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A Numerical Study of Wave Refraction in Shallow Tidal Waters

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Wave observations taken at two shallow water sites off the coast of south-west Wales, U.K. are examined. The buoys are located in a region of strong semi-diurnal tides and the observations show significant variation, at semi-diurnal frequencies, in period, amplitude and direction. The interaction between tides and waves is studied using linear theory of wave refraction by slowly varying currents and depths. It is shown that quasi-steady analytical models fail to reproduce the tidally-induced variations in wave parameters. A linear ray-tracing model, including the effects of both spatial and temporal variations in the tidal currents and elevations, improves the simulation of the observed modification of the waves. Short period waves respond to variations in current refraction over a tidal cycle; long period waves are more influenced by variations in water depth over a tidal cycle.

Keywords: waves; refraction; tides; Wales

Introduction

The interaction between ocean current systems and short gravity waves has been widely reported, particularly with regard to the dangers posed to shipping. Lavrenov (1998) described the impact of an anomalously large wave on a ship within the Agulhas current, attributing the exceptional wave heights to their modification by a counter current. *In situ* observations of wave-current interactions were obtained during the SWADE experiment when changes in directional spectra were reported as waves encountered Gulf Stream meanders (Wang *et al.*, 1994). Indirect observations are available through satellite borne synthetic aperture radar (SAR), which provides the opportunity to map the effects of wave-current interaction in two dimensions: Holthuijsen and Tolman (1991), Kudryavtsev *et al.* (1995) and Grodskii *et al.* (1997) in the Gulf Stream; Irvine and Tilley (1988) in the Agulhas; and Liu *et al.* (1994) in the Gulf of Alaska.

Unna (1942) discussed the change in wave length and velocity of wave trains entering a steady, but spatially varying, current stream. He proposed a simple linear theory to explain the shortening and steepening of waves propagating against a steady current, as observed in the Agulhas current. Longuet-Higgins and Stewart (1960) developed a non-linear theory, introducing the concept of radiation stress. This was extended by Bretherton and Garrett (1968) to show that, in a non-dissipative media, wave action is conserved. The linear theory of wave-current interaction is now well established and a comprehensive treatment may be found in Mei (1983). In particular, ray theory (geometrical optics) for the propagation of small amplitude waves within varying currents (and water depths) has been widely applied (e.g. Mapp *et al.*, 1985 Mathiesen, 1987; Liu *et al.*, 1994; Wang *et al.*, 1994).

Within coastal regions, wave-current interaction has been observed between tidal currents and waves (Barber, 1949; Battjes, 1982; Gonzales, 1984; Tolman, 1991; Masson, 1996; Wolf & Prandle, 1999). The interpretation of the observations has frequently been based on a quasi-steady approach to ray theory, in which the time of travel of the waves is assumed to be short compared to the timescale on which the current varies. However, whilst this assumption is valid for studies within large ocean current systems, it is often violated in the tidal regime. Several authors suggest that this is a minor effect and have successfully reproduced the tidally induced variations in wave properties using a quasi-steady approach (Gonzales, 1984; Masson, 1996). However, Barber (1949) observed a semi-diurnal variation in absolute wave period for swell waves arriving at the cost of south-west Britain and Wolf and Prandle (1999) observed a pronounced phase difference between the time of maximum current velocity and maximum current-induced refraction in the North Sea, both of which are inconsistent with quasi-steady

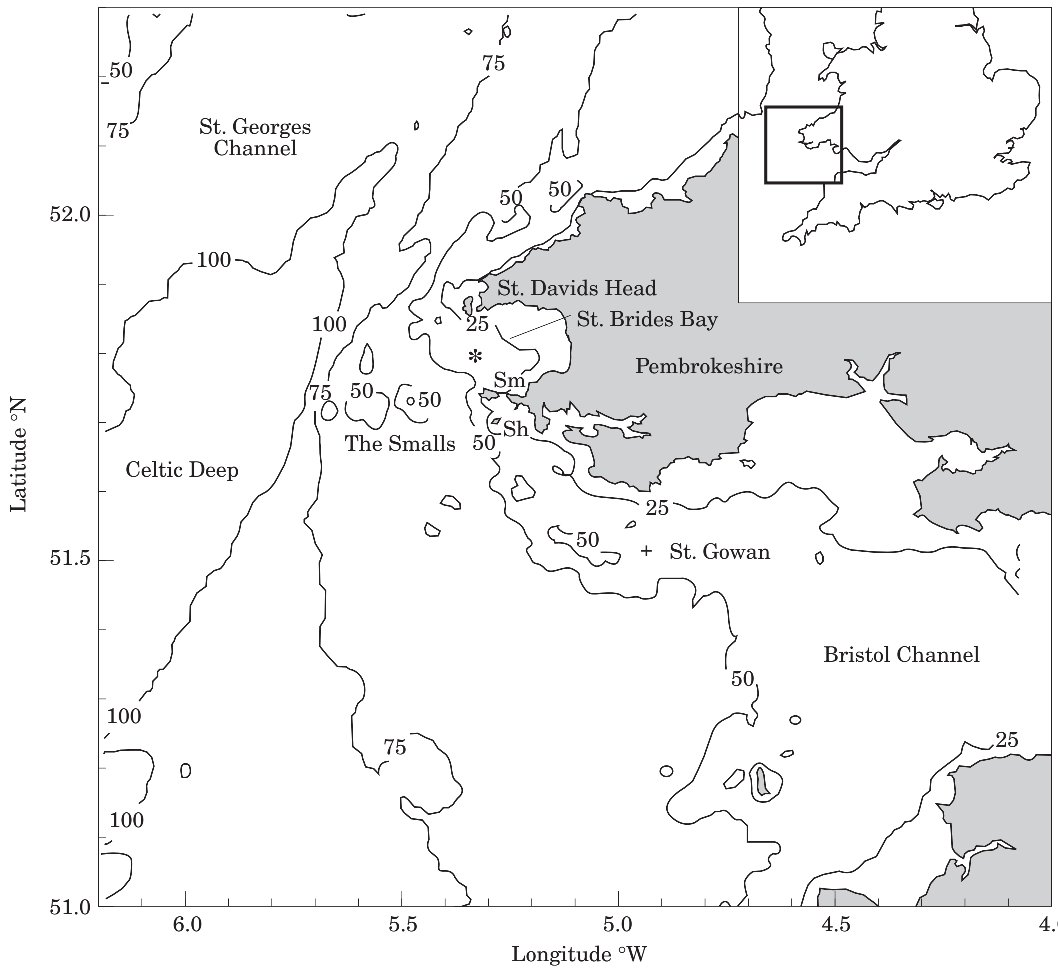

FIGURE 1. South-west Wales showing the wave observation sites at St. Brides Bay (* at 5°20′W, 51°48′N) and St. Gowan Shoals (+at 4°56′W, 51°31′N). Water depths are contoured at 25 m intervals. (Sm=Skomer, Sh=Skokholm)

theory. Furthermore, Tolman (1990) showed that, for highly simplified situations, the modification of wave parameters by tidal currents could be significantly in error if the time variation in tidal currents was not included.

In this paper, wave data obtained from two shallow water sites, St. Brides Bay and St. Gowan Shoals, off the coast of south-west Wales, U.K. (Figure 1) are described. The tidal regime in the region is dominated by the M2 constituent (Owen, 1980) with a range increasing from 3 m in the St. George's Channel to more than 11 m at the head of the Bristol Channel. Figure 2 shows the M2 tidal ellipses for the waters off south-west Wales computed using a depth-averaged finite difference numerical model (Elliott & Jones, 2000) at a resolution of 900 m. The currents are rectilinear in the Bristol Channel and St. George's Channel, whilst to the south-west of Pembrokeshire the flow becomes elliptical. Strong tidal streams (>3 knots) occur around St. David's Head and to the west of Skomer and Skokholm.

The predominant wind over coastal sites in south-west Wales is south-westerly (Chandler & Gregory, 1976). This is approximately the direction of maximum fetch for both wave observation sites and is, therefore, the direction from which the most significant wave activity is expected. Both wave buoys are situated in regions of strong, rectilinear, tidal currents with M2 amplitude in excess of 2 knots and therefore intense wave-current interactions are expected to occur. If, however we consider waves approaching Pembrokeshire from the North Atlantic, the orientation of the major axis of the tidal ellipse with respect to the waves is significantly different at the two sites. At St. Brides Bay, the major axis is aligned north-north-west to south-south-east and, therefore, Atlantic waves will approach the site at incidence angles greater than around 45° to this axis. In contrast, at St. Gowan, the major axis is aligned west-south-west to east-north-east; Atlantic waves will approach the site at angles less than 45°, with the frequent occurrence of colinear waves and currents. In this

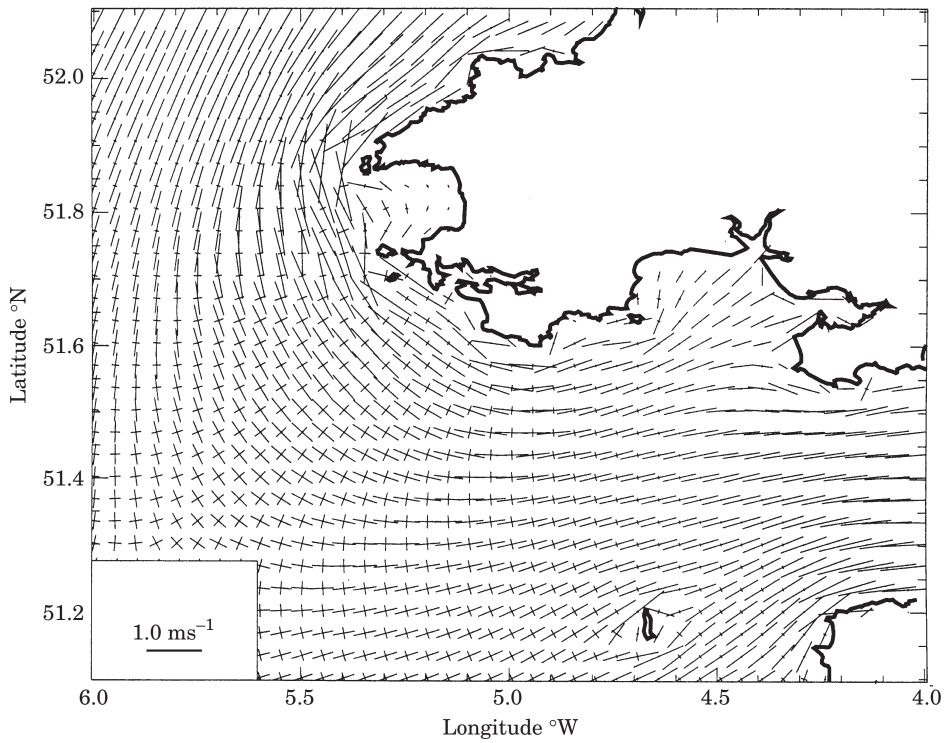

FIGURE 2. M2 tidal ellipses for the St. Georges Channel and Bristol Channel calculated using a depth integrated numerical model. Ellipses are drawn at every fourth model grid point.

paper, the observed semi-diurnal variations in wave parameters at the two sites are investigated using both quasi-steady analytical models and a ray tracing model, including the effects of spatial inhomogeneity and unsteadiness in the current field.

Observations

The St. Brides Bay wave rider, situated at 5°20′W, 51°48′N in 47 m of water, was operated by the Countryside Council for Wales and recorded directional wave spectra. Statistics derived from the spectra are available at 10 min intervals. The St. Gowan buoy, situated at 4°56′W, 51°31′N in 50 m of water, was operated by the U.K. Meteorological Office. Significant wave height and zero-crossing wave period were available at 1 hourly intervals, with a resolution of 10 cm and 1 s respectively. In this paper, data from January 1997 and February 1998, at St. Brides and St. Gowan respectively, are examined. Both data sets begin at spring tides and cover the transition to neap tides. Meteorological data were available from the St. Gowan buoy and a land-based station on Skomer Island (Figure 1).

St. Brides Bay, January 1997

Figure 3 shows the observed wind speed and direction at Skomer Island (Figure 1) between 11 and 19 January 1997. Wind speeds were near-gale force on 13 January and gale force on 17 January. Between these events, the wind fell significantly and the direction backed from south-south-westerly to south-easterly. After 17 January, the wind veered rapidly to south-westerly before backing towards south-easterly again. Figure 4 shows the observed wave parameters (significant wave height, peak period and mean direction) at the St. Brides Bay wave rider between 11 and 19 January 1997. A semi-diurnal variation is evident in all wave parameters. There is, however, no corresponding semi-diurnal variation in the wind observations.

The mean wave direction over the period was 240°, with a semi-diurnal variation of approximately 50° (peak to peak) at spring tides decreasing to 20° at neap tides. The wave direction is most northerly around 2–3 h before the time of maximum south-eastwards flowing tidal current. The semi-diurnal variation in wave direction is noticeably cnoidal, with short periods of northerly biased wave directions and longer periods of southerly biased directions.

The wave height and period varied significantly over the observation period on timescales greater than 12 h. Nevertheless, the semi-diurnal oscillation is evident in both parameters. The semi-diurnal variation in wave height is most noticeable between 13 and 17 January, during which period its magnitude decreased

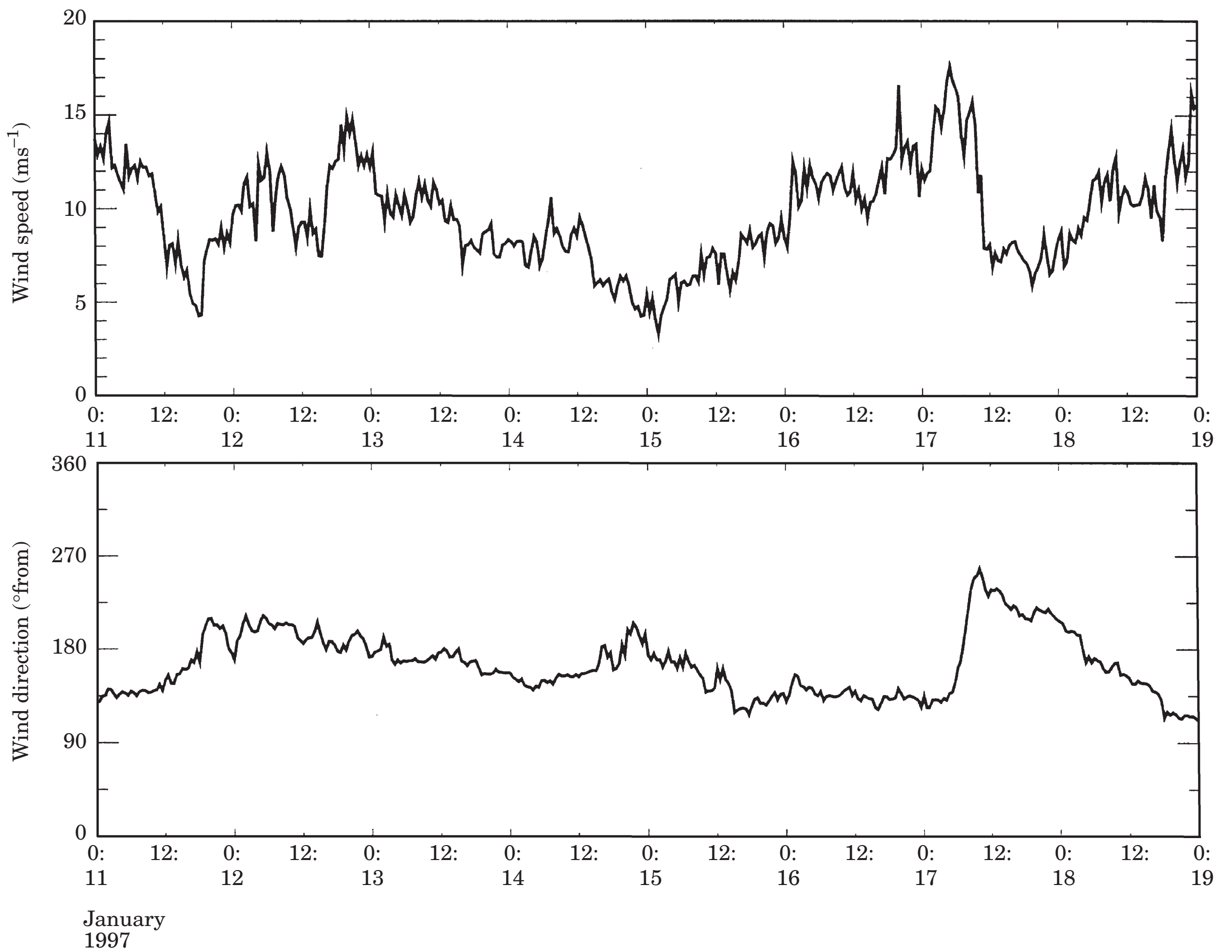

FIGURE 3. Observed wind speed and direction at Skomer Island between 11 and 19 January 1997.

from more than 50 cm to 20 cm (peak to peak). Maximum wave heights occur just after the time of maximum north-westerly currents, around 2–3 h before the time of maximum northerly bias in the wave direction. The semi-diurnal variation in (absolute) mean wave period is most noticeable between 11 and 15 January, when its magnitude was approximately 4 s. The maximum wave period coincides with the minimum wave height.

St. Gowan Shoals, February 1998

Coincident data sets at St. Gowan Shoals and St. Brides Bay were not available for this study, therefore, Figure 5 shows the observed wind speed and direction at the St. Gowan Shoals buoy between 9 and 17 February 1998. Winds were generally light to moderate (around 5 ms^{-1}) and from the south-south-west until 13 February. The wind fell to less than 1 ms^{-1} on 14 February, after which time it increased to around 6 ms^{-1} and the direction veered from south-easterly to north-westerly. Figure 6 shows the observed significant wave height and zero-crossing period at St. Gowan. Directional information is not recorded at St. Gowan. The mean wave height throughout the period was around 2·5 m, with a semi-diurnal variation of more than 1 m. The wave period oscillated, at a semi-diurnal frequency, between 6 s and 8 s. Given the limited fetch for southerly winds at the site and the low wind speed, it must be concluded that the observed waves represent swell propagating into the region from the Atlantic from the south-west. Despite the observations covering the transition between spring to neap tides, the ratio of maximum to minimum wave amplitude during a tidal cycle is approximately 1·6 throughout the entire period. Wave heights reach a maximum around 2–3 h after the time of maximum westwards flowing current; minimum wave heights coincide with the maximum eastwards flowing current. High wave

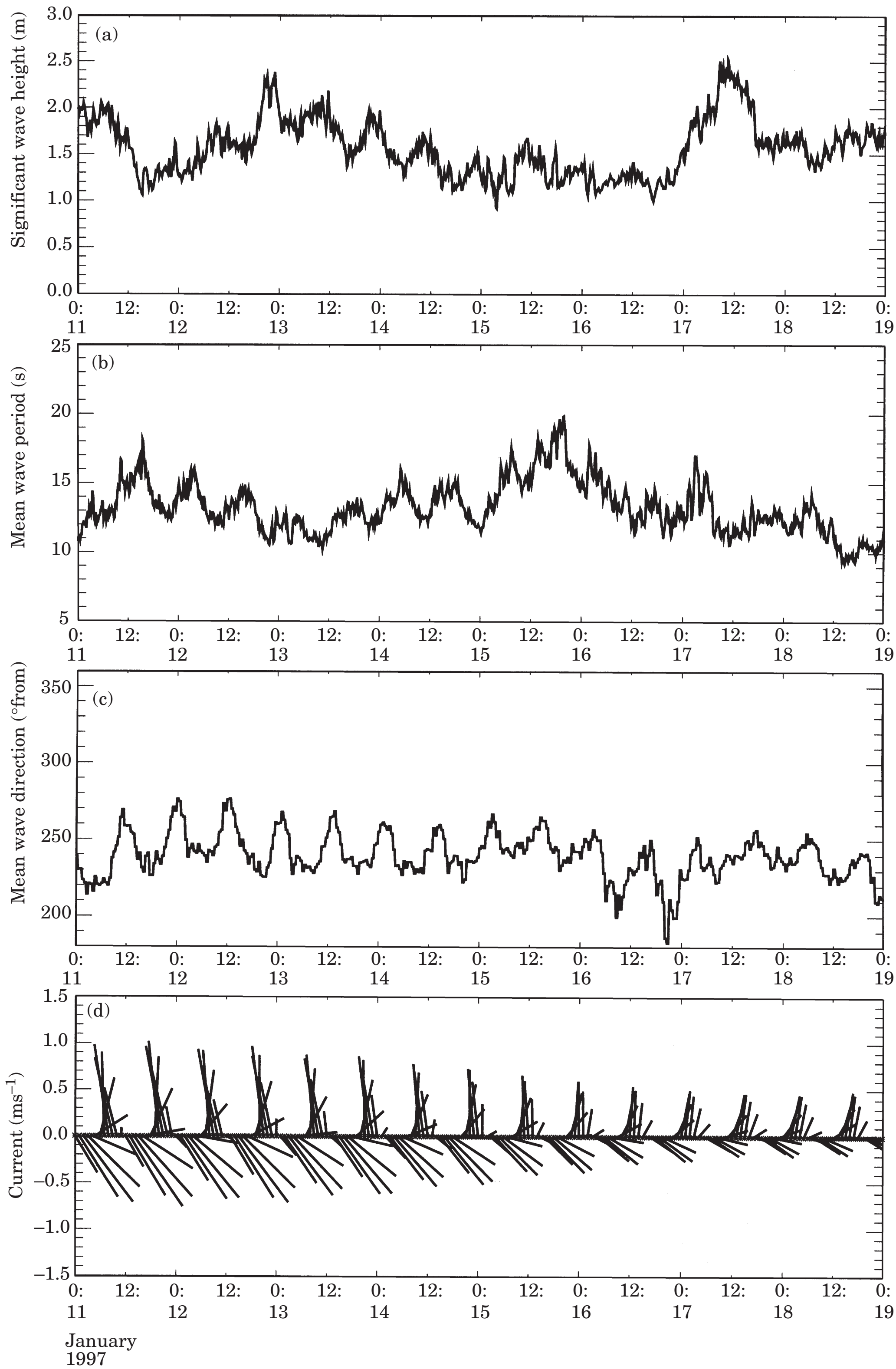

FIGURE 4. Observed wave parameters, at 10 minute intervals, at the St. Brides Bay wave rider between 11 and 19 January 1997: (a) significant wave height, (b) peak period and (c) mean direction. Modelled surface current vectors for the wave rider site are shown in (d). Vectors are drawn, at hourly intervals assuming the positive y-axis points northward, and the M2, S2, N2, K1 and O1 constituents are included.

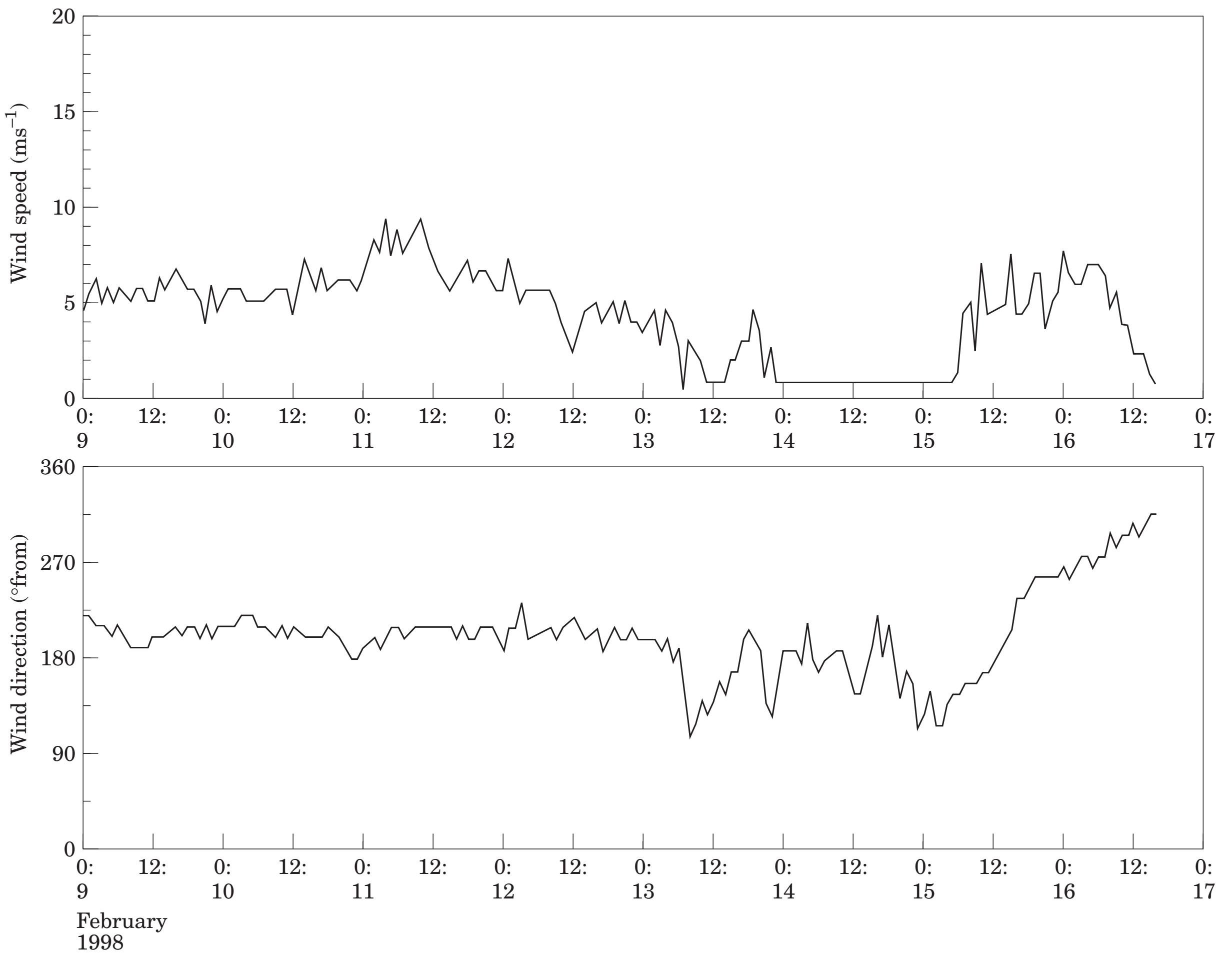

FIGURE 5. Observed wind speed and direction at the St. Gowan buoy between 9 and 17 February 1998.

periods occur 1–2 h prior to the maximum wave heights.

Theory

The general wave form for progressive waves is given by:

$$\Phi(\mathbf{x},t)=\mathrm{A}(\mathbf{x},t)\exp[iS(\mathbf{x},t)] \tag{1}$$

To apply geometrical optics theory to the propagation of waves, it is necessary to assume that the amplitude of the wave, $A(\mathbf{x},t)$, is slowly varying in space, $\mathbf{x}$, and time, t, compared to the variations in the phase function, $S(\mathbf{x},t)$. The local values of the wave vector, $\mathbf{k}$, and frequency, ω, are defined by

$$\mathbf{k}\equiv\nabla S \quad \text{and} \quad \omega\equiv-\frac{\partial S}{\partial t} \tag{2}$$

and the dispersion relation is given by

$$\omega^2=gk\tanh kh \tag{3}$$

where h is water depth and g is gravity. These equations imply that wave crests are conserved and that the wave vector is irrotational.

The above theory may be generalized to wave propagation in a moving medium (Mei, 1983). As waves are advected by a current, $\mathbf{U}$, the frequency observed by a stationary observer (the absolute frequency) is given by

$$\omega=\mathbf{k}\cdot[\mathbf{c_0}+\mathbf{U}] \tag{4}$$

where $\mathbf{k}\cdot\mathbf{c_0}=\sigma$ is the intrinsic frequency and $\mathbf{c_0}$ is the phase velocity measured relative to the moving medium. The dispersion relationship becomes

$$\sigma^2=gk\tan kh \tag{5}$$

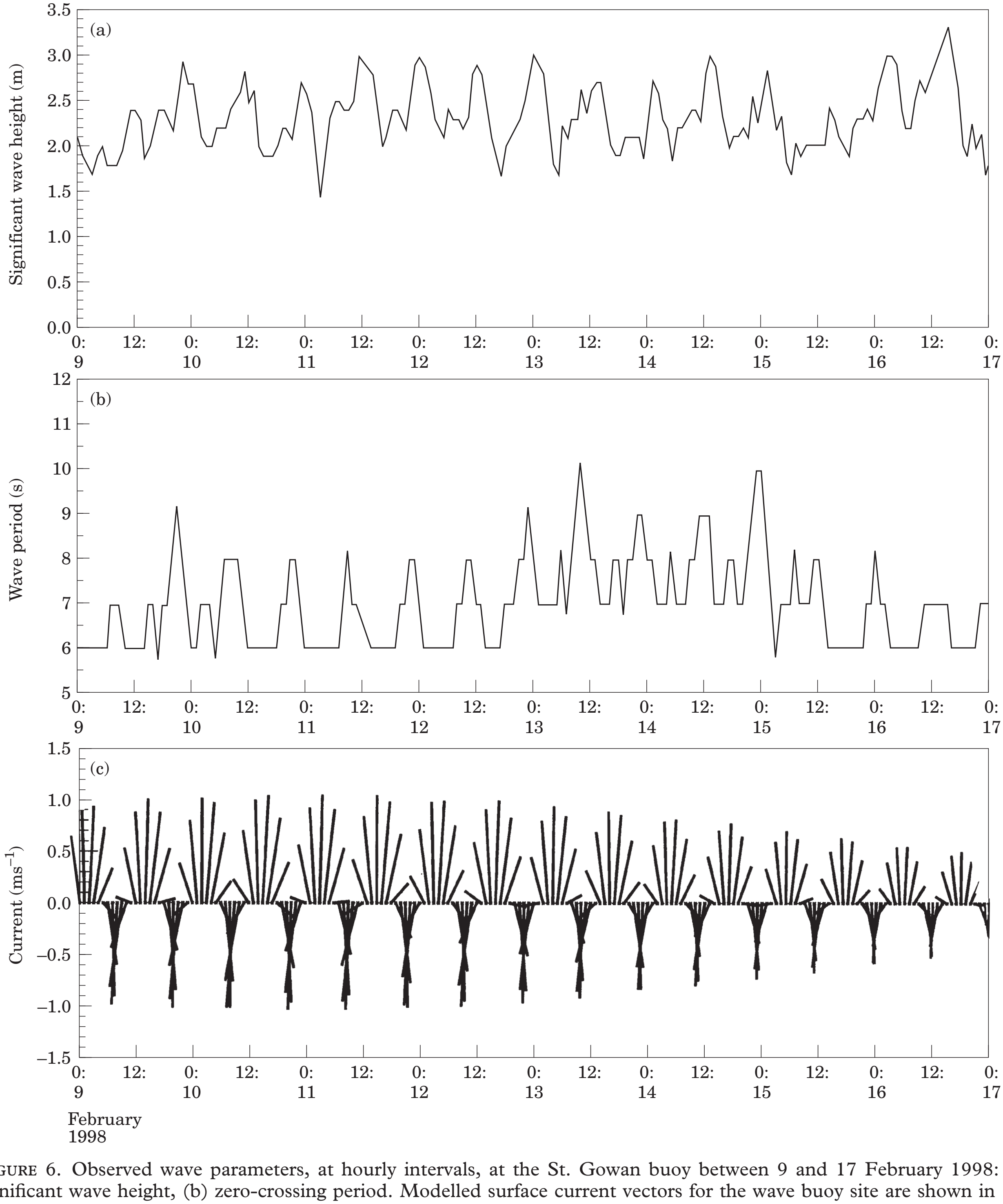

FIGURE 6. Observed wave parameters, at hourly intervals, at the St. Gowan buoy between 9 and 17 February 1998: (a) significant wave height, (b) zero-crossing period. Modelled surface current vectors for the wave buoy site are shown in (c). Vectors are drawn assuming the positive y-axis points eastward, and the M2, S2, N2, K1 and O1 constituents are included.

and the group velocity, relative to the current, is defined as

$$\mathbf{c}_g = \left.\frac{\partial \sigma}{\partial \mathbf{k}}\right|_h \tag{6}$$

Wave rays, $\mathbf{r}$, are defined as paths mapped out by points which move at the absolute group velocity, $\mathbf{c}_g+\mathbf{U}$. Wave orthogonals, defined as lines that are everywhere perpendicular to the wave crests, are interpolated from the wave vectors calculated at points along each ray. Following Mathiesen (1987)

$$\frac{d\mathbf{r}}{dt}=\frac{\partial\omega}{\partial\mathbf{k}}=\mathbf{c}_g+\mathbf{U} \tag{7}$$

where

$$\frac{d}{dt}=\frac{\partial}{\partial t}+(\mathbf{U}+\mathbf{c}_g)\nabla$$

Conservation of wave crests, given by

$$\frac{\partial\mathbf{k}}{\partial t}+\underline{\nabla}\omega=0 \tag{8}$$

can be expanded to give

$$\frac{d\mathbf{k}}{dt}=-\left.\frac{\partial\sigma}{\partial h}\right|_k\nabla h-\nabla\mathbf{U}\mathbf{k} \tag{9}$$

and Equation 7 differentiated with respect to time to give

$$\frac{d\omega}{dt}=\mathbf{k}\frac{\partial\mathbf{U}}{\partial t}+\left.\frac{\partial\sigma}{\partial h}\right|_k\frac{\partial h}{\partial t} \tag{10}$$

Bretherton and Garrett (1968) have shown that, in a moving medium, the fundamental conserved quantity is wave action defined with the intrinsic frequency, σ.

$$\frac{\partial}{\partial t}\left(\frac{E}{\sigma}\right)+\nabla\left\{(\mathbf{c}_g+\mathbf{U})\frac{E}{\sigma}\right\}=0 \tag{11}$$

where E is the wave energy. Equations 7, 9 and 10 may be solved numerically to simulate the trajectories of rays in the presence of slowly varying currents and bottom topography. In idealized situations an analytical solution may be available. The quasi-steady approach to modelling the influence of currents and depths on waves assumes that the local rate of change of the current field and water depth may be ignored. From Equation 5, this implies that the absolute frequency is constant.

The semi-diurnal variations in wave parameters at St. Brides Bay and St. Gowan Shoals clearly indicate a tidal influence. The tidal ellipses to the south-west of St. Brides Bay (Figure 2) suggest that the situation may be similar to the propagation of obliquely incident waves on a shear current, whilst at St. Gowan they suggest that the situation may be examined by assuming colinear waves and currents. In the following sections, analytical models will be developed for the two cases and the results compared to ray tracing calculations including the temporal and spatial variation in both the tidal currents and elevations in the Bristol Channel.

Waves at oblique incidence to tidal streams

Analytical model

At St. Brides Bay, the M2 tidal ellipses show approximately rectilinear currents increasing in magnitude north-eastwards to the site of the wave rider, to which waves from the south-westerly sector have oblique incidence. Therefore, the analytical model for the influence of tidal currents on waves approaching St. Brides Bay is based on a unidirectional shear current. For simplicity, the shear current is aligned with the y axis i.e. $\mathbf{U}=(O,V(x))$. The model uses the quasi-steady assumption (ω=constant) and a constant, deep i.e. $kh\gg 1$), water depth.

Equation 2 states that the wave vector is irrotational and, therefore, since $\partial/\partial y=0$ in this analytical model,

$$k_y=\text{constant}=k\cos\alpha \tag{12}$$

where $|\mathbf{k}|=k=\sqrt{k_x^2+k_y^2}$ and α is the angle of the wave vector to the y-axis. The dispersion relation (Equation 5) may then be written as

$$k=\frac{(\omega-V(x)k_y)^2}{g} \tag{13}$$

The direction of the wave can be determined from

$$k_x=\sqrt{k^2(x)-k_y^2}=\sqrt{\frac{(\omega-V(x)k_y)^4}{g^2}-k_y^2} \tag{14}$$

and

$$\alpha=\tan^{-1}\left(\frac{k_x}{k_y}\right) \tag{15}$$

Conservation of wave action (Equation 11) reduces to

$$\frac{A^2\sin\alpha}{2k(x)}=\text{constant} \tag{16}$$

where A is the wave amplitude. These equations imply that the variation in wave parameters is dependent on the current speed only and is independent of the nature of the shear current profile. Figure 7 shows the solution to Equations 12 to 16, as a function of current speed, for 5 s, 7 s and 11 s waves initially travelling at 45° to the y-axis, for currents flowing in and against the direction of the y-axis (V>0 and V<0 respectively). This is equivalent to waves from the

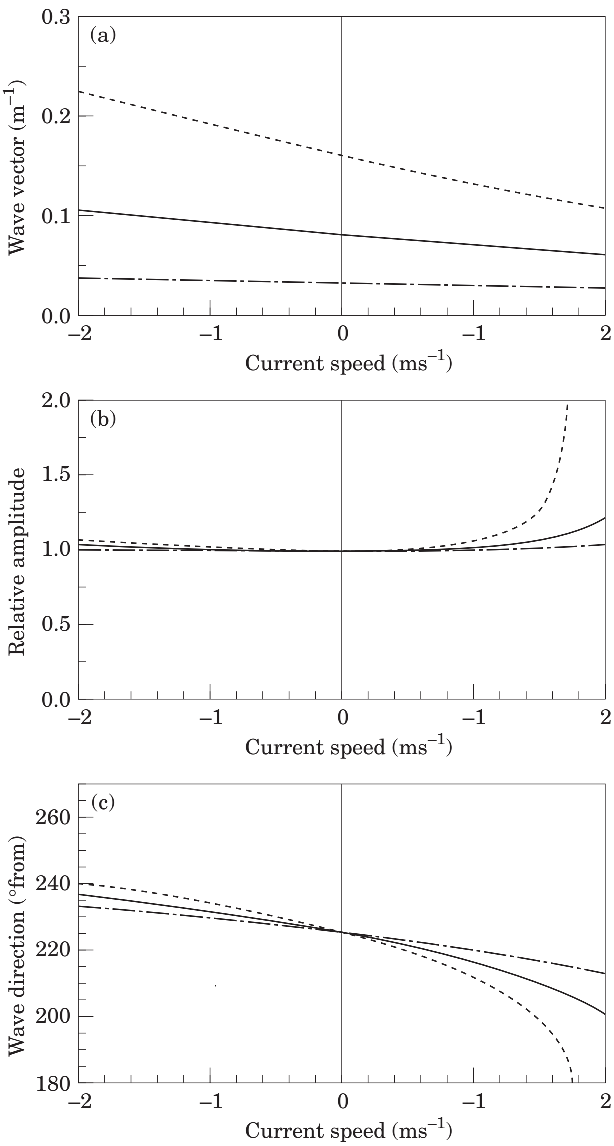

FIGURE 7. Graphical solution to Equations 12 to 16 for the refraction of 5 s (dashed line), 7 s (solid line) and 11 s (dash-dot line) waves initially travelling at 45° incidence to a shear current. Positive and negative speeds refer to currents following and opposing the waves respectively.

south-south-west approaching St. Brides Bay. For currents flowing in the direction of the y-axis, as the current speed increases, the wave vector decreases and the waves become more closely aligned to the y-axis ($a \to 0°$). Furthermore, the wave amplitude increases as the current speed increases. (Equation 16 is dominated by the decrease in the term sin a). It is evident that there is a current speed, V_c, at which the solution to Equation 14 becomes imaginary, where

$$V_c = \frac{\omega - \sqrt{gk \cos \alpha}}{k \cos \alpha} \tag{17}$$

The rays cannot penetrate the shear current and a caustic forms. At this point, the wave amplitude is predicted to become infinite and ray theory becomes invalid e.g. 5 s waves in Figure 7(b) as the current approaches $1{\cdot}75\ \mathrm{ms}^{-1}$. In this analytical solution, the maximum wave amplitude and angle of deviation coincide with the maximum current for all wave periods.

For currents flowing against the direction of the y-axis, the wave vector increases and waves are deflected towards the x-axis ($a \to 90°$). The wave amplitude increases on entering the current (increasing wave vector, $k(x)$, dominates Equation 16). However, the increase is less significant than in the previous case.

For a constant current field, the effect of the shear on waves decreases with increasing wave period. Intuitively, the faster travelling, long period waves will be least affected by a shear current of given strength. The effect of the shear also decreases as the (absolute) angle between the current and wave crests approaches 90°. Waves which travel at 90° to the current are advected downstream, but the angle of the crests i.e. the wave vector **k**, remains constant and, therefore, no change in wave amplitude or direction will be seen.

Under the quasi-steady assumption, the analytical solution described above may be applied to waves from the south-west at different states of the tide at St. Brides Bay. However, it would then be expected that the maximum northerly deviation of the observed wave direction (reported as the angle from which the waves travel) would coincide with the maximum southerly flowing current and vice versa. Furthermore, the solution predicts a maximum in the wave height record at the time of maximum northerly flowing current and a subsidiary maximum at the time of southerly flowing currents. In the observations, the maximum northerly deviation of the waves is observed 2–3 h prior to the maximum southerly flowing current and there is no double maximum seen in the wave amplitude. The solution also underestimates the magnitude of the variation in wave direction over a tidal cycle and severely underestimates the amplitude variation e.g. a tidal current of $\pm 1\ \mathrm{ms}^{-1}$ produces a directional variation of 25° in 5 s waves decreasing to 10° for 11 s waves, and a relative amplitude variation of 0·05 in 5 s waves decreasing to 0·01 for 11 s waves. Furthermore, the angle of incidence used in the analytical model (45°) is the minimum expected angle between Atlantic waves and the tidal stream at St. Brides Bay. At the time of the observations, the angle was greater than 70°. Using this angle of incidence in the analytical model would exacerbate the underestimation of the variation in wave direction and amplitude. Therefore, this highly simplified analytical model does not reproduce the observed semi-diurnal modulation of the wave parameters at St. Brides Bay.

Ray tracing

In the above analytical model, the true spatial and temporal variations in both the tidal current and elevation have been neglected. To study the semi-diurnal variation in wave parameters more fully, a ray tracing model based on Equations 7, 9 and 10 has been applied to the tidal conditions on 14 January 1997 (mid-tide, i.e. midway between spring and neap tides). The model has been applied to waves with periods between 5 s and 15 s with an initial direction of 240° (the mean observed direction at the time) along a 40 km wave front centred at 7°33′W, 51°0′N in the Celtic Deep. This is a region of comparatively weak (<0·5 knots) tidal currents and deep water and, therefore, negligible wave refraction. The model was run for tidal fields initialised at hourly intervals from 06:00h 13 January 1997 to 23:00h 14 January. (During each model run, the tidal currents and elevations, and their spatial and temporal gradients, are updated at each 10 s timestep.) Figure 8 shows a plot of 5 s rays initialized at 12:00 and 21:00h 13 January 1997. The rays reach St. Brides Bay at around 02:00 and 09:30h 14 January respectively. The deflection of the rays as they approach the site of the wave rider can be clearly seen. Figure 9(a) shows a plot of the mean direction of waves, of initial period 5 s, 11 s and 15 s, passing within 1 km of the wave rider, where the mean is taken over a 15 min period in time. The ray tracing model simulates a semi-diurnal oscillation in wave direction, with the maximum northerly deflection of waves occurring between 01:00 and 03:00h, and 14:00 and 15:00h, approximately 2 h before the time of the maximum south-eastwards flowing current. This is in good agreement with observations. Furthermore, as observed, the oscillation is cnoidal in shape, with short periods of northerly biased wave directions and longer periods of southerly biased directions. The magnitude of the oscillation is around 50° for 5 s waves, decreasing to 20° for 11 s waves. The semi-diurnal oscillation in wave direction is very weak for 13 s waves, but significantly, increases to 30° again for 15 s waves. For low period waves (<9 s), refraction by the tidal current is the most significant effect. However, for the longer period waves, ray refraction by depth variations changes significantly between low and high tide. The oscillation seen in the direction of 15 s waves is a result of wave refraction (by depth variations) over The Smalls (Figure 1) at low tide only.

The ray tracing model also predicts a semi-diurnal oscillation in wave period [Figure 9(b)]. The amplitude of the oscillation is between 1 s and 1·5 s, with the maximum wave period occurring around 2 h after the maximum northerly bias in wave direction. Whilst the timing of the maximum in wave period agrees adequately with the observations, the magnitude of the oscillation (4 s) is significantly under represented. The variation in wave amplitude over the tidal cycle in the ray tracing model output is very noisy at low period with no discernible semi-diurnal oscillation [Figure 9(c)]. However, a very large oscillation is predicted for 11 s waves which decreases again at longer periods. The maximum in wave amplitude occurs between 09:00 and 11:00h and 20:00 and 24:00h. Whilst the timing of the maximum is in agreement with the observations, a comparison of the magnitude of the observed and modelled amplitude variation in this case requires knowledge of the initial shape of the wave spectrum and is beyond the scope of this study. Nevertheless, it may be said that the wave amplitude modification is not inconsistent with the observations.

Colinear waves and tidal streams

Analytical model

At St. Gowan, waves from the Atlantic entering the Bristol Channel will propagate approximately along the major axis of the local tidal ellipses. Therefore, the tidally induced signal in the observations should be explained by considering colinear waves and currents. For simplicity, the current and waves are aligned with the x-axis i.e. $\mathbf{U}=(U(x),0)$ and $\mathbf{k}=(k,0)$. The model uses the quasi-steady assumption (ω=constant) and a constant, deep, water depth.

From Equations 4 and 5, and the quasi-steady assumption,

$$\omega = kU(x) + \varsigma = \sqrt{gk_0} = \text{constant} \tag{18}$$

where k_o is the magnitude of the wave vector where the current speed is zero. Equation 18 may be expanded to give the following quadratic expression for the wave vector as a function of current speed

$$U^2(x)k^2 - (2\omega U(x) + g)k + \omega^2 = 0 \tag{19}$$

The amplitude of the waves can then be deduced from Equation 11, which reduces to

$$A^2 \frac{(c_g + U(x))}{\alpha} = \text{constant} \tag{20}$$

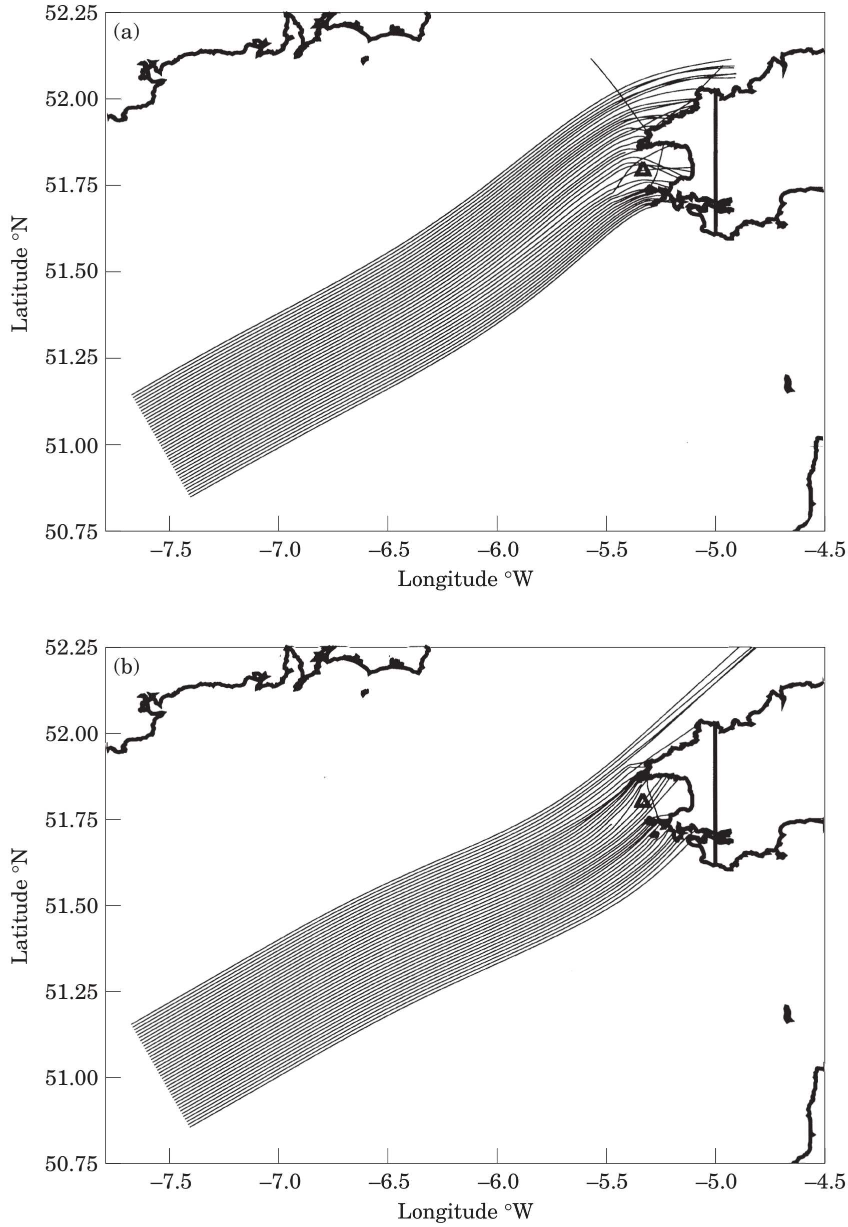

FIGURE 8. Modelled refraction of 5 s rays initialized at (a) 12:00h 13 January 1997 and (b) 21:00h 13 January 1997. The rays reach the St. Brides wave rider, denoted by a triangle, at around 02:00 and 09:30h 14 January in (a) and (b) respectively.

From Equation 19, it is evident that there is a critical current speed, U_c, at which the wave vector becomes complex and no wave propagation is possible, where

$$U_c = -\frac{g}{4\omega} \tag{21}$$

Figure 10 shows the solution to Equations 18 and 19 for 5 s, 7 s and 11 s waves in following ($U>0$) and opposing ($U<0$) currents. As in the case of oblique incidence on a shear current, the solution is independent of horizontal distance and, therefore, the wave parameters are shown as a function of current speed. Where the waves and currents have the same direction, the amplitude of the waves is decreased as the current speed increases and vice versa. For 5 s waves, the critical current speed is $-1{\cdot}95\ \mathrm{ms}^{-1}$. This is reflected in the exponential growth of the relative wave amplitude and the wave vector, i.e. a shortening of the wave length. The magnitude of the change in wave parameters decreases as the wave period increases.

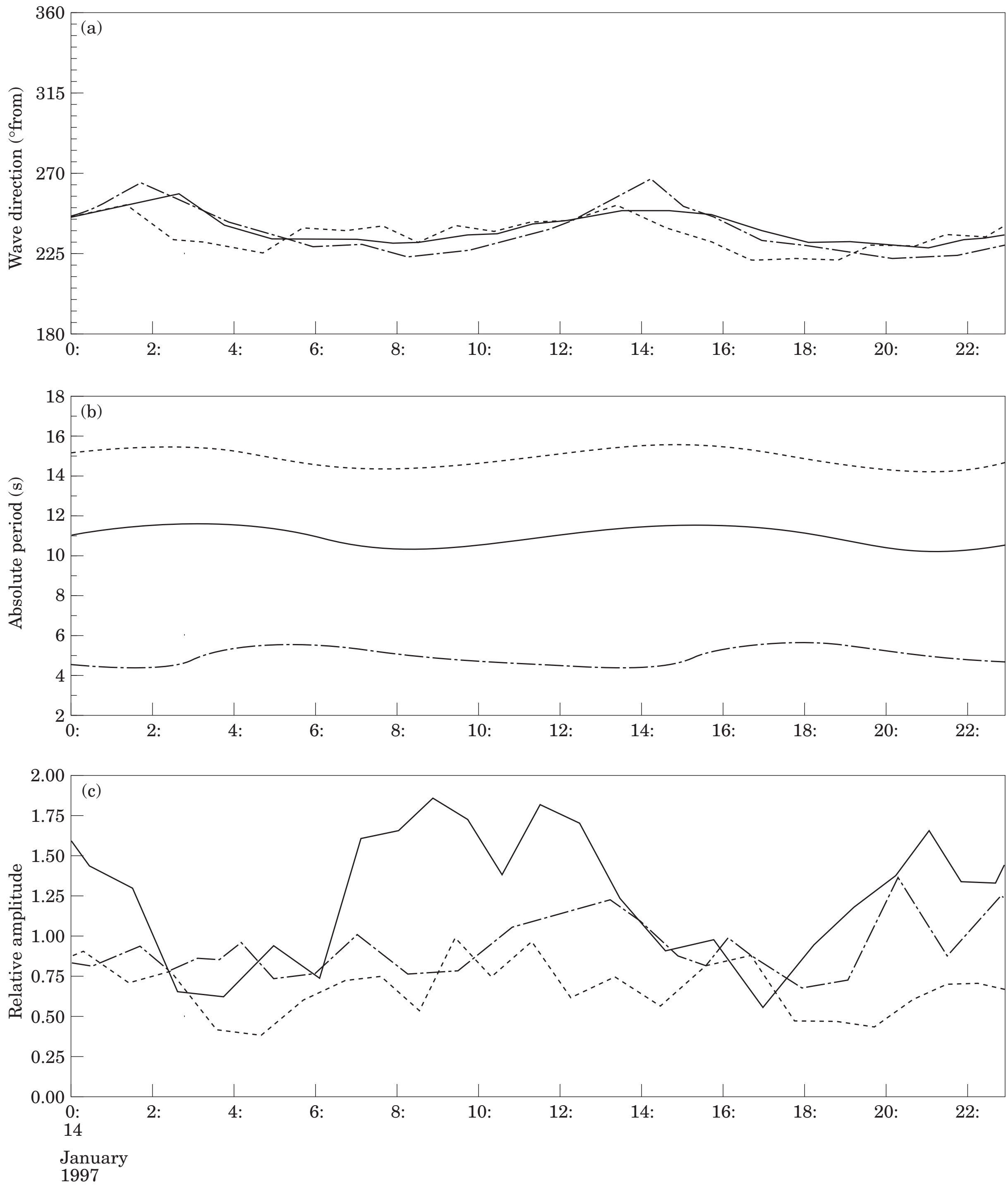

FIGURE 9. Modelled mean wave parameters at the St. Brides Bay wave rider for 14 January 1997 for waves of initial period 5 s (dash-dotted line), 11 s (solid line) and 15 s (dashed line). The mean is taken over 15 min in time and a circle of radius 1 km centred at the wave rider in space. (a) Wave direction, (b) absolute wave period and (c) amplitude relative to initial amplitude in Celtic Deep.

Under the quasi-steady assumption, the solution is assumed to be applicable at different states of the tide at St. Gowan and, therefore, the analytical model predicts that the maximum and minimum wave heights will coincide with the maximum westwards and eastwards flowing tide, respectively. The observed

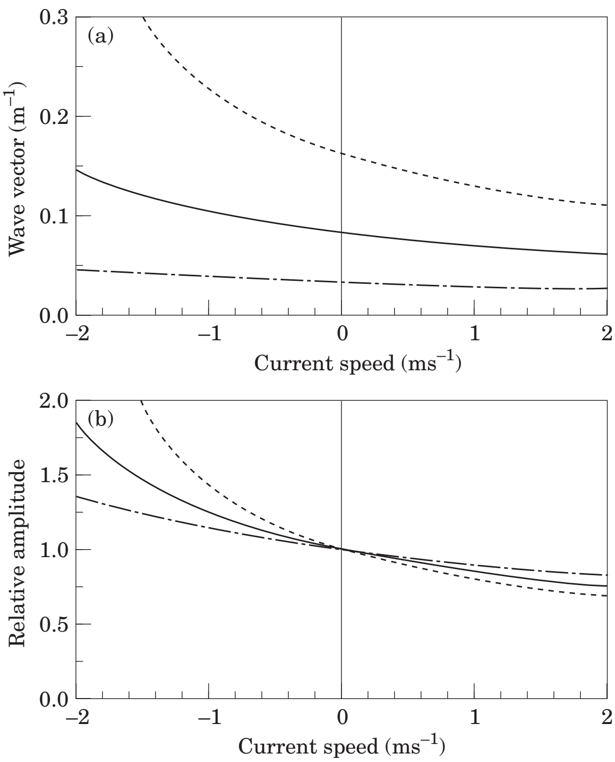

FIGURE 10. Graphical solution to Equations 19 and 20 for the refraction of 5 s (dashed line), 7 s (solid line) and 11 s (dash-dot line) waves by a colinear current. Positive and negative speeds refer to currents following and opposing the waves respectively.

minimum wave height does indeed coincide with the maximum eastwards flowing tide, however the maximum height occurs 2–3 h before the maximum westwards flow. The observed magnitude of the change in wave height is also difficult to explain using the analytical model. Significantly, the observations do not show a discernible change in the amplitude of the semi-diurnal variation in wave height over the spring-neap cycle, despite the maximum current speed falling from 1·2 to 0·5 ms^{-1}. In the observations at St. Gowan, the mean zero-crossing period was around 7 s. For such waves, the amplitude of the semi-diurnal signal in wave height is well represented by the analytical model at spring tides when the amplitude of waves, relative to slack water, is predicted to be 1·3 on the maximum ebb tide (flowing towards the west and opposing the waves) and 0·8 on the flood tide. At mid and neap tides the amplitude of the signal is under represented i.e. the wave amplitude, relative to slack water, is 1·1 on the ebb tide and 0·9 on the flood. However, there will also have been significant energy in the wave spectrum at periods both longer and shorter than 7 s. For 11 s waves, the observed amplitude of the semi-diurnal variation in wave height (1·2 to 0·9 relative to the value at slack water) is well represented at spring tides but severely underpredicted at neap tide (1·1 to 1·0 relative to slack water). Shorter period waves are predicted to experience large growth under an opposing tide and at maximum ebb at spring tide, and waves with periods shorter than 4 s cannot propagate into the Bristol Channel on the ebb tide.

The analytical solution does not explicitly predict any changes in the wave period since it is based on quasi-steady theory. However, as described above, at spring tides all energy in waves less than 3 s period will be lost since they cannot propagate upstream. Furthermore, wave breaking may occur as the waves steepen in an opposing current. Longuet-Higgins and Smith (1983) proposed that wave breaking will occur when the wave steepness, defined as H/L where H and L are the wave height and length respectively, exceeds 0·14. However, observations suggest that wave steepness rarely exceeds 0·08. Certainly, the steepness of the modulated 5 s waves in the analytical solution exceeds 0·08 at spring tides under an opposing current and wave breaking may be assumed to occur. This would lead to a shift towards longer periods for the zero-crossing period. This reasoning suggests that, as observed, maximum wave periods will occur on the maximum ebb tide.

Ray tracing

As for the case of waves at St. Brides Bay, the semi-diurnal variation in wave parameters may be studied more fully using a ray tracing model based applied to the tidal conditions on 13 February 1998 (mid-tide, i.e. between spring and neap tides). The model has been applied to waves with periods between 5 s and 15 s with an initial direction of 240° (approximately the direction of maximum fetch at St. Gowan) along a 40 km wave front centred at 7°10·4′W, 50°42·3′N in the Celtic Deep. Figure 11 shows the modelled mean wave parameters within 1000 m of the St. Gowan buoy for 5 s, 7 s and 9 s waves. The time of maximum amplitude varies from 12:00 and 24:00h at 5 s, to 10:00 and 22:00h at 11 s. The model predicts a maximum in wave height shortly after the time of maximum westwards flowing current. This represents an improvement over the analytical model, but the timing remains in error by more than an hour. The relative amplitude of the wave varies from 1·3 to 0·8, with little dependency on wave period between 5 s and 11 s. Therefore, both the overprediction and underprediction of the wave amplitude variation, seen in the analytical model at short and long wave periods

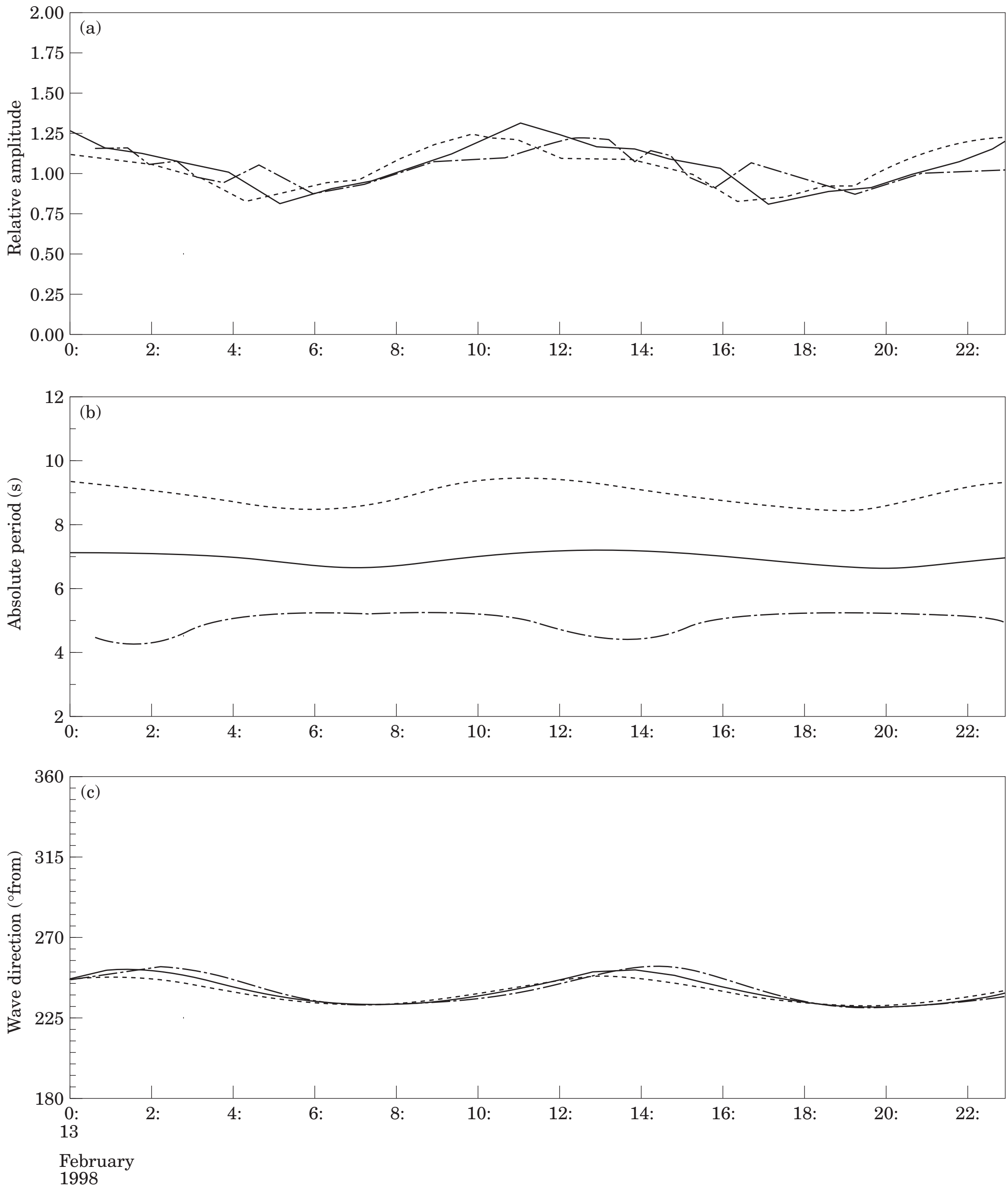

FIGURE 11. Modelled mean wave parameters at the St. Gowan buoy for 13 February 1998 for waves of initial period 5 s (dash-dotted line), 7 s (solid line) and 9 s (dashed line). The mean is taken over 15 min in time and a circle of radius 1 km centred at the buoy in space. (a) amplitude relative to initial amplitude in Celtic Deep, (b) absolute wave period and (c) wave direction.

respectively, is removed in the ray tracing model by the inclusion of non-steady effects.

The magnitude of the semi-diurnal oscillation in absolute wave period is between 1 s and 1·5 s for all initial periods. For waves with initial period greater than 7 s, the maximum period occurs between 10:00 and 11:00h, and 22:00 and 23:00h on 13 February in agreement with the observations. However, the time of this maximum is delayed by several hours at shorter periods.

As at St. Brides, the long period waves are more significantly affected by variations in the water depth than current velocities over the tidal cycle, with refraction by variations in depth more evident at low water than high water. The model predicts significant amplification of 15 s waves (a relative amplitude variation between 0·8 and 1·8 over a tidal cycle), however its effects in reality are likely to be small due to the limited energy in these wave periods.

The model also predicts a weak variation in wave direction over the tidal cycle of around 15°, but observations to verify this result are not available.

Discussion

The wave observations taken at St. Brides Bay and St. Gowan Shoals, in shallow, tidal waters show significant semi-diurnal variation in all wave parameters. In the absence of a semi-diurnal periodicity in the meteorological observations at the two sites, wave refraction by tidal currents has been investigated as the most likely cause of the semi-diurnal oscillation. The quasi-steady, analytical models, which essentially imply that the wave-current interaction is a local phenomenon in which the form of the spatial variation in wave parameters is unimportant, fail to reproduce the observed variations at either site. At St. Brides Bay, where the waves have oblique incidence to the major axis of the tidal currents, the timing of the predicted maximum deviation of the waves by current refraction is significantly in error in the quasi-steady model. At St. Gowan Shoals, where the waves and currents are locally approximately colinear, the analytical model successfully reproduces the timing of the minimum wave amplitude, but underestimates the change in wave amplitude over a tidal cycle.

Waves propagating towards the coast of south-west Wales from the Atlantic, will experience significant spatial and temporal variation in tidal current and elevation over their path. Tolman (1990) suggests that if the temporal variation in tidal currents is neglected, the variation in the wave vector, **k**, may be incorrectly predicted leading to large errors in the prediction of modulated wave height. This study provides corroboration for this conclusion, and further suggests, that for long period waves the temporal variations in water depth is important. The ray tracing model successfully captures many of the observed features of the wave observations at the two buoys. At St. Brides Bay, the timing and magnitude of the maximum deviation of the waves are well predicted; at St. Gowan, the magnitude and timing of the changes in wave amplitude are well predicted. Furthermore, it provides evidence to support the conclusion of Wolf and Prandle (1999) that the phase offset between maximum current and maximum refraction observed during the SCAWVEX Project (and seen in the observations presented in this paper) is a function of the unsteadiness of the tidal current field.

Both observation periods covered the transition from spring to neap tides. It is assumed in the ray tracing model that no wave energy growth or dissipation occurs between the Celtic Deep and the observation sites. At St. Brides Bay, two wind, and hence local wave generating, events on 13 and 17 January 1997, gave significant variation in wave height and period during the observations, clearly violating this assumption. Nevertheless, it remains possible to discern a decrease in the magnitude of the semi-diurnal variation in all wave parameters between spring and neap tides. At St. Gowan, the persistent easterly winds provided a period of approximately constant mean wave heights and periods. However, in this case, there is no discernible decrease in semi-diurnal variation in wave height and period between spring and neap tides. Whilst the quasi-steady analytical model for colinear waves and currents incorrectly predicts a large decrease in the wave height variation as the maximum surface current speed decreases, the ray tracing model correctly predicts a much less significant variation over the spring-neap cycle. Figure 12 shows the mean wave amplitude variation at the St. Gowan buoy on 10 February 1998 (spring tide), 13 February 1998 (mid tide) and 16 February (neap tide) for waves of initial period 7 s. (It should be noted that, as shown in Figure 6, there is a three hour shift in the time of slack water between 10 February and 16 February 1998). The ratio of maximum to minimum wave height during a tidal cycle was observed to be 1·6 over the observation period, and the ray tracing model predicts the ratio to be 1·6 at spring and mid tide and 1·5 at neap tide.

However, significant differences remain between the ray tracing output and the observations. The timing of the maximum wave height at the St. Gowan buoy was in error in both the analytical and ray tracing models. Furthermore, the variation in wave height at the buoy is not sinusoidal over the tidal cycle but is observed to undergo a (relatively) slow growth, followed by a more rapid decrease. This feature is not represented by the ray tracing model. Variations in the location of the initial wave front (in the Celtic Deep) produced negligible differences in the predicted modification of the waves at St. Gowan (or St. Brides). Whilst it is possible that variations in the two dimensional spectrum over the tidal cycle may account for

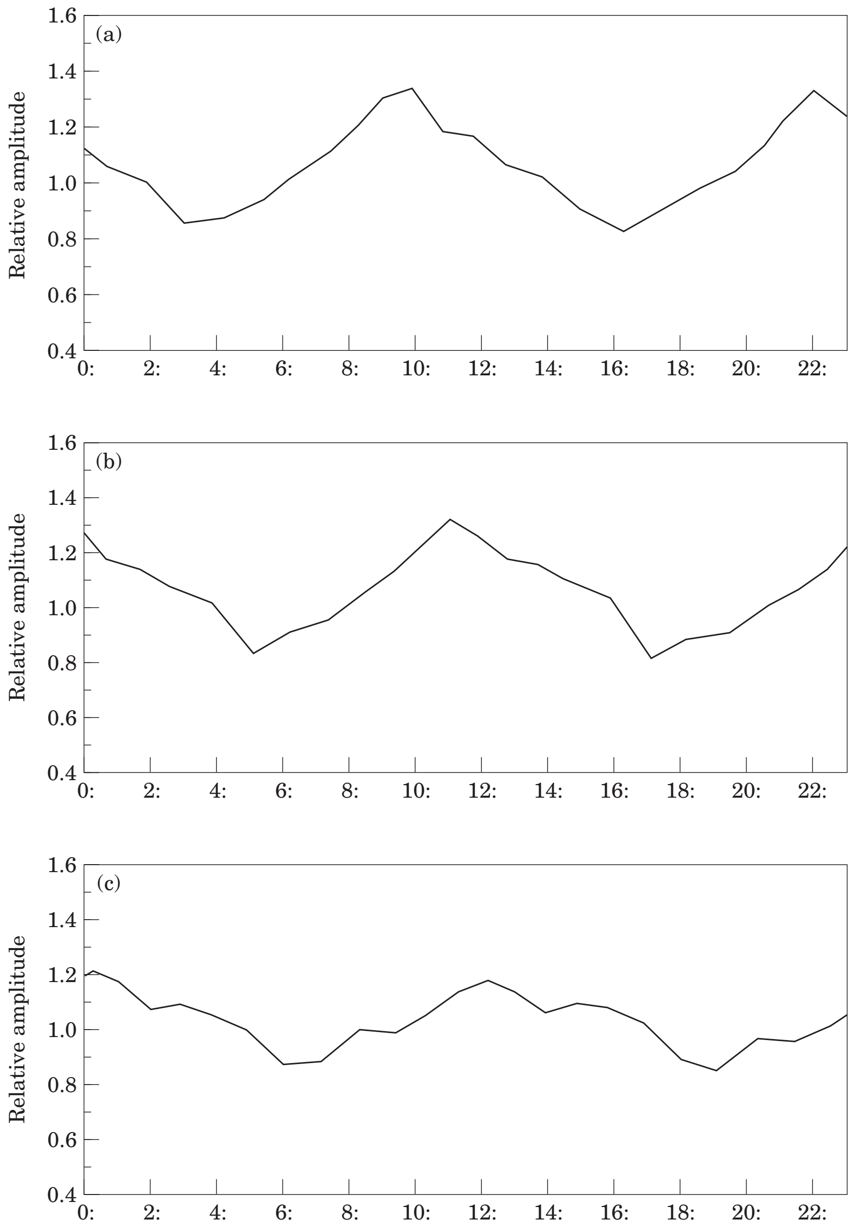

FIGURE 12. Modelled mean wave amplitude relative to the initial amplitude in Celtic Deep at the St. Gowan buoy for waves of initial period 7 s on (a) 10 February 1998—spring tides, (b) 13 February 1998—mid tides, and (c) 16 February 1998—neap tides. The mean is taken over 15 min in time and circle of radius 1 km centred at the buoy in space.

this asymmetry, it is more likely that there is an asymmetry in the tide at the buoy which is not represented by the depth integrated model. (The M2, S2, N2, O1 and K1 constituents are included in the tidal model, but no higher harmonics.) Particular difficulties arose in the interpretation of the ray tracing model results where large variations in the effects of wave refraction occurred between different wave periods e.g. amplitude variations at St. Brides Bay and period variations at St. Gowan. In these cases, a deduction of the effect of wave refraction on integrated spectral properties such as significant wave height and zero-crossing period requires a detailed knowledge of the shape of the wave spectrum in the Celtic Deep. This was unavailable to this study and the modification of the two-dimensional spectrum by tidal currents warrants further research. The role of non-linear interactions between different wave periods under conditions of strong depth and current refraction remains unknown. However, the research presented in this paper suggests that unsteady linear theory adequately represents the modification of waves by tidal currents and that non-linear effects are of secondary importance.

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