Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Ocean Modelling xxx (2005) xxx–xxx
Ocean Modelling
www.elsevier.com/locate/ocemod

A note on the decay rate of swell

L. Kantha *

Department of Aerospace Engineering Sciences, University of Colorado, Boulder, Colorado 80309, USA

Received 11 November 2004; received in revised form 9 December 2004; accepted 13 December 2004

Abstract

In this note, we examine the extraction of energy from waves by the turbulence in the upper ocean as one possible physical mechanism for the attenuation of swell as it propagates across an ocean basin. We derive a simple expression for the swell attenuation rate that is of potential use in wave forecast models.

Keywords: Swell; Wind waves; Wave–turbulence interaction; Turbulence models

1. Introduction

When surface waves and turbulence co-exist in the upper ocean, the interaction between the two can transfer energy from waves to turbulence and thus enhance mixing in the upper ocean (McWilliams et al., 1997). Kantha and Clayson (2004) presented a model for this enhanced mixing. However, this energy transfer also results in the attenuation of the surface wave, but this aspect was not considered by Kantha and Clayson (2004). This note is an attempt to correct this oversight.

Swell, the low frequency component of wind-generated waves, is known to propagate across entire ocean basins along great circle paths with very little attenuation (Barber and Ursell, 1948; Munk and Snodgrass, 1957; Munk et al., 1963). Snodgrass et al. (1966) observed long period

* Tel.: +1 303 492 3041; fax: +1 303 492 2825.
E-mail address: kantha@colorado.edu

1463-5003/$ - see front matter © 2005 Published by Elsevier Ltd.
doi:10.1016/j.ocemod.2004.12.003

swell generated near New Zealand propagate with very little attenuation from New Zealand to Alaska. This is quite remarkable since wind waves propagate not through an inviscid, frictionless ocean, but a turbulent one. This just means that the interaction between the purely potential motions due to wind waves and the turbulent field in the upper ocean is rather weak. However, it may not be completely ignored.

Wave forecasting is an inherently difficult task, more so for swell (Komen et al., 1994; Kantha and Clayson, 2000), since inadequate or excessive decay of swell can lead to significant errors in the forecast. It is not uncommon to either ignore or parameterize by ad hoc expressions, the dissipation of swell in wave forecast models, since the physical mechanism responsible for swell attenuation remains poorly understood (Phillips, 1977; Komen et al., 1994; Tolman and Chalikov, 1996; Rogers et al., 2003, but see; Wingert et al., 2001, for recent progress in predicting swell). Since the wave slope is rather small for swell, wave breaking cannot be the mechanism responsible for swell attenuation. Similarly, resonant wave–wave interactions are also quite inefficient in transferring energy from low frequency swell to higher frequency components. But energy transfer from waves to wind can occur when the wave speed is greater than the wind speed, and this mechanism is invoked in some wave models (Tolman, 2002) for parameterizing the swell decay rate. However, it is also possible that the extraction of energy from wave motions by upper ocean turbulence could be responsible for swell decay, and this has not been considered hitherto. The question is: is it possible to quantify the rate of extraction of wave energy by turbulence in the upper layers of the ocean and hence the decay rate of swell?

2. Extraction of wave energy by upper layer turbulence

Based on LES simulations of Langmuir cells in the ocean (McWilliams et al., 1997), Kantha and Clayson (2004) have recently parameterized the extraction of energy from surface gravity waves by turbulence in the oceanic mixed layer. They show that the rate of change of turbulence kinetic energy (per unit mass) can be written (see their Eqs. (4.7) and (4.8)) as:

$$\frac{\partial}{\partial t}\left(\frac{q^2}{2}\right) = -\overline{uw}\frac{\partial u_S}{\partial z} - \overline{vw}\frac{\partial v_S}{\partial z} \tag{1}$$

where $q^2/2$ is the TKE, $u_S(z)$ and $v_S(z)$ are the components of surface gravity wave-induced Stokes drift, and $\overline{uw}$ and $\overline{vw}$ are components of the turbulent shear (Reynolds) stress. It is the working of the turbulence shear stress on the shear of the Stokes drift current that extracts energy from the wave motion and transfers it into turbulence. It is rather analogous to working of the turbulence shear stress against the mean shear in transferring kinetic energy from mean currents to TKE.

The integration of Eq. (1) w.r.t. z gives the rate of increase of the total TKE in the water column due to extraction of energy from wave motions. Equivalently, this also provides the rate of decay of wave energy $E = \rho_w g(a^2/2)$, where ρ_w is water density, g is gravitational acceleration and a is the wave amplitude:

$$\frac{\partial E}{\partial t} = -D_w = -\rho_w\left(\int_{-\infty}^{0} \overline{uw}\frac{\partial u_S}{\partial z}\,dz + \int_{-\infty}^{0} \overline{vw}\frac{\partial v_S}{\partial z}\,dz\right) \tag{2}$$

where D_w is the wave dissipation term. To evaluate this term, it is necessary to appeal to a turbulence closure model (e.g., Kantha and Clayson, 1994, 2004).

From simple dimensional considerations, Eq. (2) suggests

$$D_w = \alpha \rho_w u_{*w}^2 V_S(0) \tag{3}$$

where u_{*w} is the water-side friction velocity. $V_S(0)$ is the magnitude of the Stokes drift velocity at the surface given by:

$$V_S(0) = c(ka)^2 \tag{4}$$

where c is the phase speed and k is the wave number of the surface gravity wave. However, the constant of proportionality α can only be determined through a turbulence closure model. Moreover, when the wave propagates at an angle θ to the wind direction, because of the turning of the shear stress with depth, the angular dependence may not be a simple cosine dependence as one would expect apriori.

Eq. (2) indicates that the wave is attenuated if the wave propagation is in the same direction as the wind, whereas adverse winds amplify the wave. This is rather counter-intuitive. From wind wave generation theory, following winds can be expected to add energy to the waves and adverse winds to extract. However, this is true only for the high wave number components of the wind wave spectrum, whose phase speed is smaller than the wind speed. If the phase speed is larger than the wind speed at say the half-wavelength distance above the air–sea interface, wind input of energy into the wave becomes negligible. For a swell with a period of 10–15 s, and hence phase speeds of 15–23 m s^{-1}, this condition is satisfied unless the prevailing winds are very strong. Consequently, the fact that such swell can propagate over long distances with little attenuation, implies that the physical mechanisms of swell wave energy attenuation are rather weak. For shorter period swell, whose phase speed can be comparable to or smaller than the wind speed, the situation is not as clear, since the following winds can input energy into the waves, while the turbulence generated in the water column by the wind can extract energy from the waves.

3. Determination of the proportionality constant α

The second moment turbulence closure model used by Kantha and Clayson (2004) to model turbulent mixing in the oceanic mixed layer in the presence of surface waves, was used to determine D_w for swell of a given period and amplitude for various ambient conditions. We initialized the model with a pycnocline at the mixed layer depth d to keep the turbulence within the mixed layer and integrated for two days with a given surface wave and wind conditions. Table 1 shows the results (The differences between the α values for model integrations of one and two days were less than a percent).

The model results show the complex behavior of the constant α. It is relatively insensitive to the mixed layer depth. The dependence on the angle between the wind and wave directions is not a simple $\cos\theta$ dependence, but close. The results are also rather sensitive to the wind speed and the latitude.

Eq. (2) can be readily integrated to give, for constant wind conditions:

Table 1
Proportionality constant α, β^{-1} and γ^{-1} (T–wave period, a–wave amplitude, c–wave phase speed, λ–wave length, U_{10}–wind speed, θ–angle between wind and wave directions, d–mixed layer depth)

T (s)	a (m)	c (m/s)	λ (m)	U_{10} (m/s)	Angle θ	d (m)	Lat	α	β^{-1} (day)	γ^{-1} (arcdeg)
15	1	23.4	351.2	10	0	25	30	0.3540	310	2819
15	1	23.4	351.2	10	30	25	30	0.2974	368	3355
15	1	23.4	351.2	10	60	25	30	0.1544	710	6462
15	1	23.4	351.2	10	90	25	30	0	∞	∞
15	1	23.4	351.2	10	120	25	30	−0.1414	−775	−7057
15	1	23.4	351.2	10	150	25	30	−0.2795	−392	−3570
15	1	23.4	351.2	10	180	25	30	−0.3343	−328	−2985
15	0.5	23.4	351.2	10	0	25	30	0.3457	317	2887
15	1	23.4	351.2	10	0	75	30	0.3908	281	2553
15	1	23.4	351.2	10	0	25	60	0.2840	386	3514
15	1	23.4	351.2	10	0	25	90	0.2572	426	3880
15	1	23.4	351.2	10	0	25	10	0.4359	251	2289
15	1	23.4	351.2	5	0	25	30	0.2710	2312	21,043
15	1	23.4	351.2	15	0	25	30	0.4527	83	754
10	1	15.6	156.1	10	0	25	30	0.6052	54	326
10	1	15.6	156.1	10	30	25	30	0.5160	63	382
10	1	15.6	156.1	10	60	25	30	0.2791	117	706
10	1	15.6	156.1	10	90	25	30	0	∞	∞
10	1	15.6	156.1	10	120	25	30	−0.2074	−156	−950
10	1	15.6	156.1	10	150	25	30	−0.4210	−77	−468
10	1	15.6	156.1	10	180	25	30	−0.5057	−64	−390
10	0.5	15.6	156.1	10	0	25	30	0.5814	56	339
10	1	15.6	156.1	10	0	75	30	0.6540	50	301
10	1	15.6	156.1	10	0	25	60	0.5303	61	372
10	1	15.6	156.1	10	0	25	90	0.4953	66	398
10	1	15.6	156.1	10	0	25	60	0.6825	48	289
10	1	15.6	156.1	5	0	25	30	0.5101	364	2208
10	1	15.6	156.1	15	0	25	30	0.6958	16	97
5	1	7.8	39	10	0	25	30	0.8871	4.6	13.9
5	1	7.8	39	10	30	25	30	0.7632	5.3	16.1
5	1	7.8	39	10	60	25	30	0.4284	9.5	28.8
5	1	7.8	39	10	90	25	30	0	∞	∞
5	1	7.8	39	10	120	25	30	−0.1763	−23	−70
5	1	7.8	39	10	150	25	30	−0.2647	−15	−47
5	1	7.8	39	10	180	25	30	−0.2847	−14	−43
5	0.5	7.8	39	10	0	25	30	0.8815	4.6	14.0
5	1	7.8	39	10	0	75	30	0.8978	4.5	13.7
5	1	7.8	39	10	0	25	60	0.8452	4.8	14.6
5	1	7.8	39	10	0	25	90	0.8245	4.9	14.9
5	1	7.8	39	10	0	25	10	0.9182	4.4	13.4
5	1	7.8	39	5	0	25	30	0.7515	31	94
5	1	7.8	39	15	0	25	30	0.9218	1.5	4.6

$$\frac{a}{a_0} = \exp(-\beta t) = \exp(-\gamma x); \quad \beta = \alpha\left(\frac{u_{*w}}{c}\right)^2\sigma, \quad \gamma = \beta/c_g = 2\alpha\left(\frac{u_{*w}}{c}\right)k \tag{5}$$

where $c_g = c/2$ is the group speed. The values of β^{-1} and γ^{-1} are also presented in Table 1. Because these parameters scale as the square of the (u_{*w}/c), the decay rate is much higher for low period swell. The results tend to confirm that the long period swell can propagate essentially unattenuated across entire ocean basins, whereas short period swell (e.g., 5 s swell) are attenuated in a matter of days.

The decay time and length scales (for the wave amplitude to decrease by 50%) are given by:

$$\frac{T_t}{T} = \left(\frac{\ln 2}{2\pi\alpha}\right)\left(\frac{c}{u_{*w}}\right)^2; \quad \frac{X_t}{\lambda} = \left(\frac{\ln 2}{4\pi\alpha}\right)\left(\frac{c}{u_{*w}}\right)^2 \tag{6}$$

Based on their observations of swell propagation and decay in the Pacific, Snodgrass et al. (1966) state that "... (i) Below 75 mc/s the attenuation is too low to be measured (<0.05 dB/deg), (ii) at 75 and 80 mc/s the attenuation is of the order of 0.1 dB/deg for the large events, but less than 0.05 dB/deg for the small events and background..." This means that the observed value of the constant γ in Eq. (5) is 5.18×10^{-8} to $10.35 \times 10^{-8}\ \mathrm{m}^{-1}$ for swell with a period of between 12.5 and 13.3 s. For swell with period greater than 13.3 s, γ is below $5.18 \times 10^{-8}\ \mathrm{m}^{-1}$, although the precise value is uncertain.

For 13 s swell (λ = 264 m, k = 0.028 m^{-1}, σ = 0.48 s^{-1}, c = 20.3 m s^{-1}) and following winds of 20 m s^{-1} ($u_{*w} \sim 0.03$ m s^{-1}, $\alpha \sim 0.61$) are required to yield a value of $6.3 \times 10^{-8}\ \mathrm{m}^{-1}$ for γ, within the range of Snodgrass et al. (1966) observations. The corresponding decay time and length scales are 12.5 days and 1.1×10^4 km, respectively, also reasonable. For comparison, the Earth's circumference is 4×10^4 km. However, it is important to remember that in most cases, the decay rate observed was well below $5.18 \times 10^{-8}\ \mathrm{m}^{-1}$, which could be satisfied by the use of much smaller winds. Unfortunately, the accuracy of the measurements was not enough to determine how much smaller. Nevertheless, it is clear that extraction of wave energy by turbulence in the upper ocean is a plausible mechanism for attenuation of swell.

The corresponding expressions for viscous dissipation of wave energy (Phillips, 1977; Kantha and Clayson, 2000) are:

$$\frac{\partial}{\partial t}\left(\rho_w g \frac{a^2}{2}\right) = -D_v = -2\rho_w v k \sigma^2 a^2 \tag{7}$$

$$\frac{T_v}{T} = \left(\frac{\ln 2}{8\pi^2 v}\right)(c^2 T); \quad \frac{X_v}{\lambda} = \left(\frac{\ln 2}{16\pi^2 v}\right)(c^2 T). \tag{8}$$

Assuming a value of $10^{-5}\ \mathrm{m}^2\ \mathrm{s}^{-1}$ for the background (not molecular) viscosity in the ocean, for 13 s swell, the decay time and length scales are 707 days and 6.21×10^5 km, respectively. Clearly, viscous dissipation of swell is far too weak, and inconsistent with the observations of Snodgrass et al. (1966).

The combined effect of turbulence energy extraction and viscous dissipation gives:

$$\frac{T_{1/2}}{T} = \frac{\ln 2}{2\pi\alpha\left(\frac{u_{*w}}{c}\right)^2 + 16\pi^2\left(\frac{v}{c^2 T}\right)}; \quad \frac{X_{1/2}}{\lambda} = \frac{\ln 2}{4\pi\alpha\left(\frac{u_{*w}}{c}\right)^2 + 32\pi^2\left(\frac{v}{c^2 T}\right)} \tag{9}$$

4. Concluding remarks

Extraction of wave energy by turbulence in the upper ocean is a plausible mechanism for attenuation of swell. The attenuation rate scales as $(u_{*w}/c)^2$ and hence this mechanism has a higher impact on short period swell. It can be readily built into wave forecast models, given the wind and mixed layer depth information along the great circle propagation path. However, it is possible to simplify the process and use instead Eq. (6), assuming a value for α between 0.4 and 0.8 (of course taking into account the wind direction, following or adverse) to estimate swell decay to within a factor of 2. It is unfortunate that not enough observations of swell decay have been made since Snodgrass et al. (1966). Modern satellite observations of wind waves and surface winds, and the array of wave buoys currently deployed over the global ocean may help fill in this gap. Hopefully, this note will provide the needed impetus for more accurate and more extensive observations of swell decay and improvement in swell forecasts.

This mechanism would also be valid for higher frequency waves in the wind wave spectrum. However, other dissipation mechanisms such as white capping and resonant wave–wave interactions would be overwhelming. Also, the following winds can be expected to add energy to these waves.

Note that if the winds over the ocean were more or less randomly distributed, the mechanism presented here would not be significant, since an infusion of energy is as likely as extraction of wave energy by turbulence. However, such purely random distribution is unlikely. Finally, there is a significant progress in the study of wave–turbulence interactions in recent years (e.g., Mellor, 2003; Ardhuin and Jenkins, in press), which is a welcome development since such interactions have long been ignored in dealing with upper ocean mixing and wind wave evolution. The readers are encouraged to look at these references for formal derivations of equations governing wave–mean flow-turbulence interactions.

Acknowledgements

L.K. thanks ONR for the partial support for this work through ONR grant N00014-03-1-0488.

References

Ardhuin, F., Jenkins, A.D. A practical three-dimensional formalism for the ocean, coupling random waves, mean flow and turbulence. J. Fluid Mech. (in press).

Barber, N.F., Ursell, F., 1948. Philos. Trans. Roy. Soc. London A240, 527–560.

Kantha, L.H., Clayson, C.A., 1994. An improved mixed layer model for geophysical applications. J. Geophys. Res. 99, 25, 235–25, 266.

Kantha, L.H., Clayson, C.A., 2000. Small Scale Processes in Geophysical Fluid Flows. Academic Press, p. 888.

Kantha, L.H., Clayson, C.A., 2004. On the effect of surface gravity waves on mixing in the oceanic mixed layer. Ocean Modelling 6, 101–124.

Komen, G.J., Cavaleri, L., Donelan, M., Hasselman, K., Hasselman, S., Janssen, P.A.E.M., 1994. Dynamics and Modelling of Ocean Waves. Cambridge University Press, Cambridge, p. 532.

Mellor, G.L., 2003. The three-dimensional current and surface wave equations. J. Phys. Oceanogr. 33, 1978–1989.

McWilliams, J.C., Sullivan, P.P., Moeng, C.-H., 1997. Langmuir turbulence in the ocean. J. Fluid Mech. 334, 1–30.

Munk, W.H., Snodgrass, F.E., 1957. Deep-Sea Res. 4, 272–286.

Munk, W.H., Miller, G.R., Snodgrass, F.E., Barber, N.F., 1963. Philos. Trans. Roy. Soc. London A255, 505–584.

Phillips, O.M., 1977. The Dynamics of the Upper Ocean. Cambridge University Press, p. 336.

Rogers, W.E., Hwang, P.A., Wang, D.W., 2003. Investigation of wave growth and decay in the SWAN model: Three regional-scale applications. J. Phys. Oceanogr. 33, 366–389.

Snodgrass, F.E., Groves, G.W., Hasselmann, K., Miller, G.R., Munk, W.H., Powers, W.H., 1966. Propagation of ocean swell across the Pacific. Philos. Trans. Roy. Soc. London A249, 431–497.

Tolman, 2002. Validation of WAVEWATCH III version 1.15 for a global domain. NOAA/NWS/NCEP/OMB Tech Note 213, pp. 33.

Tolman, H.L., Chalikov, D., 1996. Source terms in a third generation wind wave model. J. Phys. Oceanogr. 26, 2497–2518.

Wingert, et al., 2001. Validation of operational global wave prediction models with spectral buoy data. In: Edge, Hemsley (Eds.), Ocean Wave Measurement and Analysis. ASCE, San Francisco, pp. 590–599.