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Current effects on resonant reflection of surface water waves by sand bars

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The effect of currents flowing across a bar field on resonant reflection of surface waves by the bars is investigated. Using a multiple-scale expansion, evolution equations for the amplitudes of linear waves are derived and used to investigate the reflection of periodic wave trains with steady amplitude for both normal and oblique incidence. The presence of a current is found to shift resonant frequencies by possibly significant amounts and is also found to enhance reflection of waves by bar fields due to the additional effect of the perturbed current field.

1. Introduction

The possibility of obtaining strong reflections of incident surface water waves through interaction with undular topography has drawn attention in recent years to the mechanism's possible impact on coastal geomorphology. Davies & Heathershaw (1984) have investigated bottom topographies of the form

$$h(x) = \bar{h} + \delta(x), \tag{1.1}$$

where $h(x)$ denotes total water depth, $\bar{h}$ represents a steady mean depth and $\delta(x)$ represents a small-amplitude, rapid perturbation. 'Small amplitude' implies $|k\delta| \ll 1$ in general scaling or $|\delta/h| \ll 1$ in shallow-water scaling. In experiments to date, $\delta(x)$ has been given the simple form

$$\delta(x) = D \sin \lambda x; \quad 0 \leqslant x \leqslant L. \tag{1.2}$$

This represents a long-crested bar field confined in the region $\{0 \leqslant x \leqslant L\}$ consisting of n bars, with uniform bar amplitude D and bar wavenumber λ constrained according to $\lambda = 2\pi n/L$. The sinusoidal form is convenient in that it is physically plausible and contributes only a single wave-like perturbation in the mathematical analysis. Davies & Heathershaw experimented with normally incident waves of variable wavenumber–frequency $\{k, \omega\}$ and clearly demonstrated a strong resonance in the neighbourhood of $2k/\lambda = 1$, leading to greatly enhanced reflection. They pointed out the analogy between this resonance and Bragg-scattering in crystallography, but provided an analysis only for the case of weak reflection. Their analysis, done in the context of regular perturbations, breaks down at the resonance condition.

Mei (1985) examined the neighbourhood of the resonance directly using a resonant-interaction analysis and obtained good predictions of the maximum reflections observed in Davies & Heathershaw's experiments. Mei also examined the case of detuned interaction, where wave frequency ω is allowed to deviate from resonant frequency ω_0 by an amount Ω. No direct comparison was made between the predictions for detuned waves and data. Kirby (1986) provided an extension of the mild-slope equation to handle the general case of arbitrarily varying $\delta(x, y)$ on a slowly varying mean depth $\bar{h}(x, y)$. Numerical simulation of Davies & Heathershaw's experiments also provided good reproduction of measured reflection coefficients. Hara & Mei (1987) have extended the resonant-interaction theory to second order in $|\delta|$ and have performed additional experiments which illustrate the existence of a cutoff condition for frequency Ω, which is explained further below.

The existence of the Bragg scattering mechanism provides a possible means for constructing coastal protection devices which are relatively low in profile in comparison to local water depth. The installation of such an artificial bar field fronting a beach may provide a means for significantly reducing wave energy arriving at the surf zone. Given that any such installation would necessarily be of finite extent in the longshore direction, it is likely that the resulting localized depression in maximum set-up behind the bar field would generate a nearshore circulation pattern. Prediction of such a pattern depends on future hydrodynamic modelling. However, its result would be to introduce onshore or (more likely) offshore flows over the bar field. In order to evaluate the effectiveness of the bar field in the presence of an induced circulation, it is necessary to understand the effect of wave–current interaction over the bar field.

In this study, we examine the reduced case of the propagation of waves over a bar field $\delta(x)$ resting on an otherwise flat bottom $z = -h$. Not accounting for the bar-field perturbation, the current field $\{U_0, V_0\}$ is taken to be constant and of the order of linear wave phase speed (e.g. $O(1)$) in the perturbation analysis. In §2, we state the full problem and then give the solutions for the steady flow to $O(\delta)$, following Kennedy (1963) and Reynolds (1965). Then, in §3, we examine the current's effect on the conditions for Bragg resonance. The evolution equations for the linear wave scattering problem are constructed in §4. Solutions and various examples for normal wave incidence are examined in §5. Finally, in §6 we provide sufficient information to construct the similar solutions for the oblique-incidence case.

2. Solution for the perturbed mean flow

We first solve for the flow field and surface displacement resulting from the interaction of a uniform flow $\{U_0, V_0\}$ and bottom displacement $\delta(x)$ given by (1.2). Figure 1 illustrates the various quantities described below. The full problem for waves and current may be written in terms of surface displacement η and velocity potential ϕ according to

$$\nabla_h^2 \phi + \phi_{zz} = 0, \qquad -h \leqslant z \leqslant \eta, \tag{2.1a}$$

$$\phi_z = \epsilon \nabla_h \cdot \{\delta \nabla_h \phi\}, \qquad z = -h, \tag{2.1b}$$

$$g\eta + \phi_t + \tfrac{1}{2}(\nabla_h \phi)^2 + \tfrac{1}{2}(\phi_z)^2 = c(t), \quad z = \eta, \tag{2.1c}$$

$$\eta_t + \nabla_h \phi \cdot \nabla_h \eta = \phi_z, \qquad z = \eta, \tag{2.1d}$$

where ϵ denotes the effect of the small bottom perturbation. Noting that only linear wave motion is to be considered, we may split ϕ, η and c into time-steady parts associated with the current and time-harmonic parts associated with the waves:

$$\phi = \phi_c + \phi_w; \quad \eta = b + \eta_w; \quad c = c_c + c_w. \tag{2.2a,b,c}$$

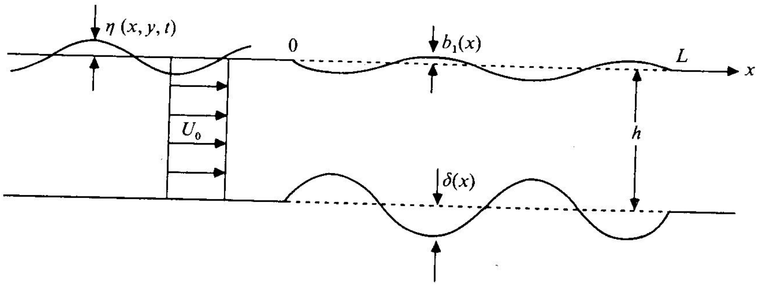

FIGURE 1. Definition sketch.

Substituting (2.2) in (2.1) and isolating time-steady terms yields

$$\nabla_h^2 \phi_c + \phi_{c,zz} = 0, \quad -h \leqslant z \leqslant b, \tag{2.3a}$$

$$\phi_{c,z} = \epsilon \nabla_h \cdot (\delta \nabla_h \phi_c), \quad z = -h, \tag{2.3b}$$

$$gb + \tfrac{1}{2}(\nabla_h \phi_c)^2 + \tfrac{1}{2}(\phi_{c,z})^2 = c_c, \quad z = b, \tag{2.3c}$$

$$\nabla_h \phi_c \cdot \nabla_h b = \phi_{c,z}, \quad z = b. \tag{2.3d}$$

We now introduce the expansion

$$\phi_c = \phi_{0c} + \epsilon \phi_{1c}, \tag{2.4a}$$

$$b = b_0 + \epsilon b_1, \tag{2.4b}$$

$$c = c_0 + \epsilon c_1. \tag{2.4c}$$

At $O(1)$ we obtain the solution for the undisturbed current field:

$$\phi_{0c} = (U_0 x + V_0 y), \tag{2.5}$$

$$b_0 = 0, \tag{2.6}$$

$$c_0 = \tfrac{1}{2}(U_0^2 + V_0^2), \tag{2.7}$$

where c_0 is chosen so as to render the $O(1)$ depth equal to h. We define Froude numbers associated with the horizontal currents according to

$$F_x = \frac{U_0}{(gh)^{\frac{1}{2}}}; \quad F_y = \frac{V_0}{(gh)^{\frac{1}{2}}}. \tag{2.8}$$

At $O(\epsilon)$, we obtain the problem considered by Kennedy (1963) and Reynolds (1965), restricted here to the case of time-steady bottoms. The solution to that problem is given by

$$\phi_{1c} = \frac{U_0 D}{2\alpha}\{\beta \cosh \lambda(h+z) + \alpha \sinh \lambda(h+z)\}\, e^{i\lambda x} + \text{c.c.}, \tag{2.9}$$

$$b_1 = \frac{-i\lambda h F_x^2 D}{2\alpha \cosh \lambda h} e^{i\lambda x} + \text{c.c.}, \tag{2.10}$$

where

$$\alpha = \lambda h F_x^2 - \tanh \lambda h, \tag{2.11a}$$

$$\beta = 1 - \lambda h F_x^2 \tanh \lambda h. \tag{2.11b}$$