufficiently simple solution was found out. A solution of the "Garden-Sprinkler" problem is proposed in the paper (Booij and Holtuijsen, 1987) for the case of wave propagation in a plane surface. It is offered to add two terms to the wave energy balance equation, allowing to correct the effects connected with the finite width of the frequency and angular bands. An inset of correction terms into the numerical wave propagation schemes requires to solve a more complex equation than the traditional wave energy balance equation. It includes the additional terms with second order partial derivatives and a solution of an additional equation for estimating the wave age, not determined locally. The solution of this problem requires additional computational time.

Effective method of "Garden-Sprinkler" effect elimination was proposed by Lavrenov (Lavrenov and Onvle, 1995; Lavrenov, 2003). Starting from the energy balance equation presented for two dimensional sea wave spectrum an advanced equation is derived taking into account limited number of frequency-angular discretization. The equation includes additional correction term presented as second derivatives over directions. It can be reduced to diffusive operator, producing some energy exchange between nearest angular direction. Numerical realization of the modified energy balance equation helps remove the "Garden-Sprinkler" effect. The same approach can be applied for the parameterization of wave diffraction around islands. In this case it describes some angular exchange between two or more angular directions, approximating diffraction. The present paper is devoted to development of this approach in spectral wind wave models.

2. Problem Formulation

The evolution of a two-dimensional sea wave spectrum $F(\omega,\beta,\varphi,\vartheta,t)$ being a function of the frequency ω, direction β (measured counterclockwise from the parallel), latitude φ, longitude ϑ and time t is described by the equation proposed in paper (Lavrenov, 2003) :

$$\frac{\partial F}{\partial t}+\frac{1}{\cos\varphi}\frac{\partial(\dot{\varphi}\cos\varphi F)}{\partial\varphi}+\frac{\partial(\dot{\vartheta}F)}{\partial\vartheta}+\frac{\partial(\dot{\beta}F)}{\partial\beta}-\mathrm{A}\frac{\varepsilon C_g}{R}\frac{\partial^2 F}{\partial\beta^2}=S \tag{1}$$

where S is source function, which is considered equal zero, C_g is a group velocity, and R is the Earth's radius, A is a parameter under consideration, value ε is defined by angular discretization as $\varepsilon=(\Delta\beta)^2/12$.

The equation of motion for a wave packet along the arc of the great circle can be written as follows:

$$\dot{\varphi}=C_g\frac{\sin\beta}{R} \tag{2}$$

$$\dot{\vartheta} = C_g \frac{\cos\beta}{R\cos\varphi} \tag{3}$$

$$\dot{\beta} = -C_g \tan\varphi \frac{\cos\beta}{R} \tag{4}$$

The main difference equation (1) from one used in the WAM model (Komen et. al., 1994) consists in the 5th term in the left hands of the equation (1). It is introduced in the equation (1) defining the spectrum variation in the direction perpendicular to direction of wave trace propagation. It is presented by an ordinary diffusive operator describing a weak "energy exchange" between the nearest angular components. The parameter A can depend on coordinate (latitude φ and longitude ϑ) and direction of wave propagation β.

Now, the problem of solving the equation (1) is connected with the correct estimation of the parameter A. A large value of this parameter causes a considerable angle smoothing resulting in anomalous isotropy of angular energy distribution. But in case of too small value of this parameter, the shadow effect cannot be eliminated.

3. Diffraction Scale Estimation

In order to get analytical estimation of the equation solution (1) an usual diffusion equation with a simplified left-hand side (1) should be considered:

$$\frac{\partial F}{\partial l} = \delta \frac{\partial^2 F}{\partial \beta^2} \tag{5}$$

where l is spatial coordinate along the ray.

$$\delta = \gamma(\Delta\beta)^2 \tag{6}$$

The elementary β solution of (5) can be written as:

$$F_m(\beta, l) = B(m)\exp(\pm im\beta - m^2 l\delta) \tag{7}$$

where $B(m)$ is a function of the parameter m determined by the initial conditions of the problem. For typical wave spectrum one can get $m = 4$. In some large distance behind island, the solution of (5) "should forget" about the island influence. It can be presented as:

$$\lim_{l\to\infty} F(\beta, l) = B(m) = \frac{1}{2\pi}\int_0^{2\pi} F(\beta, 0)\,d\beta \tag{8}$$

Characteristic scale of the process relaxation is estimated as:

$$\Delta l \approx 1/(\delta m^2) = 1/\left(\gamma(\Delta\beta)^2 m^2\right) \tag{9}$$

In such a case $\gamma \approx 1/\left(\Delta l(\Delta\beta)^2 m^2\right)$.

In order to estimate value γ, it should be pointed out that the size of wave shadow behind island can be estimated as $\Phi \approx f\left(\beta_{ab} - \beta\right)/\sqrt{\tau k}$ (Stoker, 1957), where f is periodical function of the value $\left(\beta_{ab} - \beta\right)$, which is difference between directions of wave propagation and line of strait obstacle. τ is a distance from the obstacles, and k is the wave number. It means that the wave energy in shadow varies in space as $F \approx 1/\tau$. It gives an opportunity to suppose that the scale of wave shadow behind island can be accepted equal to its size L. Thus, the value δ can be estimated as $\delta \approx 1/Lm^2$.

4. Numerical Algorithm

The numerical realization of the proposed correction of the finite angular resolution effects can be presented by a simple 1st order finite difference approximation:

$$F^{n+1}(\omega_k,\beta_l) = F^{n+1}(\omega_k,\beta_l) + \nu(F^n(\omega_k,\beta_{l-1}) - 2F^n(\omega_k,\beta_l) + F^n(\omega_k,\beta_{l+1})) \tag{10a}$$

or 2nd order as follows,

$$\begin{aligned} F^{n+1}(\omega_k,\beta_l) = F^{n+1}(\omega_k,\beta_l) + \\ \frac{\nu}{12}(-F^n(\omega_k,\beta_{l-2}) + 16F^n(\omega_k,\beta_{l-1}) - 30F^n(\omega_k,\beta_l) + 16F^n(\omega_k,\beta_{l+1}) - F^n(\omega_k,\beta_{l+2})) \end{aligned} \tag{10b}$$

where n is an indicator of the time step number in numerical integration of the equation (1), and the ν is,

$$\nu = \frac{A\varepsilon C_g}{R} \cdot \frac{1}{(\Delta\beta)^2} = \frac{AC_g}{12R} \tag{11}$$

As it is seen the value ν is not dependent explicitly on the angular resolution $\Delta\beta$. It is obvious the value ν should be small enough, at least $\nu < 0.5$. In other case, if $\nu \geq 0.5$ instead of angular spreading the operator (10) produces wave energy concentration and numerical instability.

The optimal value of the parameter ν can be defined with the help of numerical experiment. One of the algorithm testing methods consists in controlling the fact whether the total wave energy is conserved or not. In order to estimate the total energy change for all directions, the spectrum is integrated on all angles like following:

$$\frac{2\pi}{L}\sum_{l}^{L} F^{n+1}(\omega_k,\beta_l)=\frac{2\pi}{L}\sum_{l}^{L} \{F^{n+1}(\omega_k,\beta_l)+\nu(F^{n}(\omega_k,\beta_{l-1})-2F^{n}(\omega_k,\beta_l)+F^{n}(\omega_k,\beta_{l+1}))\} \tag{12}$$

Since the spectrum $S(\omega_k,\beta_l)$ is a periodic function of the directions β_l, it is possible to show that the operator (10) preserves full energy, as:

$$\frac{2\pi}{L}\sum_{l}^{L} F^{n+1}(\omega_k,\beta_l)=\frac{2\pi}{L}\sum_{l}^{L} F^{n}(\omega_k,\beta_l) \tag{13}$$

5. Results of Numerical Simulations

The values of parameter ν should be defined as a function of spatial coordinate and angle of wave propagation. Its value is equal non-zero value in nearest to island grid points, and it should be equal zero at some distance from it. It is supposed that calibration of the value ν should be made in such way that at distance larger than island size the wave shadow should be eliminated.

5.1. ***Numerical simulation design***

The WAM model cycle 4 was adapted to test the effects of diffusion parameter and to find optimal value for realization of diffraction effect behind islands. The southwestern part of Korean Peninsula is chosen for the testing domain. This area shows highly complicated coastline and many small islands near-by (Figure 1). The simulation design is set up on a fairly standard configuration. The spatial resolution shares 1/60° both in north-south 2° span and east-west 3° span on a spherical grid. The wave spectrum is resolved into 24 angle bins at 15 degree resolution and 25 frequency bins from 0.0418 Hz to 0.4114 Hz. The source terms and propagation terms are integrated every 1 minute, and the shallow water physics is applied. The sea surface wind, which is predicted from meso-scale atmospheric model, is applied in 3-hour interval (Figure 2). The atmospheric condition for this testing simulation shows strong low pressure system passing southern tip of Korean Peninsula. The wind fields show dominant northwesterly and magnitude of 14-17 ms^{-1} ranges in open sea.

5.2. ***Interpretation of numerical simulation results***

The dependence of spectral angle resolution is investigated prior to looking at the effects of diffraction parameterization. Figure 3 and Figure 4 show the control run of 24-hour prediction significant wave height fields for 30° and 15° angle resolution respectively. The contour labels were intentionally omitted to see clearly the shadows behind islands and protrusion land area. It is obvious that unrealistic long shadows appear on both control runs. Although the higher angle resolution show tendency reducing the artificial shadow intensity in some extents, we cannot increase angle resolution infinitely and need to compromise at certain level. Under these contexts, the diffusive operator describing energy exchange between nearest angular component can be a effective approach mimicking diffraction phenomena.

Several experimental runs with diffusive operator working are executed to find the optimal value of ν. In these experimental runs, the mean spectrum energy of model domain is examined (Table 1). The mean energy deviation of experimental run from control run is 6 ~ 18 %. Figure 5 and Figure 6 show the same wave parameter as of Figure 3 and Figure 4, but with the same diffusive operator (ν=0.08544) being activated. The lower angular resolution case (30°, Figure 5), which eventually has a same effect of increasing ν, appears a considerable angle smoothing resulting in anomalous isotropy of angular energy distribution. After series of experiment reducing the value ν with angular resolution of 15°, the reasonable shadow effect elimination value is presumed to be 0.08544 (Figure 6). It is seen from this figure that unreasonably long shadow behind islands (Figure 4) is properly reduced. It needs further investigation whether this value can be applied to other land-sea configurations and spatial grid resolution.

Table 1. Mean spectrum energy of model domain for different ν values with angular resolution of 15°

ν values	Mean spectrum energy (scaled to 4*SQRT(E))
Control run	2.8345
Experimental run (ν=0.08544)	2.6820
Experimental run (ν=0.05340)	2.3273
Experimental run (ν=0.02136)	2.5439

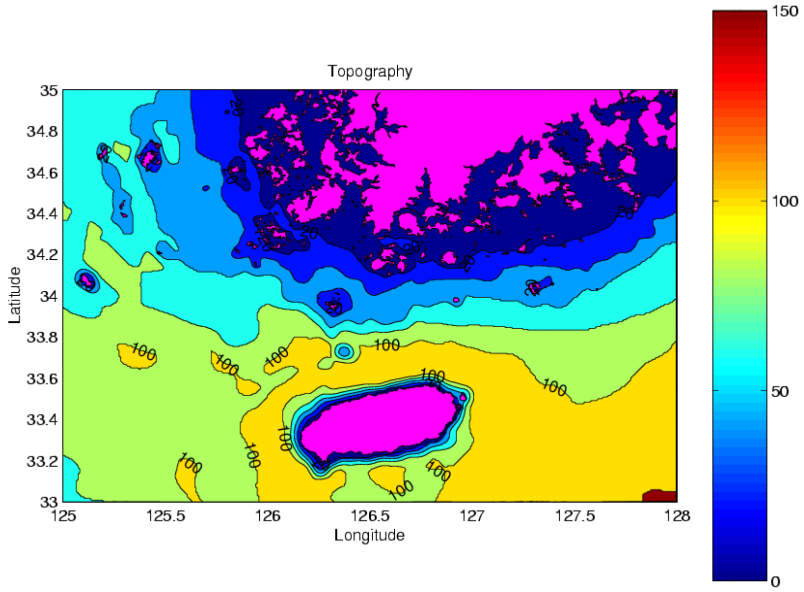

Figure 1. The model domain bathymetry (contour, meter)

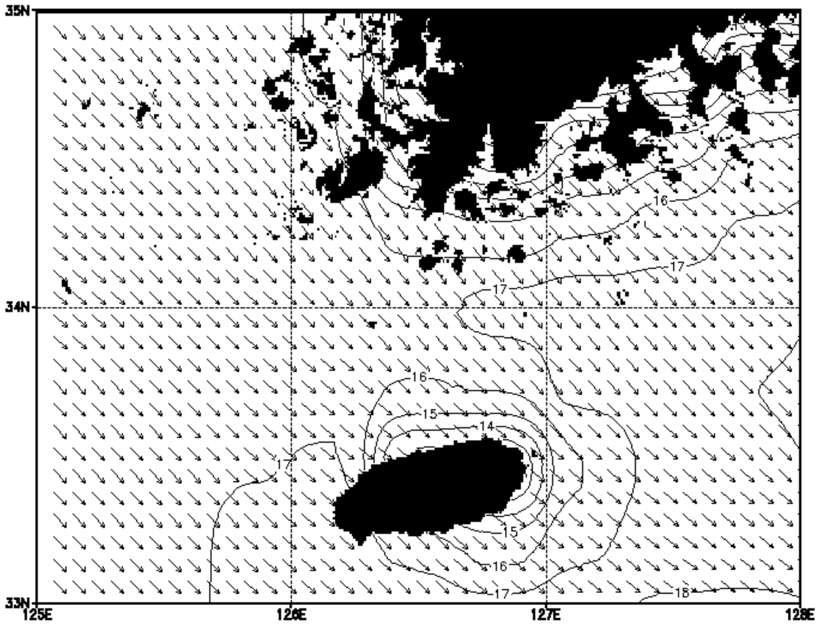

Figure 2. Sea surface wind magnitude (contour, ms^{-1}) and direction (not scaled)

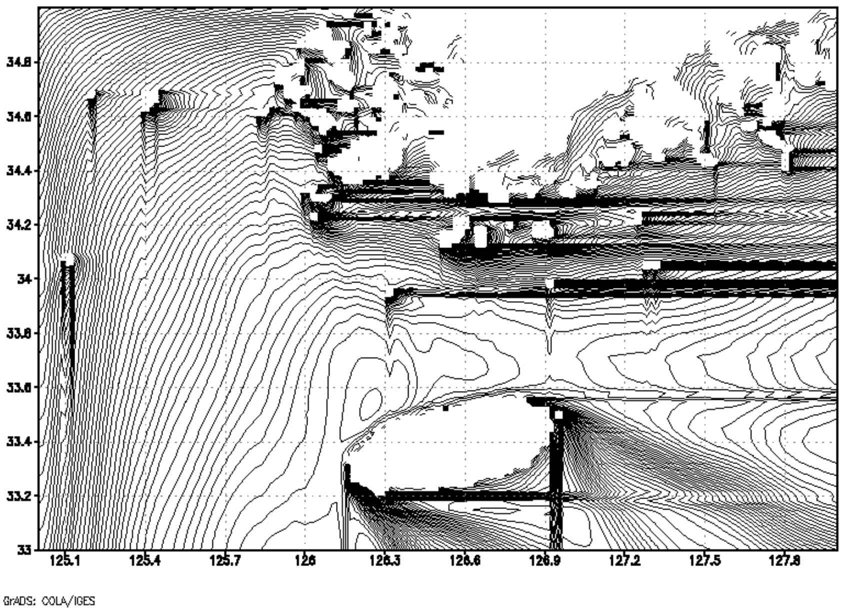

Figure 3. Significant wave height of control run with 30° directional resolution

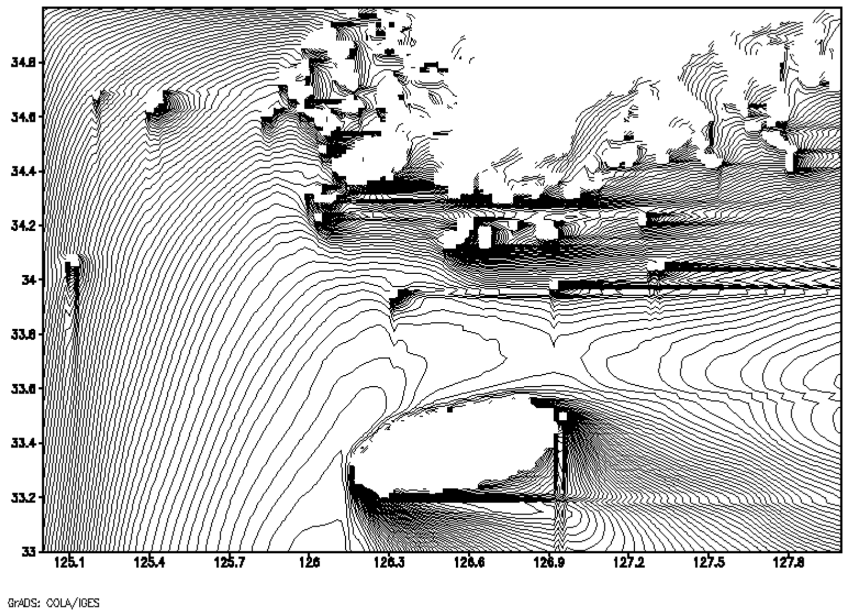

Figure 4. Same as Figure 2, but for 15° directional resolution

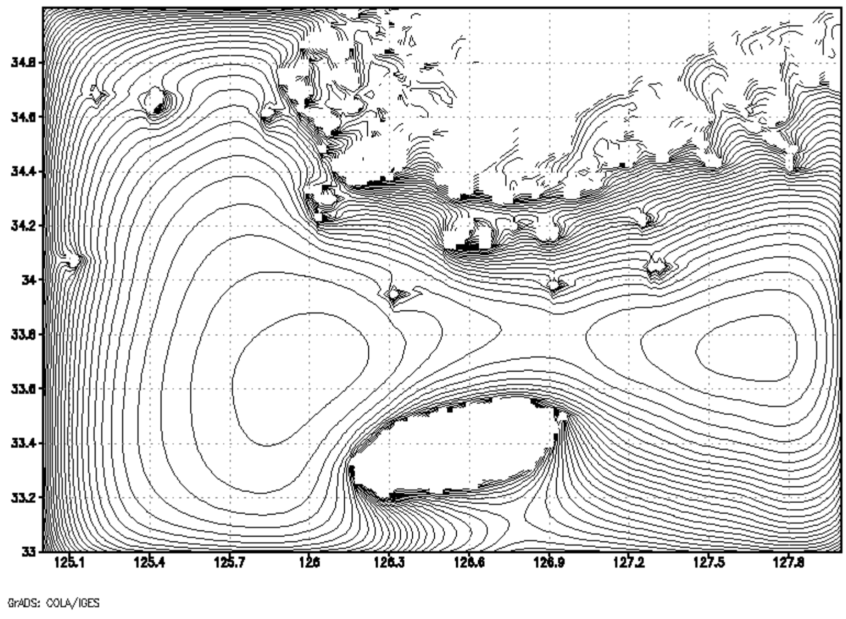

Figure 5. Significant wave height of experimental run with angular diffusive operator (ν =0.08544) and 30° directional resolution

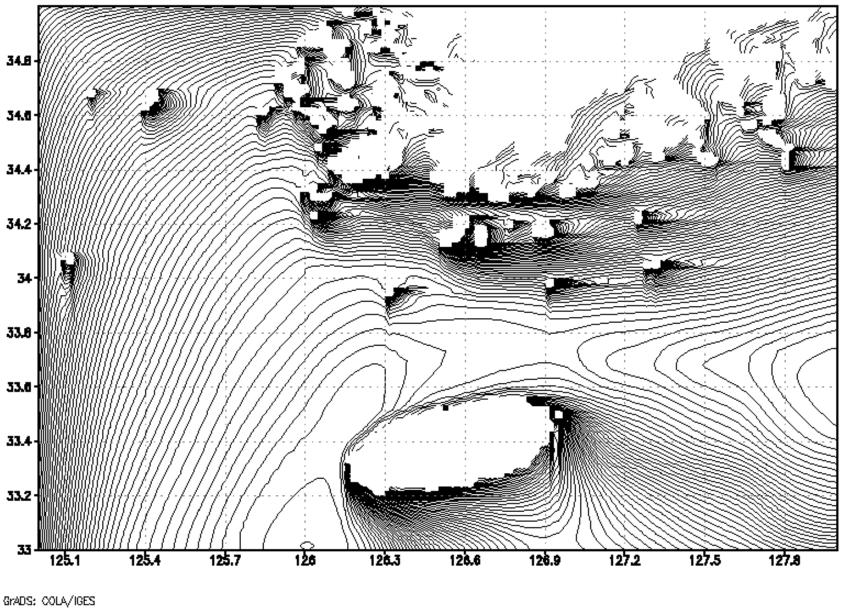

Figure 6. Same as Figure 5, but for 15° directional resolution

6. Conclusion

There are many small islands and submerged obstacles in the ocean providing the wave attenuation, dissipation and scattering. Often a size of islands can be smaller than distance between the grid points. That is why it is difficult to take into account in usual way in spectral wind wave models. Moreover the absence of diffraction in modern wind wave models produces large wave shadow behind islands in wave numerical simulations.

The proposed angular diffusive operator gives an opportunity to simulate mentioned effects by additional calibration the parameter ν around islands. The implementation of the angular diffusive operator in the energy balance equation (1) helps to improve numerical solution of wave propagation around islands and submerged obstacles.

The implementation of correction term has following advantages:

1. The implementation of the angular diffusive operator into the model is very simple. It provides energy conservation and its running does not take essential CPU times in comparison with other numerical operation, i.e. propagation source term integration, etc.
2. The angular diffusive operator is local one, taking into account only one grid point, i.e., it gives an opportunity to be easily implemented into any computer with parallelization of numerical simulations.
3. The angular diffusive operator can be used not only for simulation diffraction effects around islands but also wave energy scattering around submerged obstacles, peninsular etc. in the ocean.

Acknowledgements

The authors would like to express gratitude to Korean Meteorological Administration for providing opportunity for cooperation between Russian and Korean wave research. The presented research in this paper was conducted under the grant RFBR 04-05-64306, INTAS-01-234, INTAS-01-2156.

References

Booij, N. and Holthuijsen, L.H. (1987). Propagation of ocean waves in discrete spectral wave model. *J. of Comput. Phys*, 68, pp. 307-326.

Komen, G.J., Cavaleri, L., Donelan, M., Hasselmann, K., Hasselmann, S. and Janssen, P.A.E.M. (1994). *Dynamics and Modelling of Ocean Waves*. Cambridge University Press, pp. 532p.

Lavrenov, I.V. (2003). *Wind Waves in Ocean.* Springer-verlag, p386.

Rogers, E.W., Kaihatu, J.M., Booij, N. and Holthuijesen, L.H. (1999). *Improving the Numerics of a Third-Ggeneration Wave Action Model*. Naval Research Laboratory, Stennis Space Center, M. S., pp. 39529-5004.

WAMDI group (1988). The WAM model — a third generation ocean wave prediction model. *J.Phys.Ocean.,* (12), pp. 1775-1810.