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A note on wave energy dissipation over steep beaches

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Abstract

This paper revisits the derivation of the parametric surf zone model proposed by Baldock et al. [Baldock, T. E., Holmes, P., Bunker, S. & Van Weert, P. 1998 Cross-shore hydrodynamics within an unsaturated surf zone. Coast. Eng. 34, 173–196.]. We show that a consistent use of the proposed Rayleigh distribution for surf zone wave heights results in modification of the expressions for the bulk dissipation rate and enhanced dissipation levels on steep beaches and over-saturated surf zone conditions. As a consequence, the modification proposed herein renders the model robust even on steep beaches where it could otherwise develop a shoreline singularity.

Keywords: Wave breaking; Random waves; Steep beach; Surf zone; Rayleigh distribution

1. Introduction

Of all the processes affecting wave propagation in the nearshore across a surf zone, the instability process generally referred to as the 'breaking' of the wave is cumulatively by far the most important and at the same time probably least understood. The wide variety of appearances of breaking waves, from the gentle 'spilling' breaker on mildly sloping beaches to the violent 'shore break' on steep slopes, and the sheer complexity involved in a detailed description of the transition from smooth-surfaced macro-scale motion to one that is increasingly chaotic and involves micro-scale turbulent motions (see e.g. Peregrine, 1983), hampers a first-principle-based modeling on most any operational scale. Instead, a common approach to the modeling of wave evolution in the nearshore is to parameterize the breaking process to account for its macro-scale effects.

Many of such parametric models are based on the work by Battjes and Janssen (1978) (BJ78 hereafter), who estimate the bulk dissipation rate in a random wave field and include this as a sink term in the energy balance equation for the wave motion. The use of a balance equation incorporates the history of the propagation, thus relaxing an overly strong dependence on local bed variations as inherently present in earlier approaches (e.g. Battjes, 1972; Goda, 1975), and extending application to topography involving non-monotonically decreasing depth.

BJ78 estimate the bulk dissipation rate utilizing the analogy with turbulent bores (e.g. Lamb, 1932, article 187) while describing surf zone wave height statistics through a clipped Rayleigh distribution, truncated at a maximum wave height at which all waves are assumed to break. Although this distribution is arguably a crude representation of the broken wave height distribution (as noted by BJ78), it has proven a remarkably successful conceptual model to estimate the bulk dissipation rate in a random wave field across the surf zone. However, the use of the clipped distribution results in a transcendental equation for the fraction of broken waves Q. Further, on steep beaches, the BJ78 model predicts insufficient dissipation to counter shoaling, which BJ78 ameliorated by enforcing a saturated inner surf zone.

Although a theoretical basis for the distribution of wave heights in a dissipative surf zone is lacking, Thornton and Guza (1983) (TG83 hereafter) empirically found that wave heights across the surf zone are remarkably well described by a (complete) Rayleigh distribution, regardless whether the waves

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are breaking or not. Also, from detailed observations of breaking waves on a Pacific-exposed beach, they found that the distribution of the heights associated with broken waves has a fairly wide support, i.e. waves with a range (not a single value) of heights are breaking at a given depth. Consequently, to extend the BJ78 model to include a more realistic description of the transformation of the wave height distribution function across a dissipative surf zone, TG83 propose a smooth empirical distribution function for the breaking wave heights, written as a weighted Rayleigh distribution, with the weighting function fitted to their observations. Based on their observations, TG83 propose two different weighting functions, later augmented by a third due to Whitford (1988) who included more observations (see also Lippmann et al., 1996). The use of smoothly weighted Rayleigh distributions with a fairly wide support, allowing waves to break that are smaller and larger than some breaker height H_b, relaxes the depth limitations on the broken wave height distribution implied by the clipped distribution in BJ78. Apart from the more realistic representation of the distribution function of the breaking wave heights, the use of these smooth empirical distribution functions has several other advantages. Firstly, it conveniently results in an explicit expression for the dissipation rate, and secondly, it includes dissipation contributions associated with the breaking of waves potentially much higher than a nominal limiting wave height, which enhances dissipation levels in shallow water such that saturation needs no longer be enforced (as in BJ78).

Baldock et al. (1998) (BHBW98 hereafter) proposed another alternative for the distribution function in BJ78. The modification proposed by BHBW98 was particularly aimed at relaxing the enforced saturation condition in the original BJ78 model to improve modeling capability of wave breaking dissipation in shallow water on steeper beaches. After TG83, BHBW98 adopt the use of a full Rayleigh distribution for the surf zone wave heights. However, instead of the empirical weighting functions proposed by TG83, they propose the use of a single threshold wave height H_b, above which height they consider all waves broken. In other words, the associated wave height distribution for the breaking waves can be represented by a Rayleigh distribution weighted by a step or Heavyside function, unity for $H \geq H_b$ and zero elsewhere. As noted by BHBW98, this breaking wave height distribution is somewhat crude and notably at variance with the observations by TG83. However, similar to the empirical distributions proposed by TG83 and Whitford (1988), the wider support relaxes the depth-limitations implied by the clipping of the Rayleigh distribution in the BJ78 model, potentially enhancing dissipation levels for (over-) saturated surf zone conditions, and the dissipation rate is found in explicit form.

The present work revisits the model proposed by BHBW98. We discuss the implications of the alternative distribution function for the broken wave heights, and the assumptions implied in their model derivation. In §2 we outline the parametric modeling approach following BJ78 to derive the bulk dissipation rate, and introduce the modification proposed by BHBW98 (§2.1). We then revisit the derivation of BHBW98, correct for an inconsistency in their model, and show that if their derivation is done consistently, modeling capability actually improves for situations for which it was intended, namely steep beach topographies and over-saturated surf zone conditions (§2.2). In §3 we address shore break dissipation characteristics and compare the present model to the earlier TG83 and Whitford models. Finally, we summarize our main findings in §4.

2. Parameterization of energy loss due to depth-induced wave breaking

To describe the wave height evolution across a dissipative surf zone, BJ78 consider the energy balance equation for one-dimensional (stationary) wave propagation, which can be written as

$$\frac{d\langle \mathcal{F} \rangle}{dx} = -\langle \varepsilon \rangle. \tag{1}$$

Here $\langle \mathcal{F} \rangle$ and $\langle \varepsilon \rangle$ denote the expected value of the wave energy flux per unit span and the power dissipated per unit area respectively. For a breaking wave of height H – based on a bore analogy (e.g. Lamb, 1932, article 187) – the dissipation rate can be written as

$$\varepsilon = \frac{B}{4} \bar{f} \rho g \frac{H^3}{h}. \tag{2}$$

Here $\bar{f}$ is a representative frequency of the random wave field, g is gravitational acceleration, ρ represents the fluid density, h is water depth, and B is a (tunable) parameter that controls the intensity of the dissipation.

What remains is to estimate the expected value of the bulk dissipation in a random wave field, $\langle \varepsilon \rangle$, which comes down to adopting a suitable probability distribution function for the heights of the broken waves.

2.1. The BHBW98 estimate of $\langle \varepsilon \rangle$

To estimate $\langle \varepsilon \rangle$, BHBW98 (after TG83) – instead of the clipped Rayleigh distribution used by BJ78 – propose a full Rayleigh distribution function for the wave heights in the surf zone, written as

$$p(H) = \frac{2H}{H_{\rm rms}^2} \exp\left[-\left(\frac{H}{H_{\rm rms}} \right)^2 \right], \tag{3}$$

regardless whether H is smaller or larger than some breaker height H_b , above which the wave is assumed broken. The implied distribution of the heights associated with breaking waves can – after TG83 – be written as a weighted Rayleigh distribution

$$p_b(H) = W(H)p(H), \tag{4}$$

where $W(H) = \mathcal{H}(H - H_b)$, with $\mathcal{H}$ a Heavyside step function. The dissipation rate $\langle \varepsilon \rangle$ is then obtained from the integral

$$\langle \varepsilon \rangle = \int_0^\infty \varepsilon(H) p_b(H) dH = \int_{H_b}^\infty \varepsilon(H) p(H) dH. \tag{5}$$

The integral can be evaluated directly by inserting (2) and (3) into (5). However, before doing so, BHBW98 substitute H^2 for H^3/h in (2), which results in an explicit expression for $\langle\varepsilon\rangle$, reading

$$\begin{aligned}\langle\varepsilon\rangle^{(\mathrm{BHBW98})} &= \frac{B}{4}\bar{f}\rho g\int_{H_b}^{\infty}H^2p(H)dH\\ &= \frac{B}{4}\bar{f}\rho g H_{\mathrm{rms}}^2\left(1+R^2\right)\exp\left[-R^2\right],\end{aligned} \tag{6}$$

where $R = H_b/H_{\mathrm{rms}}$. To illustrate the nearshore behavior of this dissipation formulation, consider wave evolution across a shallow ($\bar{f}\sqrt{h/g} \ll 1$), over-saturated ($R \ll 1$) surf zone on a plane beach for which from Eqs. (1) and (6), the wave height evolution can be expressed analytically as

$$H_{rms} = H_{rms,0}\left(\frac{h_0}{h}\right)^{1/4}\exp\left[-\frac{2\bar{f}B}{\sqrt{g}h_x}\left(\sqrt{h}-\sqrt{h_0}\right)\right] \;(\mathrm{BHBW98}) \tag{7}$$

where the 0 as a subscript indicates some reference location; the h_x denotes the (constant) beach slope. Note from (7) that for $h \to 0$ we have $H_{\mathrm{rms}} \sim h^{-1/4}$, which is Green's shoaling law. Thus in the very nearshore region, dissipation is insufficient to counter shoaling, and as a consequence wave height increases without bound with decreasing depth. In fact, we can see from (7) that for these shallow-water conditions the BHBW98 model predicts increasing wave height with decreasing depth wherever

$$\frac{|dh/dx|}{\bar{f}}\sqrt{\frac{g}{h}} > 4B, \tag{8}$$

where the *LHS* of (8) is recognized as a normalized bed slope. The expression (8) indicates the transition from a dissipation-controlled to shoaling-controlled wave height evolution in the inner surf zone, which thus results in a shoreline singularity, reminiscent of the classical breakdown of conservative WKB theory. Clearly, such a singularity is unacceptable in a parametric surf zone model and needs to be addressed (as noted by BJ78).

Note that (6) is obtained from the integration in (5) with H^3/h approximated by H^2. This substitution, for which BHBW98 seeks justification in BJ78, is however inconsistent. BJ78 approximate $H_b{}^3/h$ as $H_b{}^2$ (b in the subscript designating the height above which waves are assumed broken) as 'an order-of-magnitude relationship', but this approximation is applied to the nominal breaking wave height in the given depth, which is a deterministic quantity. In other words, BJ78 use this approximation only for the lower limit of integration in Eq. (5) (along with using a delta function for $p(H)$ at $H = H_b$). In contrast, BHBW98 apply the approximation $H^3/h \approx H^2$ where H is the variable of integration in Eq. (4), before performing the integration, which is inconsistent in view of the fact that H is allowed to range according to the full Rayleigh distribution, including values that are nominally far in excess of H_b or h.

2.2. An alternative estimate of $\langle\varepsilon\rangle$

If (after BHBW98) we use the full Rayleigh distribution for the surf zone wave heights, but consistently retain H^3/h in the integral, thus performing the integration in (5) by inserting (2) and (3) directly (without further simplification), the expression for the bulk dissipation reads

$$\begin{aligned}\langle\varepsilon\rangle^{(\mathrm{present})} &= \frac{B}{4}\frac{\bar{f}\rho g}{h}\int_{H_b}^{\infty}H^3p(H)dH\\ &= \frac{3\sqrt{\pi}}{16}B\bar{f}\rho g\frac{H_{\mathrm{rms}}^3}{h}\left[1+\frac{4}{3\sqrt{\pi}}\left(R^3+\frac{3}{2}R\right)\exp\left[-R^2\right]\right.\\ &\quad\left.-\ \mathrm{erf}\ (R)\right],\end{aligned} \tag{9}$$

where erf represents the error function.

The shallow, over-saturated surf zone wave height evolution from (9) is then given as

$$H_{\mathrm{rms}} = h^{-1/4}\left[\left(h_0^{1/4}H_{\mathrm{rms},0}\right)^{-1} - \frac{\bar{f}\sqrt{\pi}B}{\sqrt{g}h_x}\left(h^{-3/4}-h_0^{-3/4}\right)\right]^{-1} (\mathrm{present}). \tag{10}$$

Note from (10) that as $h \to 0$ the wave height $H_{\mathrm{rms}} \sim h^{1/2}$, thus the wave height vanishes at the shoreline. The modification of $\langle\varepsilon\rangle$ proposed here (Eq. (9)) thus results–through a consistent incorporation of energy losses due to broken waves that are larger than the saturation height–in enhanced dissipation levels in the nearshore on steep beaches and effectively removes the shoreline singularity, i.e. dissipation is strong enough to counter shoaling effects.

2.3. Model inter-comparison and empirical verification

To illustrate the nearshore behavior of the BHBW98 model and the modification proposed herein, we integrate (1) with $\langle\mathcal{F}\rangle = \rho g\bar{C}_g H_{\mathrm{rms}}^2/8$; where $\bar{C}_g$ is the linear group speed corresponding to $\bar{f}$. The remaining free parameters in (6) and (9) are set as $B = 1$ and after BHBW98 we take the ratio $\gamma = H_b/h$ from

$$\gamma = \frac{H_b}{h} = 0.39 + 0.56\tanh(33S_0), \tag{11}$$

which is a minor modification of the expression proposed by Battjes & Stive (1985). Here S_0 denotes the offshore wave steepness, defined as $H_{\mathrm{rms}}^{\mathrm{d}}/L^{\mathrm{d}}$, the superscript d indicating deep-water values.

In Fig. 1, the evolution of the wave field over a 1:10 slope is shown in terms of normalized wave height and the fraction of broken waves Q. It can be seen that in the inner surf zone the wave height predictions strongly diverge, with the BHBW98 model (Eq. (6)) exhibiting a shoreline singularity. In contrast, the present expression (Eq. (9)) predicts a monotonically decreasing wave height, generally larger than the saturation height.

Note that the BHBW98 model and the modification thereof discussed here, only affect the predictive capability of the BJ78

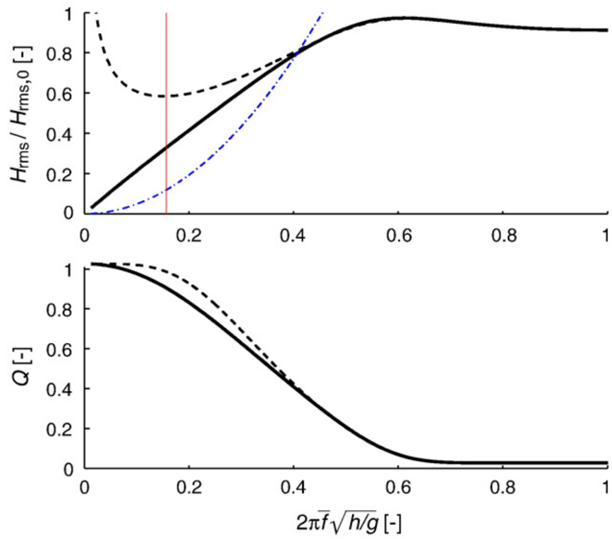

Fig. 1. Comparison of wave height (top panel) and fraction of breaking waves Q (bottom panel) predicted with BHBW98 formulation (dashed line, Eq. (6)) and the corrected form proposed here (solid line, Eq. (9)). Waves are incident on a 1:10 slope with $H^d_{\rm rms}=1.0$ m, $\bar{f}=0.2$ Hz. The vertical line in the upper panel indicates $\bar{f}\sqrt{h/g}=|h_x|/(4B)$ (Eq. (8)); the dash–dot line indicates saturation wave height $H_{\rm rms}=\gamma h$. The cross-shore integration is initialized at $\bar{f}\sqrt{h/g}=0.2$.

model for the inner surf zone on relative steep beaches, where enforced saturation, as incorporated in the original BJ78 model to circumvent a shoreline singularity, can result in underestimation of wave heights (see e.g. Cox et al., 1994; Baldock et al., 1998). On milder slopes and saturated surf zone conditions, the BHBW98 parameterization predicts similar dissipation rates as the original Battjes–Janssen model (see BHBW98). In turn, for such conditions, the present (consistent) formulation yields very similar results also. For instance, if the surf zone is saturated (such that $H_{\rm rms}=H_{\rm b}$) the present dissipation rate is roughly $3\gamma/2$ times the BHBW98 model result, which – for common γ values (say $\gamma\approx 0.7$) – renders similar dissipation levels. This is illustrated in Fig. 2. For the 1:20 slope, wave height evolution is similar throughout the surf zone except for the very nearshore, where the predictions again diverge. For the same incident wave field over a 1:100 slope, the inner surf zone is close to saturation; for this case the model predictions are similar, including the very nearshore. Thus on mild slopes and for non-saturated surf zone conditions the corrected form of the dissipation model presented here is similar to the BHBW98 model (and the BJ78 and TG83 models for that matter) but the present parameterization is more robust in the nearshore of steeper beach profiles, where over-saturated conditions are not uncommon.

Finally, to empirically verify the modeling capability of the present model on a steep beach, we compare model predictions to observations reported by BHBW98 (Fig. 3). The experiments are performed in a 50 m long and 3 m wide flume with a 1:10 slope in the surf and swash zone (for details see BHBW98). We initialize the model (Eqs. (1) and (9)) with the observed wave height at $x=-5.5$ m (with – after BHBW98 – the x-origin at the shoreline, negative in offshore direction) for cases J_2

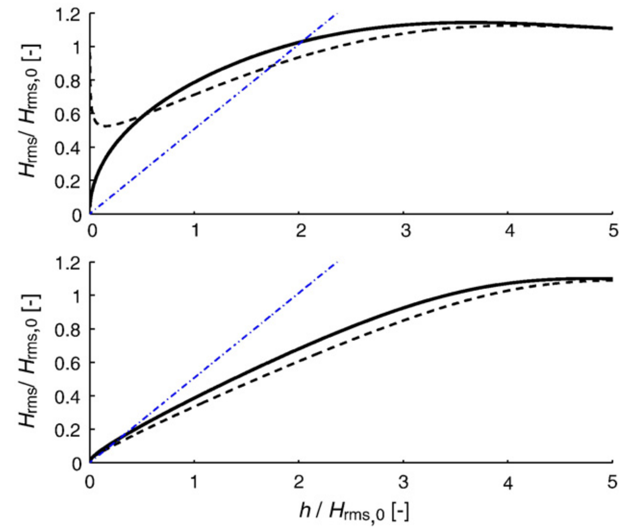

Fig. 2. Comparison of wave height predicted with BHBW98 formulation (dashed line, Eq. (6)) and the corrected form proposed here (solid line, Eq. (9)). Waves incident on a plane slope (1:20 (top panel) and 1:100 (bottom panel) respectively) with deep-water $H_{{\rm rms},d}=1.0$ m and $\bar{f}=0.1$ Hz. Dash–dot line indicates saturation wave height $H_{\rm rms}=\gamma h$. The cross-shore integration is initialized at $h/H_{\rm rms}=10.2$.

($H_{\rm rms}$,0=7.4 cm and $\bar{f}=0.67$ Hz) and J_3 ($H_{\rm rms}$,0=4.6 cm and $\bar{f}=1.0$ Hz), labeled after BHBW98. The water depth is $h=d+\bar{\eta}$, with d the quiescent depth and $\bar{\eta}$ denoting wave-induced water level corrections (set-up/down), the latter computed from a one-

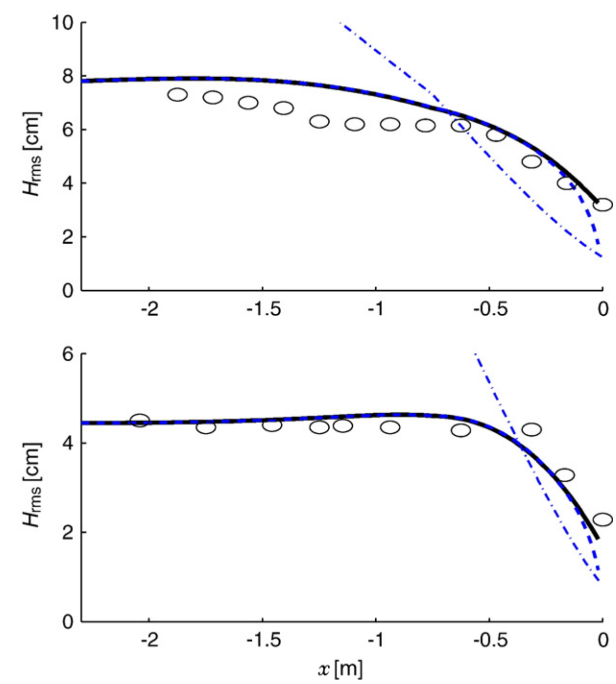

Fig. 3. Comparison of predictions (solid line) with present model (Eqs. (1) and (9)) and observations (circles) by BHBW98 of wave height evolution across a surf zone on a 1:10 beach. Dashed line represents wave heights computed without accounting for wave-induced mean water level corrections. Dash–dot line indicates saturation wave height $H_{\rm rms}=\gamma(d+\bar{\eta})$. Upper panel: case J2 of BHBW98; incident waves (at $x=-5.5$ m) $H_{\rm rms,0}=7.4$ cm and $\bar{f}=0.67$ Hz. Lower panel: case J3 of BHBW98; incident waves (at $x=-5.5$ m) $H_{\rm rms,0}=4.6$ cm and $\bar{f}=1.0$ Hz.

dimensional momentum balance (see BJ78). The water level corrections are incorporated iteratively and convergence is assumed when the $\overline{\eta}$ values at each location differ less than 0.001 mm between iterations. It can be seen from Fig. 3 that inner surf zone wave heights are indeed larger than the saturation height. The agreement between observations and predictions of wave height in the inner surf zone is good.

BHBW98 remark that measured set-up values are of the order 2−3 mm, which is considerably smaller than the computed values from second-order theory. The effect of including $\overline{\eta}$ on the wave height evolution is small (Fig. 3). The level of agreement between observations and predictions is similar regardless whether wave-induced water level corrections are included or not, apart in the very nearshore perhaps, where including the predicted set-up yields a slightly better quantitative agreement.

3. A discussion of 'shore breaks'

The use of a complete Rayleigh distribution to model over-saturated steep beach conditions as proposed by BHBW98 results in a model that is conceptually quite similar to the earlier TG83 model. In fact, with H/h consistently retained in (5) (as done in the present work), the only remaining difference between the present model and TG83 is the assumed breaking wave height distribution. To illustrate how this affects the respective representation of shore break dissipation, we consider wave height evolution in shallow water $\left(\bar{f}\sqrt{h/g}\ll 1\right)$ and over-saturated ($R\ll 1$) breaking conditions on a planar beach. In this limit, the TG83 models (i.e. their Eqs. (20) and (21) with $n=4$ and $n=2$ respectively) predict a wave height variation with depth that can be written as

$$H_{\mathrm{rms}}\sim h^{\frac{(1+2n)}{2(1+n)}},\qquad n=2,4. \tag{12}$$

The Whitford weighting function yields $H_{\mathrm{rms}}\sim h^{1/2}$, which is the same asymptotic behavior as the present model (see Eq. (10)). Thus both the TG83 and Whitford models predict a vanishing wave height at the shoreline.

To compare shore break dissipation predicted by the present, TG83 and Whitford formulation, we consider the bulk dissipation rate, which for these models (with $R\ll 1$ and $B=1$) can be expressed as

$$\langle\varepsilon\rangle\approx\frac{3\sqrt{\pi}}{16}\bar{f}\rho g\frac{H_{\mathrm{rms}}^{3}}{h}Q. \tag{13}$$

For a given root-mean-square wave height and water depth, differences in shore break dissipation levels are thus related to differences in the respective estimates for Q, the fraction of broken waves. The present model predicts $Q\to 1$, the TG83 models give $Q\sim R^{-n}(n=2,\ 4)$, and the Whitford weighting function yields $Q\to 2$ in the shore break. As a consequence, dissipation levels are thus typically higher for the TG83 and Whitford model in shore break conditions, due to the weighting function ($W(H)$) and the fraction of broken waves (Q) exceeding unity in those models. In contrast, the step-weighted Rayleigh distribution function for the breaking wave heights proposed by BHBW98, although a somewhat crude representation of the breaking wave height distribution, ensures that the breaking wave height distribution is always a subset of the Rayleigh distribution ($W(H)\leq 1$), and that the fraction of broken waves does not exceed unity ($Q\leq 1$).

However, unlike the shoreline singularity in the BHBW98 model, the fact that in some cases the TG83 and Whitford models locally predict $Q>1$, although perhaps a violation of strict realizability, does not invalidate these models as a parameterization concerned with the macroscopic effects of energy loss in the wave breaking process. This is particularly so since shore break conditions occur only very close to shore so that the cumulative effect of differences in predicted dissipation rates on the wave height evolution is limited. In fact, for most cases these parametric dissipation models are remarkably robust with respect to variations in the representation of the breaking wave height distribution (e.g. Lippmann et al., 1996), and their overall performance across natural surf zones is quite comparable (see e.g. Apostos et al., submitted for publication).

4. Conclusions

The present work revisits the parametric dissipation model by Baldock et al. (1998) (BHBW98) who, after Thornton and Guza (1983) (TG83), propose the use of a complete Rayleigh distribution function for the surf zone wave height statistics — instead of the clipped distribution assumed in the original model by Battjes and Janssen (1978) (BJ78). We discuss an algebraic inconsistency in BHBW98 and show that, if the breaking wave height distribution proposed by BHBW98 is consistently used (as proposed here), the resulting parameterization predicts vanishing wave height as the shoreline is approached, even in the presence of 'shore breaks' that can dominate the surf in the nearshore on steep beaches. For those conditions the correction presented here results in enhanced dissipation, sufficient to prevent a shoreline singularity that could otherwise develop. The present model differs from the earlier TG83 model only in the assumed distribution function of the heights associated with breaking waves, and we discuss the implications for the parametric representation wave dissipation in shore break conditions. Comparisons between observations and predictions with the present model of wave evolution on a steep beach, including the inner surf zone where the wave height exceeds the saturation height, show good agreement.

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