Trends and problems in the investigation of sea surface disturbance

YU. M. KRYLOV
Soyuzmorproyekt Institute

Okeanologiya, 1963. 3 (4) : 606–618.

IT IS well known in science that one must periodically take stock of the work and on the basis of an analysis of what has been achieved select the most promising directions for further research. We shall try to do this for investigations into sea surface disturbance. In the last two or three decades this field has grown into an independent discipline. How has sea wave research developed ? What is the present state of this research and what does the future hold ?

Four basic trends can be picked out in present work on sea surface disturbances : hydrodynamic, energetic, statistical and spectral. Four basic problems can be picked out in these fields : deep-sea waves, shallow-sea waves, waves in the coastal zone and waves in port areas.

Wave hydrodynamics

This is rightly considered to be the best developed branch of the study of sea surface disturbances. The basic results were obtained as early as the 19th century by a number of prominent mathematicians, students of mechanics and physicists [32]. Many of these results have become classical and an inseparable part of courses on hydrodynamics [12, 21]. An idea of the level of modern research in the field of hydrodynamic theory of sea waves is given by Stoker [30] and the collection of translated articles edited by Krasnosel'skii and Moiseyev [13]. In addition to the theoretical work, many experiments have also been carried out. However, the numerous articles describing laboratory experiments have not yet been published in a monograph.

The basic method of hydrodynamic wave theory consists of a mathematical study of the wave motions (ignoring friction) of an ideal liquid with a free surface [32]. This theory does not explain fully the causes for the appearance and dying away of sea disturbances nor does it explain the causes of a complex and in many ways chaotic form of a disturbed sea, although interesting attempts have been made recently [46, 58, 56].

According to the Phillips–Miles theory, wind energy is transmitted to the wave spectrum in two ways : due to the resonance between turbulent fluctuations of the wind pressure and the waves on the surface of the water (Phillips) and due to the disturbances introduced by the waves themselves into the field of pressures of the air stream (Miles).

Hydrodynamic theory basically correctly catches the internal dynamic structure of individual waves. Therefore one of the problems of hydrodynamic research has been the study of the connexions between the individual wave elements, e.g. between the elevation of the free surface and the velocity of the water particles or the pressure inside the disturbed liquid.

The theory of wave motions, just like any oscillation theory, splits into two main sections—linear theory, or the theory of small-amplitude waves, and non-linear theory, or the theory of waves of finite amplitude. The linear theory permits the simple algebraic addition of individual particular solutions, thus opening up possibilities for the solution of various complex problems, while particular solutions of the non-linear theory cannot be summed simply when a fresh solution is obtained.

Wave motion hydrodynamics is best developed on the one hand along the line of obtaining fresh solutions to equations which have already been derived, making considerable use of electronic computing techniques, and on the other hand along the line of experimental and theoretical investigation of little-studied phenomena such as the development of waves due to wind, the interaction of waves, their disintegration, wave spray round obstacles, the interaction of waves with new kinds of engineering structure.

The development of the foundations of the classical wave theory in deep water can be considered practically complete. The basics of the non-linear theory of a single wave in deep water have also been considerably developed. It is true that in a number of cases theoretical conclusions have not yet been confirmed by convincing enough experiments under laboratory conditions. It is difficult to use measurements made under natural conditions here by virtue of the inevitable distortion of the motion under study by a number of other factors.

An important theoretical problem awaiting solution is that of producing a theory for the interaction of waves of finite amplitude in deep water, a characteristic feature of the picture of wave disturbance. There is an increasing interest in this problem [52]. The turbulence of deep-sea disturbances must also be studied. The classical theory does not explain many important properties of waves because it assumes that the part played by turbulent friction is negligible. Systematization of this kind is not permissible, however, in describing the behaviour of waves over long periods of time during which turbulence may lead, for example, to complete attenuation of the wave motion. Much preliminary work has been carried out on the turbulence of sea waves.

Shallow sea wave hydrodynamics is similar to that for deep water. A number of results have been obtained later because the presence of the bottom considerably complicates the problem. This is generally shown schematically by a horizontal plane. The problems noted above for the deep-sea must also be solved for waves on the surface of shallow seas. Additional questions arise, however such as the study of steady waves above a horizontal bottom. This problem has been studied by hydrodynamic methods only for the particular case when the wave length is considerably greater than the depth [30]. When studying waves at a finite depth over long periods of time or at great distances, friction along the bottom also starts to play a role as well as internal turbulent friction and air resistance. This is a subject for further research.

Waves of the open sea (deep or shallow) as the shore is approached gradually change their external form and their whole hydrodynamic structure and are finally completely disintegrated in the surf zone. These changes are caused, to a considerable degree, by the effect of decreasing depths. No hydrodynamic theory has been devised for the transformation of waves in the nearshore zone. The basic difficulty is that a linear approximation is insufficient and a start must be made right at the beginning from the non-linear problem. Thus, in using the non-linear theory in the nearshore zone waves of finite amplitude on a sloping bottom cannot, in the first place, be steady and, in the second place, the propagation of waves in accordance with the non-linear theory is accompanied by advancing transfer of liquid, while the behaviour of the wave flow in the nearshore zone is complicated by a whole series of circumstances that are difficult to allow for. All that has been derived is the individual elements of the hydrodynamics of waves in the nearshore zone with insufficient connexion between them. Examples of these are the linear theory for progressive waves with a flat inclined bottom [30], the non-linear theories of waves of steady shape at a constant depth [30, 13] and the theory of steady long waves of finite amplitude [30]. Because the theory is not well developed a large number of laboratory investigations have been made of special cases for the propagation of waves at differing depths. A very good survey of work of this kind is Longinov's monograph [22]. The main problem of shore wave hydrodynamics is the creation of a unified theory of wave transformation starting from the deep-water zone and ending with the surf zone, which requires further laboratory and full-scale investigations. At the same time a number of special questions must be solved. The most important of these are the hydrodynamics of longshore currents and of surf.

Hydrodynamically, waves in harbours are a diffraction picture of open-sea waves distorted by the effect of the nearshore zone. This has been studied only for small-amplitude waves with a horizontal bottom. In this case the absence of wave energy losses was assumed. But even with this simple case analytical solutions have been obtained only for boundary structures of the simplest form (semi-infinite screen, slit etc.) [14]. This diffraction problem will have to be brought closer to the actual conditions, eliminating above all the assumption of the absence of wave energy losses in the port water area and compiling boundary conditions that will permit the partial disintegration of waves at the boundaries of the diffraction region. Methods must then be developed for taking into account the varying depth in the port water area and, lastly, an attempt must be made to draw up an approximate theory for the diffraction of waves of finite amplitude, however simple the types of boundary installation. All these problems can be solved only on the basis of extensive investigations into wave diffraction in the field and with models.

Problems of the interaction of waves with boundary wave-damping installations form an important part of wave hydrodynamics in harbours. Thus, the mechanism for wave damping by a compressed-air breakwater has still been very little studied despite the fact that this kind of breakwater is in use and more will certainly be built.

On the whole hydrodynamics have not yet clearly answered the basic question of wave theory : why does a mere ripple grow into a gigantic wave when acted upon by a strong wind out at sea. This question is being answered by an analysis of the wave energy balance which not only explains the causes for the growth of waves but also permits a calculation of the relationships of the wave sizes to the external wave-generating factors.

V. M. Makkaveyev, the first to provide a physical explanation for the growth and decay of sea waves [25], is rightly considered the founder of the 'energy theory.' He suggested a method which has played a decisive role in the development of sea-disturbance research. His method consists of using a general energy principle by which the change in the energy of any mechanical system is equal to the work of the external forces less the work of the internal resistance forces and internal dissipation forces. The application of this integral principle to wind waves makes it possible to derive the equation (the Makkaveyev equation) :

$$\frac{\delta E}{\delta t} + \frac{\delta}{\delta x}(V_E E) = M_v - E_0, \tag{1}$$

where t is the time, E is the energy of the wave per unit area of the sea, V_E is the wave energy transfer rate, M_v is the wave energy produced, E_0 is the amount of energy dissipated; the x-axis is plotted in the direction of travel of the wave. The quantities E, V_E, M_v and E_0 can be expressed by the wave height and its length (or another element linked with the length by the classical relationships). By combining Makkaveyev's equation with the connexion between the height and length of a wave obtained in some way a differential equation is obtained in partial derivatives with respect to the height (or length) as a function of x and t. Makkaveyev's method has stimulated the expansion of complex theoretical and experimental investigations for obtaining a whole series of formulae and graphs for the practical calculations of wave sizes. A short description of many of these can be found, for example, in Krylov's work [15].

The energy theory was based on the empirical investigations of the laws governing the growth and decay of sea disturbances. The qualitative aspect of these was established by marine geographers long before their physical interpretation was derived. Moreover numerous empirical formulae were derived for determining the sizes of waves depending on the wind speed, the duration of its action and the direction from the weather shore called the fetch (a short description of formulae of this kind is given in the same work [15]). The methods based on the solution of Makkaveyev equations for calculating the elements of deep-sea waves have basically only added more precision to empirical relationships. But the actual explanation of the relationships, discovered earlier, was the first real achievement of the energy theory. The possibilities of this method are by no means exhausted and it must be used as the basis of the development of methods for the calculation for various physico-geographical conditions.

As already pointed out, most practical methods hitherto used for calculating the elements of deep-sea waves are more or less dependent on the solution of the Makkaveyev equation. There are special manuals and reference books on their practical use [39, 61, 64, 65]. At the same time the mechanism for supplying the wind energy and scattering the wave energy as the result of various irreversible processes [15] is still not clear. This determines the solution of the Makkaveyev equation and therefore all the practical methods of calculation based on this solution. The main problem for further research in deep-sea wave energetics must therefore be the study of these mechanisms.

The energy method is applicable with varying success to both deep and shallow water. The energetics of shallow-water waves have been discussed by Braslavskii [2], Hunt [50], Khvan [34] and Kao Wen-Hsiu and Wang Hsiao-Nan [5] and practical nomograms for calculation have been produced based on the solution of the Makkaveyev equation. On the whole, the questions of the energy balance of shallow-water waves have received less study than in the case of deep water. The processes of energy dissipation caused by the small depths are particularly important for shallow seas. They are allowed for in practical calculations, but little light has been thrown on them from the physical point of view and it is important to study them.

In the nearshore zone the energy balance is considerably more complex than in the open sea (deep or shallow). On the one hand the decreasing depths concentrate the wave energy per unit area of the sea's surface and its redistribution through the depth of the water. On the other hand, the wave propagation as one approaches the shore is accompanied by continuously increasing energy losses and finally ends with complete decay of the wave in the surf zone. The quantitative expression for the change in the energy of the waves in the nearshore zone is difficult because the hydrodynamics of these waves have as yet been little studied. These circumstances taken together explain the absence of any significant results in the study of wave energetics in the nearshore zone although many important investigations have been carried out [22]. The main problem for the further study is the physical mechanism of the change in the wave energy in the conditions of the nearshore zone and the development of corresponding methods for the calculation of the wave elements in this zone.

The wave energetics in harbours have been studied even less than in the natural regions of the nearshore zone. Karaushev [9] made the only original attempt at analysing the energy changes for diffracted waves. He proposed a generalized Makkaveyev equation which allows for wave diffraction and worked out corresponding practical methods for calculating wave heights in the diffraction zone. Karaushev's equation, however, is still very incomplete and needs considerably more work and the same is true of his method of calculation. In particular, calculations made by Karaushev's method have not been compared with precise hydrodynamic solutions of diffraction problems.

Wave energetics, as a whole, encompass the property of sea disturbances consisting of a change in their force dependent on external conditions (wind speed, bottom configuration, etc.). Another important property of waves is their complexity and the known chaotic structure of the disturbed surface at any given disturbance strength. This is being studied from two points of view : on the one hand an investigation is being made as to the statistical distribution of wave elements (heights, periods, lengths) which may be observed directly; on the other a study is being made of the internal spectral structure of the surface (its so-called energy spectrum).

The statistical approach

This started historically slightly later than the energy approach but earlier than the spectral one. Its development has been considerably helped by the introduction of instrumental methods of measurement—stereophotography of waves and continuous recording of wave oscillations at a fixed point by means of various automatic wave recorders. After appropriate theoretical analysis the data obtained have made it possible to establish important statistical laws governing these observed elements. These are of great practical significance since without them no calculations or sea disturbance forecasts could be made.

The main problem of wave statistics is the distribution of the wave elements. Two kinds of such functions are distinguished [16]. Some describe a variety of wave elements at some fixed disturbance force. They are called the distribution functions for quasi-stationary disturbances or simply the distribution functions. The others characterize the mean variety of the wave elements over many years. These are called the cycle functions.

A number of fundamental conclusions has been drawn from the distribution functions for quasi-stationary disturbances. In particular it is generally accepted that the integral distribution function (ϕ) of the heights H of the wave oscillations at a fixed point on the surface in deep water is of the form (Ref. 17)

$$\phi = \exp\left[-\frac{\pi}{4}\left(\frac{H}{H_{av}}\right)^2\right] \tag{2}$$

where H_{av} is the average height. Important empirical conclusions have been drawn from analogous functions in the nearshore zone [4]. According to these there is less variety in wave heights as the shore is approached and the integral function takes the form [17]

$$\phi = \exp\left[-\frac{\pi}{4}\left(\frac{H}{H_{av}}\right)^{2n}\right], \tag{3}$$

where the number, n depends on the ratio of the average height H_{av} to the depth of the sea h at a given place in the coastal zone. This relationship obtained from empirical data [4] is shown in *Table* 1.

These results must be determined more precisely and developed first of all with respect to the distribution functions of three-dimensional waves. Only a small amount of work of an empirical nature has been carried out on distribution functions of shallow-sea wave elements and this has been chiefly based on observations from reservoirs. Unlike the deep sea there are no sufficiently

Table 1. Relationship of parameter n in formula (3) to ratio H_{av}/h

H_{av}/h	n	H_{av}/h	n
0	1	0·25	1·31
0·05	1·04	0·30	1·41
0·10	1·09	0·35	1·52
0·15	1·15	0·40	1·65
0·20	1·23	0·45	1·82

definite results. The available observations must be generalized, gaps must be filled in by fresh investigations under natural conditions, a theoretical analysis must be made and the features of the distribution functions of shallow-sea wave elements must be determined.

The cycle distribution functions are only just beginning to be studied. It has turned out that the cycle functions plotted for very widely varying sea and ocean areas have the same analytical structure [16] :

$$\phi = \exp\left[-\left(\frac{H}{\alpha}\right)^m\right], \tag{4}$$

where the parameters α and m characterize the features of the type of sea disturbance in the region in question. If the bilogarithm of ϕ is plotted on one axis of a graph and the logarithm of H on the other any such function will take the form of a straight line. *Figure* 1 shows the cycle functions

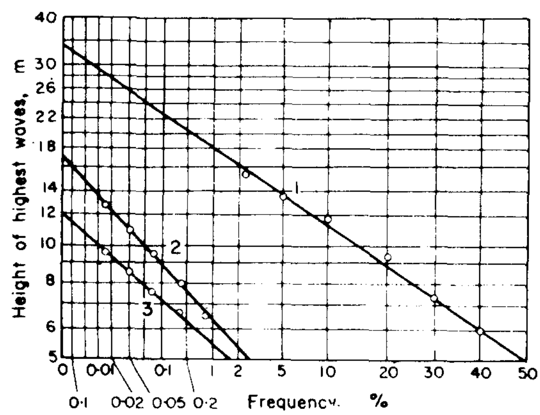

Fig. 1. Cycle distribution functions of height of highest waves : 1—Antarctic; 2—Caspian Sea, central part; 3—Caspian Sea, region of Neftyanye Kamni.

for three different regions. The first is for the central part of the Antarctic sector of the Indian Ocean. The points used for plotting it were taken from Rzheplinskii [28]. The frequency percentages on the horizontal axis relate to a statistical combination, each term of which corresponds to a height observed once in twenty-four hours. For example a frequency of 0·1% corresponds to one observation in three years. Extrapolating Rzheplinskii's calculations to low frequencies, a wave about 35 m high will be met in the Antarctic sector of the Indian Ocean once in 300 years. The two other straight lines are for the Caspian Sea. Straight line 2 characterizes the maximum height in open deep-water regions of the Caspian Sea and straight line 3 the region of Neftyanye Kamni with depths of about 15 m.

The method described is being used successfully at present by the Polish scientist Jednoral [51] in investigating the frequency over a long period of years of various hydrometeorological elements (waves, level) along the Polish coast of the Baltic Sea. Brooks *et al.* [6] have slightly different conclusions for the form of the cycle functions.

The distribution functions for harbours have not been studied systematically.

Let us now turn to the 'spectral line' of research. Sea disturbances are a complex wave process dependent on the interaction of a large number of simple waves. The energy distribution of these simple waves depending on their length and direction of propagation is called the energy spectrum of the disturbance. We obtain a stricter mathematical definition of the energy spectrum by looking at the specific wave energy (i.e. the amount per unit area of the sea's surface) belonging to elementary plane waves with frequencies from μ to $\mu + d\mu$ and directions of propagation from θ to $\theta + d\theta$. This energy is written in the form $e(\mu, \theta)\, d\mu d\theta$. The function $e(\mu, \theta)$ is called the two-dimensional energy spectrum. The amplitude of a wave with an energy E, as is well known, is

$$a = \sqrt{\left(\frac{2E}{g\rho}\right)},$$

therefore the amplitude of a wave with an energy $e(\mu, \theta)\, d\mu d\theta$ can be written in the form :

$$\sqrt{\left(\frac{2e(\mu, \theta)}{g\rho}\right)} \sqrt{(d\mu d\theta)} = A(\mu, \theta) \sqrt{(d\mu d\theta)}. \tag{5}$$

The functions $A^2(\mu, \theta)$ are also called the two-dimensional energy spectrum. Integrating the two-dimensional spectrum with respect to all θ we obtain the frequency spectrum $A^2(\mu)$ which depends only upon μ. Plotting the spectra from actual measurements is considerably more complicated than obtaining the distribution functions of directly observed wave elements and requires the use of electronic computers. This largely explains the fact that the spectral line of approach has not been widely developed. Nevertheless this approach holds much promise because its purpose is to study the very essence of the wave process from a general physical standpoint.

The chief prerequisite making possible a deep mathematical and physical analysis of the spectral structure of sea waves was the great success in the mathematical theory of random processes achieved over the last few decades. Pride of place must be given to the discovery of the Soviet mathematician A. Ya. Khinchin of the fundamental relationship connecting the energy spectrum of a process with its correlation function [35].

A number of questions on the spectral theory of sea waves are being worked on at present. The mathematical apparatus for describing the disturbed sea surface by means of the energy spectrum was first discussed by Pierson [60] who thought of using the theory of stationary random processes for this purpose. This idea has been developed by Longuet-Higgins [53] who investigated the relationship between the various statistical, surface characteristics and its two-dimensional spectrum.

An important result in the non-linear spectral theory was using the dimensionality theory [59] Phillips showed that the non-linear effects form a high-frequency section of the spectrum of the form $\alpha g^2 \mu^{-5}$ where α is a non-dimensional constant and g is the acceleration due to gravity. He also tried to draw, in general terms the physical mechanism for the formation and evolution of the frequency spectrum in time at a set wind speed on the basis of the Phillips-Miles hydrodynamic theory [58, 56]. Concrete formulae for calculating the spectrum under set wind conditions have been suggested [3, 19, 23, 27, 31, 33, 40, 41, 45, 47, 60, 62, 63]. There is, however, still not enough agreement between these formulae. On the basis of results of spectral researches a number of practical methods have been proposed for calculating sea waves : the Pierson–Neumann–James method [65], the Gelci–Cazalé–Vassal method [47], the Derbyshire method [45] and the Wen method [62, 63].

The large amount of work that has been carried out made it possible to hold the first international conference on ocean wave spectra, held from 1 to 4 May 1961 at Easton (Maryland, U.S.A.). Empirical methods were discussed for forecasting sea surface disturbances, as were also questions of hydrodynamic wave theory, wave measurements and their analysis and the use of the spectrum in solving engineering problems [42].

Despite some real successes it must nevertheless be admitted that no spectral wave theory has yet been produced to explain the laws which govern the development of the spectrum under given geographical and synoptic conditions. In particular the Gelci–Cazalé–Vassal method is the first rough method for calculating the spectrum, allowing for complex synoptic conditions in the open sea. A spectral theory would make it possible to work out methods for calculating the complex configuration of the shoreline of an area covered by water. The existing methods for calculating wind waves allow for a change in the wind wave elements only along one axis, parallel to the direction of the wind. In this case it is assumed that the dimensions of the waves do not depend on the other co-ordinate (horizontal). In actual fact this is by no means the case. Let it be required, for example, to calculate the size of waves beyond the northern part of the island of Gotland in the Baltic with a strong west wind. If we take the distance to windward of the island we obtain one height. If we take this distance from the shores of Sweden the height of the waves in the same sea area is considerably greater. The uncertainty is because it still is not known how to allow for the change in wave elements at right angles to the direction of the wind. Physically based calculations of the wave field in a harbour can likewise be carried out only on the basis of spectral theory for the reason that the port boundary installations are a kind of filter which lets through only a certain part of the disturbance of the water outside the harbour. Here the penetration of individual spectral components is complicated by their diffraction, transformation and interaction under the conditions of the varying depths of the harbour.

The main lines of investigation into sea disturbances have been outlined briefly above and some problems have been indicated for further study. Work is now being carried out, however, to unite the several approaches in order to create a unified physical theory of sea waves and on the basis of this theory to work out practical methods for calculations under specific conditions.

The idea of synthesizing wave hydrodynamics and energetics belongs to Makkaveyev, whose method depends on the one hand on the principle of the conservation of energy and on the other on the hydrodynamic formulae linking the various wave elements [25]. This question has been developed further in a number of investigations, particularly in the work of Shuleikin [37].

Wave energetics and statistics are united in Krylov's [15, 20] statistical theory of wind waves. this discusses only deep-sea waves in sufficient detail; shallow waters and the nearshore zone are discussed briefly and schematically. These same questions as applied to reservoirs have been discussed in greater detail from an empirical point of view by A. P. Braslavskii [2].

The first results on the relationships between the statistical and spectral properties of sea disturbances were obtained by Pierson [60]. Longuet-Higgins [53] has also made extensive mathematical investigations in the same field.

Interesting theoretical work in the synthesis of the energetic and spectral sides was recently published by Hasselman [49]. A general mathematical method is described for finding the two-dimensional spectrum on the basis of the Makkaveyev equation and some special cases of the problem are discussed qualitatively.

Pierson and Neumann were the first to make a bold attempt to combine all four fields (hydrodynamics, energetics, statistics and spectral analysis of waves) into a unified method of calculation [60, 65]. Although this method has serious defects, it holds out promise.

A complete survey of the work in measuring sea disturbances would take up too much space. We shall therefore limit ourselves to a general outline of the development of measurement methods with only a small amount of concrete information. As a whole considerable progress has been made in the transition from imperfect visual observations to precise measurements. Apparatus permitting the recording of short-period wave oscillations of sea level at a fixed point has been greatly developed. Wave recorders of widely varying designs have been and continue to be developed, each of which has its advantages and disadvantages. Wave records can be used, however, to obtain only information on oscillations of the sea surface at a fixed point. Wave motion is the term used for the propagation of oscillations in space. Hence, it must be recognized that a wave recorder does not really record a wave. For this reason measuring systems for several wave recorders are being developed [43]. Buoyed wave gauges have begun to be used of late. Apart from recording the rises of the disturbed surface, they also record the derivatives of this surface at the location of the buoy. In particular a buoy has been designed in Britain that records the rise and slopes of the surface in two directions at right angles to one another [42]. A. A. Sveshnikov had concluded that such an instrument was possible from purely theoretical considerations [29]. Important work has been started on the measurement of wind stresses near the disturbed sea surface in order to study directly the mechanism of supplying sea waves with wind energy [11]. At the same time spectral wave devices were developed to allow the quick measurement *in situ* of the corresponding energy spectrum on a wave recording chart [57].

Stereophotography of waves is also being perfected. Soviet researchers have succeeded in determining a wave height of 25 m in the open sea with a wind speed of 36 m/sec [28]. Use is made, on the one hand, of serial stereophotography (multiple photography) of some fixed section of the sea surface from a ship or shore and, on the other hand, of stereophotography of waves from one [8] or two aircraft [36]. In 1954 waves were photographed in the U.S.A. for the first time from two aircraft in order to obtain a two-dimensional energy spectrum [44]. Because of the great amount of work involved in processing stereophotographic data, they must be automated.

We have briefly discussed sea wave research unrelated to other processes occurring in the sea. There is also a large amount of work devoted to the study of sea waves combined with other physical phenomena in the sea such as wind mixing, currents, propagation of light, sound, radio waves, microseismic waves, movement of bottom sediments and the change in the relief of the shore zone.

It is well known that disturbances play an important part in the processes of mixing the surface layer. Interesting work on the formation of the upper isothermal layer during disturbances has been carried out by Kitaigorodskii [10] who showed the close relationship between the lengths of waves and the thickness of this layer.

Wind currents and currents of other origin may have a significant effect on waves. One of the latest works of Longuet-Higgins [54] is devoted to the theory of this question. On the other hand the formation of wind currents is due to a considerable degree to the appearance and development of surface waves since waves considerably increase the mean tangential force of the wind at the sea surface and permit the transfer of a horizontal amount of motion into the underlying layers. These questions have as yet been little studied and await examination.

The effect of waves on the propagation of light has been discussed by Shuleikin [38]. Work has also been done on the effect of waves on the propagation of sound and radio waves near the air-sea interface [24], on microseismic oscillations of the bottom of the oceans and seas caused by storm disturbance [26, 48, 55], on the interaction of waves and littoral sediments [22] and on the interaction of waves with hydraulic engineering structures [1, 7].

In conclusion one of the possible future ways of developing sea wave science is outlined in general terms. There is every reason to assume that the physical basis for future investigations will remain as before the Makkaveyev energy equation, which should be applied to each spectral component of sea disturbance. The question of finding an additional relationship between the wave heights and their length is then eliminated since the understanding of the spectral component is linked with the fixed frequency or length of a wave. The Makkaveyev equation for each spectral component must contain only one unknown function of the co-ordinates and time—the amplitude of the given component. The amplitudes of the various components will vary in different ways which also leads in concrete conditions to different disturbance spectra. From the mathematical point of view the determination of the disturbance spectrum in this case is reduced to solving a system of a large number of differential equations of the Makkaveyev type. The right-hand sides of these equations must be the sum of the incoming and outgoing energy. Both these forms of energy will depend not only on the geographical and synoptic conditions and the elements of a given component but also on the characteristics of the other components. This method approaches a solution for the problem of calculating and forecasting wave fields under various kinds of natural conditions. First of all, however, tedious and lengthy work must be done on the study of the form of the right-hand side of the equations, using various methods. Hydrodynamic research should play a large part here. Effective methods must be developed for the combined solution of these equations and obtaining, on the basis of this solution, a two-dimensional energy spectrum at each point of the sea. Knowing this spectrum it will be possible to obtain any required statistical characteristics of a disturbed surface. Clearly this line of research may lead fairly rapidly to the compilation of a general physical theory of sea disturbances. The greater the generality and depth of this theory the wider and more varied will be its practical applications.

REFERENCES

[1] Bozhich, P. K. and N. N. Dzhunkovskii (1949) Sea Disturbance and its Action on Installations and Shores (in Russian). *Mashstroiizdat.*

[2] Braslavskii, P. P. (1952) Calculation of wind waves. *Tr. Gos. Gidrol. In-ta*, **35** (89).

[3] Brovikov, I. S. (1961) Spectral representation of wind wave energy. *Met. i Gidrol.*, (7).

[4] Vilenskii, Ya. G. and B. Kh. Glukhovskii (1957) Experimental investigation of the sea disturbance process. *Tr. Gos. Okeanogr. In-ta*, (36).

[5] Kao Wen-Hsiu and Wang Hsiao-Nan. (1961) Calculation of wind waves in a sea of arbitrary depth. *Izv. Akad. Nauk SSSR, Ser. Geofiz.*, (10).

[6] DAVIDAN, I. N. (1961) Laws governing the distribution of sea waves of a long period of years and their connexion with wind speed. *Okeanologiya*, **1** (2).
[7] DZHUNKOVSKII, N. N. (1958) Basic conclusions of wave researches of the Faculty of Water Economy and Seaports of the Moscow Engineering Construction Institute. *Tr. Mosk. Inzh.-Stroit. In-ta*, (20).
[8] DUBOVSKII, B. and L. PERKIS (1956) Aerial stereophotography of sea disturbances from one aircraft. *Tr. Soyuzmorproyekt*, Coll. 3, Moscow.
[9] KARAUSHEV, A. V. (1960) Problems of the Dynamics of Natural Water Streams (in Russian). *Gidrometeoizdat, Leningr.*
[10] KITAIGORODSKII, S. A. (1961) Contribution to the theory of turbulent mixing in the upper layer of the sea in connexion with calculating the thickness of the isothermal layer. *Tr. In-ta Okeanol. Akad. Nauk SSSR*, (52).
[11] KOLESNIKOV, A. G. and G. YE. KONONKOVA (1961) Instrumental determination of the energy transferred by normal wind pressure to the surface of sea waves. *Izv. Akad. Nauk SSSR, Ser. Geofiz.*, (10).
[12] KOCHIN, N. YE., I. A. KIBEL' and N. V. ROZE (1948) Theoretical hydromechanics (in Russian), *Gostekhizdat* Pt. I.
[13] KRASHOSEL'SKII, M. A. and N. N. MOISEYEV (Editors) (1959) *Theory of Surface Waves*. Moscow.
[14] KRYLOV, YU. M. (1950) Diffraction of liquid waves. *Tr. Gos. Okeanograf. In-ta*, **16** (28).
[15] KRYLOV, YU. M. (1956) Statistical theory and calculation of wind waves. *Tr. Gos. Okeanograf. In-ta*, (33).
[16] KRYLOV, YU. M. (1961) Calculation of maximum heights of sea waves. *Tr. Gos. Okeanograf. In-ta*, (61).
[17] KRYLOV, YU. M. (1954) Theory and calculation of deep-sea wind waves. *Tr. Gos. Okeanograf. In-ta*, **26** (38).
[18] KRYLOV, YU. M. (1957) Methods of calculating elements of wind waves, their checking and application to the calculation of the maximum sizes of storm waves in the Apsheron sea area. *Tr. Gos. Okeanograf. In-ta*, (36).
[19] KRYLOV, YU. M. (1962) The energy spectrum of wind disturbance in the deep sea. *Tr. Gos. Okeanograf. In-ta*, (67).
[20] KRYLOV, YU. M. (1958) Statistical theory and calculation of sea wind waves. *Tr. Gos. Okeanograf. In-ta*, (42).
[21] LAMB, G. (1947) Hydrodynamics (in Russian). Gostekhizdat.
[22] LONGINOV, V. V. (1963) *Shore Zone Dynamics of Tideless Seas.* (in Russian). Akad. Nauk SSSR.
[23] LUKASHCHEVSKII, V. A. and O. P. SHORYGIN (1960) Investigation into the statistical characteristics of irregular wind disturbance in a limited area of water. *Tr. Okeanogr. Komis. Akad. Nauk SSSR*, (9).
[24] LYSANOV, YU. P. (1958) Theory of wave scatter on periodically uneven surfaces. (Survey). *Akust. zh.*, **IV** (1).
[25] MAKKAVEYEV, V. M. (1937) Growth and decay of waves of small length and their relationship to the distance to windward. *Tr. Gidrolog. In-ta*, (5).
[26] MONAKHOV, F. I. (1960) *Microseisms and their connexion with cyclones.* IGY Committee, Coll. of Articles : Seismic Research (in Russian). Moscow.
[27] RAKHMANIN, N. N. (1958) Empirical spectrum of sea disturbance. *Tr. Tsentr. n.-i. In-ta im. A. N. Krylova*, (126).
[28] RZHEPLINSKII, G. V. (1959) Oceanic disturbances in Antarctica. *Tr. Gos. Okeanogr. In-ta*, (48).
[29] SVESHNIKOV, A. A. (1959) Determination of probability characteristics of three-dimensional sea disturbances. *Izv. Akad. Nauk SSSR. Otd. Tekh. n., Mekhan. i mashinostr.*, (3).
[30] STOKER, J. *Waves on Water. Mathematical Theory and Applications.* (Transl. into Russian). Moscow (1959).
[31] STREKALOV, S. S. (1961) Contribution to the determination of the analytical form of the energy spectrum of developed disturbances. *Okeanologiya*, (3).
[32] SRETENSKII, L. N. (1936) Theory of Wave Motions of a Liquid. (in Russian). *ONTI NKTP SSSR.*

[33] FIRSOV, G. A. (1958) Energy spectrum of sea disturbances. *Tr. Tsentr. n.-i. In-ta im. A. N. Krylova*, (127).

[34] KHVAN, A. N. (1961) Calculation of wind wave elements in a shallow sea. *Izv. Akad. Nauk SSSR, Ser. Geofiz.*, (1).

[35] KHINCHIN, A. YA. (1938) Correlation theory of stationary stochastic processes. *Usp. Mat. Nauk*, (V).

[36] CHERKASOV, I. A. (1960) Stereophotography of sea disturbances from two aircraft : *Tr. Okeanograf. Komis., IX, Vetrovye Volny*. Moscow.

[37] SHULEIKIN, V. V. (1956) Theory of Sea Waves. *Tr. Morsk. Gidrofiz. In-ta Akad. Nauk SSSR*, (IX).

[38] SHULEIKIN, V. V. (1953) *Physics of the Sea* (in Russian). Akad. Nauk SSSR, Moscow.

[39] MANUAL FOR THE CALCULATION OF SEA DISTURBANCES AND WIND OVER THE SEA (1960) Gidrometeoizdat, Moscow. (in Russian).

[40] BRETSCHNEIDER, C. L. (1958) Revisions in wave forecasting. *Tech. Mem. Beach Eros., Bd. Washington.*

[41] BURLING R. W. (1959) The spectrum of waves at short fetches. *Deutsch. Hydrogr. Z.*, **12**, (2, 3).

[42] CARTWRIGHT, D. E. (1961) The ocean wave spectrum. *Science Progress*, (49).

[43] CARTWRIGHT, D. E. (1961) A brief survey of methods for finding the directions of sea waves. *Dock and Harbour Auth.*, **42** (492).

[44] COTE, L. I., J. O. DAVIS, W. MARKS, R. I. MCGOUGH, E. MEHR, W. J. PIERSON, J. F. ROPEK, G. STEPHENSON and R. C. VETTER (1960) The directional spectrum of a wind-generated sea as determined from data obtained by the stereo-wave observation project. *Met. Pap.*, **2** (6) New York.

[45] DERBYSHIRE, J. (1959) A further investigation of wind-generated waves. *Deutsch. Hydrogr. Z.*, **12** (1).

[46] ECKART, C. (1953) The generation of wind waves on a water surface. *J. Appl. Phys.*, **24**, (1).

[47] GELCI, R., H. VAZALÉ and J. VASSAL (1957) Prévision de la houle. La méthode des densités spectro-angulaires. *Bull. Infor. Comm. Centr. Océanogr. et Étude Côtes*, **9** (8).

[48] GUTENBERG, B. and H. BENIOFF (1956) *An investigation of microseisms.* California.

[49] HASSELMAN, K. (1960) Grundgleichungen der Seegangsvoraussage. *Schiffstechnik*, **7** (39).

[50] HUNT, M. (1956) Effets du vent sur les nappes liquides. *Houille Blanche*, **11**, (6).

[51] JEDNORAL, T. (1960) Wave phenomena appearing in the Baltic in the light of probability theory. *Tekn. Gospodarka Morska*, **10** (5–6).

[52] LONGUET-HIGGINS, M. S. and R. W. STEWART (1960) Changes in the form of short gravity waves on long waves and tidal currents. *J. Fluid. Mech.*, **8** (4).

[53] LONGUET-HIGGINS, M. S. (1957) The statistical analysis of a random moving surface. *Phil. Trans.*, Ser. A, (249).

[54] LONGUET-HIGGINS, M. S. (1961) Changes in amplitude of short gravity waves on steady non-uniform currents. *J. Fluid Mech.*, **10**, (4).

[55] LONGUET-HIGGINS, M. S. (1950) A theory of the origin of microseisms. *Phil. Trans.*, Ser. A, **243**, (857).

[56] MILES, I. (1960) On the generation of surface waves by turbulent shear flows. *J. Fluid Mech.*, **7**, (3).

[57] NAKANO, M. (1959) A wave recorder and a wave analyser recently devised in the Oceanographic Laboratory, Meteorological Research Institute Tokyo. *Preprints Int. Oceanogr. Congr., Washington.*

[58] PHILLIPS, O. M. (1957) On the generation of waves by turbulent wind. *J. Fluid Mech.*, **2**, (5).

[59] PHILLIPS, O. M. (1958) The equilibrium range in the spectrum of wind-generated waves. *J. Fluid Mech.*, **4**, (4).

[60] PIERSON, W. S. (1955) Wind generated gravity waves. *Advances in Geophys.* (2).

[61] WALDEN, H. Deutscher Wetterdienst Seewetteramt. Die Winderzeugten Meereswellen. Teil I. Beobachtungen des Seeganges undeErmittlung der Windsee aus den Windverhältnissen. Hamburg (1958).

[62] WEN, S. C. (1960) Generalized wind wave spectra and their applications. *Scientia Sinica*, **9** (3).

[63] WEN, S. C. (1961) Proposed spectra for ocean swell. *Scientia Sinica*, **10** (5).
[64] U.S. NAVY HYDROGRAPHIC OFFICE (1951) Techniques for forecasting wind waves and swell. Publ. No. 604.
[65] U.S. NAVY HYDROGRAPHIC OFFICE (1955) Practical methods for observing and forecasting ocean waves by means of wave spectra and statistics. Publ. No. 605.