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## Implementation and modification of a three-dimensional radiation stress formulation for surf zone and rip-current applications

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#### ABSTRACT

Regional Ocean Modeling System (ROMS v 3.0), a three-dimensional numerical ocean model, was previously enhanced for shallow water applications by including wave-induced radiation stress forcing provided through coupling to wave propagation models (SWAN, REF/DIF). This enhancement made it suitable for surf zone applications as demonstrated using examples of obliquely incident waves on a planar beach and rip current formation in longshore bar trough morphology (Haas and Warner, 2009). In this contribution, we present an update to the coupled model which implements a wave roller model and also a modified method of the radiation stress term based on Mellor (2008, 2011a,b,in press) that includes a vertical distribution which better simulates non-conservative (i.e., wave breaking) processes and appears to be more appropriate for sigma coordinates in very shallow waters where wave breaking conditions dominate. The improvements of the modified model are shown through simulations of several cases that include: (a) obliquely incident spectral waves on a planar beach; (b) obliquely incident spectral waves on a natural barred beach (DUCK'94 experiment); (c) alongshore variable offshore wave forcing on a planar beach; (d) alongshore varying bathymetry with constant offshore wave forcing; and (e) nearshore barred morphology with rip-channels. Quantitative and qualitative comparisons to previous analytical, numerical, laboratory studies and field measurements show that the modified model replicates surf zone recirculation patterns (onshore drift at the surface and undertow at the bottom) more accurately than previous formulations based on radiation stress (Haas and Warner, 2009). The results of the model and test cases are further explored for identifying the forces operating in rip current development and the potential implication for sediment transport and rip channel development. Also, model analysis showed that rip current strength is higher when waves approach at angles of 5° to 10° in comparison to normally incident waves.

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#### 1. Introduction

Wave-induced circulation in the nearshore has been the subject of a number of experimental studies over the last 50 years. Theoretical and analytical studies were initiated in the 60s and 70s with the works of Longuet-Higgins and Stewart (1964), Longuet-Higgins (1970) and Bowen (1969). These theories were later incorporated in numerical models that have been developed in the last 20 years. Such models are predominantly phase-averaged operating in 1-D (across the surf) or 2-D (assuming uniform along-coast bathymetry and depth-integrated). They solve the depth averaged Navier Stokes equation focusing on either simulating the development of longshore currents (Church and Thornton, 1993; Stive and De Vriend, 1994; Feddersen et al., 1998; Ruessink et al., 2001), or rip current circulation (e.g., Yu and Slinn, 2003;

Reniers et al., 2004). Phase resolving 2-D Boussinesq models (e.g., Chen et al., 1999), although considered to be more comprehensive in modeling wave evolution in the nearshore, are computationally expensive, and their use is limited at present. Lately, point-vortex models (Terrile and Brocchini, 2007; Kennedy et al., 2006) have been also used to study generation, maintenance and advection of breaking wave induced vortices which are associated with the formation of rip currents.

Overall 1-D and 2-D models provide useful information about circulation patterns but are intrinsically not able to resolve threedimensional dynamics. It is imperative to resolve the three dimensional circulation to fully investigate such processes as circulation dynamics for nearshore water quality applications, transport into and out of the surf zone, sediment transport dynamics, coastal erosion and morphodynamics. In order to fill this need, initially quasi threedimensional models like SHORECIRC (Svendsen et al., 2002) were developed. These models have been previously applied to study rip currents (Haas et al., 2003) and surf beat phenomena (van Dongeren et al., 1995) in nearshore environments. Lately, full three-dimensional wave-current coupled models have been developed and implemented

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in the coastal ocean extending their application to the wave breaking dominated environment of the surf zone. DELFT 3D flow model uses an approximate Generalized Lagrangian Mean (GLM) approach (Groeneweg and Klopman, 1998) to associate wave effects on mean currents. Wave forcing in form of dissipation of wave energy is applied as shear stress on the water surface, consistent with simplified formulations presented by Dingemans et al. (1987), as implemented by Walstra et al. (2000) and Lesser et al. (2004). Newberger and Allen (2007a,b) added wave forcing in the form of surface stress and body forces in the Princeton Ocean Model (POM), which has evolved as "Nearshore POM". Using the vortex force formalism method described in McWilliams et al. (2004), Uchiyama et al. (2010) (hereafter referred to as U10) compares model simulations (using UCLA-ROMS, see Table 1 in Shchepetkin and McWilliams, 2009) to field observations from a barred beach environment.

Mellor (2003, 2005) (hereafter referred to as M03 and M05, respectively) described depth dependent formulation for radiation stress terms, which has been implemented in publicly available version of ROMS (Rutgers-ROMS, see Table 1 in Shchepetkin and McWilliams, 2009) by Warner et al. (2008, hereafter referred to as W08). This has been used to study oblique incidence of waves on a planar beach and rip currents formed on alongshore bar trough morphology (Haas and Warner, 2009; hereafter referred to as HW09). Following Ardhuin et al.'s (2008) remarks, Mellor (2008) modified his original formulation and provided a new approach for depth dependent radiation stresses to alleviate the creation of erroneous gradients of mean currents for unforced wave conditions. Bennis and Ardhuin (in press) argued that the depth-dependent equations for radiations stress presented in Mellor (2008) when vertically integrated does not yield the expected depth averaged radiation stress formulations presented by Longuet-Higgins and Stewart (1964) and Phillips (1977). Subsequently, Mellor (2011a, in press) argued that this discrepancy occurs due to difficulty in correct representation of vertical boundary conditions in Cartesian coordinates and he suggested implementation of his method using sigma coordinate equations. The equations are now provided in a sigma coordinate system along with exclusion of some incorrect vertical gradient terms in Eqn. 51, M03 (for details see Mellor, in press-b).

In this contribution, the Mellor (2008) method including the updates presented in Mellor (2011, in press-a,b) have been implemented in ROMS (hereafter referred to as M08-11) and evaluated for an idealized situation of non-breaking and shoaling waves on a steep sloping topography (see Appendix A). These results do identify the generation of "unexpected" flow structure as in Bennis and Ardhuin (in press) although Mellor (in press-a) suggests this flow "...must be a surface contribution to the phase averaged momentum equation due to wave's intrinsic surface pressure field". Despite the ongoing scientific debate, the Mellor (2008) formulation without the updates presented in Mellor (2011, in press-a,b) is still being widely used to study wave induced flow through various models (e.g., FVCOM, Wang and Shen, 2010; CH3D, Sheng and Liu, 2011). Furthermore, the current version of ROMS publicly available at https://www.myroms.org/ still utilizes Mellor (2005). This contribution presents a modified M08-11 formulation which reduces the discrepancy in the flow structure, and we contend makes the formulation useful for engineering applications in a surf zone setting. To support this we also provide both qualitative and quantitative comparisons for three and two dimensional flow fields corresponding to conditions favorable for the development of longshore currents and rip current cell circulation (see below).

The objectives of this contribution are to provide to the community with an independent and comprehensive assessment of the performance of the Mellor (2008, 2011, in press-a,b) approach under realistic conditions and to present the implementation of a modified M08-11 formulation that includes changes in the vertical distribution of radiation stress to account for shallow water, into the

ROMS model. The performance of the new implementation is evaluated using 5 study cases. These consist of: (i) obliquely incident waves on a planar beach; (ii) obliquely incident waves on a natural barred beach (DUCK'94 experiment); (iii) uniform nearshore bathymetry with alongshore varying wave forcing; (iv) alongshore varying bathymetry with constant offshore forcing; and (v) nearshore barred morphology with rip-channels.

The outline of the paper is as follows. Modifications to the model are presented in Section 2, together with the results for the case of obliquely incident waves on a planar beach (Case 1) and barred beach (Case 2), respectively. Section 3 presents the results of the numerical experiments for the alongshore variable forcing, alongshore varying bathymetry and nearshore barred morphology with rip-channels (Cases 3, 4, and 5, respectively). The model results are compared to existing analytical solutions (Bowen, 1969), numerical solutions (Noda, 1974) and laboratory studies (Haas and Svendsen, 2002; Haller et al., 2002). Section 4 discusses the results with main emphasis on the effect of wave angle of approach to the development of ripcurrents as it is revealed through the numerical experiments and some implications for model applications related to morphodynamic development. Finally, the conclusions are presented in Section 5.

#### 2. Implementation of updated forcings

ROMS is a three dimensional, free surface, topography following numerical model, which solves finite difference approximations of Reynolds Averaged Navier Stokes (RANS) equations using hydrostatic and Boussinesq approximations with a split-explicit time stepping algorithm (Haidvogel et al., 2008; Shchepetkin and McWilliams, 2005; Shchepetkin and McWilliams, 2009). ROMS includes several options for certain model components, such as various advection schemes (second, third and fourth order), turbulence closure models (e.g., Generic Length Scale mixing, Mellor-Yamada, Brunt-Väisälä frequency mixing, user provided analytical expressions, K-profile parameterization), boundary conditions, etc. As Shchepetkin and McWilliams (2009) state, currently there are four variations of ROMSfamily codes. In this contribution we use Rutgers University ROMS which was first introduced by Haidvogel et al. (2000) and subsequently any reference to ROMS denotes this particular version.

Warner et al. (2008) improved ROMS for nearshore applications through the incorporation of the M03 and M05 radiation stress forcing methods. The model equations were presented inW08 in Cartesian coordinates (x, y, s) based on the equations originally given by Haidvogel and Beckmann (1999) and Haidvogel et al. (2008). Recently these formulations have been commented on by Shchepetkin and McWilliams (2009) who presented clarifications to the model formulations. For completeness and to avoid confusion, we elected to present the equations in horizontal, orthogonal curvilinear and vertical terrain following coordinates ( $\xi$ ,  $\eta$ , s) following the definitions and notations of Shchepetkin and McWilliams (2009).

#### 2.1. ROMS equation of motion

The horizontal momentum equations are given as:

$$\begin{aligned} \frac{\partial}{\partial t} \left( \frac{H_z u'}{mn} \right) &+ \frac{\partial}{\partial \xi} \left( \frac{u' H_z u'}{n} \right) + \frac{\partial}{\partial \eta} \left( \frac{v' H_z u'}{m} \right) + \frac{\partial}{\partial s} \left( \frac{w_s u'}{mn} \right) \\ &- \left[ \left( \frac{f}{mn} \right) + v' \frac{\partial}{\partial \xi} \left( \frac{1}{n} \right) - u' \frac{\partial}{\partial \eta} \left( \frac{1}{m} \right) \right] H_z v' \\ &= - \frac{H_z}{n} \left( \frac{1}{\rho_0} \frac{\partial P}{\partial \xi} \Big|_z \right) - \frac{\partial}{\partial s} \left( \overline{u' w'} - \frac{v}{H_z} \frac{\partial u^e}{\partial s} \right) - \frac{\partial}{\partial \xi} \left( \frac{H_z S_{\xi\xi}}{mn} \right) - \frac{\partial}{\partial \eta} \left( \frac{H_z S_{\eta\xi}}{mn} \right) \\ &+ \frac{H_z}{mn} (F_u + D_u) \end{aligned}$$
(1)

$$\begin{aligned} \frac{\partial}{\partial t} \left( \frac{H_z v^l}{mn} \right) &+ \frac{\partial}{\partial \xi} \left( \frac{u^l H_z v^l}{n} \right) + \frac{\partial}{\partial \eta} \left( \frac{v^l H_z v^l}{m} \right) + \frac{\partial}{\partial s} \left( \frac{w_s u^l}{mn} \right) \\ &+ \left[ \left( \frac{f}{mn} \right) + v^l \frac{\partial}{\partial \xi} \left( \frac{1}{n} \right) - u^l \frac{\partial}{\partial \eta} \left( \frac{1}{m} \right) \right] H_z u^l \\ &= -\frac{H_z}{n} \left( \frac{1}{\rho_0} \frac{\partial P}{\partial \eta} \Big|_z \right) - \frac{\partial}{\partial s} \left( \overline{v' w'} - \frac{v}{H_z} \frac{\partial v^e}{\partial s} \right) - \frac{\partial}{\partial \xi} \left( \frac{H_z S_{\xi\eta}}{mn} \right) - \frac{\partial}{\partial \eta} \left( \frac{H_z S_{\eta\eta}}{mn} \right) \\ &+ \frac{H_z}{mn} (F_v + D_v) \end{aligned}$$

$$-\frac{\partial P}{\partial \eta}\Big|_{z} = -gp|_{s=0}\frac{\partial \varsigma}{\partial \eta} - g\int_{s}^{0}\frac{\partial \rho}{\partial \eta}H_{z}ds$$

$$-\frac{\partial P}{\partial \xi}\Big|_{z} = -gp|_{s=0}\frac{\partial \varsigma}{\partial \xi} - g\int_{s}^{0}\frac{\partial \rho}{\partial \xi}H_{z}ds$$
(3)

where  $m^{-1}$  and  $n^{-1}$  are Lamé metric coefficients; u and v are the mean components of velocity in the horizontal ( $\xi$  and  $\eta$ ) directions, respectively; subscripts l and e define Lagrangian and Eulerian velocity;  $w_s$  is the mean component of the vertical velocity in the vertical (s) direction. Note that no vertical Stokes velocity is defined in the M08-11 method. At this juncture it is important to point out that the contribution of radiation stress in Eqs. (1) and (2) is implemented on the basis of Mellor (in press-b) and does not contain the incorrect terms corresponding to vertical gradient of radiation stress (see M03, M08 and Mellor, in press-b).

#### 2.2. Continuity equation and scalar transport

The continuity equation is written as:

$$\frac{\partial}{\partial t} \left( \frac{H_Z}{mn} \right) + \frac{\partial}{\partial \xi} \left( \frac{H_Z u^l}{n} \right) + \frac{\partial}{\partial \eta} \left( \frac{H_Z v^l}{m} \right) + \frac{\partial}{\partial s} \left( \frac{w_s}{mn} \right) = 0 \tag{4}$$

while the scalar transport is given by:

$$\frac{\partial}{\partial t} \left( \frac{H_Z C}{mn} \right) + \frac{\partial}{\partial \xi} \left( \frac{u' H_Z C}{n} \right) + \frac{\partial}{\partial \eta} \left( \frac{v' H_Z C}{m} \right) + \frac{\partial}{\partial s} \left( \frac{w_s C}{mn} \right)$$

$$= -\frac{\partial}{\partial s} \left( c' w' - \frac{v_{\theta}}{H_z} \frac{\partial C}{\partial s} \right) + C_{src}$$
(5)

finally, it should be noted that the Lagrangian velocity is related to Eulerian velocity  $(u^e)$  and Stokes drift  $(u^s)$  as:

 $u^l = u^e + u^s. ag{6}$ 

The Stokes velocities are computed based on wave properties provided by the wave model. To solve the system of Eqs. (1)-(5), the Lagrangian velocities  $(u^l)$  are replaced with their equivalent Eulerian and Stokes summation  $(u^e + u^s)$ . The advective and local acceleration terms corresponding to Stokes drift are moved to the right hand side of momentum balance equation and computed in a manner similar to a purely Eulerian advection. The ROMS model therefore solves for the Eulerian velocity as the prognostic variable. The vertical sigma coordinates vary from -1 at the bottom to 0 at the free surface; z is the vertical coordinate positive upwards with z = 0 at mean sea level;  $\zeta$  is the wave-averaged sea surface elevation;  $D(=h+\zeta)$  is the total water depth while *h* is the depth below mean sea level of the sea floor;  $H_z$  is the grid cell thickness; and f is the Coriolis parameter. Overbar indicates time average, while prime (') indicates a fluctuating turbulent quantity. Pressure is P;  $\rho$  and  $\rho_0$  are total and reference densities of sea water; g is the acceleration due to gravity;  $\nu$  and  $\nu_{\theta}$  are molecular viscosity and diffusivity;  $F_u$  and  $F_v$  are forcing terms (e.g., wind stress and thermal forcing, etc.); *C* represents a tracer quantity;

 $C_{src}$  are tracer source/sink terms; finally,  $D_u$  and  $D_v$  are diffusive terms (viscosity and diffusion) explained in details in the ROMS user guide (wikiROMS, www.myroms.org). For Cartesian coordinates (x, y and s), Lamé metric coefficients are unity and the curvilinear terms ( $v_l\partial/\partial\xi(1/n)$ ) reduce to zero.

These equations are closed by parameterization of the Reynolds stresses and turbulent tracer fluxes as:

$$\overline{u'w'} = -K_M \frac{\partial u^e}{\partial z}; \overline{v'w'} = -K_M \frac{\partial v^e}{\partial z}; \overline{c'w'} = -K_M \frac{\partial \rho}{\partial z}$$
(7)

where  $K_M$  is the eddy viscosity for momentum and  $K_H$  is the eddy diffusivity.

#### 2.3. Radiation stress formulations

In this section we discuss the radiation stress formulation presented by Mellor (2008, 2011, in press-b). Ardhuin et al. (2008) pointed out that the implementation of depth dependent radiation stress equations described by M03 and M05 is not accurate and it requires inclusion of higher order wave kinematics. Mellor (2008) attempted to address these issues and developed a modification to his original formulation for the radiation stress tensor which is given as:

$$S_{\alpha\beta} = kE\left(\frac{k_ak_\beta}{k^2}F_{\rm CS}F_{\rm CC} - \delta_{\alpha\beta}F_{\rm SC}F_{\rm SS}\right) + \delta_{\alpha\beta}E_D \tag{8}$$

where, k is the wave number and E is the wave energy, while the parameter F denotes the vertical distribution defined as:

$$F_{SS} = \frac{\sinh kD(s+1)}{\sinh kD}; F_{CS} \frac{\cosh kD(s+1)}{\sinh kD}$$

$$F_{SC} = \frac{\sinh kD(s+1)}{\cosh kD}; F_{CC} \frac{\cosh kD(s+1)}{\cosh kD}$$

$$\delta_{\alpha\beta} = \begin{pmatrix} 1 & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta \end{pmatrix};$$

$$E_{D} = 0 & \text{if } z \neq \zeta; \int_{-h}^{\zeta} E_{D} dz - E/2$$
(9)

As described in Mellor (2008), "in a finite difference rendering of  $E_{D}$ , the top vertical layer of incremental size  $\delta z$  and only the top layer would be occupied by  $\partial E_D/\partial \xi = (\delta z)^{-1} \partial (E/2)/\partial \xi$ " (hereafter this formulation is referred to as M08-11<sub>top</sub>). In the present contribution it is observed that on application of  $E_D$  as a surface stress or as a contribution at the first vertical cell, strong offshore advection occurs at the transition zone from inner shelf to surf zone, where wave shoaling occurs. This offshore advection in the wave shoaling zone occurs due to a positive gradient of cross-shore radiation stress  $(\partial S_{xx}/\partial x)$  at the surface layer, which is significantly reduced on implementing  $E_D$  as surface intensified body force (Section 2.5, Case 1). Furthermore and equally if not more importantly, the term  $E_D$  is a function of wave energy, and contains both conservative and non-conservative wave effects, which should have different vertical dependencies. Based on this understanding, U10 presented two different penetration scales for the conservative (Vortex Force and Bernoulli Head) and non-conservative (wave breaking) wave effects. In order to avoid aforementioned deficiencies in  $E_D$  formulation for application in shallow waters, we vertically distribute the forcing using a function  $(F_{ED})$  with a length that scales with the root mean square wave height  $(H_{rms})$ . We choose a distribution similar to type-III of U10:

$$F_{ED} = \frac{FB}{\int_{-h}^{\zeta} FB \cdot dz}$$
(10)  
$$FB = \cosh\left(\frac{2\pi}{H_{rms}}D(s+1)\right)$$

so that Eq. (8) is implemented as:

$$S_{\alpha\beta} = kE \left( \frac{k_{\alpha}k_{\beta}}{k^2} F_{CS}F_{CC} - \delta_{\alpha\beta}F_{SC}F_{SS} \right) + \delta_{\alpha\beta}\frac{E}{2}F_{ED}$$
(11)

and hereafter referred to as  $M08-11_{vrt}$ . In this contribution our work is focused on wave current interaction in the surf zone where the non-conservative wave effects are dominant, which justifies the modification presented in Eq. (10).

The wave fields required to compute the radiation stress terms are provided by SWAN (Booij et al., 1999), a third generation, phase averaged, wave propagation model, which conserves wave action density (energy density divided by relative frequency). The details of coupling ROMS to SWAN have been provided in W08 and are not discussed further in here.

#### 2.4. Wave roller model

In addition to the radiation stress term, spatial distribution of wave energy is affected by wave breaking processes. This is usually incorporated through the inclusion of wave rollers (e.g., Ruessink et al., 2001) that modifies the radiation stress and associated longshore and cross-shore velocities. In our application of ROMS, the roller energy ( $E_r$ ) is distributed using a vertical distribution function  $R_z$  that is added to radiation stress distribution so that Eq. (11) becomes:

$$S_{\alpha\beta} = kE \left( \frac{k_{\alpha}k_{\beta}}{k^2} F_{\rm CS}F_{\rm CC} - \delta_{\alpha\beta}F_{\rm SC}F_{\rm SS} \right) + \delta_{\alpha\beta} \frac{E}{2} F_{\rm ED} + \frac{k_{\alpha}k_{\beta}}{k} E_r R_z.$$
(12)

W08 calculated  $E_r$  using an empirical formulation based on Svendsen (1984) which depends on the fraction of breaking waves ( $Q_b$ ), significant wave height ( $H_{sig}$ ), roller area ( $A_r$ ), wave propagation speed (c) and wavelength (L).

Nairn et al. (1990) suggested that wave dissipation due to depth induced breaking contributes to creation of wave rollers. This sink term when related to local wave parameters, can be used to determine  $E_r$ . The wave rollers in this mechanism act as a storage of dissipated energy causing a lag effect in momentum transfer. This kind of surface roller model, evolving in time is termed as Roller Evolution Model (Reniers et al., 2002, 2004) and is based on the equation for roller action density  $A^r$  which is given by:

$$\frac{\partial \mathcal{A}^{r}}{\partial t} + \nabla \cdot \left( \mathcal{A}^{r} \overrightarrow{c} \right) = \frac{\alpha_{r} \varepsilon_{b} - \varepsilon_{r}}{\sigma}$$
(13)

where  $A^r$  is related to  $E_r$  as:

$$E_r = \mathcal{A}^r \sigma \tag{14}$$

and,

$$\vec{c} = u + \sigma k^2 \vec{k} \tag{15}$$

where *c* is the phase speed of the primary wave; *u* is the mean velocity;  $k (=2\pi/L)$  is the wave number; and  $\varepsilon_b$  is the wave dissipation due to wave breaking. The latter can be obtained directly from the wave model (SWAN) or can be empirically calculated externally as (Church and Thornton, 1993):

$$\epsilon^{b} = \frac{3\sqrt{\pi}}{16}\rho_{0}g\frac{B_{b}^{3}f_{p}}{D}H_{rms}^{3}\left[1 + tanh\left\{8\left(\frac{H_{rms}}{\gamma D} - 1\right)\right\}\right]\left[1 - \left\{1 + \left(\frac{H_{rms}}{\gamma_{b}D}\right)^{2}\right\}^{-\frac{3}{2}}\right]$$
(16)

where  $f_p$  is the peak wave frequency,  $H_{rms}$  is the root mean square wave height, D is the total water depth,  $B_b$  and  $\gamma_b$  are empirical coefficients dependent on the type of wave breaking.

The parameter  $\alpha_r$ , in Eq. (13), can vary between 0 and 1. A value of 1 corresponds to full wave dissipation due to breaking being used as a feeder for roller energy calculation, while 0 means no wave dissipation is used as source for roller energy. The choice of an empirical value for  $\alpha_r$  depends on the beach type and wave conditions (see U10). The roller dissipation energy is parameterized as:

$$\epsilon^{r} = \frac{gsin\beta E_{r}}{c}$$
(17)

where  $\beta$  is an empirical constant (=0.1, Reniers et al., 2004).

Eq. (13) is solved within the ROMS module for radiation stress calculation using a first order upwind scheme (Patankar, 1980), with a barotropic (fast) time stepping.  $E_r$  calculated using Eq. (14) is substituted in Eq. (12) and distributed vertically using a surface intensified distribution,  $R_z$  such that:

$$R_z = F_{ED} \tag{18}$$

where  $F_{ED}$  is a forcing function that scales with the wave height (see, Eq. (11)).

Along with creation of wave rollers, depth induced wave breaking also creates enhanced turbulence and mixing of momentum within the wave breaking zone (Feddersen and Trowbridge, 2005). These processes have been added to the local turbulence closure model by using a spatially variable empirical parameterization of wave induced eddy diffusivity based on U10 (their Eq. (59)), which has a length scale of local wave height and has the same vertical distribution as the roller energy (Eq. (18)).

#### 2.5. Case 1: Obliquely incident waves on a planar beach

The effects of updated forcing methods are examined through simulations for obliquely incident waves on a planar beach. This case has been previously discussed by HW09 using the M03 formulation (thus the results corresponding to M03 are same as HW09). The model domain has a cross-shore width (x) of 1180 m and an alongshore length (y) of 140 m. The grid resolution is 20 m for both directions. The water depth varies from 12 m at the offshore boundary to 0 m at the shoreline. The vertical domain consists of 30 equally spaced vertical layers. The boundary conditions are periodic in the alongshore (i.e., north and south boundaries), closed at the shoreline, and Chapman like radiation condition (Chapman, 1985) at the offshore end of the domain. Effect of earth rotation has not been included. The bottom stress has been formulated using a quadratic bottom drag with a C<sub>d</sub> value of 0.0015. The turbulence closure scheme is Generic Length Scale (GLS, k-epsilon) as described in Warner et al. (2005). For this simulation, wave forcing is provided by SWAN, which propagates an offshore JONSWAP wave spectrum with a significant wave height of 2 m, a peak period of 10 s and a 10° angle of incidence. The effect of wave rollers and enhanced wave breaking induced mixing has not been considered for this study, consistent with HW09.

U10 conducted similar tests on the same setup using the vortex force formalism (McWilliams et al., 2004) to compute the wave forcings. Results were compared to those in HW09, which were based on the original vertical distribution of M03. Here we compare the vertical structure of cross-shore velocity between M03 and the present model using both M08-11<sub>top</sub> and M08-11<sub>vrt</sub> in order to reveal the differences between the older and newer formulations, but also to examine the performance of the radiation stress vertical distribution shown in Eq. (10).

The cross-shore distribution of wave height, water depth and sea surface elevation after 6 h of model simulation time are shown in Fig. 1a. The free surface is very close to zero at the offshore boundary and gradually decreases landward with a maximum setdown at x = 560 m. The waves start breaking at x > 560 m as determined by



**Fig. 1.** Case 1: Obliquely incident waves on a planar beach using the original radiation stress forcing M03, the updated forcing applied at the top layer M08-11<sub>top</sub>, and with a vertical distribution as defined by Eq. (10) of M08-11<sub>vrt</sub>. Cross-shore distribution of: (a) significant wave height, water depth and sea surface elevation; The water depth and wave height have been scaled as h/20 and  $H_{sig}/10$ , respectively; (b) depth averaged Eulerian cross-shore velocity ( $\overline{u^e}$ ); and (c) depth averaged Eulerian alongshore velocity ( $\overline{v^e}$ ).

wave setdown and reduction in wave height. A comparison of the depth averaged, cross-shore and longshore Eulerian velocities for the different simulations (i.e., M03, M08-11<sub>top</sub> and M08-11<sub>vrt</sub> formulations) are shown in Fig. 1b and c. The cross-shore profile of the depth averaged cross-shore velocity (Fig. 1b) is identical for all three simulations with the maximum current occurring at 700 m. The depth averaged, Eulerian flow has similar structure and opposite sign to the wave induced Stokes drift (not shown), which when added together makes the net Lagrangian flow nil. The strength of the maximum depth averaged longshore velocity (Fig. 1c) for M08-11<sub>top</sub> and M08-11<sub>vrt</sub> is slightly weaker in comparison to M03. This reduction in longshore velocity in M08-11<sub>top</sub> and M08-11<sub>vrt</sub> is compensated for by an increase in longshore velocity further offshore.

The vertical structure of the cross-shore Eulerian velocity at five different locations across the shoreface and for each simulation is shown in Fig. 2. At the furthest offshore location (x = 100 m), the M03 cross-shore velocity profile shows offshore directed velocity increasing in strength from 0 ms<sup>-1</sup> at z = -4 m to 0.15 ms<sup>-1</sup> at z = -10.5 m. For z > -4 m, the velocity is directed onshore with maximum strength at

the surface layer. At the same location, the M08-11<sub>vrt</sub> results show no velocity at the surface, increasing towards the bed with an offshore directed velocity of 0.15 ms<sup>-1</sup>. The M08-11<sub>top</sub> simulations are similar to those of M08-11<sub>vrt</sub> except near the surface, where an offshore velocity of 0.10 ms<sup>-1</sup> is observed at the surface layer. The velocity profile at x = 300 m follows a similar trend as that for M03 and M08-11<sub>vrt</sub>, while for M08-11<sub>top</sub>, offshore advection at the surface layer is observed with a velocity on the order ~0.2 ms<sup>-1</sup>.

At the location just offshore of the wave breaking zone (i.e., x = 500 m), M03 runs have maximum offshore directed velocity (~0.2 ms<sup>-1</sup>) at the bottom layer, which decreases to 0 at the surface. For the M08-11<sub>top</sub> run, strongest offshore flow is at the bottom layer which decreases to a magnitude ~0.1 ms<sup>-1</sup> at z = 0 m. The velocity profile from M08-11<sub>vrt</sub> run has maximum offshore velocity at z = -6 m with a strength of ~0.2 ms<sup>-1</sup>, reducing to ~0.05 ms<sup>-1</sup> at the surface.

Within the surf zone (x > 500 m), the original model (M03) run predicts a strong offshore directed velocity near the bed. At the surface, the velocity is still directed offshore but with a significantly reduced strength. The M08-11<sub>top</sub> and M08-11<sub>vrt</sub> results are very similar within the surf zone. Close to the bottom boundary, velocity is directed offshore with a higher value than that observed for the M03 run. Near the surface, velocity is directed onshore as expected in the surf zone while an offshore directed undertow is developed near the bottom (also see Fig. 1, in Longuet-Higgins, 1953). This vertical segregation of flow leads to the development of a cross-shore circulation cell with a vertical velocity (not shown here) directed upwards at x~500 m and downwards close to the shoreline at x~900 m. This is generally consistent with field observations of crossshore velocity profiles within the surf zone that show similar vertical flow segregation for both barred (Garcez-Faria et al., 2000) and nonbarred planar beaches (Ting and Kirby, 1994).

Overall, the M03 formulation predicts onshore velocity for areas outside the surf zone and fails to reproduce the recirculation pattern within the surf zone. The M08-11 based simulation with stress applied to the top layer (M08-11<sub>top</sub>) works well within the surf zone but creates offshore advection of cross-shore velocity near the surface. However this offshore advection is eliminated when, implementing Eq. (10) (M08-11<sub>vrt</sub>). Furthermore, at the breaking zone; the M08-11<sub>vrt</sub> model results are qualitatively in agreement with the field observations of Garcez-Faria et al. (2000) that show slight onshore flow near the surface and offshore flows below, increasing with proximity to the bed (see Fig. 1c in Garcez-Faria et al., 2000).

## 2.6. Case 2: Obliquely incident waves on a barred beach, DUCK'94 experiment

In this case study, the M03, M08-11<sub>top</sub> and M08-11<sub>vrt</sub> formulations are further evaluated by comparing model simulated surf zone velocities to measurements obtained during the DUCK'94 experiment



**Fig. 2.** Comparison of vertical profile of cross-shore Eulerian velocity,  $u^e(x, z)$  between simulations using M03 (solid gray line), M08-11<sub>top</sub> (dashed black line), M08-11<sub>vrt</sub> (solid black line). The sea surface elevation ( $\zeta$ , dotted gray line) is also shown. Vertical black lines indicate locations of model sampling and zero value for each profile.

(Garcez-Faria et al., 1998, 2000). On 12th October, 1994, strong longshore and cross-shore currents occurred in response to passage of a low pressure storm system. These velocities were measured at 7 different surf zone locations for approximately an hour at each site, using a vertical stack of 7, two component electromagnetic current meters (ECM) which were at an elevation of 0.42, 0.68, 1.01, 1.47, 1.79, 2.24 and 2.57 m above the sea bed (Garcez Faria et al., 1998, 2000). The tidal variability during this period of data collection was minimal and the bathymetric contours were assumed alongshore uniform (Garcez-Faria et al., 2000). During this period, directional wave spectrum was measured along the shoreline at 8 m water depth using 10 pressure sensors (Long, 1996). Additionally, 11 fixed ECMs and 13 pressure sensors were used to measure the cross-shore variability of velocity and wave height in the surf zone (Elgar et al., 1998). Other details regarding data acquisition and processing can be found in Gallagher et al. (1996, 1998) and Elgar et al. (1998).

The bathymetry as well as the hydrodynamic conditions is shown in Fig. 3a. The nearshore bar is located 130 m from the shoreline. The model domain is alongshore uniform, has a cross-shore width (x) of 780 m and an alongshore length (y) of 800 m, with a grid resolution of 2 m and 80 m in x and y directions, respectively. The water depth varies from 7.26 m at the offshore boundary to 0 m at the shoreline. A tidal elevation of 0.7 m is added to the water level. The vertical domain has been distributed in 32 equally spaced vertical layers. The boundary conditions are periodic in the alongshore (i.e., north and south boundaries), closed at the shoreline, and Chapman like radiation condition (Chapman, 1985) at the offshore end of the domain. Effect of earth rotation has not been included. The bottom stress has been formulated using a logarithmic bottom drag with a bottom roughness length ( $z_0$ ) of 0.003 (Feddersen et al., 1998). The turbulence closure

scheme is Generic Length Scale (GLS, k-epsilon) as described in Warner et al. (2005). Wave forcing is provided by SWAN, which propagates an offshore JONSWAP wave spectrum with a significant wave height of 2.3 m, a peak period of 6 s and a 13° angle of incidence. The effect of wave rollers and enhanced wave breaking induced mixing has been considered in all the simulations. Wave dissipation ( $\varepsilon_b$ ) obtained from SWAN is used to feed the roller evolution model (Eq. (14)) with  $\alpha_r = 1$ .

Fig. 3 shows the significant wave height, sea surface elevation and barotropic cross-shore and longshore flows (depth averaged from three dimensional flows). The significant wave height after 3 h of model simulation shows that wave breaking occurs over the bar crest and close to the shoreline (Fig. 3a). Simulation from all the three formulations show a wave setup (Fig. 3b) at cross-shore locations with wave breaking. The sea surface elevation from M08-11<sub>top</sub> and M08-11<sub>vrt</sub> are similar, while M03 predicts slightly smaller values. The depth averaged, Eulerian, cross-shore velocity (Fig. 3c) shows same cross-shore distribution from all the three simulations. It is directed offshore and is strongest over the bar crest  $(0.14 \text{ ms}^{-1})$  and further shoreward  $(0.19 \text{ ms}^{-1})$ . It is important to point out that the depth averaged, cross-shore velocity for this case is equal to the sum of mass flux due to Stokes drift and wave roller induced mass flux, which confirms the mean continuity balance between barotropic Eulerian flows and net onshore directed mass flux. The strongest longshore velocity was measured over the bar trough during the experiment (Feddersen et al. 1998). M03 derived depth averaged longshore velocity (Fig. 3d) are weaker than the observed flows, while results from simulations using M08-11<sub>top</sub> and M08-11<sub>vrt</sub> formulation show better agreement to the observed dataset (Fig. 3d).

Measured (Garcez Faria et al., 1998, 2000) and modeled vertical distributions of cross-shore and longshore Eulerian velocities are



**Fig. 3.** Case 2: Obliquely incident waves on a barred beach (DUCK'94 experiment) using the original radiation stress forcing M03, the updated forcing applied at the top layer M08-11<sub>top</sub>, and with a vertical distribution as defined by Eqn. 10 of M08-11<sub>vrt</sub>. Cross-shore distribution of: (a) significant wave height ( $H_{sig}$ ) and water depth (h). The water depth and wave height values shown are scaled as h/10 and  $H_{sig}/2$ , respectively, for plotting purposes; (b) sea surface elevation ( $\zeta$ ); (c) depth averaged Eulerian cross-shore velocity ( $\overline{u^e}$ ); and (d) depth averaged Eulerian alongshore velocity ( $\overline{v^e}$ ).

shown in Fig. 4.  $M08-11_{top}$  and  $M08-11_{vrt}$  runs exhibit a strong vertical shear in the cross-shore velocity, with onshore directed velocity at the surface and offshore directed undertow near the bottom. This surf zone recirculation pattern is most intense over the bar crest and decreases shoreward and further offshore. Shoreward of the bar-crest, M08-11<sub>top</sub> and M08-11<sub>vrt</sub> show similar vertical profiles, while seaward of the bar crest M08-11<sub>top</sub> has a higher vertical shear than M08-11<sub>vrt</sub>. Overall, these model simulations successfully replicate the vertical structure shown by the observed data. M03 runs show relatively milder undertow at the bar crest with hardly any vertical shear. Furthermore, no onshore directed flow is simulated at the surface from M03 formulations.  $M08-11_{top}$  and  $M08-11_{vrt}$  runs show maximum longshore velocity shoreward of the bar crest (Fig. 4b) along with close agreement to measured flows. The inclusion of roller evolution model with  $\alpha_r = 1$  and enhanced mixing due to wave breaking contributes to shifting the location of peak longshore velocity from over the bar crest to further shoreward. At locations offshore of the bar crest longshore velocities from the M08-11<sub>vrt</sub> formulation are slightly weaker and show relatively better agreement in comparison to M08-11<sub>top</sub>. Irrespective of the inclusion of wave roller and wave-induced mixing, the M03 run (Fig. 4b) creates strongest longshore flow at the bar-crest. It underestimates the longshore flow in the bar trough, while further offshore it shows better agreement to observed flows.

The normalized root mean square error (defined as in Newberger and Allen, 2007b) in modeled cross-shore and longshore velocities ( $\varepsilon_u$ ,  $\varepsilon_v$ ) for simulation using M03, M08-11<sub>top</sub> and M08-11<sub>vrt</sub> formulations are (0.83, 0.30), (0.58, 0.31) and (0.55, 0.20), respectively. The errors for M08-11<sub>top</sub> and M08-11<sub>vrt</sub> are similar to the model skill shown by Newberger and Allen, 2007b (0.45–0.70 and 0.12–0.50) and U10 (0.43 and 0.09). Similarly, normalized (by the maximum value) root mean square errors (defined as in Sheng and Liu, 2011) were estimated for all seven stations (see Fig. 4) and they are listed in Table 1. The M03 simulated longshore velocities have an overall error of 30% with larger errors, in excess of 40%, occurring at stations located near the bar region, while smaller errors (<10%) are found for stations away from the bar. The M03 simulated cross-shore flows have a total error of 44% with the largest error (>100%) occurring at the furthest offshore station (Table 1). On the other hand, the errors from the M08-11<sub>top</sub> and M08-11<sub>vrt</sub> simulations are similar in magnitude for both longshore and cross-shore velocities, with the smallest error produced by the M08-11<sub>vrt</sub> simulations of longshore velocities at the two offshore station locations. The total errors in simulated flows for DUCK'94 experiment are 15.4% and 24.7%, in longshore and cross-shore velocities, respectively for M08-11<sub>vrt</sub>.

Overall, it is evident that M03 fails to create a surf zone recirculation pattern and errors in cross-shore flows from M03 run are significantly higher than for M08-11<sub>top</sub> and M08-11<sub>vrt</sub>. It is interesting to point out that for the DUCK'94 experiment wave shoaling is not observed within the computational domain. In presence of a wave shoaling region, the performance of M08-11<sub>top</sub> would deteriorate as it creates strong offshore flows at the surface layer unlike the M08-11<sub>vrt</sub> formulation (see Section 2.5).

#### 3. Nearshore circulation cell cases

Rip currents have been the subject of modeling (Bowen and Inmam, 1969; Dalrymple, 1975; Haas et al., 2000, 2003; Haller and Dalrymple, 2001; Noda, 1974; Tam, 1973) but also experimental studies in both the field (Aagaard et al., 1997; Brander and Short, 2001; MacMahan et al., 2005; Sonu, 1972) and the laboratory (Drønen et al., 2002; Haller et al., 2002). They provide a good example for testing nearshore numerical models as they invoke a number of nearshore processes and wave and current interaction patterns. In this section we have applied the M08-11<sub>vrt</sub> formulation and examine its performance on rip current development by comparing to previously published work.

Initially, two ideal cases are presented where rip current cells develop in response to alongshore variability of wave forcing (Case 3) and alongshore variable bottom bathymetry (Case 4). The former condition can be the result of temporal variability in wave group forcing (e.g., Long and Özkan-Haller, 2009) or due to the incidence of intersecting wave trains of similar frequency (e.g., Dalrymple, 1975). On the other hand, the latter condition is not uncommon in barred beach profiles. In each case, alongshore differences in wave setup, caused by alongshore variation of the wave breaking position, create an alongshore pressure gradient which in turn drives a longshore current. In both cases the creation of an alongshore gradient in wave



**Fig. 4.** Comparison of vertical profiles of (a) cross-shore ( $u^e(x, z)$ ); and (b) longshore ( $v^e(x, z)$ )Eulerian velocities; from model simulations using M03 (solid gray lines); M08-11<sub>top</sub> (dashed black lines), M08-11<sub>vrt</sub> (solid black lines) and field measurements (gray squares). Vertical gray lines indicate locations of model sampling and zero velocity value for each profile. The horizontal velocity profile data are from Garcez-Faria et al. (1998, 2000).

RMS error (normalized by the maximum value) for the simulated current velocities using M03, M08-11<sub>top</sub> and M08-11<sub>vrt</sub> formulations for Case2 (DUCK'94 experiment). Entries in "bold" show minimum errors.

| Station no | RMS error (%)        |                |                       |                |                       |                |
|------------|----------------------|----------------|-----------------------|----------------|-----------------------|----------------|
|            | M03                  |                | M08-11 <sub>top</sub> |                | M08-11 <sub>vrt</sub> |                |
|            | <i>u<sup>e</sup></i> | v <sup>e</sup> | u <sup>e</sup>        | v <sup>e</sup> | <i>u<sup>e</sup></i>  | v <sup>e</sup> |
| 1          | 49.6                 | 10.5           | 36.6                  | 62.5           | 37.1                  | 60.5           |
| 2          | 56.7                 | 41.4           | 30.2                  | 04.3           | 30.7                  | 03.7           |
| 3          | 62.5                 | 48.7           | 38.8                  | 09.7           | 40.8                  | 10.4           |
| 4          | 69.7                 | 41.0           | 16.4                  | 06.5           | 23.7                  | 07.0           |
| 5          | 60.4                 | 12.3           | 51.6                  | 21.4           | 42.6                  | 24.5           |
| 6          | 24.1                 | 15.1           | 75.4                  | 38.2           | 52.2                  | 16.7           |
| 7          | 100.9                | 12.0           | 58.8                  | 49.0           | 60.5                  | 23.5           |
| Overall    | 44.6                 | 29.7           | 23.9                  | 18.0           | 24.7                  | 15.4           |

setup leads to the development of a circulation cell like pattern in the surf zone as described in Bowen (1969) and Noda (1974).

In addition, the laboratory studies of rip currents by Haller et al. (2002) are well documented and provide an excellent set of data for comparison to numerical model results. HW09 provided a qualitative comparison of rip current formation to results from Haller et al. (2002). Expanding on this previous work, we use the modified model to simulate the formation of rip currents on an alongshore bar trough morphology (Case 5), which is a scaled-up experiment of the laboratory study conducted by Haller et al. (2002) and Haas and Svendsen (2002).

#### 3.1. Case 3: Alongshore variable wave forcing

The setup of this case study includes incidence of alongshore variable wave height distribution on a planar beach as described by Bowen (1969). Our case differs from Bowen's setup as we use spectral instead of monochromatic waves and the domain size has been increased to resemble realistic field conditions.

The alongshore uniform, planar bathymetry is analytically described by:

$$d = tan\beta \cdot x \cdot (1 + \varepsilon \cdot cos(\lambda \cdot y)) \tag{19}$$

For  $\varepsilon <<1$ , this can be approximated as  $d \approx tan \beta x$ . The beach slope  $tan\beta$  is 0.02 and the water depth (*d*) varies from 12 m offshore to 0 m close to the shoreline. The domain is 650 m in the cross-shore and 1000 m in the alongshore direction, with a resolution of 5 and 10 m, respectively. In the following discussion, results only from the area 600 m (x from 0 m to 600 m) by 1000 m (y from 0 to 1000 m) are shown, so that boundary effects are excluded. Vertically, the domain is distributed in 10 equally distributed sigma layers. Closed boundary conditions are used at the two lateral sides and the shoreline, while Neumann boundary conditions have been used at the offshore boundary. A logarithmic bottom friction is used with a roughness length of 0.005 m, a value close to those reported from field studies (e.g., Feddersen et al., 1998).

The wave model (SWAN) is run for the same grid as ROMS. The wave forcing applied at the offshore boundary is directed perpendicular to the domain, has a period of 5 s and an alongshore varying wave height described (see Eqs. (30) and (31) in Bowen, 1969) by:

$$H = \gamma \frac{(1 - K)tan\beta}{f} \times (1 + 0.2cos(\lambda \cdot \mathbf{y}))$$
(20)

where  $\lambda$  is the alongshore wave number of the wave height variability  $(2\pi/L_y, \text{ with } L_y = 1,000 \text{ m})$ , f is a scaling constant,  $tan\beta$  is the beach slope, K (a parameter which relates wave setup to slope) is calculated as  $(1 + 8/3\gamma^2)^-$  and  $\gamma$  (=0.6) is the depth-induced wave breaking constant (Battjes and Janssen, 1978; Eldeberky and Battjes, 1996). The

alongshore and cross-shore domain is scaled by a value of  $(\lambda = 2\pi/1000)$  for this case. The wave forcing is described by a directional spectrum consisting of 20 frequency bands in the range 0.04 Hz to 1 Hz, and 36 directional bins of 10° each from 0° to 360° with a directional spreading of 6°. The bottom friction used in SWAN is based on the eddy viscosity model of Madsen et al. (1988) with a bottom roughness length scale of 0.05 m. The modeling system for this case is configured in one way coupling where there is no feedback of the currents or water levels to the wave model, and in a two way coupling mode where exchange of wave and current information takes place between ROMS and SWAN at a synchronization interval of 20 s. Both model configurations were run for a simulation time of 2 h over which the computational domain achieves stability. Unlike Yu and Slinn (2003) very small differences were observed between the final results of one and two way coupling based simulations. We attribute this to a number of reasons including differences in wave forcing, bottom friction values and on the width of the rip current jet in the two cases. As Yu and Slinn (2003) mention the current effect on waves is stronger for narrow offshore rip currents as in their case, while in the present study the rip system is approximately 250 m wide. In the following sections we discuss the two way coupled results, unless stated otherwise.

The wave height distribution over the domain is shown in Fig. 5. The wave incident at the offshore boundary is alongshore variable with a maximum value of 1.5 m at the lateral boundaries and a minimum value of 1 m at the center of the domain. At the center of the domain (i.e.,  $\lambda \cdot y = \pi$ ), the incident wave height initially decreases and then increases before it starts breaking in shallower water depths. The initial decrease is due to bottom friction and depth induced dissipation and the increase after that is due to interaction of the incoming wave field with the outgoing currents. This outgoing current locally increases the wave height by a small value (0.05–0.10 m).

The depth averaged Lagrangian (Eulerian + Stokes) velocity and the associated streamlines are compared to analytically derived streamlines – following Bowen (1969) – assuming a breaking position of  $\lambda \cdot \mathbf{x} = \pi/2$  (Fig. 6). The flow patterns are symmetrical about  $\lambda \cdot \mathbf{y} = \pi$ ; therefore only the bottom half is shown and discussed here. The flow pattern within the surf zone ( $\lambda \cdot \mathbf{x} < \pi/2$ ) is onshore, offshore and alongshore directed at  $\lambda \cdot \mathbf{y} = 0$ ,  $\pi$ , and  $\pi/2$ , respectively. The longshore current within the surf zone increases from 0 to 0.2 ms<sup>-1</sup> and then reduces to 0 ms<sup>-1</sup> at  $\lambda \cdot \mathbf{y} = \pi$ . For locations outside the breaking zone



**Fig. 5.** Color shading of significant wave height distribution over the computational domain after 2 h of model simulation for two-way coupling between ROMS and SWAN. Note the significant wave height at offshore boundary is 1.5 m at the ends and decreases to a minimum value of 1 m at the center of the domain. The alongshore and cross-shore domain has been scaled by a value of ( $\lambda = 2\pi/1000$ ). Black dashed lines indicate the location of transects shown in Fig. 8, while gray dashed line shows the location of the surf zone. The horizontal, black, dotted lines indicate the location of alongshore transect shown in Fig. 17.



**Fig. 6.** Rip current simulation results for the bottom half of the computational domain (Case 3, two-way coupling) after 2 h of simulation. (a) Depth averaged Lagrangian velocity distribution  $(\overline{u^l})$ ; (b) Transport stream function  $(\psi)$  showing formation of circulation cell in the surf zone; (c) Transport stream function  $(\psi)$  calculated using the analytical solution provided by Bowen, 1969 for the present model setup. Note:  $\lambda x = \pi/2$  (shown by gray dashed line) is the location where waves start breaking.

 $(\lambda \cdot x > \pi/2)$ , the longshore current is relatively weaker and is directed from  $\lambda \cdot y = \pi$  to  $\lambda \cdot y = 0$ . Within the surf zone, the streamline patterns observed are similar for both the analytical solution (Fig. 6c) and the model simulation (Fig. 6b). It is important to note that longshore symmetry of streamlines about the center of circulation is observed, which suggests that the strength of offshore and onshore directed flow at  $\lambda \cdot y = 0$  and  $\lambda \cdot y = \pi$  are of the same magnitude. Outside the surf zone (i.e., for  $\lambda \cdot x > \pi/2$ ), the two streamline patterns differ. The model based streamlines show uniform distribution pointing at equal strength of longshore and cross-shore velocity from  $\pi/2 < \lambda \cdot x < 6\pi/5$ . The analytical solution (Fig. 6c) suggests reduction in velocity when moving further offshore (seen by increase in distances between the corresponding streamlines). These differences occur because the analytical solution includes only bottom friction as a parameter for dissipation, whereas the model simulations include additional dissipative and mixing processes, which make the velocity distribution uniform outside the surf zone. Overall, even though we use different bottom friction, turbulence closure schemes etc., qualitatively the results are comparable to Bowen (1969) (their Fig. 6) and to the results of LeBlond and Tang (1974) who included wave-current interaction in their analytical solution.

In order to examine the effect of lateral mixing on circulation pattern, we implement a sensitivity analysis based on a Reynolds number defined as  $VL_y/A_H$ , where V is the maximum longshore velocity magnitude,  $L_y$  is the alongshore wavelength of the forcing perturbation and  $A_H$  is the horizontal coefficient of viscosity (Fig. 7). Small changes in bottom friction affect the maximum velocity value but not the circulation pattern (not shown here). On the other hand, changes in horizontal mixing, affect both velocity strength and circulation pattern. As the Reynolds number increases, the solutions become more skewed, i.e., outflowing current at the location of lower waves tends to become narrow and the onshore flow broadens. This effect is shown in Fig. 7 where the stream function for different values of Reynolds number is presented. As  $A_H$  decreases from 6 to 0.5, the



**Fig. 7.** Transport stream function,  $\psi$  over the computational domain for Reynolds number (Re) values of (a) 500, (b) 125, (c) 62.5, and (d) 42. Note the solution gets skewed about the individual centers of the circulation cells with increased Re values causing a narrower outflow from shallower to deeper waters and broader inflow from deeper to shallower waters. Also note that the individual circulation cells are not exactly symmetric about the line  $y = \pi$ . The gray circulation cell in (a), (b), and (c) is same as (d); it is shown for comparison purposes. The vertical, gray dashed line shows the location where waves start breaking.

Reynolds number increases from 42 to 500, making the solution more skewed about the individual circulation cell centers. Qualitatively this solution compares well to both the theory of Arthur (1962) and the results derived by Bowen (1969) by numerically solving the non linear problem for streamline distribution.

The vertical structure of the cross-shore and longshore Eulerian velocities along three profiles at  $\lambda \cdot y = \pi/5$ ,  $\pi/2$  and  $4\pi/5$  (for locations see Fig. 5), corresponding to locations where the depth averaged cell flow is directed onshore, alongshore and offshore, respectively, are shown in Fig. 8, respectively. These results correspond to simulation runs with  $A_H = 0.5 \text{ m}^2 \text{s}^{-1}$ . The first location (Fig. 8a) corresponds to bigger waves, which start breaking further offshore ( $\lambda \cdot x \sim 0.5\pi$ ). The region onshore of location  $\lambda \cdot x = 0.5\pi$  shows a vertical segregation of the flow. The onshore flow observed at the surface layer at  $\lambda \cdot y = \pi/5$ (Fig. 8a) is stronger than the surface onshore flow at  $\lambda \cdot y = \pi/2$ (Fig. 8b). Presence of a circulation pattern in the domain reinforces current directed towards the shoreline at  $\lambda \cdot y = \pi/5$ . The offshore flow in this case is weak and is limited to the bottom boundary. The vertically integrated flow is directed onshore as shown in Fig. 6a. Outside the surf zone (i.e.,  $\lambda \cdot x > 0.5\pi$ ), the flow is predominantly weak, onshore directed ( $\sim 0.05 \text{ ms}^{-1}$ ) at the upper half of the sigma layers and gradually decreases to no flow at the bottom layer (Fig. 8a).

At the third vertical profile (Fig. 8c), the incoming waves are small and break close to the shoreline ( $\lambda \cdot x \sim 0.4\pi$ ). The flow field close to the surface is weakly (<0.05 ms<sup>-1</sup>) onshore directed as this velocity at the surface is reduced by the rip current jet directed offshore. Also the onshore flow is limited to the top layer. The offshore directed undertow is stronger in this case and occupies the largest part of the water column. The vertically averaged flow is strongly offshore directed. Outside the wave breaking zone (i.e.,  $\lambda \cdot x > 0.4\pi$ ) the velocity strength steadily decreases from 0.2 ms<sup>-1</sup> to 0.05 ms<sup>-1</sup>.

Panels d, e, and f, in Fig. 8, shows the vertical structure of longshore velocity at the same locations as in Fig. 8a, b, and c, respectively. At  $\lambda \cdot y = \pi/5$  (Fig. 8d), longshore velocity within the surf zone ( $\lambda \cdot x < 0.45\pi$ ) has a strength of 0.1 ms<sup>-1</sup> while at  $\lambda \cdot y = \pi/2$  (Fig. 8e), velocity is positive and strongest (0.2 ms<sup>-1</sup>) at the surface, gradually decreasing to 0.15 ms<sup>-1</sup> near the bed. This is reflected in the strong depth averaged longshore velocity observed within the surf zone in

Fig. 6a. At  $\lambda \cdot y = 4\pi/5$  (Fig. 8f), velocity within the surf zone is stronger than that at  $\lambda \cdot y = \pi/5$ . This occurs because the streamlines in this case are not symmetrical about the center of the circulation (Fig. 7a) and the offshore flow occurs over a smaller area in comparison to broadened onshore flow. Offshore of  $\lambda \cdot x = 0.45\pi$  (outside the surf zone), the alongshore flow is small and gradually increases to  $-0.10 \text{ ms}^{-1}$  for the rest of the vertical domain for  $\lambda \cdot y = \pi/5$ ,  $\pi/2$  and  $4\pi/5$ .

#### 3.2. Case 4: Alongshore varying bathymetry

In this case study the alongshore bathymetry of the beach is varied to produce a sinusoidal pattern according to (Noda, 1974):

$$d(x,y) = tan\beta \cdot x \cdot \left(1 + a \cdot exp\left(-\left(\frac{x}{a/3}\right)^{1/3}\right)\right) \cdot sin^{10}\left(\frac{\pi y}{L}\right)$$
(21)

where the beach slope  $(tan\beta)$  is 0.025, the wavelength( $L_y$ ) of the alongshore variation is 80 m and  $\alpha$  is a constant (Eq. (21)). This analytical expression generates a periodic beach bathymetry with channels concentrated at alongshore distances which are multiple of  $L_y$ , while it produces a straight coastline at x = 0 m.

The numerical model domain is 110 m and 560 m in the crossshore and alongshore directions, respectively, with a resolution of 2 m in both directions. Application of Eq. (21) over the domain generates 7 channel-like features. In the following discussion, results only from the central feature, over an area 100 m (x from 0 to 100 m) by 80 m (y from 240 to 320 m) is shown, so that boundary effects are excluded. Ten equally spaced sigma layers were used in the vertical direction. Closed boundary conditions are implemented in the lateral and coastline and Neumann conditions at the offshore boundary. Logarithmic bottom friction has been implemented with a roughness length of 0.005 m.

The same grid is used by the SWAN wave model and the wave forcing is a directional spectrum as that used in Case 3 but with a directional spreading of  $2^\circ$ . Wave conditions are similar to those used by Noda (1974) with a significant wave height of 0.92 m, peak wave period of 4 s and normally incident at the offshore boundary. The



**Fig. 8.** Contour plots showing the vertical structure of the Eulerian, cross-shore (a, b, c) and alongshore velocity (d, e, f) along three transects located at  $\lambda y = \pi/5$  (a, d),  $\lambda y = \pi/2$  (b, e) and  $\lambda y = 4\pi/5$  (c, f) from the southern lateral boundary (see dashed lines in Fig. 5). Solid black line corresponds to zero velocity.

other variable parameters are the same as in Case 3 (i.e., depth induced breaking constant,  $\gamma = 0.6$  and bottom friction with roughness length of 0.05 m). The ROMS-SWAN system in this case is operated in a two way coupling mode, exchanging wave current information at a 20 s interval. The results presented here are after 1 h of simulation after the model has achieved stability.

The depth averaged Eulerian velocity and wave height distribution are shown in Fig. 9a and b, while the vertical distribution of the crossshore current for two transects corresponding to y = 240 and 280 m are shown in Fig. 9c and d. The results indicate the development of rip currents and the interaction of the waves with the bathymetry which is exhibited as alongshore differences in wave breaking position (not shown in here). In addition, it is characteristic that the wave height slightly increases over the area of the rip current development (see cross-shore locations 60 to 80 m) due to the interaction of strong outgoing current with the incoming waves.

The vertical profile of cross-shore Eulerian velocity at a transect located at y = 240 m is shown in Fig. 9c. Wave breaking starts at x = 70 m as determined by a vertical shear observed in the crossshore velocity profile. Further offshore (x>70 m), the entire water column shows an onshore directed velocity due to the background circulation pattern observed in the domain (Fig. 9a). In a normal surf zone circulation pattern (see Case 1, Fig. 2), onshore flow is observed near the surface. This onshore surface flow is further enhanced in this case, due to the presence of the onshore component of the circulation cell. The offshore flow is limited to elevations close to the bottom boundary other than in very shallow waters (z < -0.5 m) where the entire water column is directed offshore. The vertical profile of crossshore velocity at y = 280 m is depicted in Fig. 9d. Wave breaking takes place at 1.5 m depth; some 60 m from the shoreline (see Fig. 9b). The rip current strength is approximately  $0.5 \text{ ms}^{-1}$  at the bottom layer and weakens close to the surface. In shallow waters (1 m), rip current strength decreases and close to the shoreline, a vertical shear in velocity is observed. The vertical structure of the cross-shore flow at y=0 and 40 m is similar to that at locations  $\lambda \cdot y = \pi/5$  and  $4\pi/5$  respectively for Case 2 and are shown in Fig. 8a and c.

The normalized stream function calculated using the depth averaged Lagrangian velocities from the model output is shown in Fig. 10, together with the stream function generated by Noda (1974). In both cases the streamlines converge at y = 40 m, creating a flow pattern from shallower to deeper waters, simulating a rip current like situation. The maximum value of stream function occurs close to x = 60 m for Noda (1974) and x = 70 m for our simulations. Both results are almost symmetrical around an imaginary line located at y = 40 m. It is worth noticing that our system of stream function is shifted slightly to the right in comparison to Noda (1974).

The depth averaged cross-shore velocity in the rip channel is approximately  $0.5 \text{ ms}^{-1}$  (Fig. 9a), a value more reasonable than that of Noda (1974), where for the same setting he predicted a rip current velocity in excess of 4 ms<sup>-1</sup>. The differences in distribution of stream function and magnitude of rip current velocity occurs because, as acknowledged by Noda (1974, see pp. 4105), his depth averaged model was rather simplified as it only accounts for pressure gradient, radiation stress and bottom friction and does not account for current-induced wave refraction and modifications of the wave field due to Doppler shift, as in the present model. Furthermore, the unrealistic rip current velocity predicted by Noda (1974) implies that the stream function might not be accurate enough for direct comparison with our model, which seems to give more realistic results.

#### 3.3. Case 5: Comparison to scaled laboratory studies

This case study investigates the dynamics for a barred beach bathymetry that develops rip currents. The application is based on a laboratory scale experiment and is similar to a case demonstrated in HW09. However there are two major differences: (i) in HW09 the



**Fig. 9.** (a) Depth averaged Eulerian, velocity (black arrows) after 1 h of simulation. Light gray lines in background depict the bathymetry contours; (b) Contour of significant wave height distribution over the computational domain. The incident wave height at the offshore boundary of the domain is 0.92 m. Contour plots showing the vertical structure of cross-shore Eulerian velocity,  $u^e(x, z)$  along two transects located at (c) y = 0 m, (d) y = 40 m from the southern lateral boundary (see gray lines in Fig. 9b). The gray dotted alongshore transect in Fig. 9b is the location at which alongshore momentum balance term is shown in Fig. 17(c).



**Fig. 10.** Transport stream function ( $\psi$ ) over the computational domain computed from the model results after (a) depth averaging the horizontal Lagrangian velocity field; (b) and from Noda (1974) paper.

wave driver was a monochromatic wave model (REF/DIF), while here we use a spectral wave model (SWAN); and (ii) the domain used in HW09 was identical to the laboratory experiments while in our simulations the domain has been scaled by a factor of 10 (kinematic similarity, Hughes, 1993) to create more realistic field conditions.

The bathymetry domain (Fig. 11) is an idealized version of that used by Haller et al. (2002) and Haas and Svendsen (2002). The scaling of the domain by a length scale,  $N_L = 10$ , leads to a maximum depth of 5 m, a nearshore bar of 0.60 m located 40 m off the coastline, cross-shore domain width of 146 m and alongshore length of 262 m. To avoid interaction of rip channel flow with the lateral boundaries, the domain was extended laterally by 40 m in either direction. Rip channels are spaced 92 m apart and the channel width is 18.2 m which makes the ratio of channel width to rip current spacing 0.2, a value consistent with those found in the field (e.g., Aagaard et al., 1997; Brander and Short, 2001, Huntley and Short, 1992). The model grid has a horizontal resolution of 2 m in both directions and consists of 8 equally spaced sigma layers. The boundary conditions at shoreline, offshore boundary and lateral ends are no flow conditions (i.e., closed boundary conditions at the coast, lateral boundaries and offshore) and are the same as the laboratory experiments of Haller et al. (2002). Bottom friction (bottom roughness of 0.015 m) similar to that of HW09 is used in our work. Our simulations were carried out



**Fig. 11.** Bathymetry for Case 5, showing the longshore bar and the rip channels. The solid black lines show the location of vertical transects at which the cross-shore velocity distribution is discussed in Fig. 14. The 4 horizontal white lines represent the alongshore transects at which cross-shore momentum balance terms are shown in Fig. 15 and alongshore momentum balance term is shown in Fig. 17(d).

with both the modified vertical distribution (Eq. (10)) of the radiation stress (M08-11<sub>vrt</sub>, see Section 2) and the original version (M03) used in HW09.

At the offshore boundary, SWAN was forced with 0.5 m waves with peak period of 3.16 s, and directional spreading of 3° propagating perpendicular to the shoreline. From these values, SWAN computes a wave spectrum based on a JONSWAP distribution. The spectral resolution is 20 frequency bands in the frequency range between 0.04 Hz and 1 Hz, and 36 directional bins of 10° each from 0° to 360°. The other variable parameters are the same as in Case 3 and 4 (i.e., depth induced breaking constant,  $\gamma = 0.6$  and bottom friction with roughness length of 0.05 m). The time steps used for ROMS and SWAN are 2 and 10 s respectively, and the coupling between the models takes place at 20 s intervals. Initial comparisons are done only for 30 min of simulation time. The model remains stable because we use a higher bottom friction coefficient and horizontal mixing than typically observed in the field.

The wave height distribution over the domain using the original and newer versions of ROMS (i.e., M03 and M08- $11_{vrt}$  formulations, respectively) is shown in Fig. 12a and b. At the location of the rip channel, the increase in wave height due to offshore directed rip



**Fig. 12.** Contours of significant wave height after 30 min of model simulation using (a) the original version of the model as in HW09 and; (b) the modified M08-11 formulations for the vertical distribution of the radiation stress (M08-11<sub>vrt</sub>). Bathymetric contours and depth integrated Eulerian, mean currents over the computational domain using (c) the original version of the model as in HW09 and; (d) the modified M08-11 formulations for the vertical distribution of the radiation stress (M08-11<sub>vrt</sub>). The black line (12c) depicts a velocity of 0.5 ms<sup>-1</sup>.



**Fig. 13.** Cross-shore variation of mean sea surface elevation at two locations corresponding to alongshore positions centered at the middle of the rip channel (black) and alongshore bar (gray).

current is lower in the M08-11<sub>vrt</sub> than the M03 simulations. The waves propagating over the bar break and generate a higher wave setup than the setup generated by waves propagating over the channel. This creates feeder currents moving from the bar towards the channel. Waves approaching the shoreline over the channel become steeper, decrease in wavelength and increase in height due to interaction with the rip current. These bigger waves break close to the shoreline creating longshore currents which move away from the channel at shallow depths. This phenomenon can be further confirmed by comparing the mean sea surface elevation over the bar and channel for M08-11<sub>vrt</sub> based simulations (Fig. 13). The elevation is lower at the location of the channel than over the bar. On the other hand, closer to the shoreline the sea surface at the channel location is higher than over the bar driving the observed flow patterns.

M03 simulated depth averaged, Eulerian, cross-shore velocity (see Fig. 12c and d) at the channel is 25% stronger than that predicted by M08-11<sub>vrt</sub>. The stronger offshore directed velocity locally increases

the wave height at the location of the rip channel in M03. Further offshore of the rip channel, the magnitude of cross-shore velocity is similar in both M03 and M08-11<sub>vrt</sub> and hence the wave height pattern is also similar. The primary circulation pattern with feeder currents exiting through the rip channel and return flow over the bar is evident irrespective of the formulation used. These circulation cells are symmetric both with respect to the rip channel and the axis of the alongshore bar.

Noticeable differences in secondary circulation patterns for M03 and M08-11<sub>vrt</sub> based simulations can be seen in Fig. 12c and d. Waves with greater wave height in the vicinity of the rip channel, for M03 formulations, drive a larger setup and stronger alongshore pressure gradient close to the shoreline in comparison to M08-11<sub>vrt</sub> formulations. As a consequence the secondary circulation pattern close to the shoreline is stronger for the M03 than the M08-11<sub>vrt</sub> based simulations.

The vertical variability of cross-shore Eulerian velocity at the center of the channel is shown in Fig. 14a and b for M03 and M08- $11_{vrt}$ , respectively. Inshore of the bar location, wave breaking induces onshore directed velocity at the surface extending all the way to the bed for M03 (Fig. 14a), while for the M08- $11_{vrt}$  simulation a return flow develops near the bed (Fig. 14b). Over the bar and shoreward the cross-shore flow structure differs between the two simulations (Figs. 14c and d). The M03 simulation (Fig. 14c) shows the development of offshore flow throughout the water column, while the improved model simulation results in an onshore flow near the sea surface with a stronger return flow near the bed. Further offshore, both simulations give similar results. These findings show that the vertical distribution of the radiation stress in M03 fails to create a surf zone vertical recirculation system, while the M08- $11_{vrt}$  run provides



Fig. 14. Vertical structure of cross-shore Eulerian velocity u<sup>e</sup> (x, z) at the center of rip channel (a and b) and bar (c and d) derived from original version of the model as in HW09 (a and c) and the M08-11<sub>vrt</sub> formulations (b and d); (e) Comparison of normalized model derived cross-shore velocity with normalized data from Haas and Svendsen, 2002 (key: symbols ● and ■ denote data at the center and 4 m off the channel, gray line (center of the channel M03), black dash dot (center of the channel M08-11<sub>vrt</sub>), blue dashed line (M08-11<sub>vrt</sub> 4 m off the channel).

#### Table 2

The RMS error (normalized by the maximum value) for the simulated current velocities using M03 and M08-11<sub>vrt</sub> formulations for Case5 (nearshore barred morphology with rip channels). Entries in "bold" show minimum errors.

| Station no | RMS error (%)    |                       |  |  |
|------------|------------------|-----------------------|--|--|
|            | M03              | M08-11 <sub>vrt</sub> |  |  |
|            | $\overline{u^e}$ | $u^e$                 |  |  |
| 1          | 30.2             | 19.2                  |  |  |
| 2          | 19.0             | 12.4                  |  |  |
| 3          | 18.9             | 16.7                  |  |  |
| 4          | 37.6             | 21.3                  |  |  |
| 5          | 62.1             | 26.6                  |  |  |
| Overall    | 20.1             | 11.7                  |  |  |

more realistic results that show qualitative agreement to field observations of cross-shore velocity profile for barred beaches (see Fig. 1c; Garcez-Faria et al., 2000).

Our scaled numerical experiment conditions correspond to Test B of Haller et al. (2002) and Test R of Haas and Svendsen (2002). Thus, we use the results of those lab experiments to provide a semiguantitative comparison between the measured and modeled vertical structure of the cross-shore velocity field. For this comparison we use all of the bin averaged velocities from Test R (Fig. 11, Haas and Svendsen, 2002) and for all reported locations (Fig. 14e). The measured and simulated velocities are normalized by the corresponding maximum cross-shore velocities at the bar crest (i.e., x = 27 m, Fig. 14e), respectively. The simulated normalized cross-shore current vertical structure from the upgraded model agrees well with the experimental data. Inside the channel, rip current speed is greatest at the level of the bar crest and decreases toward the water surface and bed. However no experimental data are available near the water surface. Just off the bar, the normalized data show the best agreement with our simulation using M08-11<sub>vrt</sub>. Such a relative agreement between data and model persists in areas further offshore of the bar location.

A normalized root mean square error analysis is presented (Table 2) by comparing the normalized, simulated flows from M03 and  $M08-11_{vrt}$  and the measured flows in the rip channel. M03 based

simulations show high error (>30%) at stations within the rip channel and at locations further offshore. On the contrary,  $M08-11_{vrt}$  shows relatively smaller errors against the measurements at all the stations. The total error in  $M08-11_{vrt}$  results is 11.7% versus 20.1% for M03.

For steady flow, the depth and time averaged cross-shore (x) momentum equation can be written as:

$$\frac{\partial}{\partial x} \left( U^2 h \right) + \frac{\partial}{\partial y} (UVh) = -gh \frac{\partial \eta}{\partial x} - \frac{1}{\rho} \left( \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \right) - \frac{\partial}{\partial x} \left( A_H \frac{\partial U}{\partial x} \right) - \frac{\tau_x^b}{\rho}$$
(22)

where *U* and *V* are the depth averaged cross and along shore Lagrangian velocities, respectively, h is the total depth,  $\rho$  is the fluid density, *S*<sub>ij</sub> represents the components of the radiation stress tensor,  $\tau_x^b$  is the component of the bottom stress acting in the x-direction and *A*<sub>H</sub> is the horizontal viscosity coefficient.

The depth averaged distribution of these terms from modified model simulations are, shown in Fig. 15. Alongshore variation of the depth averaged horizontal advection, bottom friction and gradient of alongshore radiation stress  $(\partial S_{xy}/\partial y)$  in the cross-shore direction, are shown in Fig. 15a-d for four locations (40, 30, 26 and 20 m respectively from the shoreline, see Fig. 9). Since horizontal advection and bottom stress depend on velocity magnitude and gradients, these terms become important within and in the vicinity of the rip channel as seen in Fig. 15b and c. Close to the shoreline and further offshore. bottom friction and horizontal advection become less significant. For normally incident waves,  $S_{xy}$  and  $\partial S_{xy}/\partial y$  should be 0 at all the locations, as is observed in Fig. 15a, b, c and d for all alongshore positions other than the rip channel. Local wave refraction effects due to interaction of rip currents with incoming waves lead to the development of  $\partial S_{xy}/\partial y$  within the rip channel. These terms are partially in balance with the horizontal advection terms, at locations within and outside the rip channel area (Fig. 15a, b and c).  $\partial S_{xy}/\partial y$ becomes relatively insignificant very close to the shoreline (Fig. 15d).

The alongshore variation of depth averaged horizontal viscosity, pressure gradient and radiation stress at the same transect locations as for the other terms (see above) are shown in Fig. 15e-h. At distances 40 m from the shoreline, where no wave breaking occurs,



**Fig. 15.** Alongshore variation of the depth averaged cross-shore momentum balance equation terms. Horizontal advection  $(ADV_H, \partial/\partial x(U^2h) + \partial/\partial y(UVh)$ , black line), bottom stress  $(BT, \tau_x/\rho, \text{gray line})$  and radiation stress forcing  $(\partial S_{xy}/\rho \partial y, \text{black dashed})$  terms are shown in (a) to (d). Cross-shore pressure gradient (PG,  $gh(\partial \eta/\partial x)$ , black line), radiation stress forcing  $(RAD_H, \partial S_{xx}/\rho \partial x, \text{gray line})$  and horizontal viscosity (VISC<sub>H</sub>,  $\partial(A_H \partial u/\partial x)/(\rho \partial x)$ ), (black dashed) are shown in (e) to (h). The distances at which the terms are estimated are 40 m (a) and (e), 30 m (b) and (f), 26 m (c) and (g), and 20 m (d) and (h) from the shoreline (see Fig. 11).

the gradient of cross-shore radiation stress  $(\partial S_{xx}/\partial x)$  and pressure gradient terms are insignificant. Within the surf zone,  $\partial S_{xx}/\partial x$  is balanced by the pressure gradient for all alongshore locations (Fig. 15f, g and h). As wave breaking initiates at the bar crest,  $\partial S_{xx}/\partial x$  is weaker within the rip channel (Fig. 15f, g) than over the bar. When waves propagate over the channel and break close to the shoreline, pressure gradient and  $\partial S_{xx}/\partial x$  obtain greater values than at other alongshore positions (Fig. 15h). The horizontal viscosity is always small except at locations with increased rip velocities, thus increasing the mixing within the rip channels. All these numerical simulation results are found to be qualitatively similar and in agreement with the experimentally-derived results of Haller et al. (2002).

#### 4. Discussion

Overall results presented here indicate that the modification of the M08-11 formulation with the introduction of a vertical distribution function as shown in Eq. (10) (M08-11<sub>vrt</sub>) mitigates some of the shortcomings of the original method and provides results consistent with previous solutions both in the depth averaged and the vertical distribution of the circulation patterns. In this section, our findings are explored for a more comprehensive discussion of the forces operating in rip current development. In particular we discuss the implication for sediment transport and rip channel development and also the variability of rip current strength as a function of the wave incident angle.

#### 4.1. Cell circulation and potential morphological impacts

Our Case 3 has re-affirmed how small differences in offshore wave height distribution can lead to the development of rip-current circulation patterns. However, one of the fundamental questions is the association of rip currents with bathymetry (i.e., bar-channel morphology). One suggestion from this work is that although a rip current circulation may develop due to offshore variable wave conditions, a positive feedback with the sea bed through sediment transport might lead to the bar-channel configuration that is usually associated with rip currents. In a simplified approach, we use results from Case 3 to assess the sediment transport patterns that such rip cells may create. Assuming that the combined action of wave oscillatory motion and mean current is the main mechanism for sediment resuspension and that the mean current is the advective transport mechanism (i.e., ignoring the effects of wave asymmetry), a simplified proxy for sediment erosion or accumulation can be established:

$$P_{ST} = \frac{\partial \left( U_t^2 \cdot \overline{u} \right)}{\partial x} + \frac{\partial \left( V_t^2 \cdot \overline{v} \right)}{\partial y}$$
(23)

where  $U_t$  is the total instantaneous maximum velocity, comprising the vector sum of the wave orbital velocity and mean current vector, and u and v are the cross-shore and longshore Eulerian velocities, while the overbar denotes mean values. Although this simple proxy does not account for settling of sediment and other processes important in morphological evolution (see Warner et al., 2008), it gives some indication of the trend for bed evolution under these conditions. As shown in Fig. 16, the erosion potential is greatest at alongshore location  $\lambda \cdot y = \pi$ , which corresponds to the area influenced by the outgoing rip current. The erosion potential reduces as we move towards the side boundaries  $\lambda \cdot y = 0$  and  $\lambda \cdot y = 2\pi$ . Such tendency suggests that alongshore changes in wave forcing creating a rip current cell eventually might contribute to the development of the typical bar-channel configuration.



**Fig. 16.** Contour of sediment transport proxy ( $P_{st}$ ) over computational domain for the run of Case 3 of alongshore variable wave forcing.

#### 4.2. Driving forces for rip cell circulation

As described earlier, rip cells can be developed either due to alongshore variability in the offshore forcing of wave height (Case 3) or due to variability in the nearshore bathymetry (Case 4). In this section we attempt to examine the differences in the forces that drive the cell through an analysis of the depth and time averaged alongshore momentum balance (steady state, U and V are depth averaged Lagrangian velocities):

$$\frac{\partial}{\partial y}(UVh) + \frac{\partial}{\partial y}\left(V^{2}h\right) = -gh\frac{\partial\eta}{\partial y} - \frac{1}{\rho}\left(\frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y}\right) - \frac{\partial}{\partial y}\left(A_{H}\frac{\partial V}{\partial y}\right) - \frac{\tau_{y}^{b}}{\rho}$$
(24)

These terms are plotted in Fig. 17 as a function of alongshore distance (normalized by the length scale of the offshore forcing (Case 3) or bathymetric perturbation as in Cases 4 and 5). The transects were taken well within the surf zone ensuring uniform alongshore water depth for Case 5 (Fig. 11, alongshore transect inshore of rip channel), and are located at the middle of the surf zone for Cases 3 and 4 (see dotted line in Figs. 5 and 9b, respectively). The transect location for each case corresponds qualitatively to where the alongshore flows of the circulation cell (Fig. 17a) converge to feed the main rip current. Case 3 produces an alongshore variability of the longshore current that resembles the alongshore variability of the wave forcing, but is 90° out of phase. A similar alongshore variability is observed for Cases 4 and 5, although in these cases the peak longshore feeder current is stronger than in Case 3 and located closed to the center of the rip cell.

The pressure gradient term (PG) shown in Fig. 17b, c and d cooscillates with the feeder current for each case. This indicates that pressure gradient is the dominant driver for both cases. However, within each case, the other terms exhibit similar relative behavior with the exception of the radiation stress (RAD<sub>H</sub>) term that changes sign for each case. In Case 3 (Fig. 17b), RAD<sub>H</sub> is positive to the left of the rip channel and negative to the right, while the opposite is true for Cases 4 and 5 (see Fig. 17c and d). Also it is noticeable that the absolute values of the terms for Case 3 and Cases 4 and 5 are almost an order of magnitude different, while the resulting absolute current velocities are of the same order. This increase in magnitude between the terms is attributed to the fact that in Cases 4 and 5, the undulated bathymetry creates local wave refraction effects that lead to increased values of the S<sub>xy</sub> term. This term qualitatively should be directed away from the center of the channel (location of minimum value) attaining a maximum value near the bathymetric highs. In terms of gradient,



**Fig. 17.** (a) Depth averaged, alongshore, Eulerian velocity,  $\overline{v}^e$  at alongshore transects shown by dotted line in Fig. 5 for Case 3, dotted line in Fig. 9b for Case 4 and alongshore transect onshore of the rip channel (Fig. 11) for Case 5; Alongshore variation of the depth averaged alongshore momentum balance terms for (b) Case 3 and (c) Case 4 for alongshore transect as 17(a), (d) Case 5 for alongshore transect as 17(a). The alongshore normalizing length scale (L<sub>y</sub>) used in (b), (c) and (d) are 1000 m, 80 m and 90 m, respectively, and represent the corresponding perturbation length in forcing or bathymetry (key: alongshore pressure gradient (PG, gh( $\partial \eta/\partial y$ ), black line), radiation (ADV<sub>H</sub>,  $\partial/\partial x$ (UVh) +  $\partial/\partial y$ (V<sup>2</sup>h), gray line), bottom stress (BT,  $\tau_v/\rho$ , gray dashed-dot line)).

this corresponds to zero values at the center and either side of the channel as appears to be the case in Fig. 17c and d (zero values at 0.3, 0.5 and 0.7, respectively). In Case 3, the radiation stress gradient term

is solely due to S<sub>vv</sub> and it has a small value. This increased importance of radiation stress gradient in Cases 4 and 5 is compensated by an increase in the absolute value of the pressure gradient. The latter is driven partially by increased wave setup over the shoals due to bathymetry, but also due to increased wave height caused by focusing of the waves over the shoal due to refraction (i.e., the same process that increases the importance of the radiation stress gradient term). Thus overall, independent of the conditions (i.e., variable forcing or bathymetry), alongshore pressure gradient appears to be the main mechanism for the generation of feeder currents. Any increase in the alongshore radiation stress term is compensated by a similar increase in pressure gradient so that the net forcing remains of the same order. In all the cases discussed above, the horizontal advection contribution is dominant only within the rip channel area. Of the terms  $\partial (V.V.h)/\partial y$ and  $\partial (U.V.h)/\partial x$  responsible for horizontal advection, the latter has a greater magnitude in the vicinity of the rip channel because of stronger cross-shore velocity within the channel area.

#### 4.3. Obliquely incident waves on LBT

In order to assess the effect of wave incidence angle on the development of rip current circulation, a longshore bar-trough morphology domain as in Case 5 was subjected to offshore waves with a height of 0.5 m and a period of 3.16 s incident at angles of  $0^{\circ}$ ,  $5^{\circ}$ ,  $10^{\circ}$  and  $20^{\circ}$  with respect to the shore normal. The model uses two-way coupling, allowing for interaction of waves and currents, and the results are shown in Fig. 18.

The top panel (Fig. 18) shows the depth averaged Eulerian velocity field in the rip channel for obliquely incident waves. As the incidence angle increases from 0° to 20°, the angle of exit of the rip current increases with respect to the shore normal. The trend is linear and for angles greater than 20°, the current becomes almost parallel to the shoreline. Svendsen et al. (2000) simulated rip currents on barred beaches incised by channels using SHORECIRC and observed similar behavior of strong inertia of alongshore flow and weak rip currents for high wave angle of incidence. As expected, the strength of longshore velocity increases as the wave angle of incidence increases.



**Fig. 18.** Circulation (depth averaged, Eulerian current vector, top row), significant wave height distribution (middle row) and vorticity field (bottom row) results for different wave incident angles (columns one to four correspond to angles 0, 5, 10 and 20°, respectively). The thin gray lines in top row, column one shows the alongshore transects at which relevant terms are plotted in Fig. 19. Note: The bathymetry used in this case is same as Fig. 11, but only the relevant part of the domain has been shown here.

The wave height distribution over the domain, for different wave incidence angles, is shown in the middle panel of Fig. 18. When waves are normally incident, the rip current flow makes the waves steeper at the location of the channel, locally increasing the wave height (Fig. 18 column (a)). For a wave incidence of 5°, wave steepening at the rip channel is also observed, but the increase in wave height is smaller than that observed for 0°. At higher angle of incidence (10°), wave current interaction reduces as only the component of the rip current along the direction of wave propagation interacts directly with the incoming waves. For waves coming at an angle of 20° to shore normal, the difference in wave breaking location over the bar and the channel is negligible, further hinting at the lack of substantial rip currents.

Circulation pattern at the channel location is depicted through the vorticity vector (Fig. 18, bottom panel). For normal incidence, primary and secondary circulation cell formation occurs outside the rip channel and close to the shoreline, respectively. These cells are symmetric about the rip channel center with opposite sign of vorticity indicating reverse sense of circulation. Such vortices are similar to the macrovortices formed due to wave breaking examined both analytically and computationally in Brocchini et al. (2004) and Kennedy et al. (2006). When waves are incident at 5°, the secondary circulation pattern weakens but the primary circulation pattern is reinforced as seen by increase in the magnitude of vorticity vector. Stretching and alongshore advection of vortices is also observed in this case. At a wave incidence of 10°, the secondary circulation cell close to the shoreline disappears and the vortices close to the channel become weak. The vorticity at the channel for 20° incidence shows only one circulation cell which is constrained at the original location where primary circulation was observed.

Fig. 19 (top panel) shows the Eulerian cross-shore velocity for varying angles of incidence  $(0^{\circ}, 5^{\circ}, 10^{\circ}, 20^{\circ})$  in three columns (a), (b) and (c) corresponding to alongshore transects onshore and within the rip channel (see Fig. 18a top panel, alongshore transects). Rip current velocity at these locations is stronger when wave incidence is at 5°

and 10°. Onshore of the channel, maximum offshore directed flow within the channel area occurs for 5° whereas at transects within the channel, rip current velocity is slightly higher for 10° in comparison to 5° incidence (Fig. 19, top panel, column c). Higher angle of incidence (>20°) inhibits rip currents due to inertia of alongshore motion. Aagaard et al. (1997) observed a similar increase in the rip current velocity due to oblique incidence and attributed this phenomenon to "wind enhanced longshore current". Haller et al. (2002) observed an abrupt increase in cross-shore velocity for wave incidence angle of 10° in their test F. The reason for this behavior is suggested to be due to increase in alongshore radiation stress forcing in alongshore direction created by breaking of obliquely incident waves at the bar crest.

The contribution of longshore velocity on rip current circulation pattern is determined by correlating the gradient of Eulerian longshore velocity in alongshore direction (GAV) to the rip current magnitude. A steep gradient of longshore velocity from one end of channel to other signifies a sharp change in longshore velocity. The reduction of longshore velocity feeds the alongshore momentum in cross-shore direction which intensifies the cross-shore velocity. Fig. 19 (bottom panel) shows GAV in alongshore direction for 0°, 5°, 10° and 20° angle of incidence for all three transects. The GAV values for 0° and 5° incidence show similar distributions pointing at presence of a circulation pattern whereas GAV distribution for 10° and 20° incidence are different implicating a loss of the circulation cells.

GAV is maximum for 5° at all locations except at the alongshore transect at the center of the rip channel, where this quantity is equally steep for  $10^{\circ}$  (Fig. 19, bottom panel, column c). Thus most of alongshore momentum for 5° incidence advects through the rip channel due to the inherent rip current circulation in the domain. At a higher angle of incidence, the circulation pattern is destroyed and momentum transfer in cross-shore direction reduces. This information of maximum rip current velocity for oblique incidence is useful for prediction of rip currents when waves coming at a small angle may be more hazardous.



**Fig. 19.** Eulerian cross-shore velocities (top panel) and absolute value of alongshore gradient of Eulerian alongshore velocities (bottom panel) at 3 alongshore transects located (a) 16 m, (b) 22 m, (c) 28 m from the shoreline as shown by gray lines n Fig. 18 (top panel, column (a)) for waves incident at angles 0°, 5°, 10°, 20°.

#### 5. Conclusions

A full three-dimensional, finite difference, circulation model Regional Ocean Modeling System (ROMS) coupled with spectral, phase averaged, wave propagation model SWAN has been updated to the formulations presented by Mellor (2008, 2011a,b,in press) and used to study nearshore circulation processes after a modification of the 3-D radiation stress formulation. Although the scientific debate on the applicability of Mellor's (2008) original approach is ongoing (see Ardhuin et al., 2008, Bennis and Ardhuin, in press and Mellor 2011, in press-a,b), this paper provides an independent assessment of the method for practical applications in the surf zone. The focus here was complex flow regimes, including alongshore variability in wave height and water depth, i.e., phenomenon responsible for rip current like structure formation in the surf zone.

The results indicate that the implementation of the updated radiation stress forcing (M08-11<sub>top</sub>) may create spurious flow fields in the wave shoaling region. Modified vertical distribution (M08- $11_{vrt}$ ) that incorporates wave height as a scale, significantly reduces these spurious flows. Furthermore, unlike the M03 formulation as implemented in HW09, M08-11<sub>vrt</sub> formulations successfully create a surf zone recirculation pattern (onshore flows at surface and offshore directed undertow near the bottom) for obliquely incident waves on a planar beach. Comparison of model simulated littoral velocity profiles to the energetic flow fields observed during DUCK'94 experiment further suggests the applicability of M08-11<sub>vrt</sub> formulation to study nearshore circulation. The relative rms error in velocity profiles simulated using M08-11vrt are 24% and 15.4% in cross-shore and longshore flow, respectively when compared against field observations from Garcez Faria et al. (1998, 2000). These errors are smaller than errors obtained by recent studies conducted by Sheng and Liu (2011) for laboratory measured cross-shore flows.

Comparisons of the depth integrated circulation of the threedimensional runs were found to be in agreement with the general dynamics for formation of nearshore circulation cell on normal incidence of alongshore varying wave height over a planar bathymetry (Bowen, 1969) and under alongshore variable bathymetry forced with alongshore uniform wave height (Noda, 1974). Furthermore, it has been shown that increasing the Reynolds number by decreasing the viscosity, the circulation cells become skewed with the offshore directed flow becoming narrower and faster while onshore flow broadens and becomes slower. The development of the model provided us with insights on the vertical distribution of the crossshore velocities in these circulation patterns allowing us to provide an insight into wave breaking induced flow at the surface and bottom boundary layer.

The new formulation of radiation stress forcing demonstrated a strong agreement with the scaled-up laboratory experiments of Haller et al. (2002) and Haas and Svendsen (2002). The normalized velocities within the rip channel show a relative *rms* error of 11% when compared to normalized, measured flows by Haas and Svendsen (2002). This further suggests that the model is capable of successfully representing complex flows due to changes in bathymetry.

By using a proxy for sediment transport, it is determined that rip current circulation cells formed due to differences in alongshore wave forcing may lead to formation of alongshore barred beaches interrupted by rip channels.

Finally, the effect of obliquely incident waves on rip channels is studied and it is found that rip current strength observed within the channel is stronger when waves come at angles of 5° and 10° in comparison to normally incident waves. This information may be helpful in the prediction of rip currents (Voulgaris et al., 2011).

Overall the implementation of Mellor (2008) based distribution of vertical radiation stress along with a vertical scaling as a function of wave height (M08- $11_{vrt}$ ) improves the ability of coupled ROMS-SWAN model in resolving wave and current effects in the surf zone.

This modeling tool can be used to understand the physical mechanisms driving phenomena observed in surf zone along with prediction of nearshore circulation. However, the introduction of term  $E_D$  in Mellor (2008) creates significantly "unexpected" flows for shoaling and non breaking waves on a sloping topography, which can be reduced and controlled by dissipative momentum mixing through wave breaking induced turbulence. Caution must be applied for application of Mellor (2008) formulations for studies conducted outside the surf zone, where aforementioned dissipative mechanisms are absent.

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#### Appendix A

Mellor (2003) introduced depth dependent formalism for radiation stresses to accommodate wave averaged effects on mean currents. These formulations when vertically integrated are consistent with the depth integrated solution of Longuet-Higgins and Stewart (1964). Ardhuin et al. (2008) showed that use of the M03 formulation in non-breaking wave propagation over an uneven topography produces a spurious circulation pattern at the location where  $\partial h/\partial x \neq 0$ . Subsequently, a new set of depth dependent equations for wave current interaction was presented (Mellor, 2008, 2011, in press-a,b), which is further modified and implemented in this paper for applications in the surf zone. Mellor (2008, see Section 2) suggested that for variable topography, the new set of equations would cause some errors but overall there is a good chance that these equations can be applied to shallow water environment (i.e.,  $kD \approx 1$ , where k is the wave number and D is the total water depth), when effects of viscosity and turbulence are included. In this section we test the above argument by carrying out two numerical simulations corresponding to the setup originally proposed by Ardhuin et al. (2008), Bennis and Ardhuin (in press), and to a setup that uses a milder slope as that found in Duck, NC and including friction and mixing processes. Both setups are forced with a shoaling, nonbreaking monochromatic wave with a significant wave height of 1.02 m and wave period of 5.24 s, propagating from east to west. These runs are described in some detail below.

In the setup resembling Ardhuin's et al. (2008, 2011) conditions, the bottom profile has a channel in which the water depth smoothly transitions from 6 m to 4 m ( $dh/dx_{max} = 0.0266$ ), and is symmetric about the vertical axis at the center (i.e., x = 300 m, Fig. A1a). The non dimensional water depth kD, varies from 0.85 < kD < 1 (Fig. A1b). The model domain is alongshore uniform with a cross-shore width (x) of 600 m and an alongshore length (y) of 800 m. Grid resolution is 4 m and 100 m in x and y direction, respectively. The vertical domain has been distributed in 32 equally spaced vertical layers. The boundary conditions are constant flux at east and west boundary (Neumann





**Fig. A1.** Model forcing (wave height) and non dimensional depth (a and c) and bottom bathymetry (b and d) used to test the Mellor (2008) formulation. (a) and (b) are identical as in Ardhuin et al. (2008). Forcing in (c) is the same as (a) but the bathymetry (d) has a reduced bottom slope (note differences in horizontal scale between b and d).

conditions) and closed in the north and south. Effect of earth's rotation, bottom stress and viscosity has not been included in this case. Simulations have been carried out using both M08-11<sub>top</sub> and M08-11<sub>vrt</sub> formulations.

In the absence of wave breaking, mixing and bottom friction the only dynamic effects occur due to changes in wave height. Shoaling of waves in shallower waters create divergence of the Stokes drift, which is compensated by the Eulerian mean current. The correct representation of Lagrangian velocity field (Eulerian + Stokes) for this wave field and domain setup is a flow along the direction of wave propagation  $(U_l = 0.025 \text{ ms}^{-1})$  at the surface, which decreases gradually to no flow at z = -2 m and then changes to a return flow of  $U_l = -0.01 \text{ ms}^{-1}$  close to the bottom layer. The flow field at the surface and bottom follows the bathymetric contours (see Fig. 2, in Bennis and Ardhuin, in press).

The vertical profile of Lagrangian cross-shore velocity based on M08-11<sub>top</sub> is shown in Fig. A2a. At the location where  $dh/dx \neq 0$  and where the waves are propagating upslope, spurious flow pattern is observed in the upper half of the water column showing a current along the direction of wave propagation ( $U_{lmax} = 0.15 \text{ ms}^{-1}$ ) and a compensating flow, of same strength but opposite sign, in the lower half of the water column. A reversed flow structure is established on the down-slope wave propagation region (Fig. A2a). When we use M08-11<sub>vrt</sub> based formulations (Fig. A2b), a significant part of the water column shows a weak flow,  $U_1 \approx 0.01 - 0.10 \text{ ms}^{-1}$  towards wave propagation direction, while the surface layer shows a relatively stronger flow of 0.20–0.25 ms<sup>-1</sup> in the opposite direction. The flow field is reversed when waves propagate down the slope. Irrespective of updating the formulation for radiation stresses, in an idealistic situation, M08-11 and M08-11<sub>vrt</sub> based simulations still create incorrect flow patterns for unforced waves traversing on a sloping

**Fig. A2.** Vertical distribution of Lagrangian velocity,  $u^{l}(x, z)$  (Eulerian velocity + Stokes drift) calculated using (a) M08-11<sub>top</sub> with a domain geometry as in Ardhuin et al. (2008); (b) M08-11<sub>vrt</sub> on the same domain as (a); and (c) M08-11<sub>vrt</sub> with a similar geometry but reduced bottom slope (note differences in horizontal scale), uniform vertical mixing and bottom friction. Contour line spacing is 0.01 ms<sup>-1</sup> in (a), (b) and 0.002 ms<sup>-1</sup> in (c). Note different scales in color bar used in (c).

bottom. This is consistent with Bennis and Ardhuin (in press) and Ardhuin et al. (2008).

The second setup uses a milder, more realistic slope  $dh/dx_{max} =$ 0.0066, that is an average value for continental shelf environments (e.g., Hayes, 1964), bottom friction (quadratic drag,  $C_d = 0.003$ ) and mixing (constant eddy viscosity,  $0.0028 \text{ m}^2\text{s}^{-1}$ ). The domain is also symmetric about the vertical axis at the center (i.e., x = 1200 m, Fig. A1c). The non dimensional water depth kD, is the same as before. The model domain is alongshore uniform with a cross-shore width (x) of 2,400 m and an alongshore length (y) of 800 m. Grid resolution and vertical domain remain the same as previously. In this run the Lagrangian velocity (Fig. A2c) is along the direction of wave propagation at the surface layer except at the upslope wave propagation location where small perturbations in the velocity flow field are observed. Compensating return flow in the lower half of water column is also observed. However, the strength of Lagrangian velocity is reduced by a factor of ~5 when compared to the ideal conditions (Fig. A2a and b). Also it is noticeable that velocity contours "attempt" to follow the bathymetric contours, as in Bennis and Ardhuin (in press).

The maximum velocity at the surface in Fig. A2c is twice the velocity calculated by Bennis and Ardhuin (in press) hence the flow field may be still slightly "erroneous" although Mellor (in press-a) argues that this might be expected. Bennis and Ardhuin (in press) also state that the problem is not just a question of vertical distribution of radiation stress, but one of a relatively large and spurious source of momentum. Nevertheless, our results using our modified distribution of radiation stress, realistic mixing and bottom slope reduce the "erroneous" flows (Fig. A2a) by a factor of 4. In addition, all the simulations presented in this contribution (Cases 1–5) are for surf zone conditions, where the wave breaking induced flow is an order of magnitude higher than the topography-induced flow shown in Fig. A2c (i.e., realistic shelf slope and mixing). This suggests that the errors identified by Bennis and Ardhuin (in press) might be

inconsequential for practical applications in the surf zone using our modifications. These errors will be more insignificant when injection of wave turbulence and wave roller processes are included.

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