Journal of Oceanography, Vol. 62, pp. 923 to 927, 2006

Reply

Shoaling Waves: A Discussion

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(Received 10 April 2006; in revised form 6 June 2006; accepted 7 August 2006)

Surface gravity waves are commonly observed to slow down and to stop at a beach without any noticeable reflection taking place. We assume that as a consequence the waves are continuously giving up their linear and angular momenta, which they carry with them, along with energy, as they propagate into gradually decreasing mean depths of water. It takes a force to cause a time rate of decrease in the linear momentum and a torque to produce a time rate of decrease in the angular momentum. Both a force and a torque operate on the shoaling waves, due to the presence of the sloping bottom, to cause the diminution of their linear and angular momenta. By Newton's third law, action equals reaction, an equal but opposite force and torque are exerted on the bottom. No other mechanisms for transferring linear and angular momenta are included in the model. Since the force on the waves acts over a horizontal distance during shoaling, work is done on the waves and energy flux is not conserved. Bottom friction, wave interaction with a mean flow, scattering from small-scale bottom irregularities and set-up are neglected. Mass flux is conserved, which leads to a shoreward monotonic decrease in amplitude consistent with available swell data. The formula for the time-independent force on the bottom agrees qualitatively with observations in seven different ways: four for swell attenuation and three for sediment transport on beaches. Ardhuin (2006) argues against a mean force on the bottom that is not hydrostatic, mainly by using conservation of energy flux. He also applies the action balance equation to shoaling waves. Action is a difficult concept to grasp for motion in a continuum; it cannot be easily visualized, and it is not really necessary for solving the shoaling wave problem. We prefer angular momentum because it is clearly related to the observed orbital motion of the fluid particles in progressive surface waves. The physical significance of wave action for surface waves has been described recently by showing that in deep water action is equivalent to the magnitude of the wave's orbital angular momentum (Kenyon and Sheres, 1996). Finally, Ardhuin requires that there be a significant exchange of linear momentum between shoaling waves and an unspecified mean flow, although the magnitude and direction of the exchange are not predicted. No mention is made of what happens to the orbital angular momentum during shoaling. Mass flux conservation is not stated.

Keywords:
· Shoaling waves,
· bottom force,
· linear momentum balance,
· bottom torque,
· angular momentum balance.

1. Introduction

Many different physical processes can affect shoaling waves, and Ardhuin *et al*. (2003) has given a good summary of most of them. In addition, he has presented analyses of a large wave dataset obtained at buoys off the east coast of the US and has made several hindcasts from a numerical model for comparison with the observations. Not included in his summary of mechanisms, however, is one that was more recently isolated for study by Kenyon (2004a). It is rather remarkable how close this one mechanism comes to explaining several observed features of shoaling waves as well as transport of beach material. This unexpected development has provided an opportunity to check the validity of some paradigms, which may lead to progress in the future.

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At the start of his commentary (Ardhuin, 2006) on my shoaling wave paper (Kenyon, 2004a) Ardhuin asks the interesting question: how can there be a steady state force on a sloping bottom when waves shoal other than that due to the hydrostatic pressure related to the still water level? He says this question has not been asked before within the field of near-shore dynamics, and he is probably right. Of course, the hydrostatic pressure force acts almost perpendicularly to the gently sloping bottom. Who would suspect that there is a steady force on the bottom that acts very nearly parallel to the bottom but is not related in any way to the force of friction? Ardhuin's intuition tells him that there cannot be such a mean force on the bottom.

Let us first examine a different context: surface gravity waves reflecting off a vertical wall. Here there is an embarrassingly rich history of calculations of the wave force on a wall, and there is also definite laboratory evidence for a wave force on a wall (Kenyon, 2004b). More than 25 different theoretical formulas exist for the wave force on a vertical wall, and all of them have an average value that is not zero. Note that no mean flow is involved at any stage; the momentum is exchanged directly between the waves and the wall. The wave momentum undergoes a time rate of change at the wall when the sign of the linear momentum is reversed by reflection, and by Newton's second law the wall has caused a force on the wave to reverse its momentum. Then, by Newton's third law, the wave exerts an equal but opposite force on the wall. The wave force is directed normal to the wall. This process is analogous to the force delivered to rigid surfaces by the reflection of sound and light waves.

For a sloping bottom, where the average depth gradually decreases in the direction of propagation of the incident waves, experience has shown that low frequency waves are reflected but high frequency waves are transmitted (e.g. Elgar *et al.*, 1994). The cross-over frequency between mostly reflection and mostly transmission apparently depends on the bottom slope, and for a slope of 1/30 it is about 40 cycles per kilosecond according to Munk *et al.* (1963). (There appears to be no theory available that predicts this result, however.) By analogy with the vertical wall, we expect a mean force on the sloping bottom when low frequency waves are reflected seaward. Again, no intermediate momentum exchange between waves and a mean flow is anticipated, just the straightforward transfer of momentum between waves and the bottom.

High frequency shoaling waves are transmitted shoreward and they do not return to the sea. These waves are totally absorbed in the near-shore zone. Since propagating surface gravity waves carry linear and orbital angular momentum with them (Kenyon and Sheres, 1996), they are constantly importing linear and angular momentum into the near-shore region. By the time the waves have reached the beach their linear and angular momenta have vanished. There has been a time rate of decrease of these quantities. It takes a force to cause a time rate of decrease in the wave's linear momentum and a torque to produce a time rate of decrease in the angular momentum. The sloping bottom has brought about the force and the torque, and by Newton's third law, and an extension of that law to rotary motion, the waves exert both a force and a torque on the bottom. This is the physical origin of the steady state force on the sloping bottom when waves shoal. For a gently sloping beach the wave force is directed nearly tangentially to the bottom and is oriented shoreward.

Now, it is at this point in the argument that Ardhuin says that there must be an exchange of linear momentum between the waves and a mean flow for shoaling waves. In fact, he postulates that the exchange is so complete that it is no longer possible for a significant steady-state, non-hydrostatic force to occur on the sloping bottom. All the wave's linear momentum apparently goes into the mean flow. But where does the orbital angular momentum go, since the mean flow can never have any? Even if the mean flow contained a vertical shear, that would only cause vorticity, not orbital angular momentum. Here is where our two paths part company: at the postulated prerequisite momentum transfer between the waves and a mean flow.

2. Theory

Surface gravity waves transport linear and orbital angular momentum, as well as energy, when they propagate. Angular momentum is easily pictured because it is related to the orbital motion of the fluid particles, whether those orbits are circular in deep water or elliptical in finite constant mean depths. This can be seen by watching small, neutrally buoyant floats as surface waves pass by a fixed position. Action, favored by Ardhuin (2006), has been shown to be equivalent to the magnitude of the angular momentum in deep water (Kenyon and Sheres, 1996). However, in general action is a somewhat opaque concept when applied to waves in a fluid.

The calculation of the wave force on the sloping bottom began for me as a struggle to understand how energy flux could be conserved during shoaling, as is maintained traditionally, and at the same time how angular momentum flux could be conserved too, because at first sight there is no self-evident reason why angular momentum flux should not be conserved along with energy flux. It turns out that these two conservation principles are incompatible, because they predict different wave amplitude variations in the near-shore area, and therefore both had to be abandoned together.

Mass flux is conserved in the direction normal to the beach. There can be no question about that. Waves transport mass shoreward due to the Stokes drift. What happens to this mass as the mean depth decreases and the shoreline is approached? Nearshore oceanographers do not seem to worry about this, but we have addressed the issue head-on.

We have adopted the ray equations along with the fundamental assumption upon which they are based: that the environment (the mean depth in particular) changes slowly within one wavelength. Then we must exclude from consideration interactions between the waves and small-scale irregularities in the bottom topography (Bragg scattering).

Since the ray equations do not contain the wave amplitude (Kenyon, 1971), additional information must be supplied in order to be able to compute the wave amplitude along a ray. For example, if there are no forces that operate on the waves as they propagate, the amplitude along the rays can be obtained from the linear momentum principle (Kenyon and Sheres, 2006). But there is a force on the waves, due to the presence of the sloping bottom, which slows them down and decreases their linear momentum. The force operates over a horizontal distance, causing work to be done on the waves so the energy flux is not conserved, and therefore the wave amplitude cannot be predicted from energy flux conservation either. Along with linear momentum and energy, shoaling waves also lose their orbital angular momentum when the waves slow down and stop at the beach. Consequently, angular momentum is not conserved and the angular momentum balance cannot be used to calculate the amplitude of shoaling waves. However, the constancy of the mass flux normal to the beach will determine the wave amplitude, and it forecasts that the amplitude should decrease monotonically shoreward, as swell attenuation data have shown.

A mean horizontal flow near the shore is not a necessary ingredient in our model and bottom friction is neglected. Mean sea level is taken to be flat with no set-up or set-down because sea level variations cannot explain angular momentum changes. Set-up or set-down can be added to the model in the future if needed. Wave reflection from the sloping bottom is disregarded. We focus on the wave induced force on the bottom to see how much can be explained by that one mechanism alone.

By contrast, Ardhuin maintains the position that energy flux is conserved for shoaling waves. He also uses the action balance equation. Wave action has a history in physics, going back to the classical simple harmonic oscillator, but it is a rather vague concept within the context of surface gravity waves traveling in a continuous medium.

Absorbing "wave action" into working knowledge is an arduous task for the oceanographer: he must thread his way through the details of classical Hamiltonian and Lagrangian mechanics, and then understand how to carry over these principles to a fluid continuum. Particularly hard to swallow, since they cannot be checked by independent calculations, are the required linearizations of nonlinear equations by perturbation series and the necessary averaging procedures, as well as the assumptions underlying the linearization and averaging techniques. Wave action was developed from the Hamilton-Jacobi method (Corben and Stehle, 1977) for periodic motions of systems that are slightly disturbed by some external force. However, it turns out that this complicated concept of wave action is not needed to solve the central problem of shoaling surface gravity waves in the way we have approached the problem.

Lagrangian and Hamiltonian mechanics provide a general framework for organizing certain kinds of information, but for a specific problem the physics must be provided from outside that framework. Consider the surface gravity wave. Let the researcher choose whatever generalized coordinates he pleases, set up the Lagrangian and Hamiltonian functions, and manipulate and transform these functions in any possible way. He will never end up with one feature that observations give him immediately (or linear wave theory provides, but with a bit more work): the fluid particles have an orbital motion and therefore these waves possess angular momentum with respect to the centers of the orbits.

3. Observations

More than 30 years ago Hasselmann *et al.* (1973) measured swell attenuation in the North Sea. They found significant swell attenuation but could not clearly identify the physical mechanism responsible. The normal laws of frictional dissipation did not explain the decay rates observed. This is solid evidence that conservation of energy flux does not work for shoaling swell. Energy flux conservation predicts an increase in wave amplitude as the distance to the beach decreases, everywhere shoreward of the weak maximum in the group velocity, which is just the opposite of what the data showed.

Although the observed decay rates of swell in the North Sea contradict the favored conservation of energy flux law, they are consistent with the algebraic form of my equation (14) for the wave force on the sloping bottom, in four different ways. The data showed the decay rate: 1) increased with increases in initial swell energy; 2) increased with decreases in mean depth; and 3) was independent of the swell frequency. All three of these observed features can be understood with the aid of the formula for the wave force on the sloping bottom. The first two were explained in my paper; the third was overlooked at that time. However, it can be seen that equation (14) is independent of the wave frequency in the shallow water limit, whereas the usual bottom friction force for shoaling waves (drag coefficient multiplied by the square of the velocity) does depend on frequency (higher frequency waves should be attenuated at a greater rate than lower frequency ones).

Furthermore, conservation of mass flux predicts a monotonic shoreward decrease in amplitude, which is consistent with swell attenuation measurements. This constitutes the fourth way in which swell attenuation data agree with my model.

My paper describes three features of sediment transport on beaches that are in qualitative agreement with the structure of equation (14) for the wave force on the sloping bottom. They are: 1) the concave upward profile, typical of many beaches; 2) the positive correlation between sediment size and average beach slope (larger grain sizes occur on larger slopes), which has strong observational support; and 3) the seasonal migration of sand in and out of beaches in southern California (in during summer and out in winter).

Altogether, seven observational features of swell attenuation during shoaling and sediment transport on beaches are qualitatively consistent with the algebraic form of the wave force formula on the sloping bottom, equation (14), and with mass flux conservation, equation (6).

In his commentary Ardhuin does not single out any particular comparisons between theory and observations, but his 2003 paper presents several comparisons of wave model hindcasts with swell data. Let us see how far our equation (3) can explain the swell attenuation in the data presented by Ardhuin *et al.* (2003) in his figure 7. Between the deep water station X6 and station X1, closest to shore, equation (3) predicts an amplitude ratio (X1 to X6) of 0.81 for the low amplitude 12.5 second swell out of the east on November 18, 19, 1999. During November 21–25, 1999 equation (3) predicts an amplitude ratio of 0.85 for the 11.1 sec swell with a moderately larger amplitude. In both cases equation (3) can account for about 80% of the observed decrease in the significant height of the swell at X1 compared to that at X6.

Ardhuin's numerical model obtains a much better agreement between theory and experiment. This really good agreement can be questioned, however. Nothing is mentioned about the tidal currents on the continental shelf, which could refract the shoaling waves and cause variations in the swell amplitude that are not included in the numerical model. Hasselmann *et al.* (1973) encountered tidal currents of up to 40 cm/sec in the North Sea, for example, and there is no reason to suspect that significant tidal currents do not occur off North Carolina and Virginia. Moreover, in order to reach the measurement buoys on the continental shelf, the swell must have crossed the Gulf Stream within which wave-current refraction is anticipated to be larger than almost anywhere else in the oceans. The position, speed and curvature of the streamlines of the Gulf Stream, or eddies shed by the Gulf Stream, do not appear to have been monitored in the DUCK Experiment upon which Ardhuin's paper is based.

4. Discussion

Ardhuin's review of existing analyses does not lead him to a single formula for a steady-state, non-hydrostatic force on the bottom due to shoaling waves that could be compared directly with my equation (14). This is unfortunate because if it had, then we could eventually let the observations decide between the two independent equations, when they become available (if such observations do not already exist). If both theories explain the data equally well, the simpler of the two methods is the one usually preferred in the physical sciences.

On the other hand, in my opinion, Ardhuin does not demonstrate that such a mean force cannot exist. Evidently he believes that for shoaling waves there must be an exchange of linear momentum between the waves and some sort of a mean flow. He gives no guidelines about the exact amount of momentum exchange to be expected, however, nor any discussion of the direction that this momentum flow should take (i.e. from waves to mean flow or the reverse). But then he allows that a steady force on the bottom can occur for reflecting waves without any intermediate transfer of momentum between waves and a mean flow. This seems odd because light and sound waves deliver constant forces to rigid surfaces without involvement of any mean flows, whether reflection or absorption takes place.

In near-shore regions environmental mean flows (e.g. due to tides) automatically move parallel to the beach, or approximately so, because of the impermeability of the bottom and beach material (no flow into the boundary). Due to wave-bottom refraction, shoaling waves normally propagate normal to the beach, or nearly so. How can a meaningful momentum exchange take place between the waves and the mean flow when their initial momenta are mutually perpendicular?

Just before his equation (5), Ardhuin introduces the concept of "pseudo-momentum" of waves. What I cannot ascertain is the extent to which this concept affects the conclusions that follow later. If pseudo-momentum is crucial to the main arguments, then some readers will be confused because it is not a well established idea as applied to surface gravity waves. Furthermore, "radiation stress" is listed as one of his Key Words, but I do not see how that idea fits in with his main line of thought.

My equation (14) for the steady, non-hydrostatic force of shoaling waves on the bottom is based on several assumptions. One of these assumptions is that there is negligible mean flow, which was not explicitly indicated in the paper. This was done by analogy with the case of reflected waves, where it has never occurred to authors to include an interaction in which waves and a mean flow exchange momentum.

Another assumption I made, as pointed out by Ardhuin, is that the sea surface is flat; there is no set-up, and this one is clearly stated in the paper. This assumption, adopted for simplicity as a first approximation, can be changed in the future to allow sea level variations near the shore. The theoretical reason for adopting a flat surface is that no type of sea level variation can possibly account for the disappearance of the wave's orbital angular momentum during shoaling, which is explained in my paper, because a baroptropic pressure force that would result from a sea level variation cannot cause a torque on the orbiting fluid particles.

Finally, I have neglected bottom friction based, on the experience of others. In particular, Hasselmann *et al.* (1973) were unable to relate their observations of swell attenuation to the well-known law of frictional dissipation. A quantitative comparison can be made between my shoaling wave force on the bottom and the bottom friction force, but this is left for the future. There are already sufficient qualitative disagreements between wave data and the normal bottom friction equation so that a quantitative relation does not seem worth pursuing at the present time.

Acknowledgements

Discussions with David Sheres were quite helpful and the manuscript benefited from the comments of the referees.

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