NOTES AND CORRESPONDENCE

Ekman Veering Observed near the Ocean Bottom

PIJUSH K. KUNDU

School of Oceanography, Oregon State University, Corvallis 97331

25 August 1975 and 24 November 1975

ABSTRACT

It has been shown that the phase angle of the complex correlation coefficient is a good measure of the average relative angular displacement (veering) between a pair of two-dimensional vector series. The correlation coefficient between the low-frequency (ω<0.6 cpd) components of the current vectors at 20 m and 5 m heights from the ocean bottom at a station near the Oregon coast reveals an Ekman veering of 6°.

1. Introduction

The characteristics of the bottom boundary layers in the ocean and the atmosphere have been the subject of many studies since the classical work of Ekman showed that, for the stationary horizontally uniform case with constant viscosity, the current vectors must veer counterclockwise looking down. Theoretical studies of the stationary turbulent Ekman layers (e.g., Csanady, 1967; Gill, 1968; Blackadar and Tennekes, 1968), based mostly on similarity arguments, shows that the layer can be divided into inner and outer regions, with a region of overlap in which the velocity profile is logarithmic. The inner layer, including the logarithmic part, behaves like its counterpart in a conventional nonrotating boundary layer on a flat plate, that is, the stress is nearly constant and equal to the wall shear stress τ_0 ($=\rho u_*^2$, where u_* is the friction velocity and ρ the density), and the current vectors do not veer with depth because the Coriolis effects are negligible. The outer layer, the thickness of which is of order u_*/f (f being the Coriolis parameter) has an Ekman-layer-like structure in that the Coriolis forces balance the Reynolds stress terms and the current vectors veer with depth.

The laboratory study of the turbulent Ekman layer by Caldwell *et al.* (1972) shows a veering of order 25°, most of which was above the logarithmic layer. The studies of Weatherly (1972) and of Wimbush and Munk (1970) are probably the two most important systematic observational studies of the characteristics of the bottom boundary layers that have been made in the ocean. Weatherly (1972), during a week-long experiment on the bottom boundary layer of the Florida Current, observed a mean veering of about 10° in the expected sense in the logarithmic layer ($\sim$4 m thick), above which no veering was observed, in contrast to the theoretical arguments. In a later numerical study, Weatherly (1975) found solutions to the nonstationary turbulent Ekman layer with a periodic free stream velocity of one day period, which predicted that most of the veering should occur above the logarithmic layer. He therefore suggested that the discrepancy between his observations and the theoretical-numerical studies may be due to a malfunction of the current meter above the logarithmic layer.

2. The data

The current meter data used were part of the Coastal Upwelling Experiment (CUE-2) carried out near the Oregon coast during the summer of 1973. The currents were measured with Aanderaa current meters suspended beneath subsurface floats. The direction sensors of these meters have been found to be accurate to within a degree. The acquisition and processing of the data are described by Pillsbury *et al.* (1974), while some dynamical characteristics of the flow field during this period are described in Kundu and Allen (1976). Only two moorings of the CUE-2 current meter array had measurements close to the ocean bottom. These are (Fig. 1) *Edelweiss* (measurements at 180, 160, 120, 80, 20 and 5 m from bottom; nominal water depth 200 m) and *Carnation* (measurements at 80, 60, 40, 20 and 5 m from bottom; nominal water depth 100 m). The two deeper current measurements at each of these stations are used in this study. There was no measurement closer than 20 m from the ocean bottom during the CUE-1 experiment, carried out during the previous summer of 1972.

The length of the *Edelweiss* data was 37 days, whereas that of the *Carnation* data was 59 days. The data processing included low-pass filtering the 5 or 10 min observations with a filter having a half-power point of 2 h [120 cycles per day (cpd)] to obtain an hourly time series, and refiltering this to eliminate the tidal and inertial frequencies by means of a filter having a half-power point of 40 h (0.6 cpd). We shall refer to these two series as the "high frequency" ($\omega < 12.0$ cpd) and the "low frequency" ($\omega < 0.6$ cpd) series.

Hydrographic measurements near station *Edelweiss* during this period (Fig. 2) show that the stratification was quite small within 20 m of the ocean bottom.

3. Estimation of veering: Complex correlation coefficient

Since the observed angle between the two current vectors is a function of time, some sort of time averaging is needed for estimating the mean Ekman veering. One method is to find the arithmetic average of the instantaneous veering $\alpha(t)$ between the two vectors 1 and 2 (positive if 2 is counterclockwise of 1):

$$\alpha_{av} = \langle \alpha(t) \rangle = \left\langle \tan^{-1}\frac{v_2}{u_2} - \tan^{-1}\frac{v_1}{u_1} \right\rangle$$

$$= \left\langle \tan^{-1}\frac{u_1 v_2 - v_1 u_2}{u_1 u_2 + v_1 v_2} \right\rangle, \quad (3.1)$$

where u and v are respectively the eastward and northward components of velocity and the angle braces denote arithmetic average in time. However, this simple arithmetic average of (3.1) may not be very meaningful. For example, if there is a small but roughly uniform veering for most of the length of the time series, and some large erratic veering for the remainder, then the erratic behavior will dominate the average. In practice, this large erratic veering occurs under low-speed conditions, i.e., when the direction is not very certain. What is therefore needed is a method which will weight the averaging process according to the magnitude of the instantaneous vectors.

A second method, and the one used by Weatherly (1972), is to take the average veering to be the difference in orientation of the two mean vectors, that is,

$$\alpha_{av} = \tan^{-1}\frac{\langle v_2 \rangle}{\langle u_2 \rangle} - \tan^{-1}\frac{\langle v_1 \rangle}{\langle u_1 \rangle}$$

$$= \tan^{-1}\frac{\langle u_1 \rangle \langle v_2 \rangle - \langle v_1 \rangle \langle u_2 \rangle}{\langle u_1 \rangle \langle u_2 \rangle + \langle v_1 \rangle \langle v_2 \rangle}. \quad (3.2)$$

This method has the advantage that the high-frequency (tides, inertial motions, etc.) oscillations, which are not expected to behave as in a stationary Ekman layer, would cancel out during the process of computing $\langle u \rangle$ and $\langle v \rangle$. However, in geophysical contexts the "mean" has to be interpreted as an appropriately de-

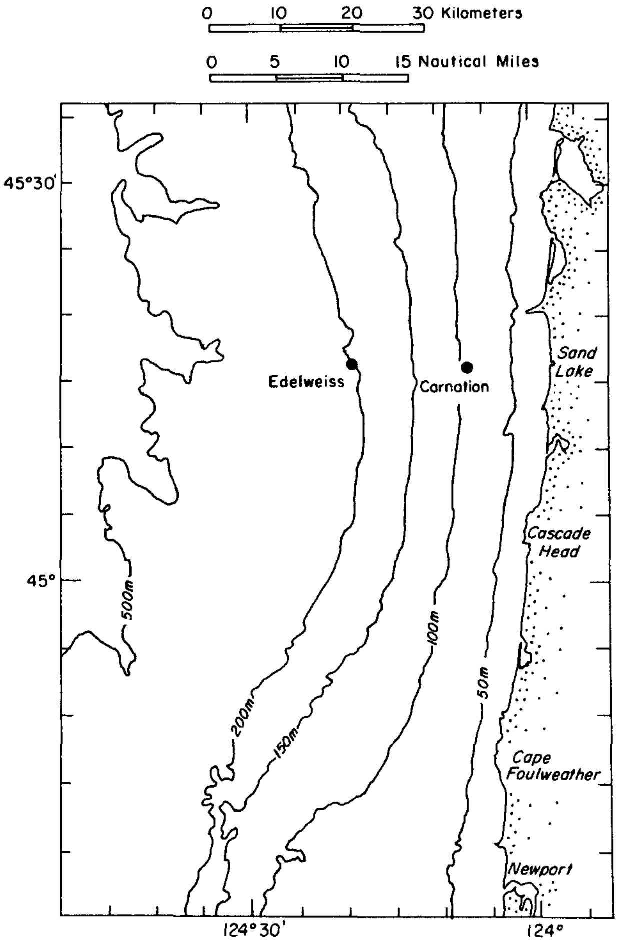

FIG. 1. Locations of current meter moorings *Edelweiss* and *Carnation*.

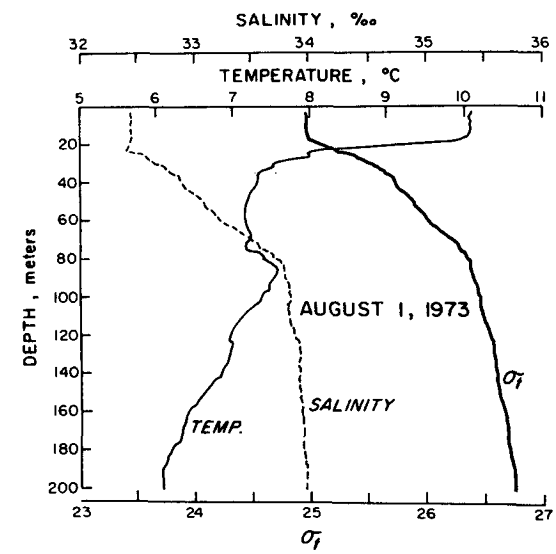

FIG. 2. Vertical profiles of σ_t, salinity and temperature at *Edelweiss* on 1 August 1973 [Cast No. 166 of Holbrook and Halpern (1974)].

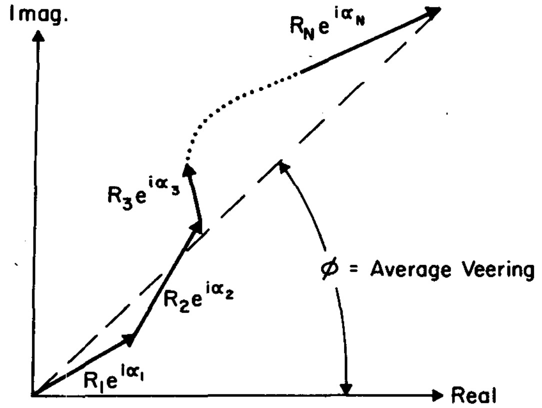

FIG. 3. Geometric interpretation of the phase angle of the complex correlation coefficient.

fined low-frequency component. In the present case, the time scale in the outer parts of a stationary turbulent Ekman layer is of order f^{-1} (Tennekes and Lumley, 1972, p. 12; Wimbush and Munk, 1970), whereas that in the inner layer is much smaller. Therefore, the low-frequency components having time scales appreciably larger than f^{-1} ($\sim$17 h in this region), for example the fluctuations left after removing the tides and inertial motions from the data, are expected to be Ekman-like. However, scheme (3.2) is evidently appropriate only for the very low frequency components whose time mean over the length of the record is appreciably different from zero. It fails, for example, in a case where the two series have a certain veering for all t, but $\langle u \rangle$ and $\langle v \rangle$ are nearly zero for both. Second, it is not easy to determine $\langle u \rangle$ and $\langle v \rangle$ with any reasonable accuracy in a case where fluctuations are much larger than the mean, as is frequently the case in oceanography, and a determination of the direction of the mean motion is even more uncertain. In the present region $\langle u \rangle$ is typically ± 1 cm s^{-1}, whereas the standard deviation of u is typically an order of magnitude larger.

In this paper a third method of finding the average angular displacement between a pair of two-dimensional vectors is proposed. This is finding the phase angle of the complex correlation coefficient between the two vector series. Let

$$w(t)=u(t)+iv(t) \tag{3.3}$$

be the complex representation of the two-dimensional horizontal velocity vector at time t, where $i=\sqrt{-1}$. Then the complex correlation coefficient between two vector series 1 and 2 is defined as their normalized inner product, namely

$$\rho=\frac{\langle w_1^*(t)w_2(t)\rangle}{\langle w_1^*(t)w_1(t)\rangle^{\frac{1}{2}}\langle w_2^*(t)w_2(t)\rangle^{\frac{1}{2}}}, \tag{3.4}$$

where the asterisk indicates the complex conjugate. The quantity ρ, which is independent of the choice of the coordinate system, is a complex number whose magnitude (<1 because of the Schwartz inequality) gives the overall measure of correlation and whose phase angle gives the average counterclockwise angle of the second vector with respect to the first. For example, if the two time series are such that $w_2 = Cw_1 \exp(i\phi)$ at all times, where C and ϕ are real constants, then $\rho=\exp(i\phi)$. The phase, of course, is meaningful only if the magnitude of the correlation is high.

A geometric interpretation of (3.3) is illuminating. The instantaneous product $w_1^*(t)w_2(t)$ can be written as $R(t)\exp[i\alpha(t)]$, where $R(t)$ is the product of the instantaneous magnitudes of the velocity vectors and $\alpha(t)$ their instantaneous veering, so that (3.4) gives

$$\rho=c\sum_{j=1}^{N} R_j \exp(i\alpha_j). \tag{3.5}$$

Here $c=1/[N\langle w_1^*w_1\rangle^{\frac{1}{2}}\langle w_2^*w_2\rangle^{\frac{1}{2}}]$ is a real constant and the subscript j refers to time t_j. Thus, the sum of the vectors $R_j\exp(i\alpha_j)$ at times t_j define the phase of the complex correlation coefficient (Fig. 3). Therefore, *estimating the mean veering by this method weights the averaging process according to the magnitude of the instantaneous vectors.*

In terms of the east-north components, (3.3) can be written as

$$\rho=\frac{\langle u_1u_2+v_1v_2\rangle}{\langle u_1^2+v_1^2\rangle^{\frac{1}{2}}\langle u_2^2+v_2^2\rangle^{\frac{1}{2}}}+i\frac{\langle u_1v_2-u_2v_1\rangle}{\langle u_1^2+v_1^2\rangle^{\frac{1}{2}}\langle u_2^2+v_2^2\rangle^{\frac{1}{2}}}, \tag{3.6}$$

so that phase angle (average veering) is

$$\alpha_{av}=\tan^{-1}\frac{\langle u_1v_2-v_1u_2\rangle}{\langle u_1u_2+v_1v_2\rangle}, \tag{3.7}$$

which should be compared with (3.1) and (3.2).

4. Results

The above method is now used to estimate the veering of the bottom currents at *Edelweiss* and *Carnation.* Some statistics of the low-frequency components of the currents at 5 and 20 m heights are listed in Table 1. The positive angles signify that the 5 m current

Note that a mean veering could also result simply from stratification without any friction. If the mean flow had positive eastward and northward components, then an upward-toward-the-coast slope of the isopycnals, as observed in this region (see, e.g., Kundu *et al.*, 1975), would cause an increase of the northward mean as one goes deeper simply by the "thermal wind" balance, resulting in a veering in the Ekman sense. However, the mean eastward flow at *Edelweiss* is negative (Table 1), and that at *Carnation* is positive but small. Thus, the effect of this mechanism would be to *oppose* the Ekman veering at *Edelweiss*, and would be negligible at *Carnation.*

The very large magnitude of the correlation (~0.97) signifies that the phase difference is fairly uniform throughout most of the record. The speed and direction of the two currents at *Edelweiss* are shown in Fig. 4, where the decrease of the speed and the veering of the direction of the current as one approaches the bottom are evident. Although the veering is of the proper sign throughout most of the record, it does show the "wrong" behavior at a few points (28 July, 1, 15, 20 August) when the speed was rather low, as was also observed by Weatherly (1972). However, this did not substantially affect the present estimate because of the proper weighting resulting from the correlation method.

Veering was also examined using the high-frequency series. At both stations the correlation coefficient was found to be roughly $0.90\exp(-i3°)$. The correlation magnitude of 0.9, although seemingly high, is actually not high enough to mean that the veering was approximately uniform throughout the record. In fact, a plot of the directions of these high-frequency series revealed that the veering changed erratically, both in sign and magnitude, throughout the records at both stations; the fairly large value of 0.9 was only a result of the fact that the two vector tips were near each other most of the time. This erratic veering of the high-frequency series suggests a behavior unlike that of stationary Ekman layers, which is expected because only fluctuations of period larger than f^{-1} are expected to be Ekman-like.

Accurate estimation of the friction velocity and the depths of the logarithmic and outer layers is not possible in the present case because of a lack of sufficient vertical resolution of the data. However, a crude order of magnitude estimation is possible. The mean "geostrophic" or free stream velocity V_g, taken to be the mean speed at 20 m at *Edelweiss*, is about 10 cm s^{-1} (at *Carnation* it is nearly the same). The ratio u_*/V_g is a weak function of the surface Rossby number

TABLE 1. Statistics of low-frequency components (ω<0.6 cpd).

	Edelweiss	*Carnation*
$\langle u \rangle$ 20 m (cm s^{-1})	−1.8	1.1
$\langle u \rangle$ 5 m (cm s^{-1})	−1.7	0.6
$\langle v \rangle$ 20 m (cm s)	7.3	2.9
$\langle v \rangle$ 5 m (cm s)	5.5	2.5
Arithmetic average veering [Eq. (3.1)]	7.5°	0.9°
Difference of orientation of mean vectors [Eq. (3.2)]	3.5°	6.9°
Complex correlation [Eq. (3.7)]	$0.97\exp(i6.3°)$	$0.98\exp(i1.6°)$

veers counterclockwise with respect to the 20 m current, looking down.

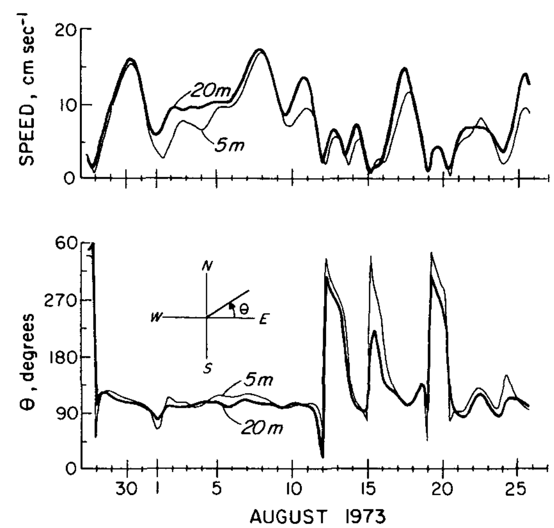

FIG. 4. Time series of speed and direction of the low-frequency components of the currents at 20 and 5 m heights at *Edelweiss*. Note the decrease of speed near the ocean bottom, and the consistent relative angular displacement (veering) of the two currents.

$Ro=V_g/fz_0$ (where z_0 is a measure of the surface roughness), u_*/V_g decreasing from 0.05 to 0.02 as Ro increases from 10^5 to 10^{10} (Monin, 1970). For the oceanic case Ro$\sim10^6$ to 10^7, for which $u_*/V_g\sim0.04$ (Weatherly, 1972), so that $u_*\sim0.4$ cm s^{-1}. The bottom stress is, therefore, $\tau_0\sim0.2$ dyn cm^{-2}, which is an order of magnitude smaller than the surface wind stresses observed in this region. The thickness of the logarithmic layer is $\sim2u_*^2/fV_g\sim3$ m, whereas the thickness of the entire frictional layer is $\sim0.4u_*/f\sim16$ m. The veering that has been observed between the 5 and 20 m heights therefore seems to be above the logarithmic layer.

The above estimates agree well with those of Caldwell (1976), based on measurements in this area in 1974. Estimates of other quantities like the dissipation rate, the Kolmogoroff microscales, and eddy diffusivities can also be found there.

Although no Ekman-like veering is to be expected above the 20 m height, a check was made to see if there was a consistent veering between the current vectors at 20 and 40 m heights. However, only erratic veering was found both at *Edelweiss* and *Carnation* for these interior currents, suggesting that this was not a frictional effect. It also adds more evidence to the suggestion that the consistent veering near the bottom is frictionally induced.

5. Summary and concluding remarks

A complex correlation coefficient has been defined, whose phase angle is a good measure of the average angular displacement between a pair of two-dimensional vector series. For the detection of stationary Ekman-like characteristics of bottom boundary layers, frequencies comparable to and higher than f should be removed from the data. A veering in the correct sense of about 6° has been observed between the current vectors at 20 and 5 m heights at station *Edelweiss*, whereas at *Carnation* it was observed to be about 2°. The veering seems to be above the logarithmic layer, whose thickness is estimated to be about 3 m. The total thickness of the frictional layer seems to be about 16 m, and the friction velocity is about 0.4 cm s^{-1}.

The accurate temperature measurements in this region in the recent work of Caldwell (1976) shows that the temperatures were uniform to within a few millidegrees in the bottom 10 m, beyond which they frequently changed abruptly, resembling a discontinuity. Caldwell (private communication) believes that Ekman veerings may be possible in certain cases in coastal waters where there are large currents not generated by the tides, but this veering is more of a discontinuous nature, with the upper layers sliding over the bottom homogeneous layer at a small angle.

Although the consistent behavior observed in this work strongly suggests a case of Ekman veering, the issue of whether there is any frictional veering in the real ocean is by no means closed. A check on the results of the accuracy tests made on the direction sensors of the current meters concerned revealed no evidence of any systematic bias or any other irregularity. However, instrumental errors can never be ruled out. It is felt that further careful experiments in the oceanic bottom layers are needed to answer some of these basic and interesting questions.

Acknowledgments. This research was supported by the Coastal Upwelling Ecosystems Analysis program (CUEA) of the International Decade of Oceanography office (IDOE) of the National Science Foundation under Grant ID071-0411.

The author is indebted to Dr. John Allen for helpful discussions, and to Drs. Georges Weatherly, Christopher Mooers, Harry Bryden and Douglas Caldwell for comments on the manuscript.

Note added in proof. More evidence of Ekman veering can be found in:

1) Wunsch, D., and R. Hendry, 1972: Array measurements of the bottom boundary layer and the internal wave field on the continental slope. *Geophys. Fluid Dyn.*, **4**, 101–145.

2) Smith, J. D., and C. E. Long, 1976: The effect of turning in the bottom boundary layer on continental shelf sediment transport. *Mem. Soc. Roy. Sci. Liege*, **8** (in press).

REFERENCES

Blackadar, A. K., and H. Tennekes, 1968: Asymptotic similarity in neutral barotropic planetary boundary layers. *J. Atmos. Sci.*, **25**, 1015–1020.

Caldwell, D. R., 1976: Fine-scale temperature structure in the bottom mixed-layer on the Oregon shelf (submitted to *Deep Sea Res.*).

——, C. W. Van Atta and K. N. Helland, 1972: A laboratory study of the turbulent Ekman layer. *Geophys. Fluid Dyn.*, **3**, 125–160.

Csanady, G. T., 1967: On the "resistance law" of a turbulent Ekman layer. *J. Atmos. Sci.*, **24**, 467–471.

Holbrook, J. R., and D. Halpern, 1974: STD measurements off the Oregon coast during July/August 1973. CUEA Data Report 12.

Gill, A. E., 1968: Similarity theory and geostrophic adjustment. *Quart. J. Roy. Meteor. Soc.*, **94**, 586–588.

Kundu, P. K., J. S. Allen and R. L. Smith, 1975: Modal decomposition of the velocity field near the Oregon coast. *J. Phys. Oceanogr.*, **5**, 683–704.

——, and ——, 1976: Some three-dimensional characteristics of the low-frequency current fluctuations near the Oregon coast. *J. Phys. Oceanogr.*, **6**, 111–111.

Monin, A. S., 1970: The atmospheric boundary layer. *Ann. Rev. Fluid Mech.*, Vol. 2, 225–250.

Pillsbury, R. D., J. S. Bottero, R. E. Still and W. E. Gilbert, 1974: A compilation of observations from moored current meters. Vol. VII, Ref. 74-7, School of Oceanography, Oregon State University.

Tennekes, H., and J. L. Lumley, 1972: *A First Course on Turbulence*. The MIT Press, 300 pp.

Weatherly, G. L., 1972: A study of the bottom boundary layer of the Florida Current. *J. Phys. Oceanogr.*, **2**, 54–72.

——, 1975: A numerical study of time-dependent turbulent Ekman layers over horizontal and sloping bottoms. *J. Phys. Oceanogr.*, **5**, 288–299.

Wimbush, M., and W. Munk, 1970: The benthic boundary layer. *The Sea*, Vol. **4**, Part 1, Wiley, Chap. 19.