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The Wave Energy Concentration at the Agulhas Current off South Africa

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Abstract. The case of a freak wave collision with the ship in the Agulhas current is described. The explanation of the appearance of the freak wave as a result of wind-wave transformation in the Agulhas current is given. Swell is captured and intensified by the counter-current and is located in the neighbourhood of the maximum value of the current velocity, as a result of which there is a great concentration of wave-energy density. The superposition of wind and sea with swell transformed by the current promotes the formation of the freak waves. Using a simple mathematical analysis, an optimal ship track is proposed which could reduce the risk of collision with a freak wave.

Key words: freak waves, Agulhas current, ship collision, wave energy, modelling of wave transformation on current.

1. Introduction

The size, power and technical equipment of modern ships would allow one to think that there is no danger for a ship to sail anywhere in the open ocean. In fact this is not true. On 13 June 1968 the tanker *World Glory* (built in the U.S.A. in 1954) under the Liberian flag while travelling along the South African coast, encountered a freak wave, which broke the tanker into two parts and led to the death of 22 of its crew members. A number of similar cases are well known (Mallory, 1974; Sanderson, 1974; Sturm, 1974; Davidan and Lopatukhin, 1985; Lavrenov, 1985a).

The term 'freak waves' (or abnormal waves, exceptional waves, killer waves, freak waves, and cape rollers) here pertains to individual asymmetric waves with a crest of an extremely high slope, in front of which appears a longer and deeper trough than compared with ordinary wind waves. The height of such waves can reach 15–20 m and more, sometimes in a relatively calm sea. Waves often appear suddenly. That is why it is practically impossible for a ship's crew to take any precautions.

Abnormal waves are often observed repeatedly in different regions of the world ocean, where there are strong currents: for example, Gulfstream, Kuroshio and others. Extremely large waves are observed near the south-eastern shore of South Africa in the Agulhas current between East London and Durban (see Figure 1). That is why the region is considered to be very dangerous.

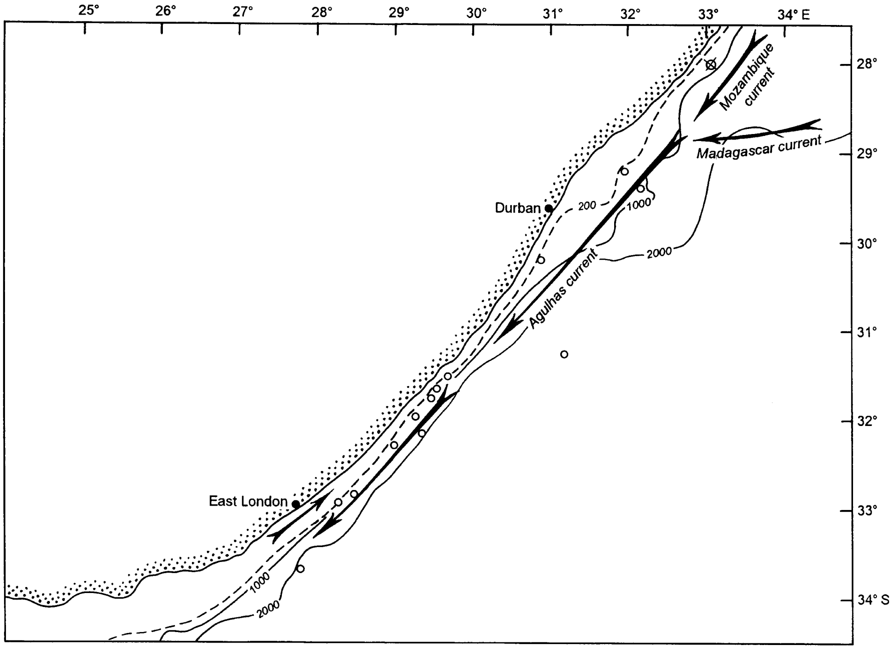

Figure 1. Map of the south-east coast of South Africa. The continental shelf, its slope and the Agulhas current are shown. Circles denote the places where the largest abnormal waves abound, according to Mallory (1974). The crossed circle is the location of the incident described in the present article.

A list of cases when abnormal waves were observed up to 1973 is given by captain Mallory (Mallory, 1974), along with an analysis of the corresponding atmospheric and oceanographic conditions which were responsible to a great extent for the appearance of the abnormal waves. In his article, Captain Mallory showed 11 cases of catastrophic ship collision with abnormal waves in the region (see Figure 1). The conditions mainly come down to the conjunction of the south-westerly swell from the southern latitudes toward the Agulhas current, and local wind waves with the passage of an atmospheric cold front. In fact, abnormal waves appeared here more often and consequences of their action were less dramatic. It can be explained by the weather conditions of the area when ships had to reduce their speed and diminish the effect of wave action.

On 27 April 1985, the same thing happened with a ship, *Taganrogsky Zaliv* of the former Soviet Union. By using the information offered by the captain of the ship we will try to analyse the situation and to produce mathematical simulations on the basis of the spectral theory of wind-wave and current interaction.

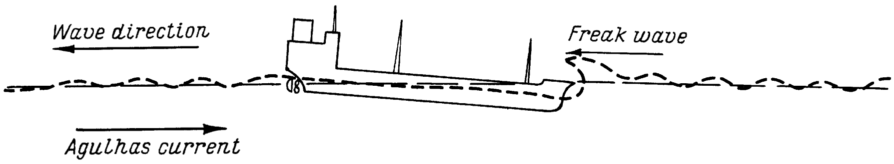

Figure 2. Collision of a freak wave with the ship according to the description of seamen of the refrigerated tanker *Taganrogsky Zaliv* (27 April 1985).

2. Description of the Accident

On 27 April 1985, the refrigerated tanker *Taganrogsky Zaliv* (length 164.5 m, deadweight 12 000 tons) was sailing from the Indian Ocean to the south-eastern region of the Atlantic Ocean.

Near the Cape of Good Hope, the possibility of encountering a weather storm is high enough, so the ship was prepared for sailing in stormy weather. The north-north east wind was blowing at a speed of 7 m/s. At 5 a.m., it changed direction to south-south west with the same force. From the previous day the atmospheric pressure was diminishing until the wind changed direction, after that it began to increase. At 8.00 a.m., the wind became stronger and at 11.00 a.m. it reached 15 m/s. By the noon of the day that everybody felt the wave impact on the ship, which tore off a lifeboat, loosened two mooring-line reels and washed them into the water.

After 12.00, the wind speed diminished to 12 m/s. Wind sea became calmer as well. The wind force didn't change during the next three hours. Wave height didn't exceed 5 m and the length was 40–45 m. To overcome the results of wave impact, the boatswain and three seamen were sent to the foredeck. The speed of the ship was diminished to a minimum that was enough for safe control of the ship's motion. The ship rode well on the waves. The foredeck and main deck were not flooded with water.

By one o'clock, the job was almost done on the foredeck. At that moment, the front part of the ship suddenly dipped, and the crest of a very large wave appeared close to the foredeck. It was 5–6 m higher than the foredeck. The wave crest fell down on the ship. One of the seamen was killed and washed overboard. It was impossible to save him. The location of the ship is shown by a cross in Figure 1 (Lavrenov, 1985).

Nobody was able to foresee the appearance of the wave as the weather was normal for ocean conditions. When the ship went down, riding on the wave, and burrowed into its frontal part, nobody felt the wave's impact. The wave easily rolled over the foredeck, covering it with more than two meters of water (see Figure 2). The length of the wave crest was no more than 20 m.

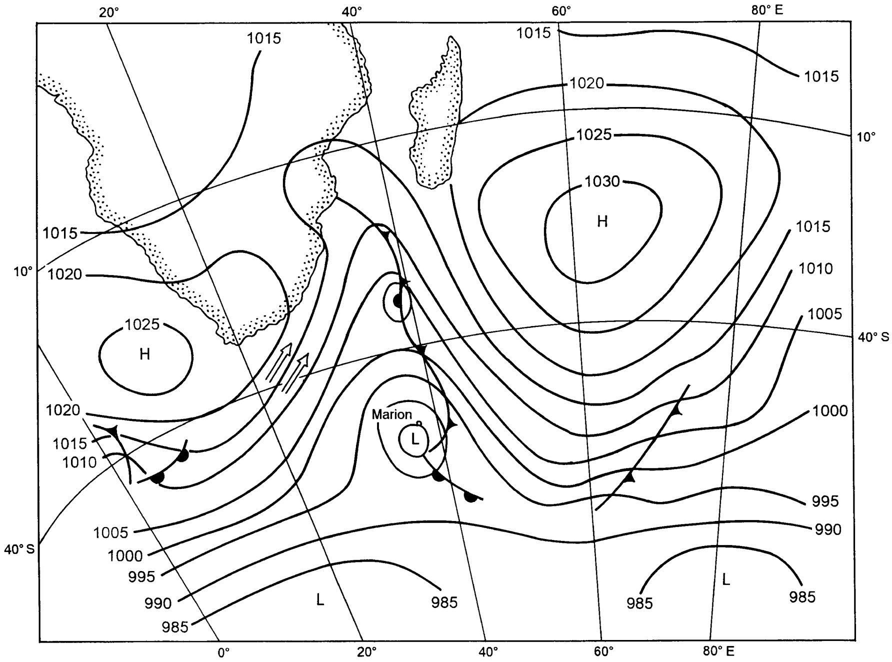

Figure 3. Synoptic map of the region at 12.00 GMT, 27 April 1985.

3. The Weather Conditions

Let us analyse the natural conditions for the appearance of the freak wave. It should be noted that the incident occurred in the area of the south-eastern coast of South Africa in the Agulhas Current, where ships often encounter abnormal waves.

By using synoptic information offered by the captain of the ship *Taganrogsky Zaliv*, we will try to analyse the situation. The meteorological situation in the area of the ship's passage was defined by the high pressure region situated offshore of the western part of South Africa and by a slowly moving cyclone whose centre was situated near Marion Island (Figure 3). The cold front was moving north-east over the area. Atmospheric pressure isobars show a south-westerly wind force nine blowing across the region of Southern Africa. The synoptic situation was approximately the same as in the cases described by captain Mallory (Mallory, 1974).

Near the regions of the south-eastern shelf of Africa, the wind waves consist of two systems: the sea wind of Southern Africa with short and steep waves generated by local wind and much longer swell waves coming from the south. The superposition of these two wave systems increases the wave heights but this is not enough to explain the behaviour of freak waves.

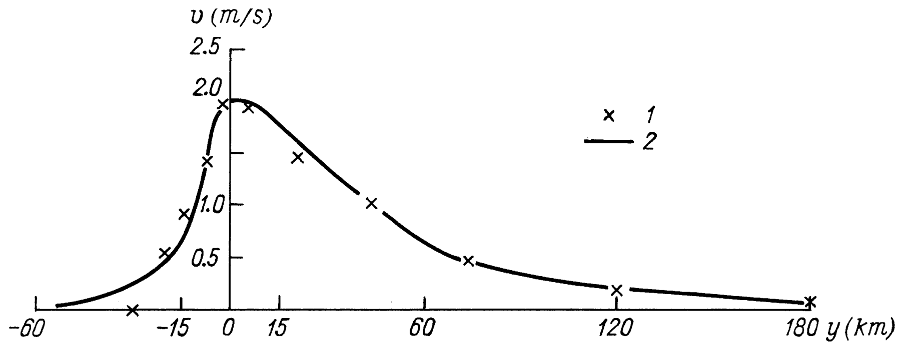

Figure 4. Distribution of the horizontal current velocity in the transverse direction: (1) from data (Schumann, 1976), (2) approximation (1). ⇒-direction of the initial wave propagation.

4. Spatial Distribution of Speed of Agulhas Current

As a rule (Mallory, 1974; Davidan and Lopatukhin, 1985; Lavrenov, 1985), abnormal waves are observed on the ocean surface where the depth coincides approximately with the two-hundred meter isobath. The latter passes parallel to the shore line and is the boundary of the continental shelf, where the depth increases sharply to 3–4 km (Figure 1).

It was therefore initially assumed (Mallory, 1974) that this abrupt change in depth was in fact the reason for the formation of abnormal waves. In reality the bottom relief at the 200 m depth can only influence waves with a length greater than 500 m. The characteristics of the horizontal dimensions for abnormal waves are much shorter. Actually, the sharp depth change at the edge of the continental shelf leads to the fact that here is where the Agulhas current, having a jet nature and directed parallel to the shore line, reaches its greatest value.

The transverse profile of the velocity distribution for the Agulhas current remains essentially unchanged over nearly the entire extent from the place where the Mozambique and South Madagascar currents merge to 34 S latitude (Figure 1). It became wider to the south (Atlas of the Atlantic and Indian Ocean, 1977).

To approximate the current speed spatial distribution, we will use a local system coordinate, where the 'ox'-axis is chosen for the direction of the current velocity (approximately parallel to the coastal line), and the 'oy'-axis has been chosen for the perpendicular direction (from the coastal line). The origin of the coordinate system coincides with the maximum current speed and the point $\{x = 0, y = 0\}$ coincides with the channel line for the current velocity at 27° E, 34° S.

In accordance with data (Schumann, 1976), the transverse profile of the velocity distribution current $\mathbf{U} = \{U(y), 0\}$ can be approximated in the local system of the coordinate region by the relation (see Figure 4):

$$U = \frac{b}{1 + cy^2}, \tag{1}$$

where b = 2.2 m/s, $c = 6.26 \times 10^{-10}$ m^{-2} for $y > 0$, and $c = 10^{-8}$ m^{-2} for $y < 0$. We will assume that, in the coordinate system, the chosen current velocity distribution (1) is valid for $x < 0$.

At more southerly latitudes, the Agulhas current diverges in a fan-shaped manner and becomes weaker. In accordance with the Atlas of the Atlantic and Indian Ocean (1977), it can be assumed that here, i.e., for $x > 0$, the horizontal components of the current $\mathbf{U} = \{U(x,y), V(x,y)\}$ are approximated by the equations

$$
\begin{aligned}
U &= \frac{b}{(1+ax^2)(1+cy^2)}, \\
V &= \frac{2abx}{(1+ax^2)\sqrt{c}}\mathrm{arctg}(\sqrt{c}y),
\end{aligned}
\tag{2}
$$

where $a = 0.6 \times 10^{-11}$ m^{-2}.

Values (2) are found from the continuity equation (for $x > 0$) in accordance with the boundary condition coming from (1) for $x = 0$.

5. The Spectral Solution of the Problem

A few studies have been reported in the literature which explain the behaviour of abnormal waves. According to the article by Ivanenkov *et al.* (1983), the occurrence of a cold front leads to the generation of the so-called ‘frontal swell waves’, which in combination with sea wind is able to produce the wave of large amplitude. However, it is not possible to explain completely the formation of abnormal waves by means of these factors.

On the basis of the theory of wave and current interaction (Longuet-Higgins and Stewart, 1961; Peregrine, 1976), the most successful explanation of the abnormal wave in the Agulhas Current was given by Smith (1976). He explained that giant waves occur in the area where the wave groups are reflected by the current. The local behaviour of the wave amplitude is modelled by the nonlinear Schrödinger equation. Wave reflection by the current amplifies wave height and modifies the usual wave form.

In fact, Smith gave a local behaviour of an abnormal wave. But the question of the initial value of wave parameters remains open. They are defined by the wave transformation on the large scale of the Agulhas Current. Here we will attempt to take into account a spatial speed distribution of the real current and to consider wave transformation by using the wave action equation in spectral form. We consider the propagation of swell waves from the southern latitudes, where there is actually no current, towards the growing Agulhas current. Here we can make use of the geometric optics approximation, since the characteristic horizontal scales for the change in the current velocity exceeds the horizontal dimensions of the waves.

It is well known that the evolution of the wave spectrum in the weak turbulence approximation is described by a kinetic equation in which the spectral density of the wave action $N(\mathbf{k}, \mathbf{r}, t)$ appears as the function sought (Komen *et al.*, 1994; Davidan *et al.*, 1995):

$$\frac{\partial N}{\partial t} + \frac{\partial N}{\partial \mathbf{r}}\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + \frac{\partial N}{\partial \mathbf{k}}\frac{\mathrm{d}\mathbf{k}}{\mathrm{d}t} = G, \tag{3}$$

where $\mathbf{k} = \{k_x, k_y\}$ is the wave vector, $\omega = \sigma - \mathbf{kU}$ is the frequency, $\mathbf{U} = \{U(x, y), V(x, y)\}$ is the current speed, $\sigma = \sqrt{gk}$, $\mathbf{r} = \{x, y\}$ is the horizontal space vector, t is time, G is the source function describing the physical processes forming the wind wave spectrum.

The Hamilton equations which are characteristics of Equation (3) and describe the propagation of wave packets can be written as follows:

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \frac{\partial \omega}{\partial \mathbf{k}}, \qquad \frac{\mathrm{d}\mathbf{k}}{\mathrm{d}t} = -\frac{\partial \omega}{\partial \mathbf{r}}, \qquad \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\partial \omega}{\partial t}. \tag{4}$$

When the source function in Equation (3) is neglected, which can be valid for swell waves, the spectral density of wave action $N(\mathbf{k}, \mathbf{r}, t)$ remains constant along the propagation trajectories for the wave packet:

$$N(\mathbf{k}, \mathbf{r}, t) = N_0(\mathbf{k}_0, \mathbf{r}_0, t_0), \tag{5}$$

where N_0 is the initial value of the spectral wave action, $\mathbf{k}_0 = \mathbf{k}_0(\mathbf{k}, \mathbf{r}, t)$, $\mathbf{r}_0 = \mathbf{r}_0(\mathbf{k}, \mathbf{r}, t)$.

Solution (5) describes the transformation of the swell wave spectrum on the large-scale horizontally-inhomogeneous flow. The problem of finding $N(\mathbf{k}, \mathbf{r}, t)$ from the initial conditions $N_0(\mathbf{k}_0(\mathbf{k}, \mathbf{r}, t), \mathbf{r}_0(\mathbf{k}, \mathbf{r}, t))$ reduces to the solutions of a system of (4).

We convert from the function $N(\mathbf{k}, \mathbf{r}, t)$ to the spectral energy density $S(\omega, \phi, \mathbf{r}, t)$ using the substitution of variables $k_x = k(\omega, \phi) \cos \phi$, $k_y = k(\omega, \phi) \sin \phi$. From (4) the value $S(\omega, \phi, \mathbf{r}, t)$ can be found at the current $\mathbf{U}(\mathbf{r})$ as a function of its initial value in the form:

$$S(\omega, \phi, U) = \frac{\partial k^2}{\partial \omega}\sigma \left[\frac{\partial k_0^2}{\partial \omega_0}\sigma_0\right]^{-1} S_0(\omega, \phi_0). \tag{6}$$

Thus, assuming the flow is steady-state, i.e., $\omega =$ const, the evolution of the wave spectrum on the flow $\mathbf{U} = \{U(y), 0\}$ (from (4) it follows that $k_x =$ const) for $x < 0$ can be written in the following form (Lavrenov, 1986)):

$$S(\omega, \phi, U(y)) = \frac{16}{(1 + \sqrt{1 - 4U\omega/g \cos \phi})^4}\frac{S_0(\omega, \phi_0)}{\sqrt{1 - 4U\omega/g \cos \phi}}, \tag{7}$$

where $S_0(\omega, \phi_0)$ is the initial value for the spectrum which is known at a sufficient distance from the region of intense flow ($\sqrt{U^2 + V^2} \approx 0$ for $x \geq 1.5 \times 10^3$ km).

We will assume that $S_0(\omega, \phi_0)$ is described by the approximation for a swell spectrum (Davidan *et al.*, 1995):

$$S_0(\omega, \phi_0) = h_0^2 \frac{8}{\pi^2} \cos^2 \phi_0 \left(\frac{\omega_m}{\omega}\right)^5 \omega^{-1} \exp\left(-1.2 \left[\frac{\omega_m}{\omega}\right]^5\right), \tag{8}$$

where h_0 is the mean height of the swell waves and ω_m = 0.5 sec^{-1} is the frequency of the spectrum maximum. We note that the initial value for the angle ϕ_0 appearing in Equation (7) is a function of the value for the parameters ω and ϕ at the point with coordinates $\{x, y\}$ being considered.

In order to find ϕ_0, it is necessary to solve the system of the Hamilton equation (2) which, in new variables, can be written as

$$\begin{aligned}
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{V + 0.5f \sin\phi}{U - 0.5f \cos\phi}, \\
\frac{\mathrm{d}\phi}{\mathrm{d}x} &= \left[\sin 2\phi \frac{\partial U}{\partial x} - \sin^2\phi \frac{\partial V}{\partial x} + \cos^2\phi \frac{\partial U}{\partial y}\right] / (U - 0.5f \cos\phi), \\
\frac{\mathrm{d}t}{\mathrm{d}x} &= 1/(U - 0.5f \cos\phi),
\end{aligned} \tag{9}$$

where

$$f = 0.5 \frac{g}{\omega} \left\{1 + \left[1 - 4 \frac{\sqrt{U^2 + V^2}}{g} \omega \cos\left(\phi + \arcsin \frac{V}{\sqrt{U^2 + V^2}}\right)\right]^{0.5}\right\}.$$

The value ϕ_{0j} corresponding to the rays arriving at the points $\{x = 0, y = y_m\}$ being considered from the region where the current is practically absent, $x > 1.5 \times 10$ km, were found numerically for the solution of the system of Equation (9) for a discrete set of frequencies ω_i (from ω_1 = 0.37 rad/sec to $\omega_{12} = 0.981$ rad/sec) and of angles ϕ_i (from 105° to 255° in step 15°).

Several of more typical rays $y = y(x, \omega, \phi)$ arriving at the point $\{y = 0, x = 0\}$ are shown in Figure 5. Oscillations relative to the x axis whose frequency increases as the rays approach the region of intense flow, are a characteristic feature of their behaviour. An unusual wave channel, a wave guide, occurs in the current, i.e. wave rays are reflected from the various caustics on different sides of the current channel line. The velocity distribution in the Agulhas current is not symmetric relative to the line for the maximum value for the velocity. The velocity gradients from the eastern side of the current line are much less than gradients from the western side (see Figures 1 and 4). The gradient from the western side is increased in addition due to the offshore counter-current (its velocity is $U \sim 0.5$ m/s), which takes place during the passage of a cold front (Mallory 1974). Such a current velocity distribution in the transverse direction leads to the fact that the frequency of the caustic arrangement

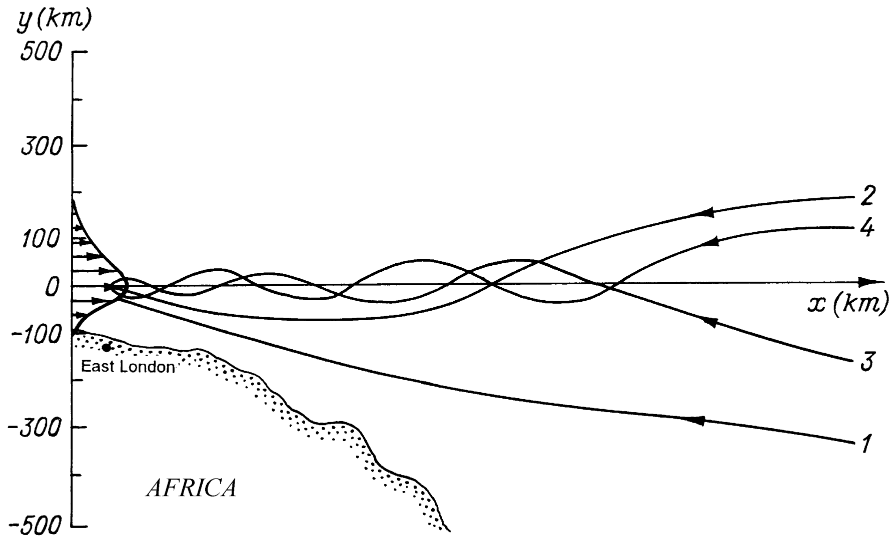

Figure 5. Behaviour of wave rays on the current (2). Rays shown are those at the point $\{x = 0, y = 0\}$ with frequencies: (1) ω = 0.20 (rad/sec); (2) ω = 0.38 (rad/sec); (at the angle $\varphi_0 = -150°$); (3) ω = 0.76; (4) ω = 0.93 (rad/sec) (at the angle $\varphi_0 = 210°$).

(for waves of different lengths and directions) from the western side is much greater than from the eastern side. For $x > 0$, the wave channel narrows as x decreases and, for $x < 0$, the width of the channel is practically unchanged. The behaviour noted for the wave rays becomes more pronounced with increasing frequency ω and decreasing angle ϕ. Thus, the unusual nature of the distribution of the current velocities leads to the capture of wave rays which, in turn, redistributes the wave energy in space.

6. Spatial Distribution of the Wave Height on the Agulhas Current

The distribution of energy or mean wave height h in the current is of particular interest. Values for the wave spectrum on the current $S(\omega_i, \phi_j, y_n)$ for $x = 0$ were obtained from the numerical solution the system of Hamilton Equations (9) and the use of spectral relations (7) and (8). Here those spectrum values corresponding to rays having no physical meaning (e.g., issuing from the shore, etc.) were assumed to be zero. The spectrum was integrated numerically with respect to frequency ω and angle ϕ by the Simpson method in order to obtain the wave energy. It should be noted that relation (8) has an integrable singularity in the line where the value $1 - 4(U\omega/g)\cos\phi$ goes to zero. The contribution from the integration in the neighbourhood of the singularity is evaluated analytically. The results of the calculation are shown in Figure 6, where the value of the ratio of the mean height on the current h to the mean wave height h_0 for the initial spectrum $S_0(\omega, \phi_0)$ in the absence of current, is plotted along the oy-axis. As seen from the calculations

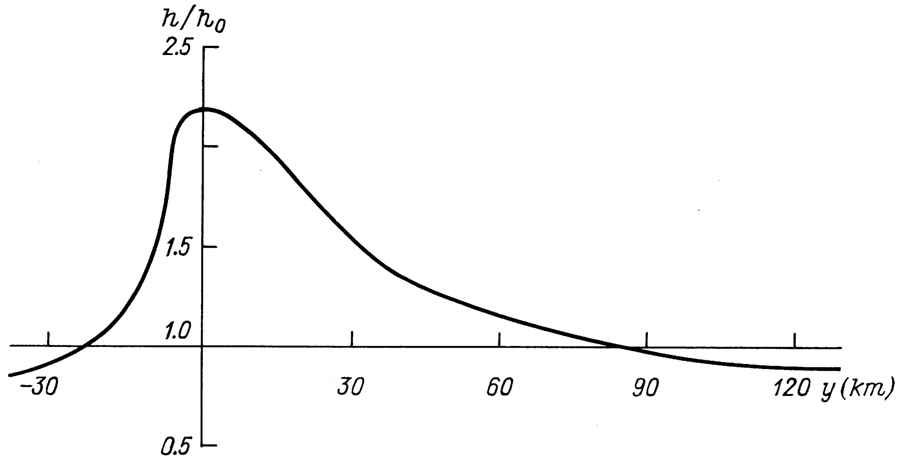

Figure 6. Distribution of mean wave heights h at the angles to the current, normalised to the initial wave height h_0.

in Figure 6, the maximum value of h/h_0, equal to 2.19, is reached in the current channel along the direction towards the shoreline, and at a distance y = 19 km from the line of maximum current velocity, the relative excess of wave height is no greater than 10%. Seaward of the current channel line, the relative height decreases to a lesser extent and the 10% excess of wave height occurs at $y = -70$ km, while for $|y| > 90$ km, even the reverse case is observed, i.e., $h < h_0$.

Thus, the spatial distribution of the wave height on the Agulhas current permits us to conclude that, in going toward the shore some distance from the line of maximum velocity, where the isobath is 200 m, the probability of observing abnormal waves is much less than going the same distance from the line of current in maximum velocity in the direction of the open sea.

7. Conclusion

The results obtained here indicate that the unusual nature of the current velocity distribution for the refraction of the south-western swell of the Agulhas current forces which propagates over a wide area of the south-western part of the Indian Ocean in such a way that it turns toward the side of the maximum in the current velocity. The swell is captured, intensified by the counter-current, and localised in the neighbourhood of the maximum velocity, propagating along the south-east coast of South Africa. As a result, there is a significant concentration of wave energy density, i.e., a focusing takes place in the area mentioned which promotes the formation of abnormal waves. The rays in the middle part of the current show that the sea surface presents here the superposition of different swell waves reflected from the two sides of the current. The superposition of these systems with sea wind promotes the shape of the wave and the suddenness of its formation.

As mentioned above, the theory presented by Smith (1976) provides some explanation of the local behaviour of giant waves, and its unusual form. Initial values to define the local wave parameters can be found from the results given above. We believe that a combination of our large scale result of the wave tranformation with Smith's theory is able to provide a well-founded explanation of abnormal wave behaviour on the Agulhas current.

The results of our study shows that the places where the wave appears approximately coincides with the spatial position of the high wave energy concentration in the current. The spatial distribution of wave heights (Figure 6) characterises the probability of observing abnormal waves in the current. So, if the weather conditions come down to the conjunction of a south-westerly swell coming from the southern latitudes toward the Agulhas current, and local wind waves with the passage of an atmospheric cold front, then the ship should go towards the shore some distance from the line of maximum velocity, where the isobath is 200 m. The probability of observing abnormal waves is much smaller than going the same distance from the maximum velocity of the line of current the direction of the open sea. The 'optimal' ship track should be as close to shore line as possible.

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