On the Effect of Ocean Waves on the Kinetic Energy Balance and Consequences for the Inertial Dissipation Technique

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ABSTRACT

For large wind speed (in practice >15 m s^{-1}) observations of the surface stress by means of the inertial dissipation technique are so close to the surface that effects of growing ocean waves on the turbulent kinetic energy budget should be taken into account. This may give rise to an increase in the surface stress of 20%.

1. Introduction

The inertial dissipation technique (cf., e.g., Edson et al. 1991 and Yelland et al. 1994) infers the dissipation rate ε from the measurement of the high-frequency part of the turbulent velocity spectrum by assuming the Kolmogorov spectrum in wavenumber space:

$$G(k) = \alpha\varepsilon^{2/3}k^{-5/3}, \tag{1}$$

(where α is Kolmogorov's constant) and applying Taylor's hypothesis, which relates wavenumber and frequency turbulent spectra. The stress in the surface layer is then obtained from the assumed balance between production and dissipation of turbulent kinetic energy.

It is argued in this note that in the presence of growing long ocean waves the energy flux associated with the wave-induced pressure fluctuations is also relevant in the kinetic energy balance. This may give rise to corrections in the surface stress over long, wind-generated ocean waves of the order of 20%. As a result, the mean Charnock parameter increases by almost a factor of 2 from 1.1 to 2.2 ($\times10^{-2}$).

2. Effect of ocean waves on kinetic energy balance

Distinguish between a mean air flow U_O and fluctuations δu and consider the kinetic energy of fluctuations $e = \frac{1}{2}\delta u_i \delta u_i$. We disregard buoyancy effects because for the large wind speeds of interest they are not important. The energy budget at a certain height z then becomes (Kaimal and Finnigan 1994)

$$\frac{\partial}{\partial t}e + \mathbf{U}_O\cdot\nabla\overline{e} = \tau\frac{\partial}{\partial z}U_O - \frac{1}{\rho_a}\frac{\partial}{\partial z}\overline{(\delta p\delta w)} - \frac{\partial}{\partial z}\overline{(e\delta w)} - \varepsilon. \tag{2}$$

We would like to apply the kinetic energy balance to the inertial dissipation method. In order to do that it is assumed that the energy balance is stationary, $\partial\overline{e}/\partial t = 0$, and that advection is unimportant. Even in the presence of ocean waves these are reasonable assumptions because the kinetic energy associated with ocean waves in the air is smaller by the ratio of air to water density compared to the kinetic energy of the water motion. Furthermore, it is standard practice to disregard the diffusion terms in the kinetic energy balance to obtain

$$\tau\frac{\partial U_O}{\partial z} = \varepsilon. \tag{3}$$

Using the logarithmic wind profile,

$$U_O = \frac{u_*}{\kappa}\ln\frac{z}{z_O}, \tag{4}$$

one then finds

$$\tau = (\kappa z\varepsilon)^{2/3}, \tag{5}$$

which gives the surface stress when the dissipation rate ε at height z is known, or, using (1),

$$\tau = (\kappa z)^{2/3}\frac{k^{5/3}G}{\alpha}. \tag{6}$$

It is pointed out now that in the presence of wind-generated ocean waves the term involving the pressure fluctuation is relevant as well. [The term involving vertical transport of kinetic energy is not because it is nonlinear in wave energy and waves have small steepness.] We denote the pressure term as

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$$I_w = -\frac{1}{\rho_a}\frac{\partial}{\partial z}\overline{(\delta p \delta w)}, \qquad (7)$$

and I_w is estimated as follows. We assume potential flow for the wave-induced motion and make use of the boundary condition that, for $z \to 0$, $\delta w = \partial \eta/\partial t$, where η is the surface elevation. We have

$$\overline{\delta p \delta w} = \int dk\, e^{-2kz}\left\langle p_k \frac{\partial \eta_k}{\partial t}\right\rangle, \qquad (8)$$

where the term within angle brackets is simply the source term for the growth of waves by wind. Hence (cf., e.g., Kinsman 1965)

$$\left\langle p_k \frac{\partial \eta_k}{\partial t}\right\rangle = -\gamma F \qquad (9)$$

with γ the growth rate of waves by wind and F the wave spectrum. The pressure term now becomes

$$I_w = -\frac{2}{\rho_a}\int dk\, k\gamma F(k) e^{-2kz}. \qquad (10)$$

In order to get some idea about the order of magnitude of I_w, a simple model for spectral shape and wind input is taken. We choose for the spectrum

$$F(k) = \begin{cases} 0, & k < k_p \\ \frac{1}{2}\alpha_p \rho_w g k^{-3}\delta(\theta - \theta_w), & k > k_p \end{cases} \qquad (11)$$

with k_p the peak wavenumber, α_p the Phillips parameter, and θ_w the mean wave direction. Furthermore, θ is the wave direction and δ is the Dirac delta function. The wind input term is taken from the observations compiled by Plant (1982) and reads

$$\gamma = \begin{cases} 0, & k < \overline{k} \\ \beta\omega\left(\frac{u_*}{c}\right)^2 \cos^2(\theta - \phi_w), & k > \overline{k}, \end{cases} \qquad (12)$$

where $\overline{k}$ is the mean wavenumber and ϕ_w is the wind direction while β is a constant. Inserting (11) and (12) in (10) gives, with $s = \rho_a/\rho_w$,

$$I_w = -\frac{\alpha_p\beta}{s}u_*^2 g^{1/2}\cos^2(\theta_w - \phi_w)\int_{k_o} dk\, k^{-1/2}e^{-2kz}, \qquad (13)$$

where k_o is the maximum of k_p and $\overline{k}$ and $k_p = g/U_{10}^2$.

It is important to note that the longer waves give the dominant contribution to the pressure term because of the presence of the exponential factor $\exp(-2kz)$. In fact, for given observation height z, only the waves with wavelength longer than $2\pi z$ will contribute.

The integral in (13) may be written in terms of the error function

$$\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^\infty dt\, e^{-t^2};$$

hence

$$I_w = -\frac{\alpha_p\beta}{s}u_*^2 g^{1/2}\sqrt{\frac{\pi}{2z}}\,\mathrm{erfc}(y)\cos^2(\theta_w - \phi_w) \qquad (14)$$

with $y = \sqrt{2k_o z}$.

Finally, the Phillips parameter α_p obeys a fetch law. We will use

$$\alpha_p = \hat{\alpha}\frac{u_*}{c_p}, \qquad \hat{\alpha} = 0.25, \qquad (15)$$

and (14) becomes

$$I_w = -\frac{\hat{\alpha}\beta}{s}u_*^3\sqrt{\frac{\pi k_p}{2z}}\,\mathrm{erfc}(y)\cos^2(\theta_w - \phi_w). \qquad (16)$$

For energy balance considerations we may rewrite (16) as follows:

$$I_w = \frac{u_*^3}{\kappa z}I_w^*, \qquad (17)$$

where

$$I_w^* = -\frac{\kappa\hat{\alpha}\beta}{s}y\sqrt{\frac{\pi}{4}}\,\mathrm{erfc}(y)\cos^2(\theta_w - \phi_w). \qquad (18)$$

Taking now the effect of gravity waves into account, the relevant energy balance equation becomes

$$\frac{u_*^3}{\kappa z}[1 + I_w^*] = \varepsilon;$$

hence, we find for the stress $\tau = u_*^2$,

$$\tau = \left\{\frac{\kappa z \varepsilon}{1 + I_w^*}\right\}^{2/3}. \qquad (19)$$

Since I_w^* is negative it is concluded that the inclusion of this wave effect will increase estimates of the stress by means of the modified inertial dissipation method. That is, the standard implementation of the inertial dissipation method using (5) results in an underestimation of the stress for developing sea. The percentage of underestimation is predicted to increase as the measurement height moves closer to the surface for a given sea state.

In order to have some idea about the magnitude of the effect of waves on the kinetic energy balance, the following values for parameters

$$\beta = \frac{0.1}{\pi}, \qquad \hat{\alpha} = 0.25, \qquad s = 1.2210^{-3} \quad \text{and}$$

$$\kappa = 0.41$$

are chosen.

Then,

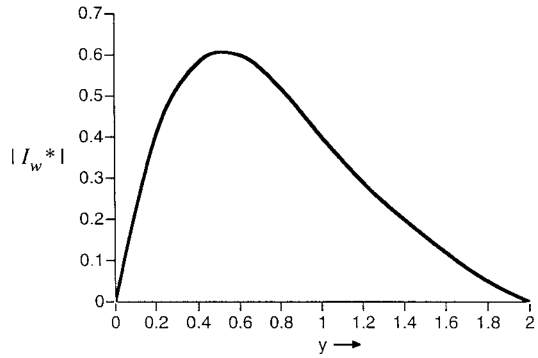

FIG. 1. Plot of I_w^* vs y.

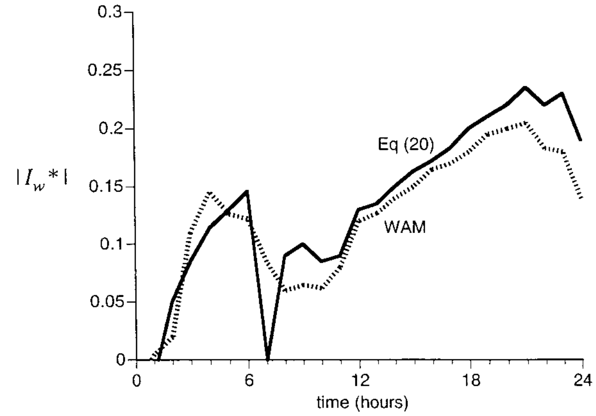

FIG. 2. Comparison of WAM result with Eq. (20).

$$I_w^* = -1.33y\sqrt{\pi}\,\mathrm{erfc}(y)\cos^2(\theta_w - \phi_w),$$

and using the inequality (Abramowitz and Stegun 1972)

$$\mathrm{erfc}(y) \leq \frac{2}{\sqrt{\pi}}\frac{e^{-y^2}}{y + \sqrt{y^2 + 4/\pi}},$$

we obtain

$$I_w^* = -2.66\frac{ye^{-y^2}}{y + \sqrt{y^2 + 4/\pi}}\cos^2(\theta_w - \phi_w),$$
$$y = \sqrt{2k_0z}. \qquad (20)$$

It is remarked that the upperbound for the error function gives in the range $y > 0.2$ an overestimate of, at most 10%.

In Fig. 1 I_w^* is plotted as a function of y for $\theta_w = \phi_w$. It is clear that when $k_oz > 1$ ($y > \sqrt{2}$) the effect of waves on the energy balance is small, but in the opposite case it may be quite considerable, giving a maximum increase in friction velocity u_* of 35%.

In closing this section it is emphasized that, using a simple model of the wind input and the wave spectrum, the effect of energy transfer from mean wind to ocean waves on the kinetic energy budget has been estimated. For long waves this effect seems to be considerable. In order to make sure that these estimates are realistic, we have estimated I_w^* [Eq. (10)] using the wind input term and wave spectrum of the WAM model. To that end, we ran the single gridpoint version of WAM with a time series for wind speed and direction provided by Yelland some time ago, and the comparison between the WAM result and Eq. (20) for an observation height $z_{obs} = 10$ m is shown in Fig 2. Noting that wind speeds for this case varied between 9 and 15 m s^{-1}, and are therefore quite low, we see that the WAM is also giving considerable corrections to the turbulent kinetic energy budget. Also, in view of the simplicity of wind input and wave spectrum, the agreement between Eq. (20) and the WAM result is remarkable, providing confidence in the use of Eq. (20) in correcting stress data obtained by the inertial dissipation method.

3. Consequences for the inertial dissipation technique

M. Yelland provided me with observations on wind speed, direction, and surface stress from a cruise that took place in the first few months of 1993 in the southern Atlantic and Southern Ocean, south of South Africa. Surface stresses were obtained with the inertial dissipation technique, including corrections for stability and an empirical imbalance term. The Kolmogorov constant was 0.55.

Yelland's dataset was collocated with results from the operational WAM model and ECMWF surface winds so that modeled stresses from WAMcy4 could be compared with observations. Comparing observed and modeled surface winds a fair agreement between the two was found, although modeled winds were somewhat lower. In addition, information on parameters such as the mean wavenumber, wave height, and wave direction was available so that I_w^* according to Eq. (20) could be evaluated. Applying Eq. (19), the observed stresses were corrected. In addition, the Kolmogorov constant was reduced to a value of 0.5, in closer agreement with the consensus value of 0.52 found in the review by Högström (1990).

Figure 3 shows a comparison of observed drag coefficient versus wind speed for corrected data and the original data. The small increase in drag coefficient below $U_{10} = 15$ m s^{-1} is mainly caused by the change in Kolmogorov constant. For large wind speed ($U_{10} > 15$ m s^{-1}) it is seen that the original data show hardly any scatter, while the corrected data give substantially larger drag coefficient and an increase in scatter. It is also noted that stresses obtained with the eddy correlation technique (cf. Donelan 1982 and Smith et al. 1992) show a considerable scatter. Part of this scatter is of a geophysical nature since both Donelan and Smith et al. argue that the drag coefficient not only depends on wind speed but also on the sea state through the wave age.

The lack of scatter in the stress data obtained with the standard inertial dissipation technique suggests that

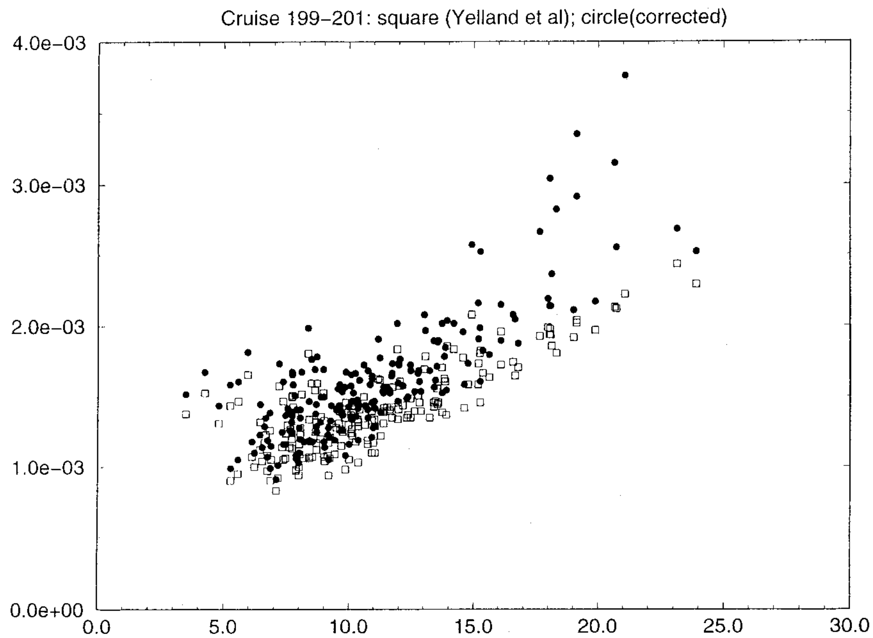

FIG. 3. Comparison of original (from Yelland and Taylor 1996) and corrected observed drag coefficient vs wind speed.

the drag does not depend on the sea state (cf. also Yelland and Taylor 1996). However, this contrasts the results obtained with the eddy correlation technique by Donelan (1982) and Smith et al. (1990).

Finally, it is also of interest to compare observed results with modeled drag. This is done in Fig. 4 where we compare modeled drag coefficient with corrected observed drag. The modeled drag coefficient was obtained from the WAM. These modeled drag coefficients are expected to be fairly accurate in view of the favorable agreement that exists between the Humidity Exchange of the Sea (HEXOS) dataset and stressed obtained with the WAM wind input source function using observed wave spectra (Janssen 1992). It is noted that in the high wind speed range modeled drag shows a considerable scatter (although it is lower than the cor-

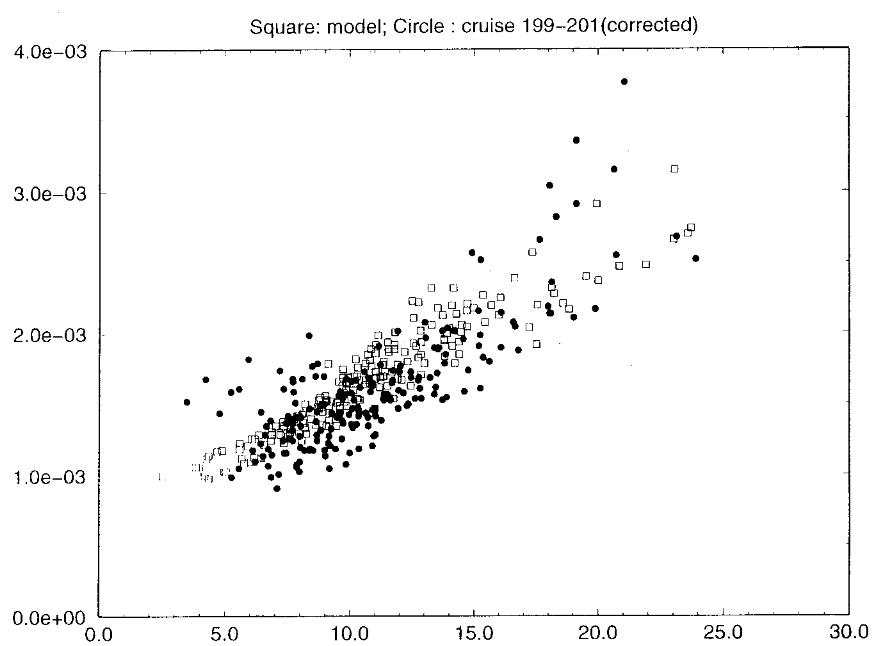

FIG. 4. Comparison of modeled and corrected drag coefficient vs wind speed.

rected data). Overall, there is fair agreement between observed and modeled drag, which also follows from a comparison between average observed and modeled Charnock parameter. Restricting the wind speed range to above 10 m s^{-1}, the average modeled Charnock parameter is found to be 2.3×10^{-2} while the observed one (corrected) is 2.2×10^{-2}. The original data only have an average Charnock parameter of 1.1×10^{-2}. This suggests that the proposed correction to the energy balance has a considerable impact on observed stresses with the inertial dissipation technique.

4. Conclusions

Recent results obtained with the inertial dissipation method (Yelland and Taylor 1996) suggest a mean Charnock parameter of about 0.01. Although the inertial dissipation technique is an appealing method in certain respects (compared to the eddy correlation technique it may be easily implemented on a ship since there are fewer problems with flow distortion), it also has its drawbacks because of the number of assumptions involved. For example, an empirical imbalance term is needed, the Kolmogorov constant varies from one application to the other, and according to this note, the assumption that growing waves have no impact on the kinetic energy balance is also not justified. Furthermore, as suggested by Yelland et al. (1996), stresses obtained with the inertial dissipation technique show no sea state dependence at all, which contrasts with results obtained by Donelan (1982) and HEXOS.

Correcting the kinetic energy budget by including the pressure term associated with growing waves gives considerable differences regarding the sea state dependence of the surface stress. For large winds, the drag coefficient increases compared to the standard inertial dissipation technique while the mean Charnock parameter is increased by a factor of 2.

It is concluded therefore that results obtained with the inertial dissipation method are uncertain. The main reason for this is that, in order to obtain viable surface stress measurements, a considerable number of assumptions have to be made. In addition, a considerable number of corrections (both of a semiempirical and theoretical nature) have to be applied. For these reasons it seems that an alternative method, namely the eddy correlation technique, seems to be preferable. The main drawback of this method is flow distortion, but during HEXOS (Smith et al. 1992) it was shown how to deal in a semiempirical way with this problem when measuring from a stable platform. Flow distortion correction around a cruising ship is, however, considerably more difficult.

REFERENCES

Abramowitz, M., and I. Stegun, 1972: *Handbook of Mathematical Functions.* Dover, 1046 pp.

Donelan, M. A., 1982: The dependence of aerodynamic drag coefficient on wave parameters. *Proc. First Int. Conf. on Meteorology and Air/Sea Interactions of the Coastal Zone,* The Hague, the Netherlands, Amer. Meteor. Soc.

Edson, J. B., C. W. Fairall, P. G. Mestayer, and S. E. Larsen, 1991: A study of the inertial-dissipation method for computing air–sea fluxes. *J. Geophys. Res.,* **96** (C6), 10 689–10 711.

Högström, U., 1996: Review of some basic characteristics of the atmospheric surface layer. *Bound.-Layer Meteor.,* **78,** 215–246.

Janssen, P., 1992: Experimental evidence of the effect of surface waves on the airflow. *J. Phys. Oceanogr.,* **22,** 1600–1604.

Kaimal, J. C., and J. I. Finnigan, 1994: *Atmospheric Boundary Layer Flows: Their Structure and Measurement.* Oxford University Press, 289 pp.

Kinsman, B., 1965: *Wind Waves: Their Generation and Propagation on the Ocean Surface.* Prentice-Hall, 676 pp.

Plant, W. J., 1982: A relation between wind stress and wave slope. *J. Geophys. Res,* **C87,** 1961–1967.

Smith, S. D. and Coauthors, 1992: Sea surface wind stress and drag coefficients: The HEXOS Results. *Bound.-Layer Meteor.,* **60,** 109–142.

Yelland, M. J., and P. K. Taylor, 1996: Wind stress measurements from the open ocean. *J. Phys. Oceanogr.,* **26,** 541–558.

——, ——, I. E. Consterdine, and M. E. Smith, 1994: The use of the inertial dissipation technique for shipboard wind stress determination. *J. Atmos. Oceanic Technol.,* **11,** 1093–1108.