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Wave refraction in ocean currents

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Abstract—Refraction of surface gravity waves in major ocean currents is investigated by applying the geometrical optics approximation to deep water waves in steady nonuniform currents. Assuming the group speed c_g' relative to the current is much greater than the current speed, the radius of curvature R of the wave rays is given by $R = c_g'/\zeta$, where ζ is the vertical component of the vorticity of the current. The magnitude of R decreases with decreasing group speed (increasing frequency) relative to the current and with increasing vorticity. The sign of R is given by the sign of ζ, and the radius of curvature is positive (negative) if the ray is concave to the left (right) when looking in the direction of energy propagation. Applications to the ocean indicate that both the trapping of surface waves in currents and the total reflection of waves by currents should be possible under the appropriate conditions. Some observations of swell propagating across the Pacific are not inconsistent with a wave–current refraction mechanism.

INTRODUCTION

THE EFFECTS of a following or opposing current on the propagation of surface gravity waves were first worked out by UNNA (1942) and SVERDRUP (1944) and their results were applied mainly to waves in tidal entrances. JOHNSON (1947) found the effects on waves which enter a uniform current at an angle and he suggested that "major ocean currents, such as the Gulf Stream, may have an appreciable effect on the height, length, and direction of waves approaching the shore and under some circumstances may cause almost complete reflection." ARTHUR (1950) investigated the combined effect of nonuniform currents and bottom topography on the propagation of shallow water waves and made an application to waves entering an intense rip current.

In the present paper the approximation of geometrical optics for an inhomogeneous moving medium is applied to surface gravity waves propagating in nonuniform steady currents. The basic ideas involved have been discussed by LANDAU and LIFSHITZ (1959), WHITHAM (1960), and BACKUS (1962). Only the kinematical effects of currents on the wave number are applied to the ocean in the present paper; the dynamical effects of currents on wave amplitude are not investigated. The applications to the ocean indicate that refraction effects of deep water surface waves in major ocean currents could be significant; in particular the trapping of waves in currents and the total reflection of waves by currents should both be possible.

THEORY

The ray equations for wave propagation in a steady inhomogeneous medium are (LANDAU and LIFSHITZ, 1959)

$$\frac{d\mathbf{x}}{dt} = \frac{\delta H}{\delta \mathbf{k}}, \qquad \frac{d\mathbf{k}}{dt} = -\frac{\delta H}{\delta \mathbf{x}} \tag{1}$$

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where $\omega = H(\mathbf{k}, \mathbf{x})$ is a known function of wave number $\mathbf{k}$ and position $\mathbf{x}$. The first equation of (1) describes the rays, which are paths traced out by points which move with the group velocity $\mathbf{c}_g = \delta\omega/\delta\mathbf{k}$. The second equation describes the change in wave number along the rays. For given initial conditions on $\mathbf{k}$ and $\mathbf{x}$ the rays and wave number along the rays are determined by simultaneous integration of equations (1). In a steady medium $\delta\omega/\delta t = 0$ and therefore $d\omega/dt = 0$ from (1) and the frequency ω is constant along the rays. Equations (1) hold under the geometrical optics approximation that the wave amplitude and frequency vary slowly over distances of the order of a wave length.

For a steady inhomogeneous moving medium the frequency is given by

$$\omega = \omega' + \mathbf{k}.\mathbf{U} = \omega_0 = \text{constant} \tag{2}$$

where ω' is the frequency relative to a coordinate system moving with the current velocity $\mathbf{U}(\mathbf{x})$ and ω is the frequency relative to a fixed coordinate system.

A useful relationship can be derived from (1) and (2) when $\mathbf{k}$ is constrained to be a two-dimensional vector and $\omega' = \omega'(\mathbf{k})$, where $k = |\mathbf{k}|$. Let $\mathbf{k}$ lie in the xy-plane. Then an approximate expression for the radius of curvature of the rays R is

$$R = c_g'/\zeta \tag{3}$$

where $c_g' = \delta\omega'/\delta k$ is the wave group speed relative to the current and ζ is the component of current vorticity in the positive z-direction. The sign of the radius of curvature is defined such that as a point moves along the ray in the direction of energy propagation the radius of curvature is positive (negative) if the ray is concave to the left (right). Equation (3) is a slightly modified special case of LANDAU and LIFSHITZ (1959, p. 261) equation (1) for propagation of dispersive waves in two dimensions. The derivation of (3) requires that the group speed relative to the current be much greater than the current speed. Equation (3) is exact when the current velocity vanishes, whether or not the vorticity vanishes, because the terms neglected in (3) are proportional to the current velocity. The radius of curvature is infinite (the rays are straight lines) where the current velocity is constant as (3) shows, but if the vorticity vanishes when the current is not constant, then R becomes large but not infinite in general due to the neglect of higher order terms involving derivatives of the current velocity. (Equation (3) can also be derived by starting with the exact expression for the radius of curvature given at the end of the next section).

Several properties of the wave–current refraction can be seen immediately from (3). The magnitude of the radius of curvature of the rays is a characteristic length scale for the refraction and it decreases with increasing current vorticity and with decreasing group speed relative to the current. Since the sign of R is the same as the sign of ζ, (3) shows that for waves which propagate with (against) a variable current the rays will bend in the direction of decreasing (increasing) current speed. Therefore, under the right conditions it is possible for waves which propagate with (against) a current to be trapped about a local minimum (maximum) in current speed. It is also possible that waves which propagate with (against) a current could be reflected from a local maximum (minimum) in current speed.

For deep water surface gravity waves the dispersion relation is

$$\omega' = (gk)^{1/2} \tag{4}$$

and equation (3) becomes

$$R = g/(2\omega'\zeta) \tag{5}$$

where g is the acceleration of gravity. Equation (5) shows that the radius of curvature decreases with increasing frequency relative to the current. Therefore, for deep water surface gravity waves refraction increases with increasing frequency.

SHEAR CURRENTS

For ocean applications it is useful to consider the propagation of deep water surface gravity waves in one-dimensional shear currents. This geometry has been discussed previously by LONGUET-HIGGINS and STEWART (1961). Let the horizontal current $\mathbf{U}(x, y)$ be for all x

$$U = \begin{cases} U(y) & y \geqslant 0 \\ 0 & y < 0 \end{cases} \qquad V = 0 \tag{6}$$

where $U(y)$, the current component in the x-direction, is an arbitrary function of y. Then the ray equations (1) show that, besides the frequency being constant (steady medium), the x-component of the wave number is also constant. Dividing one constant by the other results in Snell's law which can be written in the form (JOHNSON, 1947)

$$\frac{\sin\phi_0}{\sin\phi} = \left[1 - \frac{U}{c_0}\sin\phi_0\right]^2 \tag{7}$$

where ϕ is the angle between the wave number and the current normal (y-axis) and c_0 is phase speed of the waves where the current vanishes. The index of refraction, the right side of (7), depends on the current speed and the wave frequency ($c_0 = g/\omega_0$) and direction ϕ_0 of the wave number where the current vanishes. Equation (7) shows that for waves which propagate from still water into a following $U > 0$ (opposing $U < 0$) current the index of refraction <1 (>1) and the wave number bends away from (toward) the current normal.

Total reflection of waves entering a following current ($U > 0$) from still water occurs when the initial angle ϕ_0 is greater than or equal to the critical angle ϕ_{0c}, which is found from (7) for a given value of U/c_0 when $\phi = \pi/2$. Several examples are given in Table 1; the first column gives ϕ_{0c} and the last column gives 2 U/c_0. Total internal reflection of waves in a current can occur when the waves propagate against the current ($U < 0$). The critical angle ϕ_c for total internal reflection for a given U/c_0 is found from (7) when $\phi_0 = \pi/2$. The two critical angles ϕ_c and ϕ_{0c} are nearly equal when U/c_0 is small as can be shown from (7).

Assume now that the current (6) has constant shear s (positive or negative)

$$U(y) = \begin{cases} sy & y \geqslant 0 \\ 0 & y < 0. \end{cases} \tag{8}$$

Then the ray equations (1) can be integrated analytically once the initial conditions are specified. Let the initial conditions be that the rays pass through the point $x = 0$, $y = 0$ at time $t = 0$ with initial angle $\phi_0 = \cot^{-1} k_{y0}/k_{x0}$ at the edge of the current

$$\left.\begin{aligned} x &= 0 \\ y &= 0 \\ k_x &= k_{x0} \\ k_y &= k_{y0} \end{aligned}\right\} \quad t = 0. \tag{9}$$

The integration of equations (1) using (4), (8), and (9) gives

$$\begin{aligned} k_y(t') &= k_{x0}t' \\ y(t') &= 2R_0\,(\sin\phi_0)^{-1/2}\left[(1+t'^2)^{1/4}-(\sin\phi_0)^{-1/2}\right] \\ x(t') &= -yt'+2/3\,R_0\,(\sin\phi_0)^{-1/2}\,\{t'(1+t'^2)^{1/4}- \\ &\qquad \cot\phi_0(\sin\phi_0)^{-1/2}+2^{-1/2}[F(\Theta,K)-F(\Theta_0,K]\} \end{aligned} \tag{10}$$

where the parameter $t' = \cot\phi_0 - st$, $F(\Theta, K)$ is the elliptic integral of the first kind, $\cos\Theta = (1+t'^2)^{-1/4}$, $\cos\Theta_0 = (\sin\phi_0)^{1/2}$, and $K = 2^{-1/2}$. In (10) $R_0 = -g/2s\omega_0$ is the (exact) initial radius of curvature of the rays from (5) and (8), where ω_0 is the initial wave frequency. The x-component of the wave number and the frequency are constants of the motion.

Snell's law (7) can be put in the form $x = x(y)$ for the current (8) by using $\tan\phi = dx/dy$

$$x(y) = R_0\,(2/\sin\phi_0)^{1/2}\left[F(\Theta,K)-F(\Theta_0,K)\right] \tag{11}$$

where $\cos\Theta = \sin\phi_0(1+2y\sin\phi_0/R_0)^{-2}$, $x = 0$ at $y = 0$, and the notation is the same as above. The curves (11) are tangent to the wave numbers and perpendicular to the wave fronts.

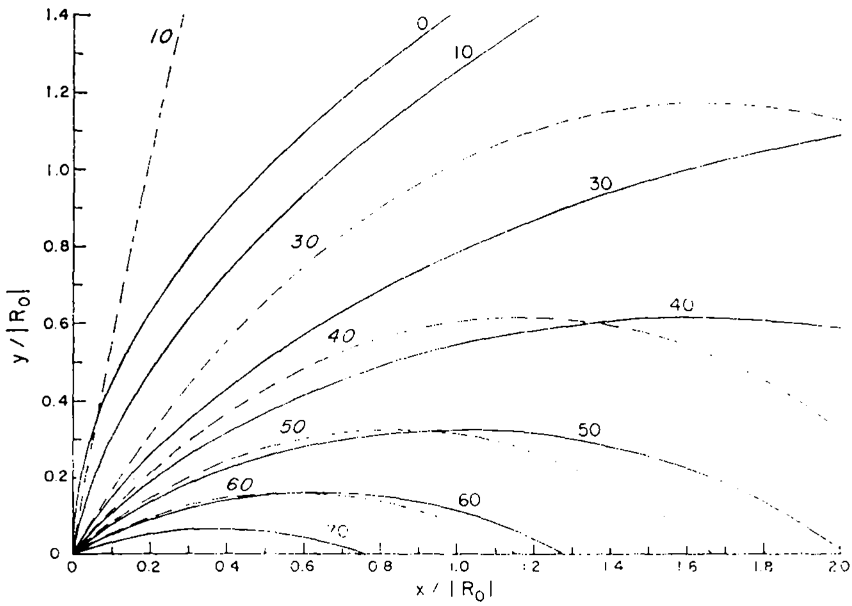

Fig. 1. Rays (solid) and curves (dashed) tangent to the wave number for deep water surface gravity waves of initial frequency ω_0 which enter a following current of constant shear s at the indicated angles. The initial radius of curvature of the rays $R_0 = -g/2s\omega_0$.

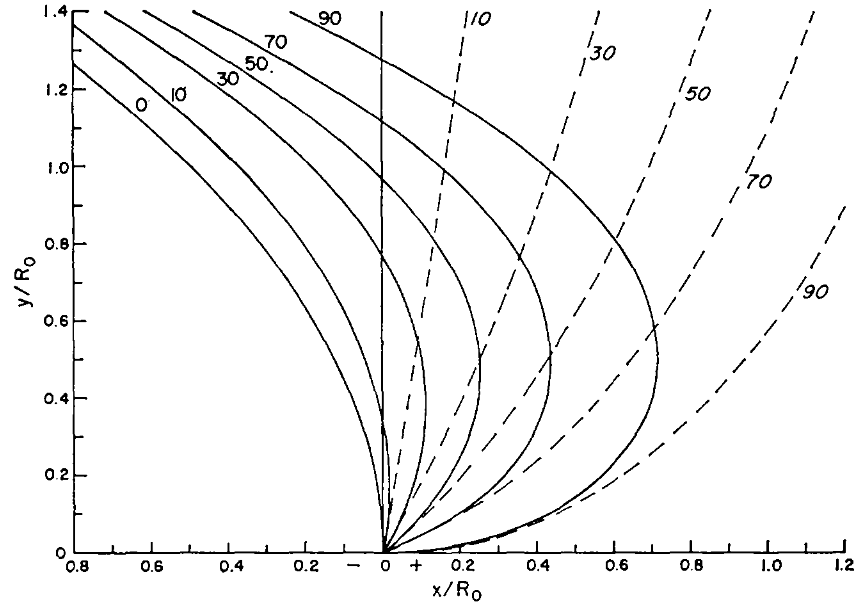

Fig. 2. Rays (solid) and curves (dashed) tangent to the wave number for deep water surface gravity waves of initial frequency ω_0 which enter an opposing current of constant shear s at the indicated angles. The initial radius of curvature of the rays $R_0 = g/2|s|\omega_0$.

The rays (solid) in (10) and the (dashed) curves (11) are shown in Fig. 1 for a following current ($s > 0$) and in Fig. 2 for an opposing current ($s < 0$) for several values of the initial angle $0 \leqslant \phi_0 \leqslant \pi/2$. Figures 1 and 2 are valid for all initial frequencies and constant shears since the coordinates are made non-dimensional by the initial radius of curvature R_0. The direction of the wave number at a point on a ray is found from the corresponding dashed curve (same initial angle) at the same value of the ordinate. It can be seen from Figs. 1 and 2 that the wave number at a point on a ray is not parallel to the rays in general, which is expected from equations (1) and (2). The rays which enter the current (8) at right angles are curved, as expected from (3), and have the form $x = -y^2/2R_0$; the wave number remains perpendicular to the current as seen from (7).

All waves which enter the following current (8) ($s > 0$) are totally reflected. For a given initial angle there is one point on the ray ($dy/dt = 0$, $dx/dt > 0$ at $t = \cot \phi_0/s$), the point of reflection, at which the wave number, current velocity, and the ray tangent are all parallel. The coordinates of this point for several initial angles are given in Table 1. The x-coordinates (column 2) depend on the current shear, whereas the y-coordinates (column 3) depend on the current speed. The values in column 3 also given the ratio of current speed at the point of reflection to the initial group speed of the waves $U/c_{g0} = 2U/c_0$. The spring length of the ray is defined to be the distance between the point where the ray enters the current and the point where the reflected ray leaves the current. The spring length is found by doubling the x values in column 2 and it decreases with increasing shear for a given initial frequency.

All rays which enter the opposing current (8) ($s < 0$) become perpendicular to the current and then bend back toward the current as shown in Fig. 2. The points

Table 1. Coordinates of maximum cross-stream penetration of rays entering a following current.

Initial angle ϕ_0 (deg)	$x/\lvert R_0\rvert$	$y/\lvert R_0\rvert$
0	∞	∞
10	23·2	6·72
20	6·17	2·43
30	2·86	1·17
40	1·63	0·617
50	1·01	0·326
60	0·636	0·160
70	0·379	0·0652
80	0·178	0·0155
90	0	0

Table 2. Coordinates of maximum up-stream penetration of rays entering an opposing current.

Initial angle ϕ_0 (deg)	x/R_0	y/R_0
0	0	0
10	0·0147	0·166
20	0·0541	0·295
30	0·110	0·381
40	0·176	0·434
50	0·251	0·467
60	0·337	0·488
70	0·438	0·499
80	0·560	0·506
90	0·714	0·507

at which the rays become normal to the current occur when the x-component of the group velocity relative to the current is equal and opposite to the current velocity (see (1) and (2)). The coordinates of these points ($\mathrm{d}x/\mathrm{d}t = 0$, $\mathrm{d}y/\mathrm{d}t > 0$) are given in Table 2.

The analytical form of the rays in (10) can be used to compute the exact radius of curvature which can then be compared to the approximate relation (5). The exact radius of curvature r is given by $r = \pm(\dot{x}^2+\dot{y}^2)^{3/2}/(\dot{x}\ddot{y}-\dot{y}\ddot{x})$, where $\dot{x} = \mathrm{d}x/\mathrm{d}t$ etc. If r and R from (5) are evaluated at the turn around point ($t = \cot\phi_0/s$) of the rays for the following current, then $R/r = \sin\phi_0/[2-(\sin\phi_0)^{1/2}]^2$, and $0 \leqslant R/r \leqslant 1$. A comparison with Table 1 shows that when the current speed is less than about 5% of the initial group speed, the error in using (5) is less than about 10%. Also it can be shown that $r \to R_0$ as $t \to 0$ so that equation (5) becomes exact when the current vanishes as mentioned above.

APPLICATION TO THE OCEAN

The refraction of deep water surface gravity waves in inhomogeneous currents is applied to wind generated waves in major ocean currents. The Gulf Stream is chosen as an example because of its large current speeds and shears. Four possible effects will be considered: (1) waves which enter the Gulf Stream from the southeast could be totally reflected and therefore not reach the coast, (2) waves which enter the Stream from the southwest could be totally reflected and therefore be trapped between the current and the coast, (3) waves which propagate against the Stream could be trapped in the current, and (4) waves could propagate through the Stream without experiencing any net changes. It is assumed that the Gulf Stream is a steady unidirectional current surrounded by an ocean environment which is at rest. The Stream is assumed to be uniform in the direction of flow, to be independent of depth, and to have a single maximum current speed.

The total reflection of waves which enter the model Gulf Stream from the southeast or southwest depends on the maximum current speed and not on the current shear. Let the maximum current speed U max be 2 m/sec. For a wave of initial period $T_0 = 2\pi/\omega_0 = 8$ sec, U max$/c_0 \approx 1/6$ and the critical angle ϕ_{0c} from (7) is about 50°. Therefore waves of period 8 sec which enter the Gulf Stream with an initial angle between the direction of propagation and the current normal which is greater than or equal to about 50° will be totally reflected. For waves of initial period $T_0 = 16$ sec, U max$/c_0 \approx 1/12$ and the critical angle is about 60°. These critical angles are smaller than might be expected from the fairly small ratios of maximum current speed to initial phase speed, and this is due to the (squared) form of the index of refraction in (7). The length scale of the reflection in the current direction depends on the current shear and can be estimated from Fig. 1 and Table 1. Let the half width of the Gulf Stream be 50 km and the maximum speed be 2 m/sec. This gives an average shear of 4×10^{-5}/sec which is assumed constant. Then the initial radius of curvature of the rays from (5) and (8) for waves with initial periods of 8 and 16 sec is 156 and 312 km, respectively. The spring length of the reflected rays for 8 and 16 sec waves (with initial angles of 50° and 60°, respectively) is about 310 and 400 km respectively, as seen from Table 1. For comparison, it is easily seen that if the radius of curvature were constant at its initial value R_0, then the spring length of the particular ray with initial angle 60° would also be R_0. Table 1 shows that for this ray the spring length is actually about 1·27 R_0, which is partly due to the increase in the radius of curvature along the ray associated with the increase in wave length and therefore increase in group speed relative to the current, as seen from (5).

Trapping of waves in the Gulf Stream can occur if the waves propagate against the current with sufficiently large angles between the wave number and the current normal. Trapping (total internal reflection) depends on the maximum current speed and not on the current shear. The critical angle ϕ_c for trapping from (7) for a wave of period 8 sec (at the edge of the current where the velocity vanishes) is about 48°, taking the maximum current speed as 2 m/sec. Therefore, all waves whose period at the edge of the current is 8 sec will be trapped if the angle between the wave number and the current normal at the position of maximum current speed is greater than or equal to about 48°. The critical angle for a 16 sec wave is about 59°. These critical angles are nearly the same as those for total reflection mentioned above since the ratio of maximum current speed to initial wave phase speed is small. The length scale of the trapping in the current direction depends on the current shear and can be estimated from Fig. 2 and Table 2. According to the one-dimensional current model assumed, in order to be trapped the waves must be generated within the boundaries of the Gulf Stream. However, since the Gulf Stream itself is actually curved it is possible that waves generated outside the Stream could also be trapped.

Waves which are neither trapped in nor totally reflected from the Gulf Stream will pass through the Stream with no net change in wave length or direction (or amplitude), assuming that no wave breaking occurs. This follows from the ray equations for the model current assumed. To the extent that the Gulf Stream cannot be represented by the model current small net changes in wave properties might be possible.

The above examples indicate that a significant fraction of the wave energy which enters the Gulf Stream from the south could be reflected by the current, and also a significant fraction of the wave energy which propagates against the Gulf Stream could be trapped in the current.

COMPARISON WITH OBSERVATIONS

The above application of wave–current refraction to surface gravity waves in the Gulf Stream indicates that major ocean currents may have an important influence in changing the propagation direction of the waves. In particular the reflection and trapping of waves by currents should be possible under the appropriate conditions. I know of no observations which are pertinent to the Gulf Stream, but the observations of MUNK, MILLER, SNODGRASS and BARBER (1963) and SNODGRASS, GROVES, HASSELMANN, MILLER, MUNK and POWERS (1966) on waves which were generated near the Circumpolar Current may be relevant.

MUNK, MILLER, SNODGRASS and BARBER (1963) and SNODGRASS, GROVES, HASSELMANN, MILLER, MUNK and POWERS (1966) made observations on ocean swell propagating across the Pacific in which directional information is available. One curious fact, which was noted in both studies, is that the storm position inferred from the direction of the waves at the recording station was typically to the left (when looking toward the storm from the recording station) by a few degrees (few hundred km) of the storm position inferred from the weather maps. This discrepancy cannot be explained by the effect of the Earth's rotation (BACKUS, 1962) nor by the Earth's oblateness (SNODGRASS, GROVES, HASSELMANN, MILLER, MUNK and POWERS, 1966). MUNK, MILLER, SNODGRASS and BARBER (1963) think the discrepancy may be produced by local refraction due to the decreasing water depth near the recording station. Since many of the storms occurred in or near the Circumpolar Current, there is a possibility that some bending of the wave rays by the current shear may have occurred. MUNK, MILLER, SNODGRASS and BARBER (1963) mention that the discrepancy in storm position is less for the lower frequencies, which is consistent with current refraction because the current shear refracts the higher frequencies to a greater extent than the lower frequencies, whereas bottom refraction affects the lower frequencies to a greater extent than the higher frequencies.

A detailed comparison of theory and measurement is not attempted because the velocity structure of the Circumpolar Current is poorly known. An idealized model of the Circumpolar Current is chosen to illustrate the possible effects of refraction. The Antarctic region is approximated by a plane tangent to the south pole. The model current in this plane is a horizontal nondivergent circular ring of current with two regions of constant shear.

$$V = \begin{cases} s(r-r_0) & r_0 \leqslant r \leqslant r_0 + d \\ s(r_1 - r) & r_0 + d \leqslant r \leqslant r_1 \\ 0 & r < r_0, \quad r > r_1 \end{cases} \tag{12}$$

where V is the azimuthal velocity component (positive in the clockwise direction), $s > 0$ is constant, r is the radius from the south pole in the plane, r_0 is the inner radius and r_1 is the outer radius of the current, and the width of the current is $2d$. The current is a maximum in the center of the ring ($r = r_0+d$), decreases linearly to zero at the edges of the ring, and vanishes outside the ring.

The ray equations (1) were integrated numerically for the current (12) and two examples are shown in Figs. 3 and 4. The following values of the constants were chosen: $r_0 = 4000$ km, $d = 500$ km, and $s = 10^{-6}$/sec, which means the maximum current speed is 50 cm/sec. (REID and NOWLIN, 1971, have observed speeds up to 40 cm/sec in the Drake passage). Figure 3 shows the rays for waves of 5, 10 and 15 sec period which all emerge from the current at an angle of 60° to the current normal. Looking in the direction from which the waves came, the dashed straight line is the projected ray based on the assumption of no current. Figure 3 shows that the current can cause the rays to deviate from the dashed line by several hundred kilometers, and the dashed line is to the left of the rays in agreement with the observations of MUNK, MILLER, SNODGRASS and BARBER (1963) and SNODGRASS, GROVES, HASSELMANN, MILLER and POWERS (1966). Figure 3 also shows that the rays of the higher frequency waves are bent more than those of the lower frequency waves.

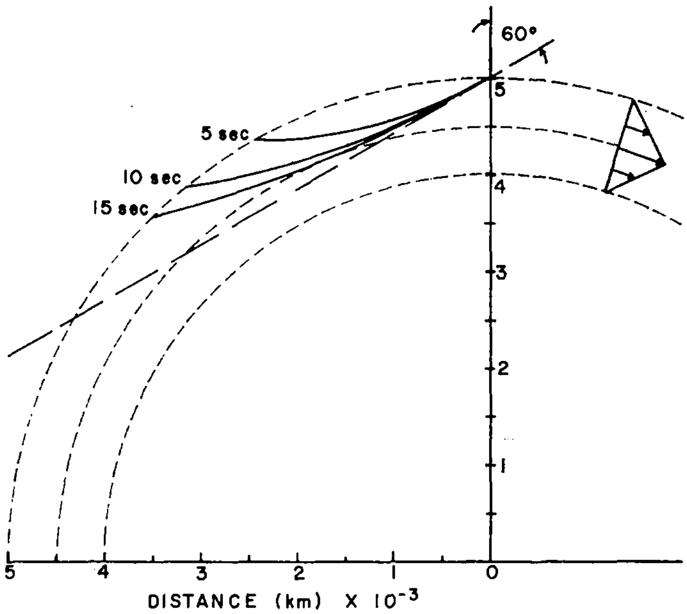

Fig. 3. Rays for surface waves of indicated periods which all emerge at an angle of 60° from the idealized Circumpolar Current in equation (12). The dashed straight line is the ray for a wave of any period projected back from the point of emergence assuming there is no current.

Figure 4 is essentially the same as Fig. 3 except that the rays emerge from the current at 55° instead of 60°. The bending of the rays of the 10 and 15 sec period waves is quite different, even though the angle of emergence has been changed by only 5°, and the change in sign of the radius of curvature is due to the penetration of the rays into the inner half of the current where the sign of the vorticity is reversed

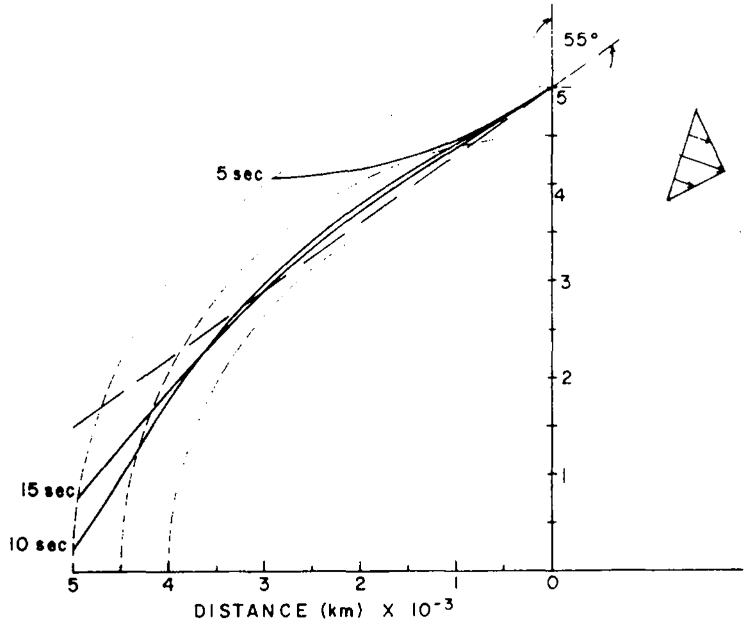

Fig. 4. Rays for surface waves of indicated periods which all emerge at an angle of 55° from the idealized Circumpolar Current in equation (12). The dashed straight line is the ray for a wave of any period projected back from the point of emergence assuming there is no current.

[see (3)]. The projected ray for zero current is still mainly to the left of the rays but the deviation is less for the lower frequencies.

SNODGRASS, GROVES, HASSELMANN, MILLER, MUNK and POWERS (1966) discuss an observation in which waves were recorded at two stations including Yakutat, Alaska, although both stations were totally shadowed. They mention that the great-circle route between the storm and Yakutat intersects the Antarctic continent south of Australia, but that a bending of the ray by only 3° would give Yakutat sufficient aperture to receive the waves. They suggest wave–wave scattering or scattering by the Antarctic Pack Ice as possible explanations. The type of bending illustrated by the rays for waves of 10 and 15 sec period in Fig. 4 might also be a possible explanation.

SNODGRASS, GROVES, HASSELMANN, MILLER, MUNK and POWERS (1966) measured the attenuation of swell which propagated along a great circle route between New Zealand and Alaska. They found that there was negligible attenuation of the low frequencies (50 to 70 mc/s, periods of 15 to 20 sec) between New Zealand and Alaska but that the higher frequencies (70 to 100 mc/s, periods of 10 to 15 sec) were attenuated within one storm diameter (1000 km). They found this lack of attenuation in the far zone and strong attenuation in the near zone to be not inconsistent with considerations involving only wave–wave scattering and island absorption. Wave refraction by currents may be another possibility. The higher frequencies are selectively refracted more than the lower frequencies and a significant bending of the rays can take place in a storm diameter as Fig. 3 shows. This might partly explain the strong attenuation of the high frequencies in the near zone in those cases in which the storm generated waves in a narrow directional beam in or near the Circumpolar Current. A qualitative explanation for the lack of attenuation in the far zone can also be given, since it is not expected that the equatorial currents, for example, would have any net effect on the waves which propagate through them. This follows from the fact that for the storm and receiving stations involved, the waves would enter the equatorial currents at too small an angle to be totally reflected and would therefore pass through with very little net effect, as was first mentioned by DRENT (1959).

DISCUSSION

The above applications suggest that refraction effects may be important when surface gravity waves propagate in major ocean currents. A quantitative comparison between the refraction model and ocean measurements has not been given. However, a qualitative comparison showed that some of the observations of MUNK, MILLER, SNODGRASS and BARBER (1963) and SNODGRASS, GROVES, HASSELMANN, MILLER, MUNK and POWERS (1966) were not inconsistent with a mechanism involving only wave–current refraction, although other physical processes mentioned by the authors may alsc be important.

Refraction effects might be important in ocean currents other than the Gulf Stream and the Circumpolar Current. For example, waves generated by the trade winds might be trapped in the Equatorial Counter Current between the North and South Equatorial Currents in the Pacific and Atlantic oceans.

Waves could be trapped by currents in situations which involve also curvature of the current, reflecting boundaries, or bottom topography. For example waves might be trapped along coasts by the shear in longshore or tidal currents in a manner analogous to the trapping of edge waves by bottom topography. If the effect of current shear were to dominate over the effect of bottom topography, then the trapped waves could only propagate along the coast in one direction for a given sign of the shear.

Currents, such as the Gulf Stream, could have a shielding effect in preventing some wave energy from reaching the coast by either reflecting or trapping waves, as was suggested by JOHNSON (1947).

In applying refraction to wind waves in ocean currents certain simple current geometries have been assumed. However, even if the velocity structure of ocean currents were known in detail, in which case the ray equations could be easily integrated numerically, the refraction effects would be expected to be qualitatively the same as for the model currents. The basic assumption of geometrical optics that the currents vary slowly over distances comparable to a wave length is probably a good one for major ocean currents (width $\sim$ 100 km) and for wind waves (wave length $<$ 500 m).

The assumption that the currents do not vary with depth may not be appropriate if ocean currents vary appreciably over the depth of penetration of the waves, which is about half a wave length for deep water waves (about 200 m for a wave period of 16 sec). The consequence of relaxing this assumption has not been investigated. The assumption is probably a good one for the Circumpolar Current, which is a deep current, but may not be as good for the Gulf Stream. Probably the largest vertical shear near the surface (order 10^{-2}/sec in the upper 100 m) is associated with the Equatorial Undercurrent (KNAUSS, 1960), but SNODGRASS, GROVES, HASSELMANN, MILLER, MUNK and POWERS (1966) did not observe any marked dissipation of energy or change in propagation direction of waves which crossed the equator in the Pacific.

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