Ocean Surface Drift Velocities

E. B. KRAUS

Cooperative Institute for Marine and Atmospheric Studies, Miami, Fla. 33149

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ABSTRACT

Symmetry considerations indicate that in steady-state conditions, and in the absence of a sea current, a horizontal interface must move in the direction of the surface geostrophic wind. With currents present, the interface velocity is the vector mean of the surface geostrophic wind and the current velocity, weighted respectively by the fluid masses per unit surface area which are affected by the viscous diffusion of vorticity upward and downward from the interface. In turbulent flow with neutral stratification, the ratio of these masses equals the square root of the density ratio. In transient conditions, the surface drift velocity has components in the direction of the isallobaric gradient and down a steeping surface slope. Wave action is responsible mainly for the spreading and the breakup of surface films.

1. Introduction

The vertical velocity and temperature structures of the upper ocean affect a wide variety of other phenomena. For practical purposes, one is concerned most often with the value of integral properties like the horizontal Ekman mass transport and its variations with time and distance or the changing heat storage. A knowledge of conditions at any particular level tends to be less immediately relevant. The sea surface is an exception to this general rule. Its temperature determines the rate of evaporation and the infrared radiation balance of the atmosphere. The direction and speed of its mean displacement governs the movement of oil spills and other surface contaminants. It also is obviously important as a boundary condition for interior atmospheric and oceanic motion.

The surface drift velocity has been derived often as a limiting value of an analysis which involves the velocity profile throughout the planetary boundary layer. A recent paper in this journal by Madsen (1977) is an example of this approach. Theoretical reconstruction of the velocity profile, however, always involves a variety of additional assumptions about the vertical stress distribution or about some form of the turbulence closure problem. Appeal has to be made to at least one universal empirical constant and the particular numerical value of the surface stress or of the corresponding surface friction velocity usually enters as a scaling factor into this type of analysis. I believe that as far as the surface drift velocity is concerned, the relevant information can be obtained with rather more generality from an elementary manipulation of the equations of motion. Detailed knowledge about turbulent processes, the stress distribution or the velocity profile is not required for this purpose. A clarification of this matter is the objective of the present note.

2. Stationary conditions

It is assumed that the turbulent processes in the water and in the air are of the same nature and that there is no slippage along the interface $\mathbf{v}_a=\mathbf{v}_w\equiv\mathbf{v}_s$ for $z=0$ (subscripts a and w refer to air and water). The steady-state motion of the interface satisfies both the following equations:

$$\left.\begin{aligned}\rho_a f\mathbf{n}\times(\mathbf{v}_s-\mathbf{v}_{ga})&=-\partial\boldsymbol{\tau}_a/\partial z\\ \rho_w f\mathbf{n}\times(\mathbf{v}_s-\mathbf{v}_{gw})&=-\partial\boldsymbol{\tau}_w/\partial z\end{aligned}\right\}, \tag{1}$$

where $\mathbf{n}$ is a vertical unit vector and $\mathbf{v}_{ga}$, $\mathbf{v}_{gw}$ are the (surface) geostrophic velocity vectors in the air and in the water. It will be seen below that the surface velocity is relatively slow ($|\mathbf{v}_s|\ll|\mathbf{v}_{ga}|$). This generally justifies the neglect of nonlinear terms in the equations of (surface) motion.

The vertical momentum flux is continuous at $z=0$, but its vertical derivative is discontinuous and opposite in direction on the two sides of the interface. Without loss of generality one can write

$$-\frac{\partial\boldsymbol{\tau}_w/\partial z}{\partial\boldsymbol{\tau}_a/\partial z}=r, \tag{2}$$

where r is a positive quantity with a value which may depend on the physical and turbulence characteristics in the two fluids.

Elimination of the stress gradient between the preceding equations yields

$$\rho_a r\mathbf{n}\times(\mathbf{v}_s-\mathbf{v}_{ga})+\rho_w\mathbf{n}\times(\mathbf{v}_s-\mathbf{v}_{gw})=0, \tag{3}$$

which is equivalent to

$$\mathbf{v}_s=\frac{\rho_a r\mathbf{v}_{ga}+\rho_w\mathbf{v}_{gw}}{\rho_a r+\rho_w}. \tag{4}$$

Eq. (4) represents the exact solution for the interface drift between two stationary Ekman layers. It only contains the parameter r. It is not affected by the Coriolis parameter or by possible variations of the geostrophic velocity along the vertical.

If the surface pressure is denoted by p_s and its height by h_s, the surface geostrophic current

$$\mathbf{v}_{gw}=-\frac{1}{f}\mathbf{n}\times\left(\frac{1}{\rho}\nabla p_s+g\nabla h_s\right)=\frac{\rho_a}{\rho_w}\mathbf{v}_{ga}+\mathbf{v}_c, \tag{5}$$

where $\mathbf{v}_c$ is the basic surface current velocity. With this consideration Eq. (4) assumes the form

$$\mathbf{v}_s=\frac{\rho_a(1+r)}{\rho_a r+\rho_w}\mathbf{v}_{ga}+\frac{\rho_w}{\rho_a r+\rho_w}\mathbf{v}_c. \tag{6}$$

In the absence of a basic current ($\mathbf{v}_c=0$), it follows immediately with full generality that the surface moves in the direction of the surface geostrophic wind. In other words for small Rossby numbers, surface films tend to drift along the surface isobars, regardless of the nature of the frictional effects.

The actual speed of the surface drift depends on the numerical value of the parameter r. Considerations of symmetry indicate that the frictional forces, when scaled appropriately, must be equal and opposite above and below the interface. If d_a and d_w are the two Ekman depths, the relation

$$d_a\partial\boldsymbol{\tau}_a/\partial z=-d_w\partial\boldsymbol{\tau}_w/\partial z$$

must always be satisfied for correspondingly scaled levels $z_a'=z/d_a$ and $z_w'=z/d_w$. For $z=z'=0$ one gets from Eq. (2)

$$r=\frac{d_a}{d_w}. \tag{7}$$

Introduction into Eq. (4) yields

$$\mathbf{v}_s=\frac{\rho_a d_a\mathbf{v}_{ga}+\rho_w d_w\mathbf{v}_{gw}}{\rho_a d_a+\rho_w d_w}. \tag{4'}$$

The quantities $\rho_a d_a$ and $\rho_w d_w$ represent the fluid masses per unit surface area which are affected by the viscous diffusion of vorticity from the interface. It follows that the surface drift velocity is simply the mean of the geostrophic wind and current velocities weighted by these two masses.

At an air–water interface, one has in general

$$\frac{\rho_a d_a}{\rho_w d_w}\ll 1. \tag{8}$$

A simpler, more readily applicable, approximate equation for $\mathbf{v}_s$ can therefore be obtained from the Eqs. (6), (7) and (8). It has the form

$$\mathbf{v}_s \approx \frac{\rho_a d_a}{\rho_w d_w} \mathbf{v}_{ga} + \mathbf{v}_c. \tag{6'}$$

We consider first the classical Ekman case of viscous nonturbulent flow in both media. One has then

$$d_a = \left(\frac{2\nu_g}{f}\right)^{\frac{1}{2}}, \quad d_w = \left(\frac{2\nu_w}{f}\right)^{\frac{1}{2}}. \tag{9}$$

In the absence of a basic current ($\mathbf{v}_c = 0$), it follows from (6′) that

$$\mathbf{v}_s = \frac{\rho_a}{\rho_w}\left(\frac{\nu_a}{\nu_w}\right)^{\frac{1}{2}} \mathbf{v}_{ga}.$$

This is the standard formula for the interface velocity in viscous Ekman flow (see, e.g., Kraus, 1972, p. 170).

In the turbulent case, for neutral conditions, one has

$$d_a \propto \frac{u_*}{f}, \quad d_w \propto \frac{u_{*w}}{f} = \frac{u_*}{f}\left(\frac{\rho_a}{\rho_w}\right)^{\frac{1}{2}}, \tag{10}$$

where u_*, u_{*w} are the friction velocities in air and water. Introduction into Eq. (6′) yields

$$\mathbf{v}_s = \left(\frac{\rho_a}{\rho_w}\right)^{\frac{1}{2}} \mathbf{v}_{ga} + \mathbf{v}_c. \tag{11}$$

The last expression suggests that the effect of a current on the surface translation tends to be relatively small, if the surface geostrophic wind speed exceeds the current velocity by a factor of 100 or more.

Eq. (11) remains applicable also in non-neutral conditions, e.g., in the case of an upward heat flux which destabilizes both the air and the water below in the same way. It ceases to be applicable in the presence of entrainment, when a surface mixed layer deepens into a stably stratified interior, because the condition of symmetry is then generally not maintained. However, the velocity change within the mixed layer tends to be relatively small in this case. If the layer is considered to move approximately like a slab, the stress must decrease linearly with distance from the interface and it is then permissible simply to equate the Ekman depth with the mixed layer depth in the preceding derivation.

3. Transient drift velocities

The preceding argument indicates that the motion of the surface is governed by the pressure and Coriolis forces. This leads to the balance which is symbolized by Eqs. (3) or (4). The ratio r of the diffusive capacities of the two fluids, which also enters these formulas, tends to remain invariant over a wide range of circumstances. This is the case not only in balanced motion, but also in the transient condition. If the characteristic time scale of the change is inversely proportional to some frequency ω, the Ekman depth—instead of being simply a function of f—becomes a function of $f-\omega$. However, as the effect is analogous in the two fluids, it will not change the value of the ratio r.

In nonstationary conditions, we obtain

$$\left(\frac{\partial}{\partial t} + f\mathbf{n}\times\right)\mathbf{v}_s = f\mathbf{n}\times\frac{\rho_a r \mathbf{v}_{ga} + \rho_w \mathbf{v}_{gw}}{\rho_a r + \rho_w} \tag{12}$$

instead of the balance equation (3).

It is convenient to replace the fraction in the last term by a single vector symbol $\mathbf{F}$. Using the same argument which led to Eq. (6′) we then have

$$\mathbf{F} \equiv \frac{\rho_a r \mathbf{v}_{ga} + \rho_w \mathbf{v}_{gw}}{\rho_a r + \rho_w} \approx \frac{\rho_a}{\rho_w} r \mathbf{v}_{ga} + \mathbf{v}_c. \tag{13}$$

Eq. (12) then assumes the form

$$\left(\frac{\partial}{\partial t} + f\mathbf{n}\times\right)\mathbf{v}_s = f\mathbf{n}\times\mathbf{F} \tag{12'}$$

which has the general solution

$$\mathbf{v}_s = \mathbf{F} - e^{-ift}\int \frac{\partial \mathbf{F}}{f\partial t} e^{ift} dt + \mathbf{F}_0 e^{ift}, \tag{14}$$

where $\mathbf{F}_0$ is an integration constant. In stationary conditions Eq. (14) is reduced to (3) or (6′). Otherwise it depends on the way the forcing $\mathbf{F}$ varies with time.

If one replaces $\mathbf{F}$ by $\mathbf{v}_{ga}$, Eq. (14) is transformed into the equation of nonsteady, frictionless, horizontal motion in the free atmosphere. The solution of that equation is discussed in standard meteorological textbooks. It involves three physically different parts: a geostrophic component, an isallobaric component which is proportional to the gradient of pressure changes and an inertial component. Analogous components appear in the solution (14) of Eq. (12′). The first and the last terms on the right-hand side of (14) represent the geostrophically balanced and the inertial motions. The middle term corresponds to an isallobaric velocity.

To illustrate this more clearly, it is convenient to convert the vector equation (12′) into two second-order, scalar differential equations for the components u_s, v_s of $\mathbf{v}_s$. The forcing vector $\mathbf{F}$ is expressed in its explicit form specified by the last term in Eq. (13). On the time scale which is of interest in the present contest, the current velocity $\mathbf{v}_c$ can usually be considered constant because its change requires oceanic mass displacements over relatively long-time intervals. With these considerations, Eq. (12′) is transformed into

$$\left.\begin{aligned}\left(\frac{1}{f^2}\frac{\partial^2}{\partial t^2}+1\right)u_s=r\left(u_{ga}-\frac{1}{f}\frac{\partial v_{ga}}{\partial t}\right)+u_c\\ \left(\frac{1}{f^2}\frac{\partial^2}{\partial t^2}+1\right)v_s=r\left(v_{ga}+\frac{1}{f}\frac{\partial u_{ga}}{\partial t}\right)+v_c\end{aligned}\right\}. \qquad (15)$$

It can be seen immediately, in addition to inertial motions, that the surface velocity is made up of three parts: one in the direction of the surface geostrophic wind u_{ga}, v_{ga} which is therefore parallel to the surface isobars, a second in the direction of the current u_c, v_c which is parallel to the contour lines of the surface, and a third which is normal to the lines of equal change of geostrophic velocity, i.e., normal to the isallobars. For example, if one considers a geostrophic wind of changing force but constant azimuth along the x direction ($v_{ga}=\partial v_{ga}/\partial t=0$, $\partial u_{ag}/\partial t\neq 0$) the second equation (15) shows that the surface velocity must have a finite component v_s along the y axis in the direction of falling pressure. Eqs. (15) show also that if inertial oscillations are supressed, the surface drift would remain in phase with a turning geostrophic wind of constant force.

The isallobaric component of the surface drift becomes relatively large if the characteristic time scale of the pressure change is shorter than half a pendulum day ($\omega/f>1$). This can be easily the case in deepening hurricanes for example. One might find there a surface drift toward the storm center in spite of a divergent Ekman transport in the opposite direction. In the initial stage of hurricane development, this may be associated with a convergence of surface water into the pressure center, as was demonstrated by O'Brien and Reid (1967) in a numerical analysis. In the case of a moving pressure trough, the area of greatest pressure decrease lies ahead of the trough line and behind the preceding pressure ridge. The isallobaric effect in this case causes a reduction of the surface drift velocity along the ridge and a speed-up along the trough line.

It is now easy to assess also the effect of a possible change in the geostrophic current velocity $\mathbf{v}_c$. In analogy to the isallobaric wind, it would produce a drift down a steepening surface slope. However, on the synoptic scale, the magnitude of this drift component, which is directly proportional to the change in slope, is very small compared to the other drift components. It may be more important on the seasonal scale.

The preceding arguments cease to be applicable when there is no symmetry in the stress distribution above and below the interface. This happens obviously at the surface of a shallow water body which is affected by bottom friction. Another case in point is the one-sided entrainment of stably stratified fluid. Surface waves may also be a cause for asymmetry. Although the classical theory of irrotational surface waves describes a motion pattern which is essentially symmetric, and although this applies also to secondary effects like the Stokes' drift, the actual generation of wind driven waves involves the presence of an atmospheric critical level, where the wave and wind velocities are equal. This level does not exist in the ocean and this must be associated with some asymmetry in the turbulent stress distribution. It is not clear, however, whether these asymmetries in the immediate vicinity of the surface can in fact produce substantial changes in the ratio r as defined by Eq. (2). Mixing by wave breaking may cause further complication, not only by introducing additional asymmetries, but by making the whole concept of a surface drift velocity rather ambiguous.

In general, wave action has probably more effect on the spreading and the eventual breakup of surface films and oil patches, than on their mean displacement. As the wave spectrum is invariably two-dimensional, it involves transient surface displacements which make an angle with the mean surface drift. Furthermore, although the resultant Stokes' drift tends to satisfy the equation of geostrophic motion (Moore, 1969), and although it can be considered at the surface as being a part of $\mathbf{v}_s$, it also has large oscillatory components which have different directions, frequencies and amplitudes for different wave-trains. This can give rise to a mean lateral momentum transport $\overline{u_s'v_s'}$. It would be interesting to determine this lateral flux as a function of the wave spectrum characteristics, but this is a problem which goes well beyond the scope of the present note.

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