MODULATION OF WIND RIPPLES BY LONG SURFACE WAVES VIA THE AIR FLOW: A FEEDBACK MECHANISM

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Abstract. The evolution of a short-wave (SW) spectrum along a long wave (LW) is studied. The evolution of the SW spectrum variation is treated in the relaxation time approximation. The variation of the SW spectrum is caused by the LW orbital velocities and by the variation of the wind stress along the surface of a LW. The latter is due to the distortion of the flow by a LW, and to the variation of the roughness induced by the modulated short waves. This introduces a feedback mechanism: more SWs give rise to a larger roughness, which by increasing the local stress stimulates the growth of more SWs. It is shown that this aerodynamic feedback effect dominates the modulation of the SW spectrum for moderate and strong winds. The feedback mechanism is most effective for SWs in the gravity-capillary range, increasing its dominance with increasing windspeed and decreasing frequency of a LW. The maximum of the SW amplitude modulation is situated at the crest of a LW. The results are in agreement with laboratory and field measurements of the short-wave modulation.

1. Introduction

The investigation of the interaction between short wind waves (SWs) and long surface waves (LWs) is of interest from at least two viewpoints. First, the modulation of SWs by LWs determines the hydrodynamical component of the radar modulation transfer function (MTF). The observation of ocean wave spectra by radar (RAR, SAR) is now done routinely (Komen et al., 1994), and better understanding of the mechanism of the radar image formation is of great practical importance. Second, the modulated SWs give rise to the variation of the roughness along the LW surface. In general this can result in an additional variation of the air flow and have an effect on the growth or decay of a LW.

Most experimental data of SW modulation by LWs are obtained via the study of radar MTF (Plant et al., 1983; Schröter et al., 1986; Hara and Plant, 1994). According to these measurements the amplitude of the SW modulation grows with decreasing LW frequency. The maxima of the SW spectrum are observed on the LW forward slope, closer to the crest than to the trough. For a given LW frequency, the amplitude of the SW modulation decreases for increasing wind speed. However, the direct observation of the SW modulation in a laboratory experiment (Miller and Shemdin, 1991) shows the growth of the SW modulation with increasing wind speed.

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Alpers and Hasselmann (1978) and Keller and Wright (1975) proposed models in which the small variation of the source in the evolution equation of the SW spectrum is described in the relaxation time approximation. These models take into account only the interaction of the SWs with the LW orbital velocities. They predict a decrease of the modulation of the SW spectrum with a decrease of the LW frequency and with an increase of wind speed. This contradicts the experimental data of Plant et al. (1983) and Schröter et al. (1986). Valenzuela and Wright (1979) proposed a theory that takes into account the modulation of the three- and four-wave interaction. Despite the considerable complexity of the model, it does not explain the SW modulation observed in real conditions.

Smith (1990) studied in detail the effectiveness of various physical mechanisms to modulate the short wind waves by a LW: the modulation of the SW by the LW orbital velocities, the effect of a varying surface roughness and stress and the effect of shear currents. The conclusion reached in this work is that the observed SW variations can not be explained by the interaction of SWs with the LW orbital velocities. The only viable cause of the observed modulation is the variation of the surface stresses with an amplitude 25 times the LW steepness in phase with the LW elevation, with the maximum of the stress located at the crest. In other words, the behaviour of the SWs is governed by the aerodynamics of the air flow above a LW. Smith (1990), and later Hara and Plant (1994) and Romeiser et al. (1994) assumed that some feedback mechanism could be responsible for large variations of the short wind waves and the surface stresses on a LW. They suggest the following qualitative description of this feedback mechanism. The short waves affect the surface roughness, which determines the stress distribution, which in turn affects the short waves. However no quantitative description of this feedback mechanism has been proposed so far.

In this paper an attempt is made to describe quantitatively this feedback mechanism. A feedback between SWs and the surface stress variations can appear only when the air flow is coupled to wind waves. Recently a few coupled models have been proposed (e.g., Janssen, 1989; Chalikov and Makin, 1991; Makin et al., 1995).

The present study is based on the approach of Makin et al. (1995). In their model the roughness parameter (and consequently the drag coefficient of the sea surface) is defined by the relation

$$z_0 = z_m \left(\frac{z_\nu}{z_m}\right)^{(1-\alpha)^{1/2}} . \qquad (1)$$

In Equation (1)

$$\alpha = \tau_w/(\rho_a u_*^2) \qquad (2)$$

is the coupling parameter, u_* is the friction velocity of the air, τ_w is the momentum flux to waves, z_m is the scale of the vertical decay of the wave-induced momentum flux, ρ_a is the air density and z_ν is the viscous roughness length defined by

$$z_\nu = 0.14\frac{\nu_a}{u_*}(1-\alpha)^{-1/2} \tag{3}$$

where ν_a is the air viscosity. The momentum flux to waves can be expressed in terms of the wave action spectrum $N(\mathbf{k})$ and the wind-wave growth rate β

$$\tau_w = \int_0^\infty \beta\omega k \cos\theta N d\mathbf{k}, \tag{4}$$

where $\mathbf{k}$ is the wave number vector, θ is its direction, $k = |\mathbf{k}|$ and ω is the wave frequency. The scale z_m is defined by

$$z_m^{-1} = \frac{1}{\tau_w}\int_0^\infty 2k\beta\omega k \cos\theta N d\mathbf{k}. \tag{5}$$

The drag of the sea surface is thus formed by the momentum flux to waves and by the viscous stress. At low wind speeds the total surface stress, which equals $\rho_a u_*^2$, is dominated by the viscous drag, whereas at high wind speeds it is mainly supported by the momentum flux to waves. In the former case the coupling parameter α is small. In the latter it is close to 1 (see Figure 3 in Makin et al., 1995) and the air flow is completely coupled to waves. In Makin et al. (1995), it was concluded that most of the momentum is transferred to decimetre and metre waves.

Equation (1) gives an explicit relation between the sea roughness (drag coefficient) and the wind-wave spectrum via the momentum flux. It plays a key role in the present study. The dependence of the wave spectrum on the wind stress implies that, in the coupled dynamical system of wind waves and air flow, a feedback mechanism can take place.

The main goal of the present paper is to give a physical description of this feedback mechanism. It is shown that SWs, modulated by the wind stress, cause a considerable variation of roughness along the LW profile, thereby further enhancing the stress variations necessary to maintain the SW modulation.

2. Short-Wave Modulation by a Long Wave via the Air Flow

2.1. Variation of the Short Wave Spectrum

The two-dimensional background spectrum of the short wave action $N(k,\theta)$ is assumed to be disturbed by a LW of a small amplitude running along the x-axis. It follows that the N modulation $\tilde{N}$ is also small: $\tilde{N}/\bar{N} \ll 1$ (hereafter a bar will denote undisturbed values).

Assuming that the frequency Ω and the wave number K of a LW are significantly smaller than the frequency ω and the wave number k of a SW, the linearized equation for the variation of the SW action spectrum in the relaxation approximation (Keller and Wright, 1975; Alpers and Hasselmann, 1978) can be written

$$\frac{\partial \tilde{N}}{\partial t} - k_x \frac{\partial u}{\partial x} \frac{\partial \bar{N}}{\partial k_x} = \tilde{\beta}\omega\bar{N} - \frac{\tilde{N}}{T}, \tag{6}$$

where u is the LW orbital velocity and T is the relaxation time. From (6) it follows that the modulation of the SW spectrum results from the variation of the growth rate $\tilde{\beta}$ (caused by variations in the airflow) and by the orbital motions of a LW. The relaxation time T is defined by

$$T^{-1} = -\frac{\partial q}{\partial N}. \tag{7}$$

The energy source function q describes the wind input, the dissipation due to wave breaking and viscosity, and the wave-wave interactions. The applicability of the relaxation approximation for short waves has been discussed by Alpers and Hasselmann (1978). If the dissipation and the wave-wave interaction terms of the source function are not known exactly the relaxation time can be defined by the known (e.g. from experiments) wind-speed dependence of the spectrum level (Kudryavtsev, 1994; Hara and Plant, 1994):

$$\omega T = \frac{1}{N}\frac{\partial N}{\partial u_*}\left(\frac{\partial \beta}{\partial u_*}\right)^{-1}. \tag{8}$$

The relaxation time can be estimated from relation (8) if the SW spectrum N and the growth rate β are specified.

2.2. Variation in the Airflow over a Long Wave

According to Equation (6) the modulation of the SW spectrum can be caused by the variation of the SW growth rate. Parameterization of Plant (1982) of the growth rate parameter

$$\beta = 0.04\frac{u_*^2}{c^2} \tag{9}$$

is used, where c is the SW phase speed. This relation shows that the modulation of the growth rate is related to variations in the surface stress that may appear due to the distortion of the air flow over a LW.

Belcher and Hunt (1993) have studied the air flow over the short slow moving waves. They distinguish two main regions above waves. In the inner region (IR) – a thin region adjacent to the wave surface – the significant disturbances to turbulence caused by waves occur and are in equilibrium with the local velocity shear. Outside the inner region – in the outer region – the wave-induced perturbations to turbulence are small and the wave-induced velocity perturbations are almost inviscid. The thickness of the IR, l, is given (Belcher and Hunt, 1993) by the implicit relation

$$Kl \sim \frac{2\kappa u_*}{|U_l - C|}, \tag{10}$$

where κ is the von Karman constant, U_l is the wind speed at height l and C is the LW phase speed. Numerical calculations of Mastenbroek et al. (1996) show that relation (10) is also valid for the air flow above fast long waves $(C > U_l)$. The depth of the IR can be roughly estimated as $Kl \sim 0.1$ (see Figure 2 in Mastenbroek et al.).

As a zero-order approximation the wind profile inside the IR is assumed to follow the logarithmic distribution

$$V(x, z) = U(x, z) - u(x) = \frac{u_*(x)}{\kappa} \ln\left(\frac{z}{z_0(x)}\right), \tag{11}$$

where $z = z' - \eta(x)$ is a vertical coordinate related to the instantaneous long wave profile $\eta(x)$, (x, z') are Cartesian coordinates, $V(x, z)$ is the wind velocity relative to the surface, u is the LW orbital velocity, and z_0 is the roughness parameter that can vary along the LW profile. As shown by Belcher and Hunt (1993), the logarithmic profile (11) is exactly valid only inside the so called inner surface layer, which is the lower part of the IR. However, the deviation of the velocity profile (11) from the exact solution in the upper part of the IR is not relevant for the present study.

We further need to define the wind-speed variation at the top of the IR. For slow moving waves, the amplitude of the wind-speed variation is $\mathcal{U}_l \sim \epsilon(\bar{U}_K^2/\bar{U}_l)$ (Belcher and Hunt, 1993), where ϵ is the steepness of a LW, $\bar{U}_K$ and $\bar{U}_l$ are the undisturbed wind velocities at height $z = K^{-1}$ and $z = l$ correspondingly. For a very fast LW with $C \gg U_K$, the amplitude of the wind-speed variation at height $z = l$ equals $\mathcal{U}_l \sim -\epsilon C$. We combine both by the relation

$$\mathcal{U}_l \sim \epsilon\left(\bar{U}_K^2/\bar{U}_l - C\right). \tag{12}$$

SWs are advected by the local orbital velocity of a LW. The amplitude of the wind-speed variation in the IR relative to the moving surface is

$$\mathcal{V}_l = \epsilon(\bar{U}_\lambda^2/\bar{U}_l - 2C), \tag{13}$$

we substituted $\bar{U}_\lambda$ instead of $\bar{U}_K$, ($\bar{U}_\lambda$ is the wind velocity at height $z = \lambda$, and λ is the LW length), to obtain a better agreement with the numerical results of

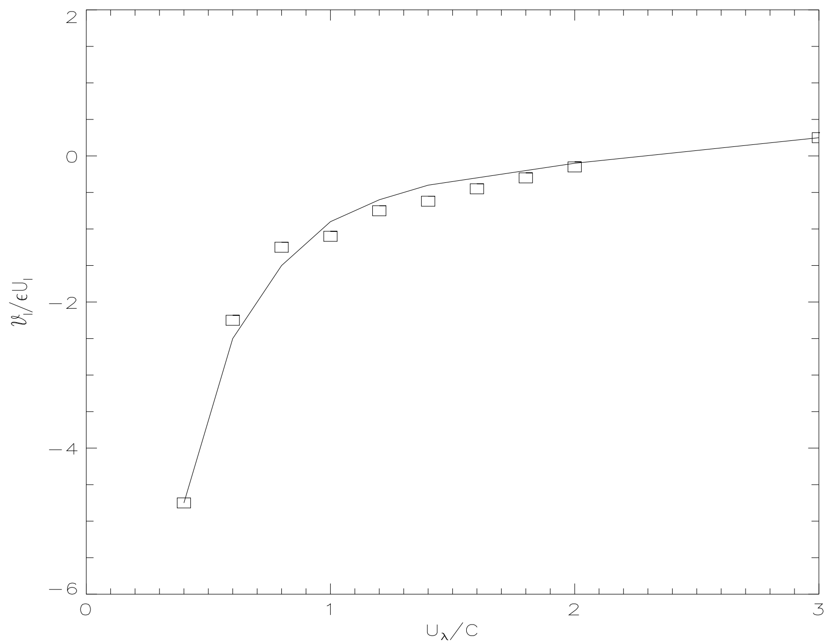

Figure 1. The amplitude of the wind speed variation over the IR relative to the moving water surface. Open squares: numerical calculation; line: parameterization (13). Dimensionless roughness parameter $z_0 K = 10^{-3}$.

Mastenbroek et al. (1996). In Figure 1 a comparison between the amplitude of the relative wind variations according to (13) and numerical calculations is shown. We find that relation (13) approximates the numerical results well. For a slow LW with $\bar{U}_\lambda/C > 2$, the maximum of the wind speed relative to the moving surface is situated at the crest of a LW, whereas for faster waves it is situated in the trough.

Disturbances of turbulence caused by the interaction of the air flow with the LW surface are located in the IR. The relative wind speed over the IR, V, is thus the external parameter that defines the momentum flux in the IR. The resistance law for the IR follows from (11)

$$u_*(x) = \left[\bar{U}_l + \tilde{V}_l(x)\right] C_l^{1/2}(x), \tag{14}$$

where $\tilde{V}_l$ is the relative wind-speed variation over the IR with amplitude defined by (13), $C_l(x)$ is the local drag coefficient related to the local surface roughness (1) by

$$C_l(x) = \left[\kappa/\ln(l/z_0(x))\right]^2. \tag{15}$$

2.3. Coupling of Short Waves and Surface Wind Stress

The surface drag coefficient (15) depends via relation (1) on the coupling parameter α and its variation $\tilde{\alpha}$,

$$\tilde{\alpha} = \alpha \frac{\tilde{\tau}_w}{\bar{\tau}_w} - 2\alpha \frac{\tilde{u}_*}{\bar{u}_*}. \tag{16}$$

The modulation of the wave induced stress $\tilde{\tau}_w$ is

$$\tilde{\tau}_w = \int_{k_{mod}}^{\infty} \tilde{\beta} \omega k \cos\theta \bar{N} d\mathbf{k} + \int_{k_{mod}}^{\infty} \bar{\beta} \omega k \cos\theta \tilde{N} d\mathbf{k} \tag{17}$$

so that the variation of the wave-induced stress is due to the variation of the growth rate parameter $\tilde{\beta}$ and the variation in the spectrum level $\tilde{N}$. The integration over k starts from k_{mod} to account for the fact that not all waves are modulated by a LW but only those with the wave number $k > k_{mod} \sim 10K$.

The equation for the variation of the friction velocity in the IR follows from the resistance law (14),

$$\frac{\tilde{u}_*}{\bar{u}_*} = (1 - \alpha\mathcal{F}) \frac{\tilde{V}_l}{\bar{V}_l} + \frac{1}{2} \alpha \mathcal{F} \frac{\tilde{\tau}_w}{\bar{\tau}_w}, \tag{18}$$

where

$$\mathcal{F} = \left[1 + \frac{\ln(l/z_m)}{\ln(z_m/z_\nu)} (1-\alpha)^{1/2}\right]^{-1}. \tag{19}$$

The accuracy of (18) is of order $O(C_l^{1/2})$. Equation (18) relates the surface stress variation to the variation in the relative wind speed and in the momentum flux to waves.

The SWs travelling along a LW experience variations of the surface stress due to variations of their growth rate. The depth of the IR is $l \sim 0.1K^{-1}$, whereas the modulated SWs satisfy the condition $k \gg K$. This means that a SW develops inside the IR of a LW and its growth is defined by the local surface stress. Variation of β follows from (9)

$$\frac{\tilde{\beta}}{\bar{\beta}} = 2 \frac{\tilde{u}_*}{\bar{u}_*}, \tag{20}$$

where u_* is defined by (18) and depends on the phase of a LW. With (20), Equation (17) for the variation of the momentum flux to waves can now be rewritten in the form

$$\frac{\tilde{\tau}_w}{\bar{\tau}_w} = 2p \frac{\tilde{u}_*}{\bar{u}_*} + p \frac{\tilde{\tau}_N}{\bar{\tau}_{wp}}, \tag{21}$$

where

$$\tilde{\tau}_N = \int_{k_{mod}}^{\infty} \bar{\beta}\omega k \cos\theta \tilde{N} d\mathbf{k} \tag{22}$$

is the variation of the momentum flux to waves due to the variation of the SW spectrum. Parameter p in (21) is defined as $p = \bar{\tau}_{wp}/\bar{\tau}_w$, where $\bar{\tau}_{wp}$ is the momentum flux to the modulated part of the SW spectrum

$$\bar{\tau}_{wp} = \int_{k_{mod}}^{\infty} \bar{\beta}\omega k \cos\theta \bar{N} d\mathbf{k}. \tag{23}$$

A closed set of equations (18), (20), and (21) describes the response of the surface stress (and therefore the SW growth rate parameter) to variations in the SW spectrum and the surface wind speed induced by a LW. The solution of these equations for the modulation of the growth rate is

$$\frac{\tilde{\beta}}{\bar{\beta}} = 2\frac{\tilde{V}_l}{\bar{U}_l}\left(1 - \frac{1-p}{p}\Phi\right) + \Phi\frac{\tilde{\tau}_N}{\bar{\tau}_{wp}}, \tag{24}$$

where Φ is the feedback parameter

$$\Phi = \frac{\alpha p}{(1-\alpha)^{1/2} + \alpha(1-p)}. \tag{25}$$

In (25) the fact that $\ln(l/z_0)/\ln(z_m/z_\nu) \simeq 1$ is used. The feedback parameter Φ relates variations of the SW spectrum to variations of their growth rate parameter. The enhancement of SWs increases the momentum flux to them from the air flow. That in turn further enhances their growth and the feedback mechanism can emerge. The feedback parameter increases if both $\alpha \to 1$ and $p \to 1$. The value of p is defined by the ratio of the LW frequency Ω to the wind-wave spectrum peak frequency (ω_p). When $\Omega \sim \omega_p$, p is close to 1, the coupling parameter α increases with the increase in wind speed. It has been shown by Makin et al. (1995) (their Figure 3) that the coupling parameter α increases from 0.7 at $U_{10} = 8$ ms^{-1} (U_{10} is the wind speed at height $z = 10$ m) to 0.95 at $U_{10} = 20$ ms^{-1}. Consequently, the feedback mechanism should be more effective for the higher wind speeds, when a large fraction of the stress is carried by waves.

2.4. The Feedback Mechanism

The normalized amplitudes $\hat{N}, \hat{\beta}$ of variations in $\tilde{N}$ and $\tilde{\beta}$ are defined by

$$\left(\frac{\tilde{N}}{\epsilon\bar{N}}, \frac{\tilde{\beta}}{\epsilon\bar{\beta}}\right) = \frac{1}{2}\left[\left(\hat{N}, \hat{\beta}\right) e^{i(Kx - \Omega t)} + \text{c.c.}\right], \tag{26}$$

where ϵ is the steepness of a LW. Substituting (26) into the action balance equation for the SW modulation (6) and taking into account the definition of the relaxation time (8) the following equation can be obtained

$$\hat{N} = -\left(\frac{1-i\mu}{1+\mu^2}\right)\frac{k}{\bar{N}}\frac{\partial\bar{N}}{\partial k} + \frac{1}{2}\left(\frac{\mu^2+i\mu}{1+\mu^2}\right)n\hat{\beta}, \tag{27}$$

where

$$\mu^{-1} = T\Omega \tag{28}$$

is the normalized relaxation time and n is the exponent in the wind-speed dependence of the SW spectrum. The latter is defined by

$$n = \frac{u_*}{N}\frac{\partial N}{\partial u_*}. \tag{29}$$

In Equation (27) the normalized amplitude of the variation of the growth rate parameter $\hat{\beta}$ follows from (24)

$$\hat{\beta} = 2\left(1 - \Phi\frac{1-p}{p}\right)\hat{V}_l + \Phi\frac{1}{\bar{\tau}_{wp}}\int_{k_{mod}}^{\infty}\bar{\beta}\omega k\cos\theta\bar{N}\hat{N}d\mathbf{k}, \tag{30}$$

where $\hat{V}_l = \mathcal{V}_l/(\epsilon U_l)$ is the normalized amplitude of the relative wind-speed variation in the IR and $\mathcal{V}_l$ is defined by (13).

The first term on the right hand side of (27) describes the modulation of SWs due to the orbital velocities of a LW. The effect of this modulation mechanism was analyzed by Keller and Wright (1975), Alpers and Hasselmann (1978) and others. The effect of the LW orbital velocities on the modulations of SWs with $\mu \gg 1$ are small. When μ decreases the amplitude of the SW modulation increases and its maximum displaces from the forward face of a LW to its crest.

The second source of the SW modulation in (27) is due to the modulation of the growth rate. There are two contributions to this modulation, described by two terms in Equation (30). The first one is defined by the variation of the relative wind speed over the IR of a LW. Parameterization of this variation is given by Equation (13). Such a mechanism was analyzed by Smith (1990) and Grodsky et al. (1991). Smith (1990) has prescribed the variation of the surface stress with the maximum at the crest of a LW. He found that one has to assume a rather large amplitude for this stress variation ($\sim$ 25 times the LW steepness) to explain the observed amplitude and phase of the SW modulation. The cause of this large stress variation remains unclear, which indicates the possibility that some undiscovered mechanisms of the wind-wave interaction should be involved. Grodsky et al. (1991) have assumed the wind-speed variation similar to (13) and found SW variations that are not consistent with observations.

The second term on the right hand side of (30) describes the contribution to the variation of the growth rate parameter due to the modulation of the SW spectrum along the LW profile. This term is responsible for the feedback mechanism. When the feedback parameter Φ is of order O(1) the positive feedback emerges: an area with more SWs induces a larger local stress, which will stimulate the growth of the SWs in exactly the same area.

The value of Φ determines the effectiveness of the feedback mechanism. The value of Φ is determined by the coupling parameter α and by the contribution of the modulated part of the SW spectrum to the total momentum flux to waves (parameter p). For low winds ($U_{10} < 5$ ms^{-1}) the total stress is supported mainly by viscous drag ($\alpha \ll 1$). The modulation of the SWs will result in a limited stress modulation. Likewise, if a LW is relatively short and modulates only a small part of the spectrum ($p \ll 1$), the stress modulation will also be limited. As can be seen from (25), in both cases Φ will be small and will reduce the influence of the second term in (30). Thus the feedback mechanism will be most effective for strong winds and if the LW frequency is comparable with the frequency of the wind-wave spectral peak, i.e. $\alpha \simeq 1$ and $p \simeq 1$.

The feedback mechanism is efficient for those components of SWs that have a small relaxation time (parameter $\mu \gg 1$). These SWs are completely adjusted to the local surface stress (see eq. (27)). The SWs with a large relaxation time (parameter $\mu < 1$) are disturbed by a LW adiabatically with their amplitudes increased at the LW crest. If these waves contribute considerably to the total momentum flux to waves the maximum of the stress will be also at the LW crest. This will enhance very short wind waves just at the LW crest.

Note, that the SWs modulation due to the modulation of the growth rate parameter (the second term in (27)) depends also on the exponent n, which describes the wind-speed dependence of the SW spectrum (Equation (29)). In the short gravity and capillary-gravity range of the wind-wave spectrum this exponent is close to 2 (e.g. Keller and Plant, 1985). Therefore the feedback mechanism will effectively enhance the wave modulations in this spectral range.

3. Modulation of Short Waves by a Long Wave

3.1. Role of the Feedback Mechanism

In this section we shall illustrate the influence of the feedback mechanism on the modulation of SWs by a LW. We use the same model of a wave spectrum $S(\mathbf{k})$ as was adopted in Makin et al., (1995). This wave spectrum model is based on Donelan et al. (1985) and Donelan and Pierson (1987).

The undisturbed spectrum corresponds to the conditions of a fully developed sea ($U_{10}/c_p = 0.83$, where c_p is the phase velocity of the spectral peak of wind waves). Calculated amplitudes and phases of the SW modulation $M \equiv \hat{N}$ by a long

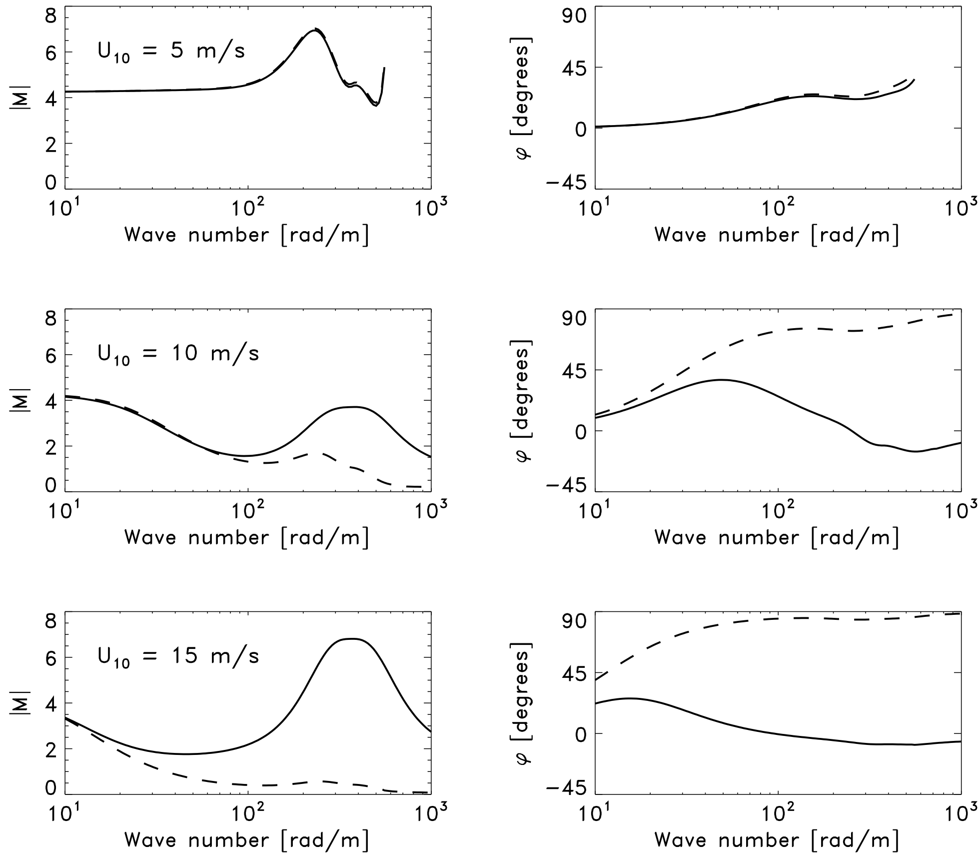

Figure 2. Calculated amplitudes (left column) and phases of the SW modulation $\hat{N}$ by a long wave with $U_{10}/C = 1.5$: full line, calculation including modulation growth rate; dashed line, modulation of SWs due to orbital velocities only.

wave are shown in Figure 2. The long wave is specified by the inverse wave age parameter $U_{10}/C = 1.5$. Calculations are based on Equation (27) that describes two modulation mechanisms: due to the modulation of the orbital velocities (first term in (27)) and due to the modulation of the growth rate parameter (second term in (27)). To show the importance of the latter mechanism the reference calculations are done where only modulations due to the orbital velocities are accounted for.

In low winds ($U_{10} = 5$ ms^{-1}) the μ parameter is small for all modulated SWs. Only modulations due to the orbital velocities are important in this case. The SWs are modulated almost adiabatically and enhanced at the crest of the LW.

When wind speed increases the μ parameter also increases. At wind speed $U_{10} \geq 10$ ms^{-1} its value exceeds 1 for the wave components $k > 50$ rad m^{-1}. These components are modulated mainly due to modulations of the growth rate parameter. With increasing wind speed the amplitude of the modulation increases and the maximum shifts to the crest of the LW. When only modulations due to orbital velocities are accounted for the amplitude of the modulation for these components decreases and its maximum is shifted towards the forward face of the LW.

At high wind speeds the feedback mechanism plays the dominant role. The coupling parameter α is close to 1 (see Makin et al., 1995) and the feedback parameter Φ is large enough to cause the feedback. Makin et al. (1995) have shown that the drag of the sea surface is formed by decimetre to metre waves. These waves are modulated adiabatically by the LW with the maximum energy located at the crest of the LW. This leads to the enhancement of the local wind stress at the crest of the LW. The short gravity and capillary- gravity waves ($k \sim 10^2 - 10^3$ rad m^{-1}) are adjusted to the local wind stress and as the maximum of stress is situated at the crest of the LW they are mostly enhanced also at the crest (see Figure 2, right column, U_{10} = 10 m s^{-1} and 15 m s^{-1}). This description explains the feedback mechanism between modulated waves and the air flow.

Calculations are also performed for the case when the wind speed is kept constant at U_{10} = 10 m s^{-1} and the frequency of the long wave is changed from 0.13 Hz to 0.5 Hz. Calculations are shown for three values of the inverse wave age parameter U_{10}/C of the LW in Figure 3. If only the influence of the LW orbital velocities is accounted for, the SW modulation in the range $k > 10^2$ rad m^{-1} is increased when the frequency of the LW increases. This is due to a decrease of the μ parameter.

When the modulation due to the growth rate parameter is taken into account, the feedback mechanism emerges. The maximum of SW modulations ($k > 10^2$ rad m^{-1}) occurs at the crest of the LW and their enhancement is larger for the LW with a smaller frequency. At wind speed $U_{10} = 10$ ms^{-1} the coupling parameter $\alpha \sim 0.8$. The feedback parameter Φ depends on the α and p parameters. The p parameter characterizes the ratio of the momentum flux to the modulated part of the wave spectrum to the total momentum flux to waves. The range of the modulated waves is related to the LW frequency Ω, $k > k_{mod} = 10\Omega^2/g$. It increases when the LW frequency decreases. This means that $p \to 1$ and the feedback mechanism becomes more efficient when waves are modulated by the LW with a smaller frequency.

Calculations show that for moderate to high winds the feedback mechanism plays the dominant role in the modulation of SWs. The modulations are enhanced and phases are shifted towards the crest of the LW. However to cause the feedback mechanism the LW should be long enough ($U_{10}/C < 3$). Otherwise, as well as at low winds, the modulations of the SWs are determined by the orbital velocities of the LW.

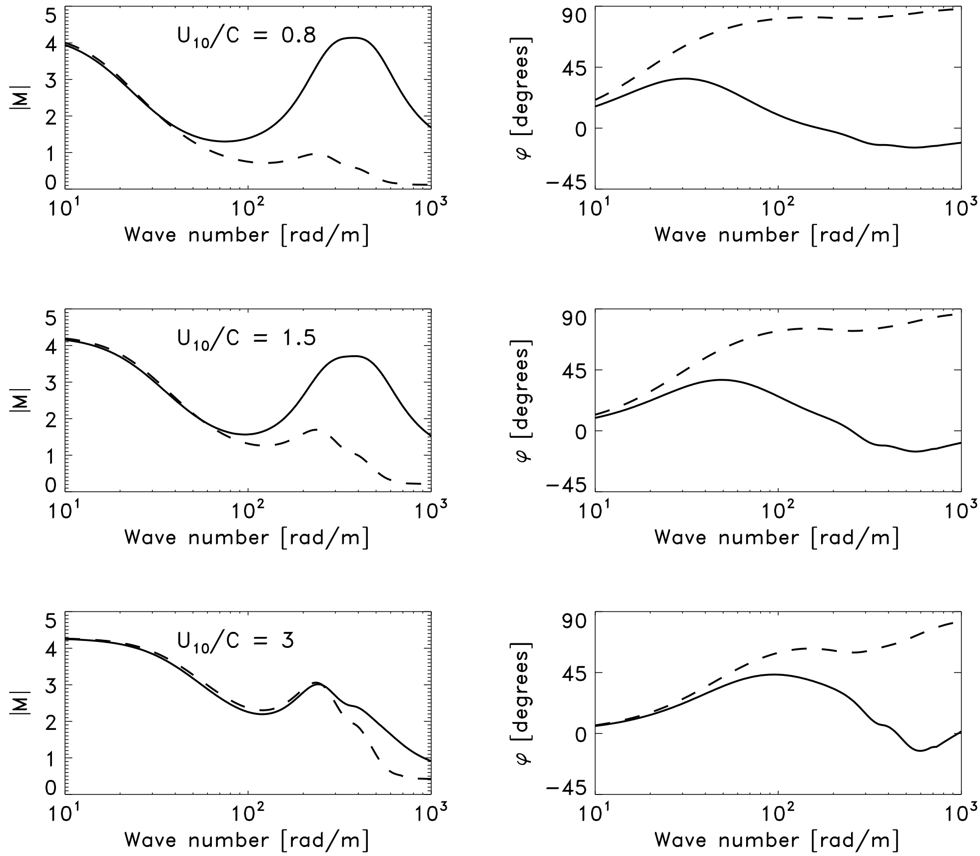

Figure 3. Calculated amplitudes (left column) and phases of the SW modulation $M \equiv \hat{N}$ by a long wave for $U_{10} = 10$ ms^{-1} for different wave ages: full line, calculation including modulation growth rate; dashed line, modulation of SWs due to orbital velocities only.

3.2. Comparison with Laboratory Measurements

Miller and Shemdin (1991) studied the modulation of wind SWs by a mechanically generated wave with a period of 2 seconds in a wave tank for different wind speeds. The SW modulation was obtained from measurements of the surface slope using a laser-optical slope sensor. The observations were transformed from frequency to wave number space taking into account the Doppler shift due to the long wave orbital velocities. The frequency of the observed SWs ranged from 14 to 30 Hz ($k \simeq$ 300 to 700 rad m^{-1}). For a given wind speed, the observed modulation amplitudes and phases show little dependence on the SW wave number. Therefore we averaged the results of the different wave numbers, and compared them to the modulation calculated for a SW with $k = 500$ rad m^{-1} (Table I). The 10 metre wind speed U_{10}

Table I

Experimental results from Miller and Shemdin (1991) for the modulation of wind SWs by a mechanically generated wave with the period 2 sec.. Column U_{10}/c_p gives the age of the spectrum of wind waves. The modulation amplitudes $|M|$ and phases are averaged over SWs with k=300 to 700 rad m^{-1}. Wind-speed units are m s^{-1}.

U_∞	U_{10}	U_{10}/c_p	$\|M\|$	phase
4	5.2	11	4.5	47°
6.4	8.3	13	3.7	39°
9	13.0	18	6.5	5°
10	14.4	20	6.6	7°

is calculated from the reported wind speeds U_∞ in the middle of the tank and an estimate of the drag.

In Figure 4 the amplitude of the SW modulation is plotted against the wave age U_{10}/C of the long wave. The spreading of the observations around the average value due to the spreading in the SW frequencies is indicated with an error bar.

Since the fetch in a wave tank is limited, in this case 24 metre, the wind-wave spectra are underdeveloped. The inverse wave age parameter, defined as the ratio of the 10 metre wind speed and the phase speed c_p in the peak of the spectrum U_{10}/c_p, ranges from 11 to 20 (third column in Table I).

For the laboratory condition the frequency of the modulating LW differs significantly from the frequency of the spectral peak of the modulated SW. Hence, all SWs that extract momentum flux from the air flow are modulated by the LW (parameter p equals 1), and the efficiency of the feedback mechanism depends only on wind speed.

The amplitude of the modulation, calculated when only the influence of the LW orbital velocities are taken into account (dashed lines in Figure 4), decreases with increasing U_{10}/C. This clearly contradicts the observations, which show a marked increase. This mechanism also fails to reproduce the trend observed in the phase of the SW modulation. In the observations, the maximum of SW amplitudes is found at the crest for high wind speeds, whereas the action of the LW orbital velocities shifts them towards the forward face of the LW. Due to the feedback effect the calculation, taking into account the modulation of the growth rate (full lines), reproduces both trends. For low wind speeds the feedback mechanism has little effect. When the wind speed is increased, the feedback mechanism becomes more efficient and starts to increase the amplitude of SW modulations, shifting the region with the maximum SW energy toward the LW crest.

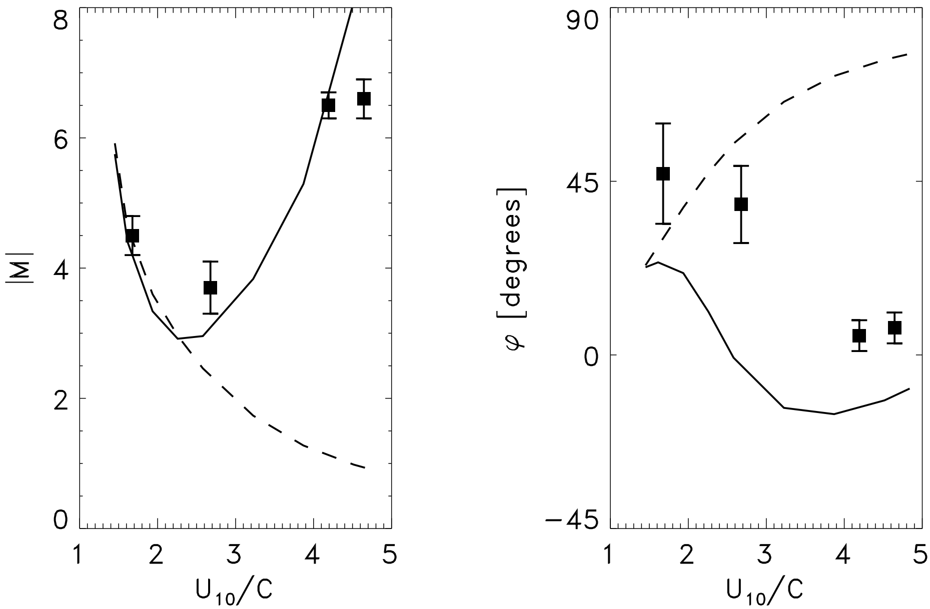

Figure 4. Amplitudes and phases of the modulation of SW's $M \equiv \hat{N}$ in a wave tank experiment (Miller and Shemdin, 1991) compared with calculations: full line, effect of a modulated growth rate included; dashed line, effect of orbital velocities only.

3.3. Comparison with field microwave measurements

Hara and Plant (1994) presented estimates of the hydrodynamic modulation transfer function (MTF), extracted from field microwave radar measurements. The hydrodynamic MTF describes variations of a microwave return along the LW profile caused by modulation of a SW spectrum on the Bragg wave number k_b. In the notation of the present paper the hydrodynamic MTF is defined as $M = \hat{N}(k_b)$. Compiled data of the amplitude and phase of the hydrodynamic MTF from Hara and Plant (1994) (their Figure 7) as a function of wind speed are plotted in Figure 5 together with model calculations. Frequencies of the observed LWs are in the range of $0.25 < f < 0.3125$ Hz and the Bragg wave number k_b is 277 rad m^{-1}. Model calculations are for a LW of 0.3 Hz frequency. The model wind-wave spectrum is for fully developed waves ($U_{10}/c_p = 0.83$) corresponding to the observed spectra. Accounting for the feedback mechanism we find a good agreement of model calculations with the observed data, see Figure 5. The remarkable feature of the feedback mechanism is that it shifts the phase of SW modulations to the crest of the LW. Model calculations which account for the influence of the LW orbital velocity only (dashed line in Figure 5) fail to reproduce this observed feature of the SW modulation.

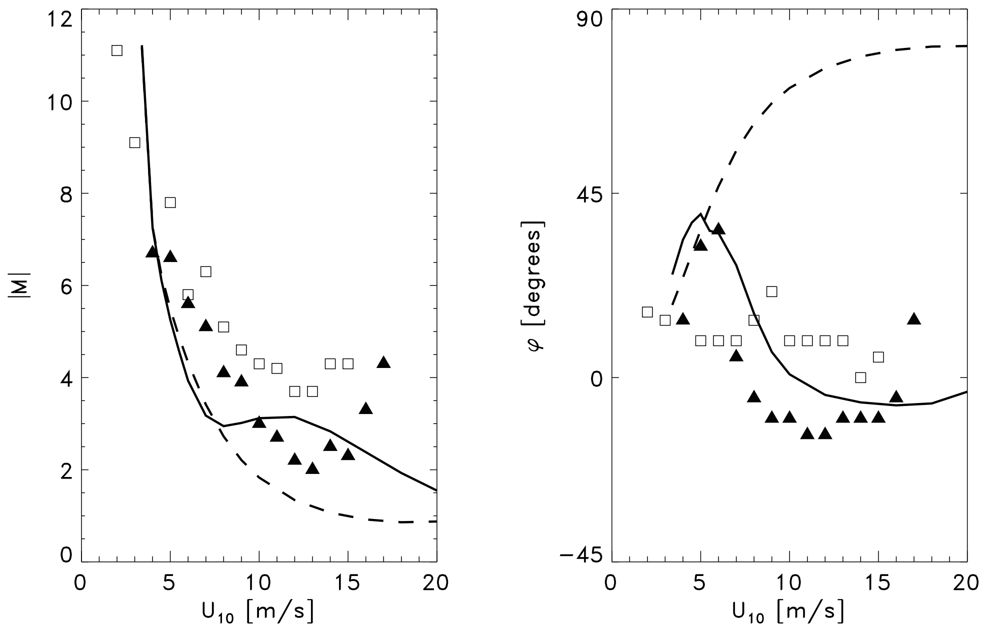

Figure 5. Amplitude and phase of the hydrodynamic MTF for the X-band as a function of wind speed. Marks represent the hydrodynamic MTF extracted from SAXON-FPN (square) and Marsen (triangle) experiments (compiled from Hara and Plant (1994), their Figure 7). Solid lines are model calculations accounting for modulation of the SW growth rate. Dashed lines represent modulation of SW due to the influence of the LW orbital velocities only.

Note that the amplitude of the modulation, observed in the wave tank by Miller and Shemdin (1991) (Figure 4), increases with U_{10}/C. This seems to contradict its decrease with wind speed found in the microwave measurements (Figure 5). This can be explained with the feedback mechanism. In the laboratory experiment the frequency of the mechanically generated LW is significantly smaller than the frequency of the wind-wave spectral peak. Therefore the full wind-wave spectrum can be effectively modulated by the LW. The larger the wind speed, the larger the resulting modulation of the stress will be, since the contribution of the waves to the surface stress increases with wind speed. The effectiveness of the feedback mechanism will increase with the increase in wind speed for a given long wave. In other words, the long wave modulates the total wave induced stress ($p = 1$), and the coupling parameter α increases with wind speed. This will lead to the increase of the feedback parameter Φ (see equation (25)), which is favourable for the feedback mechanism.

In the microwave measurements of Hara and Plant (1994) the LW frequency is comparable to the frequency of the spectral peak of wind waves for low wind speeds ($U_{10} < 5$ ms^{-1}), which means the p parameter is close to 1. But because the coupling is weak ($\alpha < 0.5$) the feedback mechanism does not work. For moderate and high winds the given LW frequency is several times more than the frequency of the spectral peak of wind waves. Therefore only a limited part of the wind waves is modulated by this LW. For this case the coupling parameter α is close to 1, but the p parameter is reduced that means the reduction of the modulation in the wind stress. This results in the decreasing of the feedback parameter Φ and thereby the efficiency of the feedback mechanism with the increase in wind speed.

The present model is able to explain the apparent contradiction of laboratory and field measurements of the SW modulation by a LW.

4. Conclusions

The 'classical' approach to describe the modulation of short waves (SWs) by a long wave (LW) accounts only for the interaction of SWs with the LW orbital velocities and fails to explain the laboratory and field radar measurements.

It has been suggested (e.g., Wright et al., 1980; Smith, 1990) that the modulation of SWs is governed by the aerodynamics of the air flow above a LW. However no quantitative description of this phenomenon has been proposed.

In the present paper we presented a model of the SW modulation by a LW that accounts for a feedback mechanism between wind waves and the air flow. The coupling of wind waves with the atmospheric boundary layer is described using the approach of Makin et al. (1995) to calculate the drag of the sea surface.

The SW modulation feels the variation of the air flow above a LW via the variation of the SW growth rate. The variation of the growth rate is caused by the wind speed variation inside the inner region, and by the variation of the drag coefficient along the LW profile. The variation of the drag coefficient is induced by the modulation of the momentum flux to wind waves, which provides the positive feedback mechanism: an area with more SWs induces a larger local stress that stimulates the growth of SWs. The efficiency of this feedback mechanism depends on the ratio of the wave-induced stress to the total surface stress. Therefore the feedback mechanism is most efficient for moderate to strong winds and results in the enhancement of SWs at the crest of a LW.

The agreement between the model results and the direct observations of the SW modulation in a laboratory (Miller and Shemdin, 1991) and field observation of the hydrodynamic MTF (Hara and Plant, 1994) is encouraging. The feedback mechanism seems to offer a physical explanation of the SW behaviour on a LW obtained in these experiments.

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