# Wave transformation

# J.W. Kamphuis

Department of Civil Engineering, Queen's University at Kingston, Kingston, Ont. K7L 3N6, Canada (Received March 28, 1990; accepted after revision August 31, 1990)

#### ABSTRACT

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Wave transformation is studied for a new data set collected on a mobile bed beach in a hydraulic model. Irregular waves are simulated on a beach with parallel contours. It was found that wave refraction is well described by Snell's Law, that wave shoaling is slightly overpredicted by linear theory, that bottom friction effect is small, even in hydraulic models which exhibit large bed forms, that percolation effect is small, even though the model grain size is quite large compared to the wave dimensions and that wave attenuation by wave saturation must definitely be considered.

INTRODUCTION

Wave transformation and breaking have been discussed in many papers and the engineer who has to use such information to determine design breaking wave conditions is faced with a number of expressions that are at variance with each other.

Some of the difficulties result from differences in basic definitions. For instance, some authors use regular wave parameters, some use various definitions for irregular waves; some papers include wave set-up in the definition of depth of water, others do not.

Some expressions are complicated by the introduction of higher-order wave theory expressions (for example, Shuto, 1974). Such expressions in themselves are more elegant representations of wave motion and useful for research; in practice, however, the characteristics of waves are difficult to measure, particularly in the breaking zone, and one would not expect higher-order expressions to be necessary.

Finally, much confusion must be attributed to combining the wave transformation and breaking processes into one single formula relating breaking wave parameters directly to deep water wave parameters (for example, Svendsen and Hansen, 1976; Singamsetti and Wind, 1980). Such 'leap frog' expressions were initially introduced when iterative solutions involving sep-



Fig. 1. Wave basin layout.

arate wave transformation and wave breaking calculations were tedious to perform. They were convenient short-cuts. With the advent of powerful calculators and micro-computers such methods may have outlived their usefulness.

Solution of practical engineering problems requires a clear understanding of basic principles which must be expressed only with a degree of sophistication that matches the accuracy of the input data and the output results. To provide this type of basic understanding, the present paper analyses wave transformation only using a completely new set of experimental results. A companion paper (Kamphuis 1991b, this issue) addresses wave breaking separately from wave transformation.

It will be seen, from this extensive set of carefully performed experiments, that wave transformation may readily be described by well-known and commonly used expressions.

The experimental results quoted are from a large series of three-dimen-

#### WAVE TRANSFORMATION

sional mobile bed hydraulic model tests performed at the Queen's University coastal processes basin (see Fig. 1). The tests are described in detail in Kamphuis and Kooistra (1990). The purpose of the tests was to determine wave transformation, wave breaking, nearshore currents, sediment transport rates and distributions, and beach morphology simultaneously. It is recognized that these hydraulic model tests may contain scale effects and that the best possible results would be direct field observations. But these model tests were performed specifically to obtain a comprehensive and coherent data set of wave-beach interaction, impossible to obtain in the field. Valid field data are very difficult to obtain, impossible to control and not repeatable. Field results may be used to enhance the understanding gained from these comprehensive model tests, particularly with respect to any scale effects that might be present in the model.

When the sediment transport rate results obtained from this set of model tests were compared with available field data, it was found that scale effects were minimal (Kamphuis, 1991a). It is expected that scale effects in the wave transformation and breaking processes are similarly small and that the present results may be applied to field conditions without much error.

It is of particular interest that this is the first time such a comprehensive set of data obtained on a deformable beach as distinct from a fixed beach of constant slope is compiled. Experiments are still ongoing. Because of modelling restrictions, all the beach profiles contained a single offshore bar and the breaking waves were spilling-plunging or plunging.

#### DESCRIPTION OF THE TESTS

Both regular and irregular waves were studied and the test conditions are summarised in Table 1. For the irregular waves, a Jonswap incident wave spectrum with  $\gamma = 2.3$  was used and the random wave signal typically repeated itself after 200 waves.

An initial plane beach slope of 1:10 was prepared for each test. The depth of water in Table 1 was measured at the toe of the sloping beach. Waves of constant wave height, period and incident wave angle were generated for approximately seven hours. Each test was divided into one-hour segments. Wave heights, wave angles, longshore current velocity distributions, sediment transport rates and distributions, and beach profiles were measured in an hourly cycle.

The incident wave spectrum was measured offshore. The peak period of the offshore wave spectrum was used for the subsequent calculations. Wave heights were measured at 0.2-m intervals from offshore of the breaker, through the surf zone, into the swash zone and the significant and r.m.s. wave heights were calculated by zero crossing analysis. Only significant wave heights are discussed here.

# TABLE 1

Test summary

Test	Generated wave						Grain size
	H <sub>s</sub> (m)	$T_{p}$ (s)	d (m)	Wave type	Groupiness factor	Incident angle	D (mm)
IA	0.045	1.15	0.50	irr	0.8	10	0.105
#IB	0.063	1.15	0.55	irr	0.8	10	0.18
#IC	0.088	1.15	0.55	irr	0.8	10	0.18
#ID	0.117	1.15	0.55	irr	0.8	10	0.18
IE	0.063	0.92	0.50	irr	0.8	10	0.105
IF	0.063	1.15	0.50	irr	0.8	10	0.105
IG	0.063	1.39	0.50	irr	0.8	10	0.105
II	0.078	1.38	0.50	irr	0.8	10	0.105
#IJ	0.090	0.92	0.55	irr	0.8	10	0.18
#IK	0.084	1.38	0.55	irr	0.8	10	0.18
#IL	0.122	1.15	0.55	irr	0.8	20	0.18
#IM	0.094	1.15	0.55	irr	0.8	20	0.18
#IN	0.063	1.15	0.55	irr	0.8	20	0.18
#IO	0.063	1.15	0.55	irr	0.8	30	0.18
#IP	0.080	1.15	0.55	irr	0.8	30	0.18
#*IQ	0.124	1.15	0.55	irr	0.8	30	0.18
#*IR	0.084	1.15	0.55	irr	0.2	30	0.18
#*IS	0.084	1.15	0.55	irr	1.4	30	0.18
#*IT	0.127	1.00	0.55	irr	0.8	40	0.18
#*IU	0.139	1.50	0.55	irr	0.8	40	0.18
#*IV	0.094	1.20	0.55	irr	0.8	40	0.18
RC	0.074	1.15	0.55	reg		10	0.18
RE	0.045	0.92	0.50	reg		10	0.105
RG	0.045	1.39	0.50	reg		10	0.105
RI	0.060	1.38	0.50	reg		10	0.105
RM	0.084	1.15	0.55	reg		20	0.18
*RP	0.051	1.15	0.55	reg		30	0.18
*RT	0.132	1.00	0.55	reg		40	0.18

\*Breaking angle recorded with overhead video.

"Test result included in the wave shoaling, friction, percolation and saturation analysis.

Breaking wave angles were measured using a video camera mounted 8.5 m above the model. Before the first test, a large rectangular grid was placed on the dry beach and photographed from above. This grid was projected onto the screen of a monitor and thin lines were taped directly to the screen over the projected grid. This grid of tape lines on the monitor served as a reference grid for all the wave angle measurements.

Overhead video was taken of eight tests with larger incident wave angles, as indicated in Table 1. Records were taken at approximately one-hour intervals and each record was 3.5 min long. This resulted in 39 good records. Each

record contained approximately 200 waves; the length of the repeating pattern in the irregular wave train.

As the tests progressed, it became clear that the regular wave tests were much more difficult to control than the irregular tests and that their results were more scattered and less reliable. Hence, with the exception of the discussion on wave refraction, the present paper is based on the irregular wave test results only, which are of more direct practical interest anyway. In addition, there was some problem with accurate determination of the offshore wave conditions for the tests with the 0.105 mm sediment and these test results could not be used for the present analysis. The tests included in the wave shoaling, friction, percolation and saturation analysis are indicated in Table 1.

#### WAVE REFRACTION

Wave refraction was studied, using the wave breaking angle observations. For seven of the eight videotaped tests, the record taken about three hours into the test was analysed to determine the breaking angle of each wave in the record. The average and r.m.s. value of the breaking angle was determined for each of these records; only the average angles are discussed here. Since measuring 200 waves for each record was tedious, every 10th breaking wave was also analysed in the same manner for the seven records and Fig. 2 shows that the two sets of results did not differ significantly. Hence for the remaining 32 records, the breaking angle of every 10th wave was measured.

Figure 1 shows that a sediment feeder was placed in the model. Its purpose was to ensure that the beach was not subjected to net erosion and that the beach contours would remain essentially straight and parallel to simulate an infinitely long prototype beach. This operation was successful in that the contours were indeed approximately parallel. Nevertheless, the breaking angle



Fig. 2. Breaking angle comparison.

was determined with respect to the offshore contours, the contours of the bar and the still water line. The differences were small and the angles with respect to the still water line gave slightly better correlation with theory; they will be discussed.

The depth of breaking was determined from the simultaneously measured wave height profile (as shown in Fig. 3). The intersection of the wave transformation curve (coming from offshore) and the wave height decay curve (coming from onshore) defined the point of breaking. The details of the method are described in Kamphuis (1991b, this issue).

For a straight beach such as in the present tests, the theoretical expression that describes wave refraction is Snell's Law. When the measured values of the breaking wave angles were compared with Snell's Law, it was found that the measured angles exceeded the calculated ones as shown in Fig. 4. The line of best fit is:



Fig. 3. Determination of breaking location and height.



Fig. 4. Calculation of breaking angle using Snell's Law.

 $\alpha_{\rm b} \,(\text{measured}) = 1.24 + 1.03 \,\alpha_{\rm b} \,(\text{calculated}) \tag{1}$ 

Re-examination of the results showed that the actual breaking angle was impossible to identify from the video and that the wave crest immediately prior to breaking was used. This would mean that all the measured values in Fig. 4 would be slightly too large to represent actual breaking angles and that this difference would increase with the amount of refraction taking place. This would explain both the intercept and the slope of 1.03 in Eq. 1. It was concluded from this part of the analysis that for practical computations Snell's Law describes refraction adequately up to breaking.

What is striking about Fig. 4 is the large range of measured results at any particular calculated value. The error in each individual angle measurement is estimated to be about one degree. Since each point in Fig. 4 represents an average of about 20 values, the spread cannot be a result of measurement errors in wave angles. Figure 4 also shows that the spread is not related to wave period.

For the example given in Fig. 3, it is possible to determine the sensitivity of the calculated breaking angle to incorrect estimates of the breaker location and hence the depth of breaking. Figure 5 gives the relevant sensitivity diagram and it is seen, for instance, that an error of 0.3 m in breaker location results in an error in breaking wave angle of  $1.5-2^{\circ}$ . Differences in the dynamics of wave breaking for each test, resulting from long wave action in the test basin could possibly cause some further spread in the results. Thus it may be concluded that the large variation in the observed breaking angle is the result of variation in observed depth of breaking.

The above discussion on wave refraction may be summarised as follows: These carefully controlled experiments, in which the actual breaking conditions were measured, exhibit quite large variations in measured breaking angle. Thus for practical calculations, in which the breaking conditions are de-



Fig. 5. Breaking angle sensitivity.



Fig. 6. Wave shoaling using linear wave theory.

termined from a generally formulated "breaking criterion", Snell's Law should suffice to calculate wave refraction, even if minor irregularities in beach contours occur.

#### WAVE SHOALING

The wave heights measured prior to breaking for the tests in Table 1 marked with '#' are plotted in Fig. 6 against wave heights calculated, using a combination of Snell's Law of Refraction and Linear Shoaling Theory.

It is seen that linear theory overpredicts the wave heights. This is surprising since it is generally accepted that linear theory underpredicts shoaling and that a higher order, shallow water wave theory is required to predict shoaling correctly. The same tendency for linear theory to overpredict wave shoaling has also been demonstrated for field results by Hughes and Miller (1987).

The overprediction was always less than 20% and there are three major causes for the measured wave heights to be smaller than the calculated values.

The present results were obtained on a natural beach as opposed to an impervious, plane, solid beach. This means there was bedform which causes wave attenuation. Percolation into the beach causes additional wave attenuation. Finally, for the irregular waves tested, some wave breaking took place offshore of the breaking zone, particularly for the steeper waves in the spectrum (saturation), causing a further wave height reduction. Each of these aspects is discussed below.

#### **BOTTOM FRICTION**

The bedform geometry in the model conformed with the relationships reported by Mogridge and Kamphuis (1972) and updated by Kamphuis (1988). The relationship for bedform height from these studies is:



Fig. 7. Wave attenuation by bottom friction. (a) Wave period as parameter. (b)  $(H_s/L_p^{3/4})_g$  as parameter.

$$\Delta/D = 0.14 \ (a/D)^{1.05} \tag{2}$$

where  $\Delta$  is the bedform height, D is the particle size and a is the wave orbital amplitude at the bottom. The bottom roughness k is estimated using:

$$k=2\Delta$$
 (3)

resulting in a wave friction factor,  $f_w$ :

$$f_{\rm w} = 0.8 \ (a/\Delta)^{-0.75} \tag{4}$$

The attenuation relationship of Kamphuis (1978) was used:

$$\frac{\mathrm{d}a}{\mathrm{d}x} = \frac{2Ka^2k^2f_{\mathrm{w}}}{n\sinh(2kd)\sinh(kd)} \tag{5}$$

where x is distance in the direction of wave propagation, K is a function of the phase difference between orbital motion and shear stress; its usual value is 0.18, k is the wave number  $(=2\pi/L)$  and:

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$$n = \frac{1}{2} \left[ 1 + \frac{2kd}{\sinh(2kd)} \right] \tag{6}$$

Figure 7 indicates that introduction of bottom friction improves the prediction but even for these tests in which bedform was quite pronounced, the correction is small.

# PERCOLATION

The model beach was porous, hence percolation needs to be considered. Liu and Dalrymple (1984) provide a thorough discussion of wave percolation. The relevant equation is:

$$a = a_0 \,\mathrm{e}^{-bx} \tag{7}$$

and their work indicates that for D=0.18 mm, b<0.005. Thus over the 5 m of model beach offshore of the breaker, the maximum possible wave height decrease resulting from percolation would be less than 2.5%. This is of the same order as the bottom friction correction.

# WAVE SATURATION

During the tests, it was noticed that some of the steeper waves broke offshore of the breaking zone, which would indicate that spectral saturation took place. The simplified expression derived by Hughes and Miller (1988) was introduced. Hughes and Miller state that once a wave has reached saturation:

$$H_{\rm s}/L_{\rm p}^{3/4} = {\rm constant}$$
 (8)

Figure 7a was redrawn in Fig. 7b showing the generated value of  $H_s/L_p^{3/4}$  as parameter. If complete saturation of the generated wave is assumed, i.e., Hughes and Miller's relationship is written as:

$$H_{\rm s}/L_{\rm p}^{3/4} = (H_{\rm s}/L_{\rm p}^{3/4})_{\rm g}$$

as in Fig. 8, it is clear that for most tests, wave saturation overpredicts wave attenuation. This is reasonable since the generated waves were not breaking directly off the generator and broke only marginally for a few of the tests. Of interest is the behaviour of the test with the largest value of  $(H_s/L_p^{3/4})_g$ . For this test, extensive offshore breaking was noted and indeed, Hughes and Miller's wave spectral saturation correction brings the measured wave heights for this set of points on the 45 degree line. Since for the present data set, the recorded wave trains were not saved, further work has now been initiated to investigate wave spectral saturation in more detail.



Fig. 8. Wave saturation effect on wave height.

CONCLUSIONS

The above analysis indicates that for these carefully collected experimental results:

- (1) Wave refraction is adequately described by Snell's Law for straight shorelines or shorelines with minor irregularities.
- (2) Linear wave shoaling relationships overpredict wave heights as they approach breaking. The overprediction was always less than 20% in the present tests.
- (3) Wave attenuation by bottom friction is small, even for the pronounced bedform present in the model.
- (4) Wave attenuation by percolation is small, even though the model particle diameter is large, relative to the wave motion.
- (5) It appears that the only process able to match calculated with observed wave heights is wave saturation.
- (6) Wave spectral saturation needs to be taken into account when waves break offshore. Preliminary results indicate that the approximation by Hughes and Miller (1987) may be useful.
- (7) For practical calculations the combination of Snell's Law and linear shoaling should yield adequate results, since the breaking process (particularly the depth of breaking) is normally poorly defined.

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