Lagrangian and surface-following coordinate approaches to wave-induced currents and air–sea momentum flux in the open ocean.

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Abstract

A short review is presented of selected aspects of modelling the mean velocity profile in the atmospheric and oceanic boundary layers, taking account of the effects of surface waves, using coordinate systems which follow the free surface. The relative merits of Lagrangian and other, more general, coordinate systems, are discussed; also the Generalised Lagrangian Mean formulation of Andrews and McIntyre. Attention is given to the problem of providing a consistent parameterization of wave energy dissipation in order that spectral wave models and models for the ocean current may be coupled together correctly.

1 INTRODUCTION

To include the effects of water waves in a coupled model for the atmospheric and oceanic boundary layers, it is either necessary to resolve individual waves, a procedure which is usually computationally uneconomic, or to use some kind of averaging procedure. The large variations in atmospheric and oceanic properties (fluid velocity, temperature, composition etc.) in the region very near the air–water interface, over vertical distances which may be much smaller than the wave amplitude, will not be properly resolved if the averaging, temporal, spatial (horizontal), or over an ensemble, is performed for fixed vertical co-ordinates (x_3). Much better resolution across the interface will be obtained if a coordinate system is used in which the interface corresponds to a coordinate surface.

2 COORDINATE SYSTEMS

2.1 Types of Coordinate System

An infinite variety of surface-following coordinate systems is, of course, available, the choice of which is according to the convenience of the user. If one can assume that the waves on the interface are of fixed form, e.g. sinusoidal, then the system may be made approximately time-independent by transforming to a moving reference frame [1]. At the other end of the spectrum of possibilities, one can use a Lagrangian formulation, in which the fluid particles have fixed coordinate labels [2, 3]. In general, one can use a time-dependent curvilinear coordinate system: if the coordinate system is such that the mean fluid velocity at a particular coordinate location is equal to the mean drift velocity of a fluid particle passing through the location, the coordinate system corresponds to that of the generalized Lagrangian mean (GLM) formulation of Andrews and McIntyre [4].

2.2 General Formulation

It is advantageous to write the hydrodynamic equations in conservation-law form, also in the case of curvilinear coordinate systems [5]. A general treatment which encompasses a large variety of coordinate systems is described by Jenkins [6, 7]: a brief presentation follows here. The notation is similar to that used by Andrews and McIntyre [4]. The fixed (Cartesian) coordinate system is denoted by $\mathbf{x} = (x_1, x_2, x_3)$, and the curvilinear system by $\mathbf{y} = (y_1, y_2, y_3)$. Vector components in the curvilinear coordinate system continue to be referred to the original Cartesian coordinate directions. Superscripts $(\cdot)^{\mathbf{x}}$ and $(\cdot)^{\mathbf{y}}$ are applied to variables in order to state which coordinate system is being referred to. Partial differentiation with respect to spatial coordinates and time is represented by $(\cdot)_{,\alpha}$, where α may be 1, 2, 3, or t. We assume that the system satisfies the following momentum and continuity equations:

$$\rho^{\mathbf{x}}\left[u^{\mathbf{x}}_{j,t} + u^{\mathbf{x}}_l u^{\mathbf{x}}_{j,l} + \Phi^{\mathbf{x}}_{,j} + 2(\mathbf{\Omega}\times\mathbf{u}^{\mathbf{x}})_j\right] - \tau^{\mathbf{x}}_{jl,l} = 0, \quad (1)$$

$$\rho^{\mathbf{x}}_{,t} + u^{\mathbf{x}}_l \rho^{\mathbf{x}}_{,l} + \rho^{\mathbf{x}} u^{\mathbf{x}}_{l,l} = 0, \quad (2)$$

where ρ is the fluid density, $\mathbf{u} = (u_1, u_2, u_3)$ is the velocity, $\mathbf{\Omega}$ is the rotational angular velocity vector of the Cartesian coordinate system, Φ is a force (e.g. gravitational) potential and τ is a tensor which incorporates both pressure $p = -\frac{1}{3}\tau_{ll}$ and shear stress. The Einstein summation convention is used: repeated indices are summed from 1 to 3.

If we assume that the coordinate transformation $\mathbf{x}^{\mathbf{y}}$ is invertible and differentiable sufficiently many times, with its Jacobian determinant J having co-factors K_{jl}, we may write the momentum equation (Eq. 1) in the following form:

$$P_{j,t} - T_{jl,l} = S_j, \quad (3)$$

where $P_j = \rho^{\mathbf{y}} J u^{\mathbf{y}}_j$ is the 'concentration of x_j-momentum in $\mathbf{y}$-space',

$$T_{jl} = \left[\tau^{\mathbf{y}}_{jm} - \rho^{\mathbf{y}} u^{\mathbf{y}}_j \left(u^{\mathbf{y}}_m - x^{\mathbf{y}}_{m,t}\right)\right] K_{ml} \quad (4)$$

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is minus the flux of x_j-momentum across y_l-surfaces, and

$$S_j = -\rho^{\mathbf{y}} \mathbf{\Phi}^{\mathbf{y}}_{,l} K_{jl} - 2\rho^{\mathbf{y}} J (\mathbf{\Omega} \times \mathbf{u}^{\mathbf{y}})_j \tag{5}$$

is a source function representing the potential (gravity) and Coriolis forces. If ρ is constant, the potential force term can be removed from S_j and incorporated into T_{jl} as an additional term, $-\rho^{\mathbf{y}} \mathbf{\Phi}^{\mathbf{y}} K_{jl}$.

The derivation of Eqs. 3–5 becomes quite straightforward if we employ the four-dimensional coordinate systems $(x_1, x_2, x_3, x_4 = t)$ and $(y_1, y_2, y_3, y_4 = t)$ (Ref. [7] section 2*b*).

2.3 Specific coordinate systems

Obviously, if $x^{\mathbf{y}}_{j,t} = 0$, $J = 1$, and $K_{jl} = \delta_{jl}$, we recover the usual Eulerian formulation of the hydrodynamic equations. If we set $u^{\mathbf{y}}_j = x^{\mathbf{y}}_{j,t}$, we obtain the Lagrangian hydrodynamic equations. If we choose $\mathbf{x}^{\mathbf{y}}$ so that we can simultaneously decompose $\mathbf{x}^{\mathbf{y}}$ and $\mathbf{u}^{\mathbf{y}}$ into mean and fluctuating parts:

$$\mathbf{x}^{\mathbf{y}} = \mathbf{y} + \xi^{\mathbf{y}}, \qquad \overline{\xi^{\mathbf{y}}} = 0; \tag{6}$$

$$\mathbf{u}^{\mathbf{y}} = \overline{\mathbf{u}^{\mathbf{y}}} + (\mathbf{u}^{\mathbf{y}})', \qquad \overline{(\mathbf{u}^{\mathbf{y}})'} = 0; \tag{7}$$

in such a way that

$$(u^{\mathbf{y}}_j)' = \xi^{\mathbf{y}}_{j,t} + \overline{u^{\mathbf{y}}_l}\,\xi^{\mathbf{y}}_{j,l}, \tag{8}$$

we recover the GLM equations, with $\overline{\mathbf{u}^{\mathbf{y}}}$ as the generalized Lagrangian mean velocity.

2.4 A Coordinate System for Both Air and Water

The GLM representation is very elegant and powerful: unfortunately, singularities may appear in the fluctuating fields $\xi^{\mathbf{y}}$, $(\mathbf{u}^{\mathbf{y}})'$ etc., for example, at critical levels, where the mean velocity resolved along the wavenumber direction of a wavelike component of the flow is equal to its phase propagation speed, even where there is no singularity in the flow itself. To avoid this singular behaviour it can be valuable to take the more general approach of section 2.2. In the case of wind blowing over surface waves, there may be critical levels in the atmospheric boundary layer, where the wind speed is equal to the phase speed of a wave component, and also in the near-surface region of the water column, if there is wave breaking, or if capillary rollers or bores are formed near the wave crests. It can then be useful to employ a coordinate system, such as that shown in Fig. 1, which is determined by the form of the interface, but not necessarily directly by the flow field either in the air or in the water column.

2.5 Reynolds Stress

The quantity T_{jl} in Eqs. 3–4 has the function of a stress: if the quantities $\tau^{\mathbf{y}}$, $\mathbf{u}^{\mathbf{y}}$, etc. are split into mean and fluctuating parts, we may obtain equations similar to

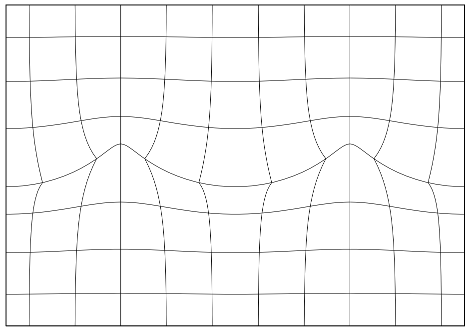

Figure 1: Example of a coordinate system, above and below a wave surface. In this particular case we have $\mathbf{x} = \big(y_1 - ae^{-k|y_3|}\sin(ky_1 - \omega t), y_2, ae^{-k|y_3|}\cos(ky_1 - \omega t)\big)$. Above the interface, the coordinate system is isomorphic: below the interface $J = 1 + O\left((ak)^2\right)$.

those involving Reynolds stress for incompressible fluids in fixed coordinate systems. However, the equivalent to the Reynolds stress is normally more complicated, with many more terms, including some which involve fluctuations in the coordinate system transformation. An example, from a model which applies perturbation theory to $O\left((ak)^2\right)$, is shown in Fig. 2, in which we see that the total mean stress, which is equal to the mean turbulent shear stress $\overline{\tau_{13}}$ at the top of the boundary layer, has at the water surface a large fraction supported by the pressure-slope covariance $\overline{p'\zeta_{,1}}$. The wave-induced apparent Reynolds stress $-\rho\overline{u'w'}$ makes a moderate contribution at intermediate levels, but it can be seen that the coordinate-transformation-dependent terms $\overline{u'\zeta_{,t}}$ and $U\overline{u'\zeta_{,1}}$ make large (and oppositely-directed) contributions.

In the particular case considered here, of wind blowing over waves, it was found necessary, for simplicity in the calculation, to neglect contributions to the mean ('eddy') viscous shear stress which are of second and higher order with respect to the wave slope ak. A similar simplification was found to be necessary by Groeneweg and Klopman [8] when applying the GLM theory to wave–current interaction under the influence of viscous and/or turbulent shear stresses.

2.6 Remark on perturbation theory

Equations of motion in Lagrangian and other time-dependent curvilinear coordinates, particularly if they involve second derivatives of the velocity field, are complex, and perturbation expansions contain very many terms. Xu and Bowen [9], when treating the problem of wave-induced currents in finite water depth, avoided this problem by retaining the Eulerian representation.

An alternative method was applied by Jacobs [10] in his analysis of wind over waves. He applied the theory of domain perturbations [11], in which modified dependent variables satisfy the same equations in $\mathbf{y}$-coordinate space as the original variables do in the fixed Cartesian $\mathbf{x}$-coordinate

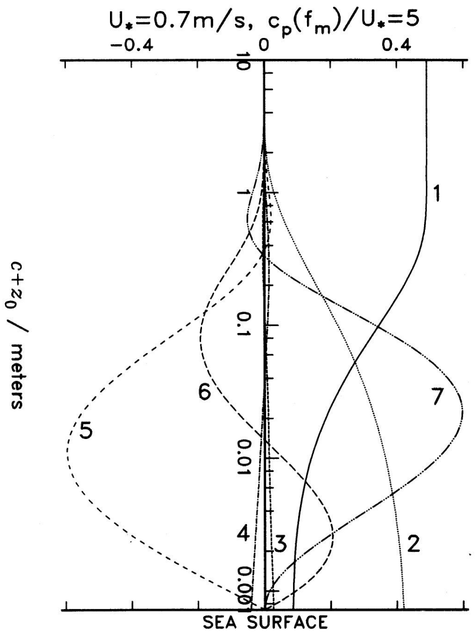

Figure 2: Computed vertical profile of the various contributions to the downward momentum flux over wind waves, calculated by the quasi-linear eddy-viscosity-based model of Jenkins [7]. 1, $\overline{\tau}_{13}/\rho$; 2, $\overline{p'\zeta_{,1}}/\rho$; 3, $-\overline{\sigma'_{11}\zeta_{,1}}/\rho$; 4, $\overline{\tau'_{13}\xi_{,1}}/\rho$; 5, $\overline{u'\zeta_{,t}}$; 6, $-\overline{u'w'}$; 7, $U\overline{u'\zeta_{,1}}$. Notation: c is the vertical curvilinear coordinate, with $c = 0$ being the water surface—see Fig. 1; ξ and ζ are the horizontal and vertical coordinate displacements; U is the mean horizontal velocity; u and v are the horizontal and vertical velocity components; $\sigma_{jl} = \tau_{jl} + p\delta_{jl}$ is the traceless stress tensor; overbars and primes denote mean and fluctuating values with respect to the curvilinear coordinate system. (Reprinted from *Journal of Physical Oceanography*, © 1992 American Meteorological Society.)

space. The solutions to the equations are then transformed to represent the original variables by adding terms involving the perturbation expansion of the coordinate transformation. This domain perturbation method may reduce the amount of algebraic manipulation necessary, at the cost of being somewhat more difficult conceptually.

3 NEAR-SURFACE WAVE-INDUCED CURRENTS

3.1 Introduction

The computation of drift currents induced by wind and waves, in the presence of (eddy) viscosity and rotation, has been the subject of numerous studies (e.g. [1, 3, 9, 12–19]). I will restrict myself here to discussing currents near the sea surface induced only by local wind and wave action, and discuss two problems in particular: (1) the relation between viscous and inviscid wave-induced currents in the presence of rotation; (2) the interrelationships been air–sea momentum flux and the wave generation and dissipation.

3.2 Wave-Induced Ekman Spirals and Inertial Oscillations

According to Ursell [13], it is impossible for a steady mean drift current to be generated by irrotational surface gravity waves in an inviscid, rotating ocean. This result is apparently inconsistent with the necessity of a mean drift current (Stokes drift) for irrotational waves in a non-rotating reference frame [12]. This paradox was resolved in an elegant way by Pollard [14], who found an exact solution of the Lagrangian hydrodynamic equations which was a sum of Gerstner waves [20], which are rotational but which have no mean drift, and depth-dependent inertial oscillations, so that at one phase in the inertial cycle the flow is irrotational, but Coriolis force subsequently deflects the current away from the wave direction.

Figure 3 shows what happens to the drift current due to surface waves of a single wavenumber when a small (eddy) viscosity is added to the system—the inertial oscillations, which start when a wave field propagates into the system, are gradually damped out [16]. It is conceivable that a situation like this may arise if the ocean is calm and significantly stratified. If the eddy viscosity is increased, the drift current behaves more like that in a classical Ekman layer [21, 22], with a spiral hodograph for the mean current, and Fredholm spiral time dependence for the the current at given depths. Since no wind forcing is applied, the momentum flux which drives the current is generated by the 'viscous' damping of the wave field, which, though specified to have a constant wave height for $t > 0$, must decay along the wave propagation direction. Note that if the ocean is initially at rest, and waves propagate into the area at time $t = 0$, the drift current 'immediately' increases to the appropriate 'Stokes drift' value.

The model which was used in these simulations is based upon a perturbation expansion of the Lagrangian hydrodynamic equations. Because of the similarity of the Lagrangian coordinate system, for short drift intervals, to the sub-surface coordinate system shown in Fig. 1, I anticipate that the same results would be obtained from an analysis based upon the latter coordinate system, or, indeed, from a GLM formulation of the problem.

3.3 Wave Generation/Dissipation and Coupling of Wave and Current Models

As discussed above, the flux of momentum into the wave field from wind forcing, and the flux of momentum from the wave field into the current when waves dissipate, must be taken into account. It is also possible under certain circumstances, such as when swell propagates in light-wind conditions, for the waves to be damped by atmospheric forcing and for the wave momentum to drive an airflow in the near-surface boundary layer [23, 24, 25]. The prob-

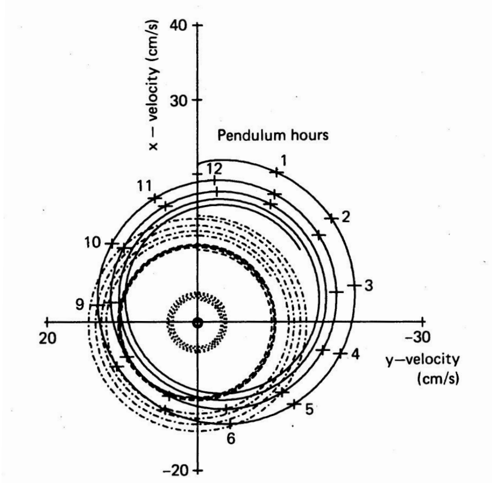

Figure 3: (After Jenkins [16].) Development in time (0 to 50 pendulum hours) of the mass transport velocity at depths of 0, 1, 2, 5, and 10 metres, after a monochromatic wave field of wave height 1.76 m propagates into the area of interest. Eddy viscosity = 10^{-5} m^2 s^{-1}. (Reprinted from *Journal of Physical Oceanography*, © 1986 American Meteorological Society.)

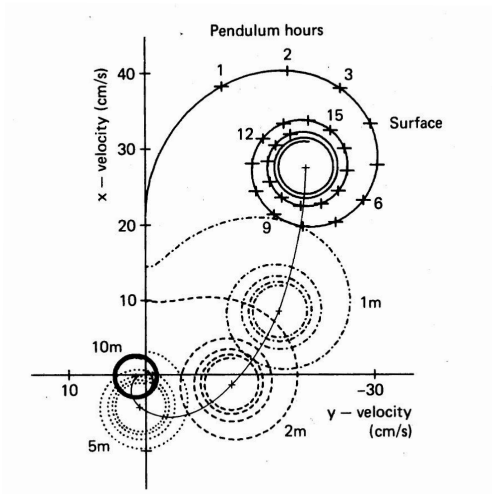

Figure 4: (After Jenkins [16].) Development in time of the mass transport velocity due to the same wave field as in Fig. 3. Eddy viscosity = 10^{-5} m^2 s^{-1}. (Reprinted from *Journal of Physical Oceanography*, © 1986 American Meteorological Society.)

lem of wind-wave generation has been discussed by many authors (e.g. [7, 10, 26–35]); in this section I will primarily touch on the other problem, that of wave damping or dissipation.

The simplest wave dissipation case to deal with is when the (eddy) viscosity is constant. In this case, in linear deep-water gravity wave theory we may decompose the oscillatory motion into an irrotational part, which decays with depth (increasing negative y_3) as e^{ky_3}, and a rotational component which decays rapidly with depth, as $e^{\gamma y_3}$, with $\gamma = \sqrt{\omega/(2\nu)}$, where ω is the wave angular frequency and ν is the (eddy) viscosity. In the absence of other forcing, the wave amplitude will then decay with time [36] as $\exp\left(-\frac{2\nu k^2}{\omega}t\right)$, and the wave momentum will be transferred into the water column with an apparent source at the surface.

In the case where we may regard the waves as being damped by an eddy viscosity ν which varies with depth, the situation becomes more complex: we still have an irrotational oscillatory flow with an e^{ky_3} depth dependence, but the rotational component extends to greater depths [17]. Under the assumption that ν is constant within the vorticity layer which extends downwards from the surface with an e-folding depth of γ^{-1}, Jenkins [17, 37] determined that the waves decay with time according to

$$a \propto \exp\left[-\frac{2k^2}{\omega}\left(\int_{-\infty}^{0} 2k\,\nu(y_3)\,e^{2ky_3}\,dy_3\right)t.\right]$$

He also found that the momentum was then transferred from waves to the current partly at the surface, at a rate given by the surface value of ν, and the rest from a diffuse source distributed within the water column as $\nu_{,3}e^{2ky_3}$.

It is then tempting to simulate the wave-dissipation effects of, for example, wave breaking and whitecapping (e.g. [38]), by employing a vertically-varying eddy viscosity which has the same wave-frequency-dependent wave-damping effect. Unfortunately, it is impossible to use the same eddy viscosity to damp the wave energy as one uses for the diffusion of momentum within the current field: the former must be much smaller than the latter. This can be understood in terms of the fact that the current will be affected by turbulent eddies and other motions such as Langmuir circulations, which have time scales too great to respond to ocean waves sufficiently rapidly to exert a wave-damping effect. It is therefore necessary to employ timescale-dependent eddy-viscosity profiles in order to use this approach, and this method was applied for the first time, to my knowledge, by Jenkins [39], the results being reproduced here in Figs. 5–6. It was found necessary to adjust the results of the wave model used in this simulation to ensure that its formulation of energy, momentum, and wave action conservation was consistent throughout the whole wave spectrum.

More precise formulations of this problem necessitate the analysis of detailed laboratory and/or field experiments and also realistic wave-resolving numerical simulations (e.g., [40–47]).

4 CONCLUSION

I have given here what is necessarily a very brief description of the atmosphere–wave–sea momentum flux problem, from the point of view of a modeller of near-sea-surface processes. Notwithstanding the difficulties which may arise in the analysis, the use of surface-following coordinates has its advantages, since such coordinates enable

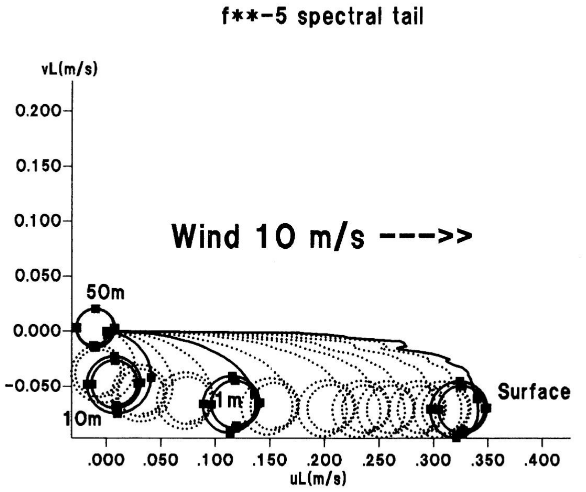

Figure 5: (After Jenkins [39].) Evolution of the surface drift current when the wave field is calculated using a spectral wave model (a 1-point version of WAM [48]). The significant wave height is initially zero, increases to 1.5 m after 9 hours and to 1.8 m after 36 hours. The rapid oscillations in the current during the first few hours may be due to the numerical properties of the version of the wave model used, and reflect the oscillations in the Stokes drift shown in Fig. 6. (Reprinted from *Deutsche Hydrographische Zeitschrift*, © 1989 Bundesamt für Seeschifffahrt und Hydrographie.)

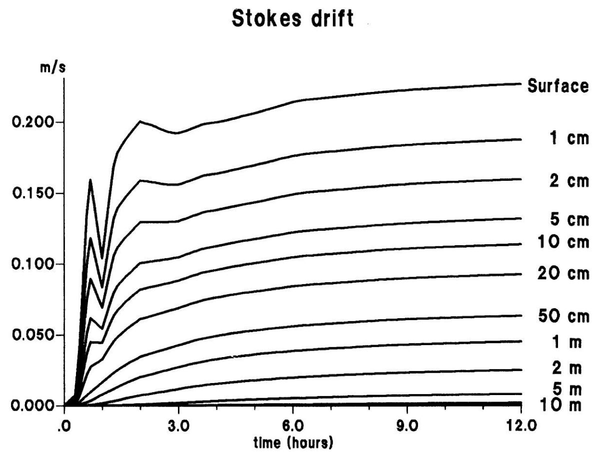

Figure 6: (After Jenkins [39].) The Stokes drift computed from the wave model results used in the coupled model runs for Fig. 6. (Reprinted from *Deutsche Hydrographische Zeitschrift*, © 1989 Bundesamt für Seeschifffahrt und Hydrographie.)

a fine resolution of what are expected to be large gradients in the dependent variables in the cross-interface direction. This is particularly important when considering heat and mass flux problems, since such large gradients are indeed observed [47, 49–52].

5 ACKNOWLEDGEMENTS

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