SURFACE WAVES AND LOW FREQUENCY NOISE IN THE DEEP OCEAN

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Abstract. Low frequency (1-5 minute) pressure fluctuations have been observed in the deep sea beneath 4000 m of water. This paper proposes a model connecting the beat of surface gravity waves to the observed pressure fluctuations. The model depends on the presence of nonlinear surface gravity wave packets.

Introduction

Long period pressure disturbances (1-5 minutes period) were observed at a depth of 4000 m in the Pacific Ocean by Sutton et al. (1965). The intensity of the noise was observed to increase during periods of enhanced surface wave activity and they observed signals ranging from 3 to 300 microbars of pressure. The noise was in both crystal hydrophone records and in records from long period seismometers.

In this note I show that noise of the magnitude observed can be related to the existence of a coherent pocket of surface waves. A coherent packet may be defined as a group of surface waves that propagate as an entity and move at the group velocity of a carrier wave. An example is the modulated wave train discussed by Lake and Yuen (1977). For this discussion the detailed nonlinear behavior of surface waves is not important, however it is required that surface waves form coherent entities composed of a number of individual waves. Two simplified assumptions are used in this study; the wave packet is to propagate at the group velocity at the carrier wave and the wave packet is sinusoidal in shape.

Analysis

Assume that a carrier wave of wave number, k, and frequency, ω, exists. Let this carrier wave be modulated in space and time. Let the wave amplitude be a and $\varepsilon = ak$ be a small parameter describing the slope of the carrier wave. Then if spatial and temporal scales of the modulation, X, T, are εx, εt, where x and t are space and time, a wave modulation is governed by the nonlinear Schrödinger equation. Zakharov (1968) derived this equation for deep water. Benny and Roskes (1969) and Davey and Stewartson (1974) derive the equations for the modulation based on the assumption that $\varepsilon kh << 1$. The assumption that $\varepsilon kh << 1$ is violated in 4000 m of water. In the deep ocean εkh is large compared to unity but not infinite as was assumed in Zakharov's (1968) analysis.

A complete development of the equations for arbitrary εkh is not required in order to understand the effects of a modulated wave train and is omitted in this discussion. The modulation equation is a variant of the nonlinear Schrödinger equation and it has been shown by Longuet-Higgins (1976) that this equation passes uniformly from the shoaling depths considered by Benney and Roskes (1969) and Davey and Stewartson (1974) to the deep water equations as derived by Zakharov (1968). Since this discussion centers on a single fourier component of the modulation envelope the modulation equation is not required.

Consider a wave train of amplitude, a, propagating on a fluid of depth, h. Introduce non-dimensional length and time scales $(x^*, z^*) = (kx, kz)$, $t^* = \omega t$, and scale the actual water surface displacement η on the peak amplitude a, then the non-dimensional surface displacement $\zeta = \eta/a$ is of order unity. The velocity potential is measured in units of ac where c is the phase speed of the carrier wave. A non-dimensional gravity is defined $g^* = gk/\omega^2$. For a freely propagating linear gravity wave in deep water $g^* = 1$.

In terms of the non-dimensional variables, the mathematical problem may be stated

$$\nabla^2\phi = 0 \qquad -D < z < \varepsilon\zeta \tag{2.1}$$

$$\phi_z = 0 \qquad z = -D = -kh \tag{2.2}$$

$$2g\zeta + 2\phi_t + \varepsilon(\phi^2_x + \phi^2_z) = 0 \quad z = \varepsilon\zeta \tag{2.3}$$

$$\phi_z = \zeta_t + \varepsilon\phi_x\zeta_x \qquad z = \varepsilon\zeta \tag{2.4}$$

It is convenient to approximate the boundary conditions (2.3) and (2.4) by expanding the equations about z=0. This leads to the boundary conditions

$$2g\zeta + 2\phi_t)_{z=0} = -\varepsilon(2\zeta\phi_{tz})_{z=0} + \phi^2_x)_{z=0}) + O(\varepsilon^2) \tag{2.5}$$

$$\zeta_t - \phi_z)_{z=0} = -\varepsilon\frac{\partial}{\partial x}(\phi_x\zeta) + O(\varepsilon^2) \tag{2.6}$$

Now seek the solution of the above problem in terms of an expansion in ε. The leading terms are:

$$\zeta(x,z,t;0) = \alpha e^{i\chi} + \alpha^* e^{-i\chi}, \quad \alpha\alpha^* = 1, \quad \chi = x - t \tag{2.7}$$

$$\phi(x,t;0) = -ig\frac{\cosh(z+D)}{\cosh D}(\alpha e^{i\chi} - \alpha^* e^{-i\chi}) \tag{2.8}$$

with the dispersion relation $g\sigma = 1$ where $\sigma = \tanh kh$.

The solution of the nonlinear problem is expressed in a power series in ε the leading terms of which are

$$\zeta = \alpha_o(X,T;\varepsilon) + \alpha(X,T;\varepsilon)e^{i\chi} + \alpha^*(X,T;\varepsilon)e^{i\chi} + \ldots \tag{2.9}$$

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$$\phi = \phi_o(X,z,T;\epsilon) + \phi(X,z,t;\epsilon)e^{iX} + \phi^*(X,z,T;\epsilon)e^{iX} + \ldots \qquad 2.10$$

Direct substitution into the governing equations yields the equations for the sub-harmonic terms,

$$g\alpha_o = \epsilon\phi_{oT} = \epsilon\alpha\alpha^*(1-g^2) \qquad z = 0, \qquad 2.11$$

$$\epsilon\alpha_{oT} - \phi_{oz} = -2g\epsilon^2 \frac{\partial}{\partial X}\alpha\alpha^* \qquad z = 0, \qquad 2.12$$

$$\phi_{ozz} + \epsilon^2\phi_{oXX} = 0 \qquad -D \quad z \quad 0, \qquad 2.13$$

$$\phi_{oz} = 0 \qquad z = -D. \qquad 2.14$$

Because Laplaces equation 2.13 involves ϵ it is tempting to expand this equation in powers of ϵ. Such an expansion leads to the equations derived by Benney and Roskes (1969) and Davey and Stewartson (1974) and represents a solution valid in the limit $\epsilon kh << 1$. This condition is not satisfied in deep water so an alternative approach is needed. In deep water $g = 1$ thus eliminating the forcing in equation 2.11. However any coherent structure $\alpha\alpha^*$ propagating with the group velocity of the carrier wave will force 2.12 even though the water is deep.

Assume,

$$\alpha\alpha^* = \cos B(X - \tfrac{1}{2}T) \qquad 2.15$$

where B is the length scale of a harmonic component of a wave pocket. The approximation that the modulation propagates at the group velocity introduces the factor 1/2 T. In deep water, this is half the phase velocity and is accurate to order ϵ. A solution to the system 2.11 to 2.14, valid for large ϵkh is

$$\phi_o = \frac{-2\epsilon}{(\sigma_o - \frac{\epsilon B}{4})} \frac{\cosh \epsilon B(z+D)}{\cosh \epsilon BD} \sin B(X-1/2\ T) \qquad 2.16$$

$$\alpha_o = O(\epsilon^2), \qquad \sigma_o = \tanh \epsilon BD \qquad 2.17$$

The message delivered by 2.17 is that in deep water the water level is not affected by the passage of a wave. In shoal water $\sigma_o = \epsilon BD$ and the solution (2.16) reduces to that given by Davey and Stewartson (1974). In shallow water α_o is of order ϵ and sea level is affected by a beat in the waves.

Pressure fluctuations at the sea floor are determined by:

$$P = -\rho\phi_t + \text{nonlinear terms} \qquad 2.18$$

and the nonlinear terms are small compared to $-\rho\phi_t$. Substituting (2.16) into (2.18) and using dimensioned variables the magnitude of the pressure fluctuation at the sea floor is

$$P \sim \frac{\rho 2\epsilon^2 a\omega BC}{\cosh \epsilon Bkh}. \qquad 2.19$$

In deep water, $\epsilon kh > z$, $\sigma_o = 1$ and for any wave packet $\epsilon B/4 << 1$, thus $(\sigma_o - \epsilon B/4)$ is unity.

For example consider a wave whose amplitude is 6 m and whose period is 15 seconds, then $\epsilon = .1$. For a wave packet of 10 waves, B is of order unity. In waters of 4000 m depth $\epsilon Bkh = 7$. Thus although the velocities and pressures associated with the carrier wave are negligible, fluctuations due to the beat of the waves are significant at great depths. At the bottom, pressure fluctuations of about

$$\frac{\delta P}{\rho} \sim .003\ m^2/sec^2 \qquad 2.20$$

can be found. This corresponds to 30 microbars of fluctuations. For a train of modulated waves such that every 10th wave is a large wave the period of the signal at depth is

$$T = \frac{2\pi}{\epsilon kB(\frac{c}{2})} \qquad 2.21$$

which is about 2.5 minutes.

These numbers fit with the few observations of which the author is aware. The main object of this calculation is to show that a modulated wave train is capable of forcing low frequency disturbances that are measurable at the sea floor.

Phillips (1969) presents another type of argument which is useful in interpreting the results of this paper; although care must be taken in applying the averaging scheme introduced by Phillips to this problem. Phillips integrates the vertical momentum equation from the sea floor to the surface. Then on averaging in the horizontal and to a first approximation he finds that bottom pressure fluctuations will have the magnitude.

$$\bar{P} = \rho_o \frac{\partial}{\partial t} \overline{\zeta\omega_o}.$$

Consider the passage of a wave packet. During the time when the wave amplitude is increasing, i.e. subsequent waves are of increasing amplitude then the correlation of $\zeta\omega_o$ increases. The reverse happens during the decay stage of a wave packet. Longuet-Higgens and Stewart, (1963) use an argument based on radiation stress to show that the fluid pressure is least beneath high waves. This is also true in the model presented in this paper. The new feature presented in this paper is the connection between the beat of nonlinear wave trains, and noise of the sea floor.

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